Study of the LHCb pile-up trigger and the $B_s \to J/\psi \phi$ decay

Zaitsev, N.Y.

Citation for published version (APA):
Zaitsev, N. Y. (2000). Study of the LHCb pile-up trigger and the $B_s \to J/\psi \phi$ decay
CHAPTER 3

Test beam results of prototype silicon detector

The vertex detector is a crucial element of the LHCb detector. It is used for the precision measurement of tracks and decay vertices, both off-line and on-line. In this thesis we will describe in detail two possible additional applications, i.e. the vetoing of event with multiple interactions and the luminosity monitoring. The development of the detector requires an understanding of the detector operation, the requirements for its mechanical precision and experience in the analysis of the data.

In this chapter we discuss test beam results obtained with a prototype vertex detector and compare them with Monte Carlo simulations.

The use of the detector for tasks relevant to this thesis are described in subsequent chapters, i.e. pileup trigger algorithm (Chapter 4) and luminosity monitor (Chapter 5).

8 Silicon detector

The LHCb experiment requires interaction and decay vertices to be determined with a precision of the order of 200 μm or better. To achieve this a vertex detector is used, which is very close to the interaction point. The high radiation conditions at this position and the fact that the detector has to operate inside a vacuum tank lead us to a design using silicon strip detectors. As described in Chapter 2 the detector will be used in the:

- offline analysis to determine with high precision vertices and impact parameters of tracks for the flavour tag;
- Level-1 and -2 triggers to tag on the presence of a secondary vertex;
- Level-0 pileup trigger for the fast identification and localization of the primary vertex and the testing of events with multiple primary vertices (see Chapter 4).

Silicon detectors are known since the beginning of the 50’s for their excellent energy resolution [22]. They are also characterized by simplicity of handling due to the rigid, self supporting structure and by simplicity of operation. In the last 20 years they have been increasingly used for precision coordinate measurements. The first silicon strip detector was used at CERN in 1983 by the NA32 collaboration [23]. Since then various improvements have been made (i.e. pixels, production technology and readout optimization) but the main principle of a thin, efficient and highly partitioned detector has
Test beam results of prototype silicon detector

not changed.

In the following we give a brief description of charged particle detection with silicon detectors. For a detailed description of semiconductor devices we refer to [24].

When a charged particle crosses a silicon detector it deposits a fraction of its energy in the detector. The LHCb intends to use 150 μm thick silicon detectors. For this thickness the most probable energy deposited for a minimum ionising charged particle is 46 keV. Fluctuations in the energy deposition result in the so-called Landau distribution as shown in Figure 10 for the LHCb silicon detector.

The principle of a silicon strip detector is shown schematically in Figure 11. A voltage across the detector creates a depleted region with a uniformly distributed volume charge. This creates an electric field almost linearly changing over the depth of the depletion region. When a particle crosses the detector a tube of ionization plasma is formed along the track in the depleted part of the detector. Electrons and holes created by the ionization are pulled in opposite directions and start to drift to the strip and ground side, respectively. In doing so they diffuse, can recombine and can be captured by the imperfections in the material. The number of imperfections increases due to radiation damage. This damage therefore increases with time and is proportional to the integrated flux of particles passing through the detector.

The signal current on the strip is induced by the net motion of electrons and holes. The typical current value is so small that it must be amplified before a reliable measurement (or digitization) can be done. Both, the amplification and the digitization of the signal, are usually implemented within a single readout chip.

Obviously, the overall detector performance depends on the operation regime of the silicon and the parameters of the readout chip. In this chapter we will discuss the results from the LHCb prototype silicon detector in a test beam set-up at the CERN-SPS. In this test the hit and vertex resolutions are measured. The effect of charge sharing over several strips is demonstrated and
Test beam results of prototype silicon detector

described with a simple model. Simulated data are generated for the detector set-up and compared with the experimental results.

9 Beam test

9.1 Setup and data acquisition

9.1.1 Experimental setup

The test beam setup consists of 6 detectors measuring the radius ($r$) and 6 detectors measuring the azimuthal angle of a track hit ($\phi$). There are three pairs of $r$, $\phi$-detectors on the left side (SL1-3) and three on the right side (SR1-3). Each detector is a ~70 degree sector with inner radius of 1 cm and outer radius of 5 cm. One $r$-detector and one $\phi$-detector form a station with a distance between them of about 2-5 mm. The distance between stations is about 4 cm, see Figure 12.

![Figure 12: The set-up of detectors. It consists of detectors (left: SL1-3 and right: SL1-3), scintillators (C1-3, V) and targets (T1-12).](image)

The strip configuration is optimized so that good track/vertex resolution is obtained using a minimum of readout channels. Therefore, the strip pitch varies with radius. Their values are shown in Figure 13. This strip layout is close to that given in the TP of LHCb [19].

In order to reproduce a situation as expected in LHCb operation (with primary and secondary vertices) a layered copper target was designed, fabricated and installed in front of the
Test beam results of prototype silicon detector

Si-detectors with a distance between the most downstream target and the first detector of about 7.5 cm. The set of targets consists of twelve thick targets of 300 μm thickness (T1-T12) and two thin targets of 100 μm thickness (T4 and T8'). Their radius is 1 mm. The targets are glued to 10 μm kapton foils and are mounted in a supporting tube. The separation between the thick targets is 1 cm, while the thin target T4'(T8') is placed in the middle between targets T4 and T5 (T8 and T9). Figure 14 shows a photograph of the target setup.

The beam is bunched with a 2.37 sec bunch length and with a 14.4 sec bunch interval. Each bunch contains $10^5$-$10^6$ pions with a momentum of 120 GeV/c. The transverse size of the beam is $\sigma_{x}\times\sigma_{y} \sim 4\times7$ mm$^2$.

Signals from four scintillators define the trigger for the testbeam setup (see Figure 15).
Test beam results of prototype silicon detector

Figure 15: Data Acquisition scheme of target run. Trigger scheme of parallel tracks run included signals from $C_2$ or $C_3$ depending on the tested side.

They are:

- $C_1$: a scintillator with a cross section of $2.5 \times 2.5$ mm$^2$ and a thickness of about 4.5 mm.
- $V$: a scintillator with a cross section of $20 \times 20$ mm$^2$, a thickness of 2.5 mm and a central hole with a diameter of 2 mm.
- $C_2$ and $C_3$: scintillators which are 12 cm wide and 1 cm thick and have a shape at the top corresponding to the 72° silicon sector.

The counters $C_1$ and $V$ were readout with 0.3 inch photomultipliers (PM). $C_2$ and $C_3$ were readout with 2 inch diameter PM tubes. The analogue signals of the PM’s were converted, if exceeding a threshold value, into logical ones. The signal length was 50 ns (100 ns) for the $C_1$ and $V$ ($C_2$ and $C_3$) counters. Two forward scintillators, $C_1$ and $V$, were mounted on a rail connected to the detector frame.

Two different trigger combinations were used for the pileup algorithm study:

1. **Target run**: to select interactions of the beam particles with one of the targets an anti-coincidence of $C_1$ and $V$ was required in coincidence with either $C_2$ or $C_3$, i.e. $C_1 \& \overline{V}-(C_2+C_3)$. Figure 15 illustrates the trigger scheme. The pileup algorithm was applied to these data.

2. **Parallel tracks run**: the beam counters ($C_1$ and $V$) and the set of targets were removed to allow the beam to pass one of the two sides of the detector setup. A signal from the rear scintillators ($C_2$ or $C_3$, depending on which side was tested) triggered the DAQ. These data were used to determine the resolution of the detectors.
Test beam results of prototype silicon detector

Data were collected with a readout system consisting of a read-out chip on the silicon detector, VME modules to readout the detector signal and software. The readout required a minimum of 12 ms. This corresponds to the readout of 12000 channels in sequence at a 1 MHz readout rate. A Software Busy flag denied readout of any subsequent triggered event until all data were transmitted to the computer.

The counting rate of C2 or C3 (used as trigger for the parallel tracks run) was \(-5 \times 10^5\) per bunch whereas on average an event was stored every 15.21 ms [25]. In the target run due to the more restrictive trigger scheme we had about 70-80 triggers per bunch. This corresponded to a dead time of \(-30\%\) and accordingly to \(-50\) events per bunch stored on the tape.

A bias voltage of 70 V was applied to the Si-detectors in order to ensure the best possible performance in terms of hit resolution, signal to noise ratio and charge collection efficiency. With this bias voltage the silicon detector is fully depleted. The value was determined in the laboratory on the basis of the volt-ampere characteristics of the detector [25]. The charge integration time was about 1.4 \(\mu\)s. It is large compared to the charge collection time of about 70 ns \(^{*}\). The analogue amplitude from each detector was digitized using a 12 bit ADC. The raw hit information (event number/strip number/ADC count) was stored on tape and analyzed offline.

The detectors were aligned mechanically by means of special alignment equipment. This measured the positions of targets and silicon detectors through mechanical touching of reference points. A separate procedure established the relative positions of the detector planes and the reference points. The measurement was done several times in the lab before and after the tests took place.

9.2 Offline analysis

9.2.1 Introduction

For the off-line analysis we have to develop the tools for searching clusters of hits, finding and fitting tracks and vertices, determine the resolutions of the detectors and develop a MC program that is able to reproduce the data.

The resolution of the detectors is first determined from the parallel tracks data run, in particular the subsample in which there is only one beam track. Events where more than one track is found constitute another part of the sample. These data are used to develop our pattern recognition and track fit procedure. The quality of the track fit can be verified by comparing the resolutions found in this multiple track sample with those determined in the single track sample.

With the tools developed for track finding and fitting we process the data from the target sample. This leads to the development of a procedure for vertex finding and fitting and gives us information about the resolution of reconstructed vertices.

\(^{*}\) When the detector is fully depleted the generation current is almost a constant function of the voltage. It is known that this current is proportional to the depleted volume of the detector.

\(^{**}\) A 70 ns corresponds to the collection of roughly 99\% of all electrons and holes generated by an ionizing particle. In normal conditions the collection time for holes is 3 times larger than for electrons.
Test beam results of prototype silicon detector

9.2.2 Cluster search

The first stage in the data analysis is finding clusters of signal strips in the detector corresponding to the passing of ionizing particles.

The cluster search is done in two steps. First, for every detector the signal pedestal and noise levels are determined for the group of neighboring channels which are connected to the same readout chip. These groups consist of 64 to 256 channels. For a group it is assumed that the pedestal is a linear function of the strip number, i.e., and can be approximated as \( <p> + s \cdot i \), where \( s \) is the slope. We start in the first event with the calculation of the average pedestal value \( <p> \), taking the ADC counts of all strips in the group. A first estimate of the noise level is given by the \( \sigma \) of the ADC count fluctuations. Using these first estimates of the pedestal value and the noise level, channels are eliminated which exceed by \( 3 \sigma \) the pedestal level. For the remaining channels the slope \( s \) is determined. After three iterations \( <p> \), \( s \) and \( \sigma \) have converged. In the subsequent events \( <p> \), \( s \) and \( \sigma \) are continuously updated. After 50 events it is believed that the procedure for determining pedestal and noise level has stabilised and the real cluster search starts. The pedestal and noise level nevertheless have to be continuously updated because of effects as low frequency oscillations of the biasing voltage and because of pick-ups in the readout chain. The algorithm of the cluster search looks for groups of signal strips, where the ADC count exceeds the pedestal value by \( 3\sigma \). The position of a found cluster is calculated using the center of gravity algorithm.

9.2.3 Track fit

A track candidate is formed by a combination of six clusters, i.e., one cluster per detector. The clusters are fitted to a straight line, i.e., assuming no multiple scattering. The track fit results in four track parameters (Appendix B):

- \( x_0, y_0 \) - the intercepts of the straight line with the plane \( z=0 \) and
- \( s_x, s_y \) - the slopes of the track in \((xz)\) and \((yz)\)-planes respectively.

The track parameters are obtained by minimization of a \( \chi^2 \)-function. This function is defined as the sum of the squares of the weighted distances between the predicted position of the track at the detector and the measured cluster position. The weights are proportional to the measurement errors, \( \varepsilon = \text{width of strip} / \sqrt{12} \), where \( \text{width of strip} \) hit by the track. The \( \chi^2_{\text{meas}} \) depends on the track parameters, \( t \), as:

\[
\chi^2_{\text{meas}}(t) = \sum_{i=1}^{6} \frac{d^2_{\text{meas}}(t)}{\varepsilon_i^2}
\]

The track selection always requires 6 clusters per track \( (t=1.6) \). The distances, \( d \), are determined as the distance of closest approach to the average position of the cluster.

The \( \chi^2 \) minimization utilizes the MINUIT package [26]. An estimate of the initial values of the track parameters is necessary as the \( \chi^2 \) function has a very deep and narrow minimum.

A simple and robust iterative procedure (track estimator) was developed to find the starting values for the track parameters for any combination of clusters. It uses the fact that the detectors are grouped in stations with a small distance between \( r \) and \( \phi \)-detectors (about 2 mm). The procedure starts with an estimate of all track parameters using 3 \( r \) and 3 \( \phi \) measurements where the distance between \( r \)- and \( \phi \)-detectors is assumed to be zero. This provides three 3D-points where \( z \) is given by the position of the \( r \)-detector. The least square method (LSM) returns track parameters, which are used to estimate the radius measurement at the true position of the
Test beam results of prototype silicon detector

Φ-detectors. The resulting improved 3D-points are fitted to a straight line in a similar manner to the previous step. Track parameters obtained with the estimator are very close to those found with MINUIT: the $\chi^2$ of the estimator is only 2-3 times larger than the minimum $\chi^2$ obtained with MINUIT. Good track candidates are selected by the $\chi^2$ value given by MINUIT: tracks with $\chi^2 < 200$ are accepted.

9.2.4 Alignment

The alignment of the detector was checked by minimizing (for selected tracks), in both the parallel tracks and target samples, the sum of the individual $\chi^2$ of the track fits, i.e. the following $\chi^2$ was minimized as a function of the detector positions $\hat{R}$:

$$
\chi^2(\hat{R}) = \sum_{i=1}^{\text{tracks}} \chi^2_{\text{tracks}}(\hat{R}).
$$

Instead of the track fit described in 9.2.3 results of a faster analytic track fit were used developed by V.Chabaud [27]. The detector positions found by this alignment procedure are shifted by less than 50 µm in the transverse and less than 100 µm in the longitudinal (beam) direction, and rotated by less than 50 mrad with respect to the positions measured mechanically.

Since the track fit of V.Chabaud involves approximations which negatively affect the hit resolution we checked the alignment using the track fit described in section 9.2.3. For this we looked for the difference between the prediction of the position of a track, using our track fit, and the cluster. A possible misalignment leads to a radius residual $\Delta r$ which is a function of $\phi$ and is given by:

$$
\Delta r = r_i \cos \phi + r_j \sin \phi - \frac{\chi^2_i + \chi^2_j}{2r}.
$$

where $x_i$ and $y_j$ are the transverse offsets. In deriving this formula it is assumed that the longitudinal positions of the detectors are correct and that the detectors are perpendicular to the beam. Figure 16 shows $\Delta r(\phi)$ for the SL2 $r$-detector. The left distribution is the result of introducing a shift of the detector by 100 µm in the x- and 50 µm in the y-direction. After correcting the detector position Figure 16B is obtained. The remaining slope in the distribution corresponds to an effective y-shift of 2.6±0.9 µm. Looking at all detectors this appears to be consistent with statistics.

Since the pile-up trigger algorithm requires detectors to be precisely aligned it is important that the alignment procedure can be done in a reasonably short time under actual LHCb running conditions. The alignment procedure is estimated to take, because of time-consuming $\chi^2$ minimization, about 30 minutes using the CPU-farms proposed for the Level-1 trigger. This would still be acceptable as the typical run period will be 10 hours. A faster procedure, moreover, seems possible using equation (23).

9.2.5 Resolution

To determine the detector resolutions the single track sample obtained in the parallel tracks run was used. The resolution of the position measurements was estimated by analysing the residual of clusters in single detectors. Considering 3 equidistant detectors and assuming an equal hit resolution for all of them, the single hit resolution is equal to the RMS of the residual measured in the middle of the setup (SL2 or SR2) multiplied by a factor $\sqrt{3}/2$. Figures 17 and 18 show these residuals obtained after the track fit with MINUIT. Table 5 summarizes the results.
Test beam results of prototype silicon detector

![Figure 16: Residuals of radius cluster positions as a function of φ. Left plot (A) shows the residuals where the shift of detector position in x and y. A fit with eq.23 reproduces this shifts. The right plot (B) shows the residuals after applying the corrections from the fit.](image)

Tracks found in the parallel tracks run correspond to monoenergetic particles with momentum of 120 GeV/c, while in the target run secondary tracks have momenta, which are mostly in the range 0.2-0.5 GeV/c. For the beam tracks multiple scattering is negligible while for the secondary tracks in the target sample it is significant. Compared to the parallel beam tracks an average deterioration of the resolution by 14% is observed in the target run sample. It amounts to 11% for tracks with polar angle between 0.04 rad and 0.15 rad and to 26% for polar angles above 0.15 rad. This reflects the correlation between momentum and polar angle for secondary particles produced in the beam-target interaction.

9.2.6 Event reconstruction

Track finding

For studies of the pileup trigger we have to find in every event all possible tracks and all vertices. The tracks are determined from fits to all 6-cluster combinations in either left or right side detectors. Clusters are assigned to tracks in three successive steps:

1. all combinations of 6 clusters are fitted to a straight line using the procedure described in section 9.2.3. A loose quality cut of $\chi^2<200$ is applied;
2. combinations satisfying this loose cut are sorted according to their $\chi^2$;
3. those combinations with a unique set of clusters and the best $\chi^2$ are selected.

The hit resolutions obtained with all tracks found in the parallel tracks run are equal to those found in the single track subsample of this run, see Figure 19 and Table 6. Finally, good tracks are selected by a $\chi^2$ probability $>1\%$. The background of combinatorial origin contributes to this sample at $3.2\pm0.1\%$ level.
Test beam results of prototype silicon detector

Figure 17: Track radius residuals for middle (SL2 and SR2) detectors. Residuals are given for inner and outer areas in both types of detectors.

After having results from the track fit it is possible to study the effect of the center of gravity algorithm on the calculation of the cluster position. The algorithm introduces a bias in the 2-strip cluster position. This is shown in Figure 20. This bias was parametrised and corrected for in the cluster reconstruction. This improves the cluster position resolution by 1 μm.

**Vertex finding**

For the vertex reconstruction a minimum of two tracks is required one on the left and one on the right side of the detector. The right side tracks are combined with the left side tracks yielding two-track vertices. The two-track vertex position is found by:

\[ \vec{r}_V = \vec{r}_2 + \frac{\vec{s}_2^2 (\vec{R} \times \vec{s}_1) \cdot \vec{s}_2}{s_\perp^2} \]

\[ s_{1,2} = \frac{(s_x, s_y, 1)}{\sqrt{1 + s_x^2 + s_y^2}} \]

\[ \vec{R} = \vec{r}_1 - \vec{r}_2 \]

\[ \vec{s}_\perp = [\vec{s}_1 \times \vec{s}_2] \]
Test beam results of prototype silicon detector

![Graphs showing track residuals for middle (SL2 and SR2) detectors. Residuals are given for inner and outer areas in both types of detectors.](image)

**Figure 18:** Track $\phi$-residuals for middle (SL2 and SR2) detectors. Residuals are given for inner and outer areas in both types of detectors.

<table>
<thead>
<tr>
<th>Detector/Area, units</th>
<th>Resolution at Left side</th>
<th>Resolution at Right side</th>
</tr>
</thead>
<tbody>
<tr>
<td>radius/inner, microns</td>
<td>6.2±0.1</td>
<td>5.9±0.1</td>
</tr>
<tr>
<td>radius/outer, microns</td>
<td>11.6±0.1</td>
<td>11.5±0.1</td>
</tr>
<tr>
<td>angle/inner, milliradians</td>
<td>0.93±0.01</td>
<td>0.93±0.01</td>
</tr>
<tr>
<td>angle/outer, milliradians</td>
<td>0.31±0.01</td>
<td>0.32±0.01</td>
</tr>
</tbody>
</table>

**Table 5:** Detector resolutions obtained in the parallel tracks sample.

where $(r_1,s_1)$ and $(r_2,s_2)$ are the parameters of the left side track and right side track, respectively; $r_V$ is the 3D position of the vertex. The vertex is accepted if the distance of closest approach between tracks is less than 1 mm. The vertex positions from two tracks combinations were grouped around the expected target positions (found from vertices in the sample with only...
Test beam results of prototype silicon detector

Figure 19: Radius hit residuals after the pattern recognition. The two upper plots are the residuals from the inner (left) and outer (right) regions of the $SR2 r$-detectors, respectively. The two lower plots are residuals from the inner (left) and outer (right) regions of the $SL2 r$-detectors.

<table>
<thead>
<tr>
<th>Detector/Area, units</th>
<th>Resolution at Left side</th>
<th>Resolution at Right side</th>
</tr>
</thead>
<tbody>
<tr>
<td>radius/inner, $\mu$m</td>
<td>6.3±0.1</td>
<td>5.8±0.1</td>
</tr>
<tr>
<td>radius/outer, $\mu$m</td>
<td>12.2±0.5</td>
<td>12.0±0.2</td>
</tr>
</tbody>
</table>

Table 6: Detector resolutions after pattern recognition.

one track per side, i.e. two tracks per event) and then averaged to provide the event vertex. The effect of correlations due to the use of the same track in different vertices was not taken into account. The target profile, is the distribution of event vertex positions along the beam-axis, shown in Figure 21.

9.2.7 Monte Carlo simulation

The test beam results were compared to a Monte Carlo simulation, which uses the GEANT 3.21 package [28] for track propagation through the detector set-up, and a simplified track fit. The simulated set-up consists of a set of 14 copper targets and six wheel detectors of which three
Test beam results of prototype silicon detector

**Figure 20:** Residual (top plot) of the radius measurement as a function of the position on the strip. Statistics (middle plot) and mean position of the cluster (bottom plot) are also shown. Plots are constructed using a periodic window with a width of 3 strips, i.e. strips 4,5,6 (7,8,9...) are the same as 1,2,3. Points are fitted to trigonometric functions.

**Figure 21:** Distribution of reconstructed interaction points (Left). Target profiles (Right) were fit to a Lorenz (or Breit-Wigner) distribution assuming a flat background. The FWHM is taken as the vertex resolution. Vertices reconstructed between targets correspond to beam-air interactions.
are at \( z = 7, 11 \) and 15 cm, and the other three at \( z = 7.1, 11.1, 15.1 \) cm. The track propagation included the interaction of particles with material, multiple scattering and decays of particles. The characteristics of the detector response such as hit clustering and noise were not included, i.e. 100% hit efficiency was assumed. The comparison of absolute event rates was impossible as the trigger information was not monitored.

The analysis of the data is done as an application in ROOT [29] and is written in C++. The GEANT simulation is written in FORTRAN. A simplified track fit procedure is used in the simulation, in which the track hits are points in euclidean coordinates, \((x,y,z)\) and not in polar ones as in the real set-up. The fit also assumes that there is no correlation between \( r \) and \( \phi \) measurement. Therefore, the least square method can be used to fit separately hits in the \((x,z)\) and \((y,z)\) plane. The hit positions are smeared using Gaussian distributions with widths taken from the resolutions found in the data analysis, see Table 5. The uncertainty of the \( x \)-coordinate is dominated by the resolution in the \( r \)-measurement (\( \delta r = \delta x \)) and the uncertainty of the \( y \)-coordinate by the resolution in the \( \phi \)-measurement (\( \delta y = r \delta \phi \)). The resulting track parameters are defined in the same way as in the data analysis. No pattern recognition was performed.

During the development of the simulation program it was found that the default generator of hadron interactions in GEANT, GEISHA, poorly reproduces the angular distribution of tracks coming from the primary interaction. This is due to the switching between two generator regimes around a hadron momentum of 2 GeV/c (in order to speed up the algorithm). Since the momentum, \( p \), and the angle of the secondaries, \( \Theta \), are correlated as \( \Theta \sim 1/p \) we could see this effect in the angular distribution of tracks. The FLUKA generator [30] is a more detailed simulation and better describes the test beam data, see Figure 22. In order to simulate the measurements in the LHCb silicon detector both programs can be used, despite their difference, because the contribution of hadronic interactions to the errors of the track measurement is very small compared to multiple scattering.

### 9.2.8 Vertex resolution

To test the performance of the pileup trigger algorithm the positions of the targets and vertex resolutions have to be known. For this purpose, we selected events with a maximum of two tracks on a side and searched for vertices. The peaks in the vertex distribution (see Figure 21) are fitted to a Lorentz function. The contribution due to the target thickness is subtracted. The half width at half maximum (HWHM) is taken as the vertex resolution. The HWHM is related to the sigma of a Gaussian by \( \text{HWHM} = (2 \ln 2)^{1/2} \sigma \sim 1.177 \sigma \). The right plot in Figure 21 shows two target profiles, T1 and T12, with HWHM equal to \( 232 \pm 9 \) \( \mu \text{m} \) and \( 621 \pm 16 \) \( \mu \text{m} \), respectively. Figure 23 shows the vertex resolution as a function of the target positions.
Test beam results of prototype silicon detector

resolutions obtained from a Monte Carlo simulation are also given. The relative difference between data and simulation is on average $-0.6\pm1.4\%$, see Figure 23(B).

Figure 23: The vertex resolution versus the position of the vertex (A): Minuit fit and MC refer to the resolutions obtained from the test beam data and the Monte Carlo simulation; MC, no MS refers to resolutions obtained from the test beam simulations without multiple scattering and SICB refers to results from the LHCb Monte Carlo simulation for the $J/\psi \rightarrow \mu \mu$ decay. The relative difference of the beam test resolutions (MC and Minuit fit) is on average $-0.6\pm1.4\%$ (B).

The vertex resolution has contributions from the non-zero hit resolution and from multiple scattering. The multiple scattering is less for high momentum particles like those expected at LHCb. The average momentum of the secondary particles produced in the pion-target interactions of the test beam is about 500 MeV/c, while the particles reconstructible by the LHCb spectrometer are expected to have momenta above 2 GeV/c, peaking at 10 GeV/c and with a mean value of about 30-40 GeV/c. The LHCb situation therefore can be reproduced by using our test beam Monte Carlo without multiple scattering. This yields a vertex resolution (HWHM) of $\approx 70 \mu m$ at T1 (the target closest to the detectors). This is compared to the vertex resolution of a typical two-body decay, $J/\psi \rightarrow \mu \mu$, which is simulated with the full LHCb Monte Carlo program, see Figure 23 ("SICB, $J/\psi \rightarrow \mu \mu$" points).

The target positions and two tracks vertex resolutions obtained from the off-line analysis are given in Table 7. These values will be used in the study of the pile-up algorithm performance.

9.2.9 Charge distribution

One of the factors influencing the hit resolution is the charge collected on the strips in the cluster. To study this effect of charge sharing we used tracks with a $\chi^2$ probability $>1\%$ and considered the size of the $r$-clusters of SL2 and SR2.

The charge distribution over the strips of the detector is a complex function. It depends on the applied voltage, the detector thickness and the temperature. The charge collected on a strip may be distorted by effects of non-linearity of the electric field, dependence of the charge mobility on the electric field, traps etc. The charge distribution without diffusion is a simple step function extending from the entrance to the exit detector point. It is the largest contribution to the track.
Test beam results of prototype silicon detector

<table>
<thead>
<tr>
<th>TN</th>
<th>z, cm</th>
<th>σ, cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.554±0.002</td>
<td>0.039±0.001</td>
</tr>
<tr>
<td>2</td>
<td>-1.603±0.002</td>
<td>0.045±0.002</td>
</tr>
<tr>
<td>3</td>
<td>-2.610±0.002</td>
<td>0.047±0.002</td>
</tr>
<tr>
<td>4</td>
<td>-3.625±0.002</td>
<td>0.056±0.002</td>
</tr>
<tr>
<td>4'</td>
<td>-4.114±0.003</td>
<td>0.056±0.003</td>
</tr>
<tr>
<td>5</td>
<td>-4.628±0.002</td>
<td>0.057±0.002</td>
</tr>
<tr>
<td>6</td>
<td>-5.642±0.003</td>
<td>0.072±0.002</td>
</tr>
<tr>
<td>7</td>
<td>-6.652±0.002</td>
<td>0.074±0.002</td>
</tr>
<tr>
<td>8</td>
<td>-7.675±0.003</td>
<td>0.087±0.003</td>
</tr>
<tr>
<td>8'</td>
<td>-8.189±0.004</td>
<td>0.057±0.005</td>
</tr>
<tr>
<td>9</td>
<td>-8.679±0.003</td>
<td>0.090±0.003</td>
</tr>
<tr>
<td>10</td>
<td>-9.691±0.003</td>
<td>0.081±0.003</td>
</tr>
<tr>
<td>11</td>
<td>-10.696±0.004</td>
<td>0.102±0.004</td>
</tr>
<tr>
<td>12</td>
<td>-11.722±0.003</td>
<td>0.098±0.003</td>
</tr>
</tbody>
</table>

Table 7: Positions of targets (T1-T12) and the vertex resolutions.

resolution. The noise added to the strip amplitude can bias the true position of the hits due to fluctuations in the amplitude. Moreover, if not subtracted it can increase the cluster width. The ratio of signal to noise is between 20 and 40 [25].

In order to reproduce the charge distributions on the strip we use the following formulas from Damerell's review on silicon detectors [24]:

- Einstein relation:
  \[ D = \frac{\mu kT}{q} \]
  where \( D \) is the diffusion coefficient, \( \mu = 1350 \text{ cm}^2/(\text{Vs}) \) is the mobility of the electron, \( kT \) is the Boltzmann temperature and \( q \) is the charge (we set it to 1).

- The effect of diffusion on the width of the charge distribution, which is proportional to the drift time of the charge, \( \tau_d \):
  \[ \sigma^2_{\tau_d} = 2D\tau_d \]  \hspace{1cm} (25)

- The variation of the drift time as a function of the distance:
  \[ \tau_d = \frac{d}{2\mu V} \ln \left( \frac{d-l}{d} \right) \]
Test beam results of prototype silicon detector

due to the fact that the electric field is a linear function of the distance \( l \) to the strip side: \( E(x) = -2\mu V(d - l)/d^2 \) with \( d \) the thickness of the detector and \( V \) the applied voltage.

Combining these formulae into a single expression we find the charge distribution on the strip side\(^1\) of the detector. It is a convolution integral of gaussian distributions with sigmas \( \sigma_d \) given by (25):

\[
I(x) = \int_{x_{in}}^{x_{out}} dx_0 \exp \left\{ - \frac{(x - x_0)^2}{2A \ln \left[ \frac{x_0 - x_{out}}{x_{in} - x_{out}} \right]} \right\}
\]

where \( x_{in} \) and \( x_{out} \) are the entry and exit point of the particle in the detector and \( A = (kT d^2)/V \). Definitions of some variables are sketched in Figure 24. The factor \( A \) includes all detector dependent constants.

The integration is done numerically. Results for the charge distributions of tracks with different inclination angles are shown in Figure 25. Despite the obvious asymmetry in the
distribution its mean value corresponds exactly to the middle between the entry and exit points

\[^1\)we have neglected the hole component, because it only shows up in the time evolution of the signal on the strip and must be equal to zero over infinitely long collection times. This is the case for 1.4 \( \mu \)s integration time and 70 ns collection time of the holes.\]
Test beam results of prototype silicon detector

of the track. Therefore, no bias is introduced by diffusion.

Using this simple model we can explain the cluster size, i.e. the number of strips in a cluster, as a function of incident angle (α), see Figure 26. This plot uses tracks found in the target run with a $\chi^2$ probability >1% and considers the $r$-clusters of SL2 and SR2. For this purpose the $I(x)$ distribution was integrated over individual strips (digitization).

The noise cut is essential because it determines not only the cluster size at $\alpha=0$, but also the transition between the constant and linear part of the function shown in Figure 26. For instance, the model without noise cut gives a cluster size of 2.5 at $\alpha=0$ for a 40 μm strip. The cluster size remains constant up to 0.1 radians. The slope of the linear part is independent of the value of the noise cut. For our model a noise cut equivalent to 6 ADC counts is applied. It is consistent with the noise level measured in the data [25].

The model describes the data well at small polar angles, while at large angles the predicted cluster size is somewhat larger than the observed one. The size of the bias in determining the cluster position as described in the Section “Track finding”, page 37, and shown in Figure 20 is also reproduced with this model.

10 Summary

A silicon detector test set-up was placed in a 120 GeV/c pion beam at the CERN SPS. For the data analysis a track fit procedure was developed using MINUIT. It was found that coordinates are measured with average resolutions of 6 (11.5) microns in radius and 0.9(0.3) milliradians in angle for the inner (outer) parts of the detectors. Charge sharing effects are observed and reproduced by a simple model, which includes charge diffusion. For the analysis with the pileup trigger algorithm, reconstructed tracks are selected on the basis of $\chi^2$ probability, keeping 99% of the real tracks. The combinatorial background is 3.2±0.1%.

Monte Carlo simulations were developed for the track fit and vertex reconstruction. By parametrizing the hit resolution with values obtained from the analysis the simulation reproduces the vertex resolution. It is shown that a two track vertex measured with the test beam has a resolution (HWHM) of 230 μm. When extrapolating this value to the conditions expected in LHCb this value becomes 70 μm.

Summary

46