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Snel, J.G.

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DETECTION OF CARPAL BONE CONTOURS FROM 3-D MR IMAGES
Planar contours of the carpal bones, the radius, and the ulna.
Detection of the Carpal Bone Contours from 3-D MR Images of the Wrist Using a Planar Radial Scale-Space Snake

J.G. Snel\(^1\)  H.W. Venema\(^{1,2}\)  C.A. Grimbergen\(^{1,3}\)

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Abstract

In this chapter we consider the problems encountered when applying snake models to detect the contours of the carpal bones in three-dimensional (3-D) magnetic resonance (MR) images of the wrist. In order to improve the performance of the original snake model introduced by Kass et al. [19], we propose a new image force based on one-dimensional (1-D) second-order Gaussian filtering and contrast equalization. The improved snake is less sensitive to model initialization and has no tendency to cut off contour sections of high curvature, because 1-D radial scale-space relaxation is used. Contour orientation is used to minimize the influence of neighbouring image structures. Due to 1-D contrast equalization, an intensity insensitive measure of external energy is obtained. As a consequence, a good balance between internal and external energetic contributions of the snake is established, which also improves convergence. By incorporating this new image force into the snake model, we succeeded in accurate contour detection, even when relatively high noise levels were present and when the contrast varied along the contours of the bones.

Keywords

contour detection, image segmentation, scale-space, snakes

Author affiliations:
1) Department of Medical Physics at the Academic Medical Center, Amsterdam
2) Department of Radiology at the Academic Medical Center, Amsterdam
3) Department of Measurement and Control, Faculty of Mechanical Engineering and Marine Technology at Delft University of Technology
2.1 Introduction

The wrist is a complex anatomical region containing eight carpal bones, with numerous articular surfaces, ligaments, tendons, and neurovascular structures. Wrist pain, with or without dysfunction, is a common diagnostic problem, but the clinical signs are often subtle. Conventional imaging (plain film radiography, scintigraphy, conventional tomography, and arthrography) frequently proves to be inadequate or equivocal in determining the source of the pain. With their high-resolution, high-contrast, and multi-planar imaging capabilities, magnetic resonance imaging (MRI) and computed tomography (CT) can be used to overcome this problem.

In many cases, the source of wrist pain is due to dislocation or luxation of the carpal bones. In an attempt to assist radiological diagnosis we developed a segmentation algorithm for the detection of the planar contours of the carpal bones in a sequence of successive slices of a 3-D image of the wrist. After detection, a stack of planar contours is obtained for each bone, which describes its boundary surface and from which the orientation and location of the bone can be determined. Our aim is to use this information for visualization, and also for quantification of the location and orientation of the carpal bones when the wrist is placed in different postures.

In this study we chose MRI for 3-D imaging of the wrist, using an imaging protocol developed at the MRI department of our hospital (Academic Medical Center, Amsterdam), to obtain optimal contrast between bones and soft tissue. Technical details are given in Section 2.2. Figure 14 shows a region of interest (ROI) of nine slices of a 3-D MR image of one of the wrists used in this study.

Our segmentation algorithm was based on the active contour model, introduced by Kass et al. In this model a deformable curve called a snake is pulled by external forces towards desired image features, such as contours in our application. This snake is controlled by internal (spline) forces, which give it resistance to elastic deformation and bending.

Many authors have tried to improve the original model of Kass et al. with regard to sensitivity to the required model initialization, derivation of the image force, adjustment of model parameters, convergence, and numerical approximation of the model. This resulted among other things in finite difference methods (FDM), finite element methods (FEM), dynamic programming algorithms, and local approximation algorithms.

When we applied different FDM and local approximation models to detect the contours of the carpal bones, it appeared that a number of problems still remained.
Figure 14: Nine slices (ROIs covering 64 × 64 pixels of 1 mm²) of 1 mm thickness with a distance of 4 mm of a 3-D MR image of the wrist. The 3-D spoiled-gradient echo technique with frequency-selective fat saturation was used. The eight carpal bones, hamate (H), capitate (C), trapezoid (Zd), trapezium (Zm), triquetrum (T), lunate (L), scaphoid (S), and pisiform (P), five metacarpal bones (MC₁ to MC₅), radius (R), and ulna (U) are displayed dark on a brighter background, which is composed of soft tendons such as cartilage, and muscles.
The main shortcomings we encountered are the following:

- sensitivity to model initialization
- improper contour detection when the contrast varies along the contour
- part of the snake is sometimes attracted to the contour of a neighbouring carpal bone
- the snake has a tendency to cut off contour sections of high curvature.

The purpose of the present study was to find a solution for these problems. We achieved this by using a new image force based on 1-D second order Gaussian filtering in combination with 1-D contrast equalization.

2.2 Materials

Four wrists were imaged in different postures using MRI (1.5-T Siemens Magnetom 63 SP 4000). Since the standard Siemens protocols for imaging of the wrist did not meet our requirements, a new protocol was developed at the MRI department of our hospital: 3-D Fast Low Angle Flash (FLASH) (spoiled gradient echo), with frequency selective fat saturation (FATSAT), TR = 50 ms, TE = 10 ms, $\alpha = 40^\circ$. Using this technique, the MR signal originating from the bones was suppressed and high contrast between bones and soft tissue could be achieved. However, in order to obtain a good fat suppression over the whole region of interest, a good field homogeneity is required. Due to the irregular anatomy of the wrist and hand, much effort had to be put into obtaining this homogeneity. The wrists were imaged in a reasonable scan time (6 min) with high resolution ($128 \times 128 \times 64$ cubic voxels of 1 mm$^3$).

A graphical workstation (Sun SPARC 20 at 66 MHz) was used for image processing and visualization. We developed software in C to implement the snake algorithm and triangulation algorithms for surface reconstruction from contour stacks, and built an infrastructure for contours and surfaces. To display images and contours, we used the image processing package SCILIMAGE, which was developed at the TNO (Netherlands Organization for Applied Scientific Research) and at the Faculty of Mathematics and Computer Science of the University of Amsterdam. The visualization package AVS (Advanced Visual System Inc., Waltham, MA, USA) was used for 3-D surface rendering.
2.3 Methods

2.3.1 Snake model

In the original definition of the snake due to Kass et al. [19], contour detection leads to an energetic variational problem. The snake is defined as a 2-D curve $\vec{v}(s) = (x(s), y(s))$ in the plane, dependent on the arc length $s \in [0, S]$, where $\vec{v}(0)$ and $\vec{v}(S)$ denote the snake’s head and tail, and $S$ its length. A total energy $E[\vec{v}]$ is associated with the snake as a combination of internal energy $E_{\text{int}}$ and external energy $E_{\text{ext}}$. $E_{\text{int}}$ accounts for elasticity and curvature of the snake; $E_{\text{ext}}$ is composed of potential energy $P(x, y)$, dependent on the kind of image features to be detected, and external constraint energy $E_{\text{con}}$, which is used for interactive purposes. We left $E_{\text{con}}$ out of consideration here and as a result $E[\vec{v}]$ becomes

$$E[\vec{v}] = \int_0^S \left[ w_{\text{ela}}(s) \| \vec{v}_s(s) \|^2 + w_{\text{curv}}(s) \| \vec{v}_{ss}(s) \|^2 + w_{\text{im}}(s) P(\vec{v}(s)) \right] ds ,$$

(2.1)

where $w_{\text{ela}}$, $w_{\text{curv}}$, and $w_{\text{im}}$ denote the weights for the elasticity, curvature, and potential energy; the brackets ‘$\|$’ denote Euclidean normalization and subscript ‘$s$’ denotes differentiation with respect to $s$.

In our application the image features to be detected were the contours in the image $I(x, y)$. For contour detection the potential energy $P(x, y)$ was derived from $I(x, y)$, such that the contours corresponded with minima in $P(x, y)$. The Gaussian gradient filter $\nabla G_\sigma$ was used for this purpose

$$P(x, y) = -\| \nabla (G_\sigma(x, y) \ast I(x, y)) \| ,$$

(2.2)

where $\sigma$ denotes the width of the 2-D Gauss filter $(1/2\pi\sigma^2) e^{-((x^2+y^2)/2\sigma^2)}$ (‘$\ast$’ denotes convolution). An example of $P(x, y)$ is given in Fig. 15 where the potential energy was calculated from a ROI of an MR image slice containing the carpal bones.

The minimum of $E[\vec{v}]$ represents the minimum energetic state of the snake and is considered to be an optimal approximation of the contour given the elasticity and curvature constraints. The snake that minimizes $E[\vec{v}]$ is the solution of the Euler equation derived from Eq. (2.1)

$$-(w_{\text{ela}}(s)\vec{v}_s(s))_s + (w_{\text{curv}}(s)\vec{v}_{ss}(s))_{ss} = -w_{\text{im}}(s) \nabla P .$$

(2.3)

The terms of the left side of Eq. (2.3) represent the elastic and curvature forces, imposing resistance upon elasticity and bending; the gradient $\nabla P$ at the right side represents the image force which attracts the snake to the contour.
Figure 15: (a) MR image slice (ROI containing 64 × 64 pixels of 1 mm²). (b) Potential energy \( P(x, y) \) derived from (a), using a Gaussian gradient filter (\( \sigma = 3 \) pixels). The contours of (a) give rise to low values in potential energy (displayed dark in (b)).

Kass et al. [19] solved the snake by modifying Eq. (2.3) into a time-dependent partial differential equation (PDE). They introduced a damping term \( \gamma \vec{v}_t(s, t) \) to dissipate potential energy, where the snake \( \vec{v}(s, t) \) is dependent on space \( s \) and time \( t \) and \( \gamma \) denotes the damping density (viscosity factor); subscript \( t \) denotes differentiation with respect to time. In this way the minimum of \( E[\vec{v}] \) is found dynamically, starting from an initial snake \( \vec{v}(s, 0) \) which is a rough estimate of the contour.

Subsequently, to improve convergence, Leymarie and Levine [26] modified Eq. (2.3) into an Euler-Lagrange equation of motion by introducing an additional inertia term \( \mu \vec{v}_{tt}(s, t) \)

\[
\mu \vec{v}_{tt}(s, t) + \gamma \vec{v}_t(s, t) = \left( w_{\text{ela}}(s) \vec{v}_s(s, t) \right)_s + \left( w_{\text{curv}}(s) \vec{v}_{ss}(s, t) \right)_{ss} = -w_{\text{im}}(s) \nabla P(x, y). \tag{2.4}
\]

The mass density \( \mu \) together with damping density \( \gamma \) determine the snake’s kinematics. In this way a more stable minimum \( E[\vec{v}] \) is found by converting potential energy into kinetic energy, which is dissipated through damping. When all kinetic energy has been dissipated, the snake has reached its steady state, which solves Eq. (2.3).

### 2.3.2 Discrete snake

Leymarie and Levine [26] solved the snake numerically using the finite difference method (FDM). In short, \( S \) is divided into \( N_s \) segments \( \Delta s = (S/N_s) \) and the time \( t \) is divided into time intervals \( \Delta t \)

\[
s_i = (i - 1)\Delta s \quad (i = 1, \ldots, N_s),
\]

\[
t_j = j\Delta t \quad (j = 0, \ldots). \tag{2.5}
\]
The snake $\tilde{v}(s,t)$ is defined in $N_s$ points, called snaxels, which are stored in a vector $V_j$ at point of time $j\Delta t$

$$V_j = (\tilde{v}_{0,j}, \tilde{v}_{1,j}, \ldots, \tilde{v}_{N_s-1,j}) , \quad (2.6)$$

where $\tilde{v}_{i,j} = \tilde{v}(s_i,t_j)$. For each snaxel the image force $\vec{f}$ is calculated from the potential energy $P(x,y)$

$$\vec{f}_{i,j} = -\nabla P(\tilde{v}_{i,j}) \quad (i = 1, \ldots, N_s). \quad (2.7)$$

The potential energy is calculated from a discrete image $I(kx,ly)$ where $(k,l)$ denote the pixels of size $\Delta x \times \Delta y$. In an arbitrary point the gradient $\nabla P$ is approximated by interpolation of forward or central differences of a discrete potential energy $P(kx,ly)$.

The weighted image forces $w_{\text{im}}(s_i)\vec{f}_{i,j} (i = 1, \ldots, N_s)$ are stored in a vector $F_j$ at point of time $j\Delta t$. After discretization the Euler-Lagrange equation becomes [26]

$$(M + \frac{1}{2}C + K)V_j = F_{j-1} + 2MV_{j-1} - (M - \frac{1}{2}C)V_{j-2} , \quad (2.8)$$

where $M$, $C$, and $K$ denote the mass, damping, and stiffness matrices, dependent on respectively $\tilde{\mu} = (\mu/\Delta t^2)$, $\tilde{\gamma} = (\gamma/\Delta t)$, $(w_{\text{ela}}(s_i)/(\Delta s)^2)$, and $(w_{\text{curv}}(s_i)/(\Delta s)^4)$ $(i = 1, \ldots, N_s)$.

At each point of time $j\Delta t$ a new snake $V_j$ is calculated by solving the linear system of equations [Eq. (2.8)], which can be done conveniently by Choleski decomposition [39]. For initialization $V_0 = V_1 = V^{\text{init}}$ is set, where $V^{\text{init}}$ is an initial snake supplied externally. This snake is iteratively improved and, when a convergence criterion has been satisfied, iteration is terminated.

We note that the average displacement of the snaxels for each iteration step is determined by the factors $\tilde{\mu}$ and $\tilde{\gamma}$, as these factors incorporate $(\Delta t)^2$ and $\Delta t$, respectively. When $\tilde{\mu}$ is increased with a factor $\phi^2$ and $\tilde{\gamma}$ with a factor $\phi$, the average displacement per iteration step is decreased with a factor $\phi$.

### 2.3.3 Scale-space relaxation

Kass et al. [19] proposed a scale-space relaxation method to enlarge the capture region in which the snake is attracted to an image feature (a contour of a carpal bone in our case). This region is large when a high value of scale $\sigma$ is used in the calculation of the potential energy in Eq. (2.2). Consequently, the choice of the initial snake becomes less critical to its exact positioning, thus improving contour detectability. By using a low value for $\sigma$, small image details are preserved, thus improving contour localization. Hence, a series
of smoothed potential images is used for decreasing $\sigma$ to optimize the snake through (discrete) scale-space. The snake model is started by setting $\sigma$ equal to an upper bound $\sigma_{\text{max}}$. The value of $\sigma$ is reduced gradually to a lower bound $\sigma_{\text{min}}$. For each new value of $\sigma$, the optimized snake from a previous scale is used as the new initial snake.

### 2.3.4 Problems and limitations of snake models

Initially we tried to apply the snake model as described above to the problem of contour detection of the carpal bones. We encountered, however, a number of problems:

- **The tuning of model parameters.** The snake model possesses a number of parameters which have to be chosen properly. The weights $w_{\text{ela}}(s_i)$, $w_{\text{curv}}(s_i)$, and $w_{\text{im}}(s_i)$ had to be tuned in order to balance the internal and external forces for each snaxel. When the internal forces dominated, shrinking of the snake or extreme smoothing of its curvature could happen. When the external forces dominated, the coherence between the snaxels was reduced. In this way clusters of snaxels could arise at corners of the contour [37], or – even worse – the snake acted like a collection of snaxels moving independently. The dynamical behavior of the snake is determined by the factors $\tilde{\mu}$ and $\tilde{\gamma}$. If $\tilde{\mu}$ or $\tilde{\gamma}$ were chosen too small, the snake was deformed too strongly and could be displaced out of the region of capture. When the value of $\tilde{\mu}$ was chosen too high, the snake could keep oscillating around the contour and no convergence was obtained. To improve convergence, we increased the value of $\tilde{\gamma}$. However, the snake could be slowed down too much in this case and never reached the contour.

- **Sensitivity to model initialization when applying scale-space relaxation.** The upper bound $\sigma_{\text{max}}$ (see Sec. 2.3.3) can be chosen high in order to enlarge the region of capture. We empirically found that in some cases the snake did not converge to the contour when $\sigma_{\text{max}}$ was chosen too high, even when we reduced the value of $\sigma$ by very small steps in scale-space. In addition, we found that the snake had a bias to cut off regions of high curvature when $\sigma_{\text{max}}$ was chosen too high, independent of the settings of the internal weights $w_{\text{ela}}(s_i)$ and $w_{\text{curv}}(s_i)$. Another problem was that in the images of the wrist a number of adjacent contours were present. If the initial snake was not positioned carefully, part of the snake could be attracted to the wrong contour. Even when we positioned the initial snake entirely within the area surrounded by the contour, part of the snake still converged to the wrong contour when the strength of the
adjacent contour was stronger and \(\sigma_{\text{max}}\) was chosen too high. In these cases \(\sigma_{\text{max}}\) had to be reduced and we had to supply a better initial snake.

- **Varying contrast along the contour.** When the contrast along the contour varied, the snake was attracted towards regions of high contrast, which could cause the snake to be pulled away from regions of low contrast. In this case clustering could not be removed by increasing all the elasticity weights \(w_{\text{ela}}(s_i)\), since this also increased the pull in areas of low contrast.

- **Clipping of the image force.** In accordance with Leymarie and Levine [26] we balanced the image force with the internal forces by clipping \(\nabla P\) to a global saturation value \(\| \nabla P \|_{\text{sat}}\) when \((\| \nabla P \| / \| \nabla P \|_{\text{sat}}) > 1\). The choice of \(\| \nabla P \|_{\text{sat}}\) appeared to be critical. If \(\| \nabla P \|_{\text{sat}}\) was underrated, the snake could keep oscillating around the contour; if \(\| \nabla P \|_{\text{sat}}\) was overrated, the internal behavior of the snake could dominate, giving rise to shrinking of the snake and smoothing of its curvature.

Many authors tried to solve these problems. Leymarie and Levine [26] updated \(w_{\text{ela}}(s_i)\) and \(w_{\text{curv}}(s_i)\) during iteration to prevent shrinking and to obtain a snake with a desired curvature at each snaxel. Furthermore, they proposed a *steady support* convergence criterion instead of a *steady state* criterion. The former is solely based on external energy contributions to improve convergence and to prevent shrinking. However, this criterion does not provide a solution that minimizes the energy in Eq. (2.1). In this way iteration of Eq. (2.8) may be terminated at a stage at which the snake is still far from equilibrium.

In an attempt to avoid problems with model initialization, Cohen [25] introduced balloons by adding a radial inflation force by which the snake expands until it reaches the contour. However, the balloons may also be attracted to noisy structures and may penetrate weak contours.

Lobregt and Viergever [37] found that the snake does not reach an acceptable energetic optimum simultaneously for both internal and external energetic contributions. In their view only radial deformations of the snake contribute to the detection of the contour. Consequently, they proposed a new active contour model by using a new curvature force and radial projection of the image force. They omitted the shrinking elastic force and preserved coherence between the snaxels by using a special resampling method. Their dynamic contour model [37] is local in the sense that local deformations influence the curve only locally, whereas in Kass’ global model [19] the entire curve is influenced by local deformations. We also tried Lobregt and Viergever’s active contour model and we empirically found that radial projection of the image force improved the performance of the model significantly. In addition, by the absence
of elastic forces and the radial definition of curvature forces the tendency of
the snake to shrink – as present in Kass’ global model – was completely re-
moved. However, we have chosen for a global snake, because we found that
it was less sensitive to initialization and therefore produced more reproducible
and stable results, in contrast with the local model which sometimes even self-
intersects. The advantages of Lobregt and Viergever’s approach motivated us
to incorporate some of their ideas into our solution, which is described in the
next section.

2.3.5 Outline of our approach

We introduce a new image force by following an essentially 1-D radial ap-
proach rather than radial projection of a 2-D image force. The potential energy
in Eq. (2.2) is modified into a 1-D radially oriented potential-energy function.
The accompanying radial image force is obtained using a 1-D radial second-
order Gauss filter \( G_\sigma^\prime \) where \( r \) denotes the radial co-ordinate. The parameter
\( \sigma \) determines the amount of radial smoothing which is used in the calculation
of the image force. Tangential smoothing is omitted since this gives the snake
a bias to cut off contour sections of high curvature.

In the original snake model [19] the potential energy is expressed as the
norm of the image gradients [Eq. (2.2)]. Consequently, no information on the
direction of the image gradient is used in the detection of the contours. How-
ever, in the derivation of the 1-D radial potential-energy function information
on contour direction is used, because that is advantageous when adjacent con-
tours are present.

1-D radial scale-space relaxation is used to optimize the snake through
discrete scale-space, using a series of 1-D Gauss filters for decreasing scale.

The response of the second-order Gauss filter \( G_\sigma^\prime\prime \) is proportional to the
image contrast. When no provisions are taken, areas with high contrast attract
the snake more strongly than areas of low contrast. This motivated us to use
(1-D) contrast equalization to level the contrast along the snake. We show
that this improves contour detection in areas of low contrast. Due to contrast
equalization we are able to obtain an intensity insensitive measure of external
energy. Consequently, we obtain a good balance between internal and external
forces using global settings for the model weights: \( w_{\text{ela}}(s_i) \equiv w_{\text{ela}}, w_{\text{curv}}(s_i) \equiv w_{\text{curv}}, \) and \( w_{\text{im}}(s_i) \equiv w_{\text{im}}. \)
2.3.6 A radial image force based on 1-D second-order Gaussian filtering

We change the co-ordinate system \( \bar{x} = (x, y) \) to a local system \((r, \tau)\) for each snaxel \(i\) using

\[
\bar{x} = \bar{v}_i + r\hat{r}_i + \tau\hat{\tau}_i, \tag{2.9}
\]

where \(\bar{v}_i\) denotes the location of snaxel \(i\) and \(\hat{r}_i\) is a unit vector perpendicular to \(\hat{r}_i\). For convenience we set \(\hat{r} = r\hat{r}_i\) and \(\hat{\tau} = \tau\hat{\tau}_i\). The image \(I(x, y)\) can then be expressed in \((r, \tau)\) co-ordinates

\[
I(r, \tau) \equiv I(\bar{v}_i + \hat{r} + \hat{\tau}) . \tag{2.10}
\]

Using \((r, \tau)\) co-ordinates, the potential energy \(P(x, y)\) becomes

\[
P(r, \tau) = -\left\| \frac{\partial}{\partial r} G_\sigma(r, \tau) * I(r, \tau) + \frac{\partial}{\partial \tau} G_\sigma(r, \tau) * I(r, \tau) \right\| . \tag{2.11}
\]

Subsequently, we modify Eq. (2.11) by omitting both the tangential derivative and Euclidean normalization, which leads to a ‘radial’ potential energy function

\[
P(r, \tau) = -\frac{\partial}{\partial r} G_\sigma(r, \tau) * I(r, \tau) , \tag{2.12}
\]

with accompanying radial image force amplitude

\[
f(r, \tau) = -\frac{\partial P}{\partial r} = \frac{\partial^2}{\partial r^2} G_\sigma(r, \tau) * I(r, \tau). \tag{2.13}
\]

Note that the image intensity \(I(r, \tau)\) is convoluted with a kernel composed of a radial second-order Gauss filter \(G''_\sigma(r)\) and a tangential smoothing filter \(G_\sigma(\tau)\). This can be seen from the orthogonal filter decomposition

\[
\frac{\partial^2}{\partial r^2} G_\sigma(r, \tau) = G''_\sigma(r) * G_\sigma(\tau) , \tag{2.14}
\]

with \(G_\sigma(\xi) = (1/\sqrt{2\pi\sigma})e^{-(\xi^2/2\sigma^2)}\) (\(\xi = r, \tau\)). Since tangential smoothing gives the snake a bias to cut off curved regions, we omit the tangential dependency and use the image force amplitude

\[
f(r) = G''_\sigma(r) * I(r) , \tag{2.15}
\]

where \(I(r) \equiv I(r, 0)\).

\(^1\)The direction of \(\hat{r}_i\) is chosen radially outward when detecting dark objects on a bright background and radially inward in the reverse case.
Figure 16: The effect of omitting normalization. (a) The initial snake (rectangle) is superimposed on the image containing dark objects A and B on a bright background. The contour surrounding object A has to be detected. The snaxel under consideration is indicated by a black square in the middle of a black line segment in the radial direction of the snake. (b) The image intensity $I$ and the potentials of Eq. (2.11) and Eq. (2.12), indicated by $P_{2,11}$ and $P_{2,12}$, are compared along this line segment. The contour of the neighbouring object ($r_B$) gives rise to repulsion using Eq. (2.12), hence the snake is attracted to the proper contour ($r_A$). The use of Eq. (2.11) causes the snake to be attracted to the wrong contour ($r_B$).

As a consequence, we introduce a new image force at snaxel $i$ which is essentially radial

$$\vec{f}_i = f(0)\hat{r}_i,$$  \hspace{1cm} (2.16)

where $f(0)$ denotes the 1-D radial image force amplitude at $r = 0$.

The radial definition of the potential energy and the image force has the advantage that normalization is no longer necessary, so that both attraction (to a contour with the right orientation with respect to $\hat{r}_i$) and repulsion (from a contour with the wrong orientation with respect to $\hat{r}_i$) are obtained. The effect of omitting Euclidean normalization is illustrated in Fig. 16.

### 2.3.7 Radial scale-space relaxation

1-D radial scale-space relaxation is used to optimize the snake starting from a rough estimate of the contour. To eliminate the need for adjustment of the model weights ($w_{ela}$, $w_{curv}$, and $w_{im}$) when $\sigma$ is decreased in (discrete) scale-
space, a normalized filter \( \hat{G}_\sigma''(r) \) is used

\[
\hat{G}_\sigma''(r) = \frac{\sigma^2}{4} \sqrt{2\pi\sigma} G_\sigma''(r) = \frac{1}{4\sigma} \left( \frac{r^2}{\sigma^2} - 1 \right) e^{-\left(\frac{r^2}{2\sigma^2}\right) + (1/2)}, \tag{2.17}
\]

with

\[
\int_{-\infty}^{\infty} |\hat{G}_\sigma''(r)| dr = 1. \tag{2.18}
\]

Using \( \hat{G}_\sigma''(r) \), the maximum amplitude of the image force remains approximately constant when \( \sigma \) is varied (for a step edge it is constant) for each snaxel. In a conventional approach the image force is derived from a series of intermediate potential energy images for various \( \sigma \), using Eq. (2.2). The radial image force presented here is directly applied by filtering the image intensity along the snake with a series of filters \( \hat{G}_\sigma''(r) \). The value of \( \sigma \) is decreased from \( \sigma_{\text{max}} \) to \( \sigma_{\text{min}} \) by step size \( \Delta\sigma = \left( (\sigma_{\text{max}} - \sigma_{\text{min}}) / (N_\sigma - 1) \right) \), where \( N_\sigma \) denotes the number of scales.

### 2.3.8 Numerical approximation of the image force

The amplitude of image force corresponds to a 1-D convolution of the image intensity \( I(r) \) along a line \( l \) through a snaxel \( \vec{v}_i \) in the direction of \( \hat{r}_i \)

\[
f(0) = \hat{G}_\sigma''(r) * I(r) \bigg|_{r=0} = \int_{-\infty}^{\infty} \hat{G}_\sigma''(r) I(-r) dr \tag{2.19}
\]

The convolution integral in Eq. (2.19) is approximated by using equally spaced samples \( I(k\Delta r) \), with \( k \in \mathbb{Z} \), separated by a distance \( \Delta r \) along \( l \) (Fig. 17). The sample value \( I(k\Delta r) \) is calculated by linear interpolation of adjacent pixels. Using zero-order polynomial interpolation of the image samples \( I(k\Delta r) \), we obtain after integration

\[
f(0) = \sum_{k=-\infty}^{\infty} K_\rho(k) I(-k\Delta r), \tag{2.20}
\]

where the ratio \( \rho = (\sigma / \Delta r) \) is used. \( K_\rho \) is the (piecewise) integrated kernel \( \hat{G}_\rho'' \)

\[
K_\rho(k) = \frac{\sqrt{\rho}}{4\rho} \left( \left( k - \frac{1}{2} \right) e^{-\left( (k-1/2)^2/2\rho^2 \right)} - \left( k + \frac{1}{2} \right) e^{-\left( (k+1/2)^2/2\rho^2 \right)} \right). \tag{2.21}
\]

Since \( K_\rho(k) \) decays rapidly with \( k \), we truncate the series in Eq. (2.20) when \( |k| > (N_k / 2) \) (\( N_k \) even). Hence the convolution is applied to line segments pointing radially along the snake having length \( N_k \Delta r \) and containing \( N_k + 1 \) samples. \( N_k \) depends on decay of \( K_\rho \) which is determined by \( \rho \). We chose \( N_k \approx 6\rho \).
2.3.9 1-D contrast equalization

The amplitude of the image force in Eq. (2.15) is proportional to the contrast along the radial line segments at each snaxel; therefore the snake is less attracted in areas of low contrast, which can result in poor contour detection in these areas. This observation motivated us to level the contrast along the snake by normalizing the amplitude of force $f(0)$ by a local radial contrast estimate to obtain a ‘contrast-independent’ snake. The line segments define a radial local neighbourhood around each snaxel. From this neighbourhood a contrast estimate $C_r$ is derived by first-order differences

$$C_r = \max_{|k| < (N_r/2)} \left( |I(k\Delta r) - I((k-1)\Delta r)| / \Delta r \right). \quad (2.22)$$

The value of $C_r$ may change considerably when the contrast along the contour suddenly changes, e.g. in gap regions of the image contour or when a snaxel moves rapidly. We reduced these variations by averaging $C_r$ over a time interval of 10 iterations. The normalized amplitude $\hat{f}$ is expressed as

$$\hat{f}(0) = \frac{f(0)}{C_r}, \quad (2.23)$$

where $\bar{C}_r$ denotes the time-averaged radial image contrast estimate.
2.3.10 Turning off the image force

In two cases the image force is turned off. First, to avoid problems with the normalization as proposed in Sec. 2.3.9 when \( C_r \) is very small, or even zero, we regard the image intensity to be constant when the contrast \( C_r \) is lower than a preset threshold \( T_c \) and we set \( \hat{f}(0) \) to zero.

Secondly, when a snaxel is located relatively far outside the boundary region (on the other side of the 1-D potential well in Fig. 16) it may be pushed away instead of being attracted towards the boundary. This is especially the case when two adjacent contours are very close, which often occurs in our application. We prevent such repulsion by checking the sign \( s' \) of the 1-D first\(^2\) and the sign \( s'' \) of the second order filtered radial image intensity. We set \( \hat{f}(0) \) to zero when \( s' < 0 \land s'' > 0 \). It turned out to be sufficient when this check was applied in the first scale (\( \sigma_{\text{max}} \)) during scale-space relaxation.

In both cases the movement of a snaxel is influenced by internal forces only.

2.3.11 Resampling of the snake

To decrease the dependency on elasticity we adopt the resampling method proposed by Lobregt and Viergever [37] using a target distance \( \Delta s \) between the snaxels by fusing two snaxels when their distance is smaller than \( \frac{1}{2} \Delta s \) and by inserting a new snaxel between two snaxels when their distance is larger than \( \frac{3}{2} \Delta s \). We apply this resampling during iteration of Eq. (2.8) with a periodicity of \( N_{\text{res}} \) iterations. Repeating of the Choleski factorization is needed when the number of snaxels changes.

2.3.12 Parameter settings

In the snake model a number of parameters have to set. Table I lists these parameters, which are divided into three classes: spatial, temporal, and miscellaneous parameters. The spatial class is split into model parameters and parameters used for calculation of the image force. In order to achieve that the snake is only influenced by relative changes in the weights \( w_{\text{ela}}, w_{\text{curv}}, \) and \( w_{\text{im}} \), each weight is divided by the sum \( w_{\text{ela}} + w_{\text{curv}} + w_{\text{im}} \) when they are used in Eq. (2.8).

We use a separate set of weights \( (w_{\text{ela}}^{\text{init}}, w_{\text{curv}}^{\text{init}}, \text{and } w_{\text{im}}^{\text{init}}) \) for the first scale (\( \sigma_{\text{max}} \)) in scale-space. In this initial set the value of the weights for elasticity and curvature are increased in order to smooth the initial snake and to att-

\(^2\text{The first order filtered radial image intensity equals } G'(r) \ast I(r)\).
Table I: Parameter settings used for the test images with pixel size [0.5 × 0.5 mm$^2$] (A) and the MR images with pixel size [1.0 × 1.0 mm$^2$] (B). *For the first scale in scale-space ($\sigma_{\text{max}}$) different values for $w_{\text{ela}}$ and $w_{\text{curv}}$ are used: $w_{\text{ela}}^{(\text{init})} = 0.5$ and $w_{\text{curv}}^{(\text{init})} = 2.5$.

Table: | Parameter       | A       | B       | Description                                               | Sec. |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{\text{ela}}^*$</td>
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<td>0.1</td>
<td>elasticity weight</td>
<td>2.3.1</td>
</tr>
<tr>
<td>$w_{\text{curv}}^*$</td>
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<td>0.1</td>
<td>curvature weight</td>
<td>2.3.1</td>
</tr>
<tr>
<td>$w_{\text{im}}$</td>
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<td>1.0</td>
<td>weight for the image feature</td>
<td>2.3.1</td>
</tr>
<tr>
<td>$\Delta s$</td>
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<td>0.5</td>
<td>target snaxel distance [pixel units]</td>
<td>2.3.1</td>
</tr>
<tr>
<td>$\Delta r$</td>
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<td>0.5</td>
<td>spatial step size [pixel units]</td>
<td>2.3.8</td>
</tr>
<tr>
<td>$N_{\sigma}$</td>
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<td>4</td>
<td>number of scales used in scale-space</td>
<td>2.3.7</td>
</tr>
<tr>
<td>$\sigma_{\text{max}}$</td>
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<td>2.0</td>
<td>upper bound used in scale-space [pixel units]</td>
<td>2.3.7</td>
</tr>
<tr>
<td>$\sigma_{\text{min}}$</td>
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<td>0.5</td>
<td>lower bound used in scale-space [pixel units]</td>
<td>2.3.7</td>
</tr>
<tr>
<td>$\tilde{\mu}$</td>
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<td>normalized mass density constant</td>
<td>2.3.1</td>
</tr>
<tr>
<td>$\tilde{\gamma}$</td>
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<td>2.3.1</td>
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<tr>
<td>$\epsilon$</td>
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<td>0.01</td>
<td>lower bound for average snaxel displacement [pixels per iteration]</td>
<td>2.3.12</td>
</tr>
<tr>
<td>$N_{\text{res}}$</td>
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<td>5</td>
<td>resample periodicity</td>
<td>2.3.11</td>
</tr>
<tr>
<td>$T_c$</td>
<td>1</td>
<td>1</td>
<td>contrast threshold (grey value ranging from 1 to 255)</td>
<td>2.3.10</td>
</tr>
</tbody>
</table>

tract the snaxels for which the image force is turned off to the contour (see Sec. 2.3.10).

The spacing ($\Delta r$) of the radial image samples determines the accuracy of the calculation of the image force. It must be chosen smaller than the distance between the contours in the image. The spacing ($\Delta s$) of the snaxels is based on the resolution of the contour in the image. The upper bound $\sigma_{\text{max}}$ determines the size of region of attraction. Its value is set in accordance with the expected distance of the initial snake to the contour. To improve contour localization the value of $\sigma_{\text{min}}$ is set to a low value.

The values $\tilde{\mu}$ and $\tilde{\gamma}$ determine the dynamical behavior of the snake and are chosen in such a way that the average displacement of the snaxels is small with regard to the value of $\sigma_{\text{max}}$. We stop the iteration when the average displacement per snaxel decreases and becomes lower than a threshold $\epsilon$.

The threshold $T_c$ is set to a low value with regard to the minimum contrast in the image.
Detection of Carpal Bone Contours from 3-D MR Images

Figure 18: (a) MR image slice (ROI covering $64 \times 64$ pixels of $1 \text{ mm}^2$). (b) A binary test image ($128 \times 128$ pixels of $0.25 \text{ mm}^2$) derived from (a) by thresholding and smoothing. Superimposed is a set of coarse initial snakes. (c) A second set of initial snakes. (d) Intermediate sets of initial snakes obtained from the sets in (b) and (c) by morphing.

2.3.13 2$\frac{1}{2}$-D contour detection

We use the snake described above to detect the contours of the carpal bones by tracking the contours from slice to slice in the 3-D MR image, which produces a stack of contours for each bone as a result.

We start with an initial snake in an image slice in the central region of the carpal bone which is supplied by the user. For the other slices the optimized snake from the neighbouring slice is used as initialization. Since 2-D information is thus used to solve a 3-D problem, this hybrid method is called 2$\frac{1}{2}$-D contour detection.

2.4 Results

2.4.1 Experiments—Contour detection in test images

We tested the snake in situations of varying image noise and contrast. A binary image ($128 \times 128$ pixels) was used for this test, as shown in Fig. 18(b). Using interpolation, thresholding, and smoothing, this image was derived from a region of interest ($64 \times 64$ pixels of $1 \text{ mm}^2$) of a MR image slice containing seven carpal bones, shown in Fig. 18(a). Since the location of the contours in a binary representation is exactly known, this binary image was suitable for a quantitative evaluation of the quality of the match between the snake and the contour. We added Gaussian noise to this image and also manipulated the image contrast to observe the snake’s performance.

To observe the sensitivity to initialization and the reproducibility of our model we used a series of 10 coarse initial snakes for each carpal bone to be detected. Each series was created by transforming two coarse initializations into
Chapter 2

each other using a morphing technique to transform curves [Fig. 18(b)–(d)]. The quality of contour detection was quantified by calculating the average radial distance $d_{avg}$ and the maximum radial distance $d_{max}$ of the snaxels to the contour in the binary image. In the captions of the relevant figures $d_{avg}$ and $d_{max}$ are presented, which are average values for all series of initializations for seven carpal bones.

The parameter settings are indicated in Table I. In Fig. 19 the contour detection is shown for the ideal case and for images with decreasing signal-to-noise ratio. For all signal-to-noise ratios all initial snakes converged to a virtual identical definitive snake, which described the desired contour very well except at the highest noise level (SNR = 1). At this very high noise level we prevented the snakes from shrinking at the initialization scale $\sigma_{max}$ by setting $w_{ela}^{(init)} = 0.05$. Since the movement of the snaxels due to elastic forces was slowed down at the initialization scale some of the snaxels for which the image force was turned off converged too fast. As a consequence, we had to increase the iteration time by setting $\epsilon = 0.01$ pixel per iteration. In addition, the snakes tended to get a jagged appearance [Fig. 19(d)]. In this case smoother snakes could be obtained by increasing $w_{curv}$ (not shown).

When we added various contrast variations to the noisy image with SNR = 3, it appeared that the quality of the contour detection was unaffected by the presence of even considerable contrast gradients along the contour. When we turned off contrast equalization the internal forces dominated in areas of low image contrast and the snakes became locally very smooth or even tended to shrink. In addition, snaxels kept oscillating in areas of high image contrast. We conclude that the equalization of image contrast improves the accuracy and convergence of the contour detection substantially.

Model convergence

Besides spatial accuracy our method gives also temporally stable results since convergence was obtained within 100 iterations on the average, during which the average snaxel velocity $\bar{v}_t$ and average image force amplitude $\bar{f}$ decayed gradually. We show this for the case of one contour in the noisy image of Fig. 19(c). The graphs of $\bar{v}_t$ and $\bar{f}$ are plotted as functions of the number of iterations in Fig. 20.

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\textsuperscript{3}We define the signal-to-noise ratio SNR by the ratio of the image intensity range and the standard deviation of the Gaussian image noise.
Detection of Carpal Bone Contours from 3-D MR Images

Figure 19: Contour detection in noisy images. The upper panels show the binary image and three versions in which Gaussian noise has been added (SNR = 3 (b), 2 (c), and 1 (d)). The lower panels show the snakes in each case. Distance statistics $[d_{\text{avg}}, d_{\text{max}}]$: (a) $[0.19, 0.61]$, (b) $[0.22, 0.87]$, (c) $[0.39, 1.76]$, (d) $[1.44, 8.76]$.

Figure 20: Example of average snaxel velocity $\bar{v}_t$ (pixels per iteration) and the average image force (arbitrary units) amplitude $\bar{f}$ as a function of the number of iterations $t$ for the contour detection of a bone of Fig. 19(c). After initialization for each scale, $\bar{v}_t$ first increases and then decreases as the snake converges. The image force gradually decreases to a constant value which reflects the balance with the internal forces at equilibrium. The scales $\sigma$ are shown at the top of both graphs.
2.4.2 Experiments—Contour detection in a MR image slice

We applied the snake to detect the contours of the MR image slice using the series of initial snakes from the previous experiments. Since the size of the pixels in the MR image is twice the pixel size in the binary test image we decreased $\Delta s$, $\sigma_{\text{max}}$, $\sigma_{\text{min}}$, $\epsilon$, and the speed of iteration by a factor of 2 (Table I). Fig. 21 shows the contour detection for different values of the internal weights $w_{\text{ela}}$ and $w_{\text{curv}}$. For the complete set of initial snakes the variation in the definitive snakes was very small.

2.4.3 Contour detection of the carpal bones

Two 3-D MR images were obtained by imaging two wrists in the neutral position of the hand. Six 3-D MR images were obtained of two other wrists by imaging each wrist in three different postures (ulnar deviation, neutral position, and radial abduction). A medial slice is shown in Fig. 22 (top) for each posture of one wrist.

We applied the snake to detect the planar contours of the carpal bones from a sequence of successive slices in the eight 3-D images, using $2\frac{1}{2}$-D contour detection (see Sec. 2.3.13). As a result, eight carpal bone stacks were produced for each 3-D image. This is shown in Fig. 22 (middle) for one wrist in three postures, where each contour of the stack represents a contour in a slice of the sequence.

We used the internal model parameters that we considered to be optimal
Figure 22: Contour detection in 3-D MR images of one wrist in three different postures (a), (b), and (c). The upper panels show a medial slice (128 × 128 pixels of 1 mm²) of each image. The contour stacks of the carpal bones are shown in the middle and are visualized at the bottom by rendering their corresponding triangulation with Gouraud shading.
**Figure 23:** Contour detection in 3-D MR images of 4 different wrists in the neutral position. Each of the four horizontal panels show a ROI (64 × 64 pixels of 1 mm²) in six slices of a wrist, separated by a distance of 4 mm, on which the snakes are superimposed.

for contour detection in the MR image slice of the previous experiment (Table I).

On our computer, for each slice the snakes converged within 100 iterations and an average of about two seconds was needed for iteration and resampling per contour. The contours detected by the snakes (Fig. 23) were located very close to the ‘anatomical contours’ as indicated by a radiologist.

Finally, we visualized the carpal bones by rendering the triangulation of the contour stacks using Gouraud shading [40], which is shown in Fig. 22 (bottom) for the three postures of one wrist.

2.5 **Discussion**

We have shown that accurate contour detection can be obtained – by incorporation of the new image force into the snake model – in images of varying noise and contrast, and in the presence of adjacent contours.
Our method involving a new radial image force is simple to implement and computationally inexpensive, as it is obtained by convolution and forward differences in 1-D. Besides, the computation requires little computer memory since no intermediate potential images $P(x, y)$ have to be stored for different values of $\sigma$ in (discrete) scale-space.

The amplitude of the image force is obtained by convolution of a second-order Gauss kernel and the image intensity. When this convolution is determined numerically, usually the kernel is discretized and the convolution is performed discretely. We approximate the image force by integrating the product of the continuous kernel and the interpolated image intensity. In this way a more accurate approximation is obtained.

In the conventional 2-D approach the potential valleys may become very flat in curved regions of the contour due to tangential smoothing. As a consequence, the snake may be unable to follow the curved region and converges to a local minimum. Due to the 1-D radial approach – in which tangential smoothing is omitted – the sensitivity to the setting of the upper bound in scale-space ($\sigma_{\text{max}}$) is reduced and the snake converges to the desired contour for a large set of different initializations. This feature is demonstrated in Fig. 24 where we compare contour detection using 1-D radial scale-space and the conventional approach based on 2-D isotropic scale-space using a radial projection of the potential gradient $\nabla P$ as proposed by Lobregt and Viergever [37].

By applying 1-D contrast equalization an image intensity insensitive measure of external energy is obtained. As a consequence, for each snaxel a good balance between internal and external forces is established. Besides we can use a simple convergence criterion by terminating iteration when the average displacement of the snaxels decreases and is lower than a threshold $\epsilon$. In other studies [26, 28] energetic criteria were used. In [28] it was reported that these criteria sometimes lead to erroneous convergence.

By incorporating Lobregt and Viergever’s resampling technique [37] we preserved the coherence between the snaxels and the influence of elasticity was reduced but maintained to a certain extent to prevent the snake from penetrating weak edges or small gaps in the contour (cf. Fig. 21).

A possible drawback of contrast equalization may arise from the fact that in some situations it counteracts the capability of bridging larger gaps. When these are present the snake may partially be attracted to a faint edge or noise structure close to the gap. In these cases, which occur rarely in our application, we normalized the radial image force in Eq. (2.23) by a radial contrast estimate averaged over all snaxels at each iteration. As a result, the snake was able to bridge larger gaps in the contour at the expense of a more accurate boundary fit in lower contrast areas.
Figure 24: Comparison of contour detection based on 1-D radial scale-space versus 2-D isotropic scale-space using two initial snakes: a coarse one $V_{a}^{(init)}$ (image upper-left) and a more accurate one $V_{b}^{(init)}$ (image upper-right). The snake converges to the desired solution using 1-D radial scale-space for both initial snakes and a value of $\sigma_{\text{max}} = 2.0$ (a). In (b) and (c) contour detection based on 2-D scale-space is shown. For $\sigma_{\text{max}} = 2.0$ the snake converged to the solution shown in (b) for both initial snakes. For these cases the snake is unable to follow the area of relatively high contour curvature. Only when $\sigma_{\text{max}}$ is diminished to 1.0 the snake is able to follow the curved region starting from $V_{b}^{(init)}$, as shown in (c).
Another important class of contour detection models related to snakes are dynamic programming algorithms, which are not envisaged in this chapter. Tagare applied such algorithms to detect the contours of the carpal bones in 3-D CT images [34, 35]. Our main objection to use a dynamic programming framework is that large search windows are needed at the expense of numerical speed, when coarse initializations are used. Moreover, these algorithms are not easily extended into 3-D.

We noted three situations (2% of all cases) in which user interaction was needed when applying 2½-D contour detection to 3-D MR images of the wrist:

- in some situations the snake converged to an extraneous edge detail nearby (e.g. a blood vessel)
- when the diameter of the contour was small with regard to the value of $\sigma_{\text{max}}$ (which is often the case when applying the snake to the upper and lower slices through the bone in the 3-D MR image), the snake sometimes shrunk into a point due to elasticity and smoothing
- if the initial snake was located far from the contour, the snake sometimes folded itself into loops, due to wrong radial direction on the loops. This happened when the snake was applied to successive slices of the 3-D MR image in which the topology of the contour clearly changes, which is illustrated in Fig. 25.

The first problem could be solved, as noted above, by normalizing the image force with an average radial contrast estimate. The last two problems were solved by reduction of the value of $\sigma_{\text{max}}$ and improvement of model initialization. A better solution, however, can be obtained by a 3-D generalization of the snake model instead of using a 2½-D method.

Cohen and Cohen [30] generalized the original model by Kass et al. [19] to a true 3-D snake using the finite element method (FEM). The radial image force as presented in this chapter, is easily extendible into 3-D. We are presently investigating the 3-D generalization of our model.

### 2.6 Conclusion

The new radial image force introduced in this chapter improves the performance of the snake model significantly. The snake is less sensitive to i) initialization, ii) the choice of the upper bound used in (discrete) scale-space ($\sigma_{\text{max}}$), and iii) varying contrast along the contour, due to the use of 1-D radial scale-space and contrast equalization. Furthermore the bias to cut off contour sections of high curvature is removed. By using directional information we eli-
minated problems in situations where adjacent contours were present. Our model proved to be a powerful tool to detect the planar contours in 3-D MR images of the wrist.

2.7 Acknowledgments

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