Hybrid resonators for light trapping and emission control

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Chapter 8

Controlling nanoantenna polarizability through back-action via a single cavity mode

The polarizability $\alpha$ determines the absorption, extinction and scattering by small particles. Beyond being purely set by scatterer size and material, polarizability can also be affected by back-action: the influence of the photonic environment on the scatterer. As such, controlling the strength of back-action provides a tool to tailor the (radiative) properties of nanoparticles. Here, we control the back-action between broadband scatterers and a single mode of a high-quality cavity. We demonstrate that back-action from a microtoroid ring resonator significantly alters the polarizability of an array of nano-rods: the polarizability is renormalized as fields scattered from — and returning to — the nano-rods via the ring resonator depolarize the rods. Moreover, we show that it is possible to control the strength of the back-action by exploiting the diffractive properties of the array. This perturbation of a strong scatterer by a nearby cavity has important implications for hybrid plasmonic-photonic resonators and the understanding of coupled optical resonators in general.
8.1 Introduction

The scattering, absorption and extinction cross-section of small scatterers is often attributed to the dielectric properties of the particle, i.e., the scatterer’s volume, shape and its refractive index with respect to the host medium [119]. Central to this argument, for scatterers with a physical size much smaller than the wavelength, is the so-called polarizability, which contains the frequency-dependent susceptibility that quantifies the strength of the dipole moment induced in the scatterer by an incident field. A rather subtle notion is that the polarizability also depends on the mode structure offered by the photonic environment (Fig. 8.1). To illustrate this, consider that extinction, i.e., the total power that a scatterer extracts from an incident beam [119] is directly proportional to the imaginary part of polarizability. According to the optical theorem [120], this power is distributed over Ohmic heating and scattering, with the contribution of scattering being proportional to the squared magnitude of polarizability and the local density of states (LDOS) [12]. The fact that LDOS, i.e., the number of available photonic modes for the scatterer to radiate into, enters the polarizability is known as back-action: a correction on the imaginary part of polarizability. According to the optical theorem [120], this power is distributed over Ohmic heating and scattering, with the contribution of scattering being proportional to the squared magnitude of polarizability and the local density of states (LDOS) [12]. The fact that LDOS, i.e., the number of available photonic modes for the scatterer to radiate into, enters the polarizability is known as back-action: a correction on the total field that drives a polarizable scatterer. This correction is neglected in standard (Rayleigh) scattering theory [119]. However, even for a single scatterer placed in free-space, back-action leads to additional damping (depolarization) and thus needs to be integrated in a self-consistent description of any system [120, 158]. For a scatterer coupled to a cavity mode, this back-action can lead to a strong modification of the polarizability near the cavity resonance, as discussed in Section 2.4.2. Although back-action effects on quantum emitters [19] have been routinely studied, very few studies exist that probe back-action on plasmonic scatterers. First, Buchler et al. [136] revealed that the spectral width of a nanoantenna’s plasmon resonance can be modulated when the antenna approaches a reflector, whereas more recently Heylman et al. [137] demonstrated that the absorption cross-section of a single nanoantenna can be modified via coupling to a microtoroid cavity. It was

![Figure 8.1: Cavity-induced modification of antenna polarizability](image)

(a) A single polarizable scatterer, probed in a transmission experiment. (b) A simple Fabry-Pérot cavity modifies the local density of states and alters the scattering properties of a plasmonic scatterer. (c) A spectrally narrow cavity mode can suppress the imaginary part of the polarizability $\alpha$ of a plasmonic scatterer.
also shown by Zhang et al. [295] that, under specific driving conditions, coupling of a single molecule to an antenna can lead to similar modification of the antenna response, which demonstrates the generality of this phenomenon. While back-action on a single antenna is perhaps the most intuitive example to study, one is not limited to a single antenna or resonator to observe back-action. For any resonating system that is coupled to a bath of modes, the properties of this bath will influence the susceptibility (polarizability) of the resonating system. Crucially, this change in susceptibility carries information on the properties of the bath, and a measurement of the modified susceptibility thus provides a non-invasive method to obtain information on the bath. It has been proposed [118] that if the bath is represented by the single mode of a cavity, the modified susceptibility would, in principle, allow access to the Purcell factor [14] of the cavity mode.

Here we experimentally investigate back-action on polarizability in a hybrid antenna-cavity system, demonstrating a strongly modified extinction response of an array of gold nano-rods due to back-action imparted by a single whispering-gallery mode (WGM) of a microtoroid ring resonator. At conditions where the cavity offers a high mode density for the scatterers to radiate into, the nano-rods’ susceptibility to an incoming field is suppressed: the cavity mode density thus effectively depolarizes the nano-rods (Fig. 8.1c), yielding an experimental signature that relates to electromagnetically induced transparency [296]. A unique feature of the array, as our experiments reveal, is that it is possible to control the strength of the measured back-action by careful tuning of a diffraction order of the array, phase-matching its wavevector with the WGM of the cavity. Using a coupled-oscillator model we retrieve an antenna-cavity cooperativity and provide a lower bound on the cavity Purcell factor [14] at the lattice origin. Our results have strong relevance in the context of recent proposals on hybrid plasmonic-photonic resonators [100, 105, 109, 110, 112, 113, 118, 297, 298] as a unique venue for huge Purcell factors [14] and quantum strong coupling with single emitters. While the most intuitive consideration for such a proposal is to assess how scatterers perturb cavity resonances [130], in fact, this work shows that one rather has to ask what opportunities the cavity offers to control antenna polarizability.

8.2 Experimental methods

An ideal experiment to probe cavity-induced back-action would directly measure the complex-valued polarizability $\alpha$ in presence and absence of the cavity. This is not a trivial task: polarizability is not a directly measurable quantity in optics. Instead one has to rely on far-field measurements of extinction and scattering cross sections to deduce $\text{Im}[\alpha]$ and $|\alpha|^2$ respectively. Such quantitative polarizability measurements are challenging even for scatterers in an uniform environment [299, 300]. The proximity of the cavity further compli-
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cates the task of strictly probing the scatterers only. Practically, this means that
direct excitation of the cavity mode by the incident beam, as well as radiation
from the cavity directly into the detection channel, should be prevented, as
both would contaminate the interrogation of the scatterer’s response. We ap-
proach these constraints by a combination of experimental techniques. First,
we use a WGM resonator that only allows in- and outcoupling of light under
select wavevector matching conditions. Second, we use an array of antennas,
as opposed to a single antenna, to obtain a strong extinction-like signal that
can be probed in specular reflection with a nearly collimated plane wave,
again using wavevector conservation to separate the extinction channel from
all other scattering channels. Crucially, we demonstrate that the use of an
array allows tailoring of the coupling strength between cavity and array via
wavevector matching, controlled by the angle of incidence. Note that our
choice for an array results in a measurement probing back-action on the lattice
polarizability [134].

8.2.1 Sample fabrication

We study reflection from an array of gold nano-antennas, coupled to a
whispering-gallery mode in a silica microtoroid. A high Q silica microtoroid
(diameter ≈ 36 µm) is fabricated on the edge of a silicon sample (see Fig. 8.2a).
For the fabrication protocol we largely followed methods that have been
previously reported in for example [301, 302]. In this work, spin-coating
(ma-N 2410) and subsequent cleaving of the sample enabled targeted e-beam
lithography of the cavity on the edge of the sample. The toroid supports
whispering-gallery modes (WGMs) of high quality factor $Q$, and for the
remainder of this chapter we will focus on a fundamental TE-polarized mode
(linewidth $\kappa/2\pi \approx 30$ MHz) at resonance frequency $\omega_c/2\pi \approx 194.4$ THz.

Gold nano-antennas are fabricated in an array (150 µm by 150 µm) on
a glass coverslide of 170 µm thickness. A positive resist (ZEP-520) layer of
130 nm thickness is spin-coated on the coverslide, and nanoantennas are
declared using electron beam lithography. The antenna width and thickness

Figure 8.2: Silica microtoroid and gold nano-antennas. (a) SEM image of a silica
microtoroid on a Si substrate. Note that the toroid in this work is smaller (30 µm
diameter). (b) SEM image of the nano-antenna array. (c) Transmission measurement
of the antenna array, showing a resonance at 208 THz.
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were designed to be 120 and 40 nm and the length is approximately 400 nm (see Fig. 8.2b). The pitch along the short axes of the antennas is 1500 nm, with a pitch along the long axes of 800 nm. Gold is evaporated thermally at 0.05 nm/s. We characterize the spectral properties of the array in a transmission measurement (under normal incidence) using Fourier-Transform Infrared spectroscopy, obtaining a broadband resonant response at resonance frequency $\omega_a/2\pi \approx 208$ THz and linewidth $\gamma/2\pi \approx 55$ THz (see Fig. 8.2c).

8.2.2 Measurement setup

Our experimental system is sketched in Fig. 8.3a. A more detailed diagram is shown in Fig. 8.3b. We perform narrowband spectroscopy by scanning the frequency of a tunable laser (New-Focus TLB-6728, 1520-1570 nm, <100 kHz linewidth). The antennas are illuminated through a high-NA objective (Nikon, CFI Apo TIRF 100x, NA ≈ 1.33, used with index-matching oil) with an incident field polarized (s-polarization) along the principal dipole axis of the rods, which themselves are oriented to match a high-Q TE-polarized mode of the microtoroid. Focusing the incoming laser beam onto the back-focal-plane (BFP) of the objective gives precise control over the angle of incidence of the drive field. The position of this focus, i.e. the angle of incidence, is controlled using a translation stage. For scatterers arranged in a periodic array, scattering takes the form of diffraction into well-defined angles (wavevectors, Fig. 8.3c). We discard the ($-2$) and ($-1$) diffraction orders propagating back into the substrate using Fourier-filtering such that our detector is only sensitive to the specular reflection signal. In addition we employ a real-space filter, selecting a circular area of $\sim 4.5 \mu$m in diameter, to reduce background signals not originating from antennas coupled to the cavity. A tapered fiber is used to directly excite the cavity mode (in a separate experiment) and obtain information on the cavity mode profile and polarization of the cavity mode. We also use the fiber to check that, with the cavity positioned in front of the glass substrate away from the antenna array, we do not directly excite the cavity mode with the incident drive field (and associated wavevectors) used in our experiment. Cavity and tapered fiber position are controlled using 3-axis piezoelectric actuators.

To illustrate our experimental arrangement, Fig. 8.3d displays an overlay of Fourier-space data obtained by BFP imaging (without Fourier-filter) on an infra-red camera (Allied Vision Goldeye P008). We identify 1) the radiation profile of the two propagating cavity modes, obtained by direct excitation of the cavity using an evanescently coupled tapered fiber (color scale), and 2) the position of the three diffraction orders of the array (indicated by arrows). In the experiment, the incoming wavevector is chosen such ($k_y = 0$ and $k_{\parallel}/k_0 \approx 0.8$) that the ($-2$) diffraction order of the array (which is evanescent in air) overlaps with one of the propagating whispering-gallery
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Figure 8.3: Experimental setup and measurement scheme. (a) Cartoon of the hybrid antenna-cavity system. (b) Experimental setup. The input laser is focused in the back focal plane (BFP) and can be moved to change the incoming angle at the sample. The reflection passes through a Fourier filter to select the specular reflection, and through a real-space filter to select only antennas close to the cavity. An infra-red incoherent source (EPI) is used in EPI-illumination for navigation and to determine the objective NA. (c) For some incoming field $E_{in}$, the $(-2)$ diffraction order associated with the array evanescently couples to the toroid. back-action from the cavity on the array is measured in the specular reflection signal. (d) An overlay of Fourier-images obtained via back-focal-plane imaging. The transparent white blobs indicated by arrows are diffraction orders, with 0 indicating the specular reflection. The $(-2)$ order overlaps with one of the cavity modes (indicated with the color scale). As such, we excite the cavity mode via the antenna-array. The inner and outer white circles indicate the edge of the light cone ($k_{||}/k_0 = 1$) and the objective NA, respectively. These were obtained from a reflection measurement where illuminate the BFP homogeneously using an incoherent source, and are slightly displaced due to a sample tilt. Cavity mode intensities are not equal because only one of the modes is excited directly by the taper.

modes in the microtoroid, allowing the incoming field to efficiently scatter to the cavity mode via the antennas. Our system thus allows for a proper back-action measurement: the antennas can couple to the cavity, yet the detected signal is exclusively a probe of antenna polarizability. Any change in detected signal upon approaching the cavity can thus be directly attributed to cavity-mediated back-action fields that renormalize the antennas response.
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8.2.3 Measuring extinction

In the experiment we probe the antennas through zero-order reflectance at a small (around 30°) incident angle, where zero-order reflectance is a direct measure of extinction, i.e., \( \text{Im}[\alpha] \) \cite{303}. Since extinction is usually associated to zero-order transmittance and not reflectance, this claim requires substantiation. To predict the lineshape in reflectance, we quote an expression from De Abajo \cite{134} for the specular reflection signal \( r' \) that one expects from a particle array (with real-space lattice area \( A \)) in free space. The observed reflection depends on the self-consistent single particle polarizability \( \alpha_E \), corrected by the lattice-summed Green’s function \( G_{xx}(0) \), and reads

\[
r' = \frac{2\pi ik/A}{\alpha_E^{-1} - G_{xx}(0)},
\]

(8.1)

where \( k \) is the wavenumber. In our case, the antennas lie on a glass-air interface which in itself is reflective. Introducing the lattice dressed polarizability \( \alpha^{-1} \equiv \alpha_E^{-1} - G_{xx}(0) \) and the background reflection \( r_{\text{glass}} \) of our interface (see \cite{304} for an elaborate discussion on properties of plasmonic nanoantenna arrays on interfaces) we arrive at

\[
r' = r_{\text{glass}} + \frac{2\pi ik\alpha}{A}.
\]

(8.2)

Generally, the Fresnel coefficient \( r_{\text{glass}} \) is real valued. Using this notion one can continue to write the specular reflectance \( |r'|^2 \) as

\[
|r'|^2 = r_{\text{glass}}^2 - r_{\text{glass}} \frac{4\pi k}{A} \text{Im}[\alpha] + \frac{4\pi^2 k^2}{A^2} |\alpha|^2,
\]

(8.3)

which evidences that the imaginary part of the polarizability \( \text{Im}[\alpha] \) leads to a reduction in specular reflectance, whereas the scattering term scaling with \( |\alpha|^2 \) results in an increased reflectance. Alternatively one can express Eq. (8.3) as a function of the extinction cross section \( \sigma_{\text{ext}} = 4\pi k \text{Im}[\alpha] \) and scattering cross section \( \sigma_{\text{scat}} = \frac{8}{3} \pi k^4 |\alpha|^2 \), which gives

\[
|r'|^2 = r_{\text{glass}}^2 - r_{\text{glass}} \frac{\sigma_{\text{ext}}}{A} + \frac{3}{2} \frac{\pi}{A^2 k^2} \sigma_{\text{scat}}.
\]

(8.4)

From this expression we learn that a reduction in reflectance, with respect to the background signal coming from the interface, can be associated with extinction. This conclusion is supported by calculations using a full electrodynamic model, discussed in Section 8.4.2. For a plasmon particle or array such a reduction occurs over a wide frequency range that is commensurate with its bandwidth (an example is shown in Fig. 8.4a). In addition, Eq. (8.4) shows

*Note that in \cite{134} and in Eqs. (8.1) to (8.4), \( \alpha \) is given in CGS units. To convert to SI units, \( \alpha \) should be multiplied by \( 4\pi\epsilon \), with \( \epsilon \) the permittivity in the host medium.
that dilute lattices, such as ours, result in a more pure extinction measurement than dense lattices, for which the scattering term contributes more strongly to the observed signal as a result of the larger proportionality factor \((A^2k^2)^{-1}\).

In essence, destructive interference causes a reduction in reflectance, similar to the textbook scenario of extinction measurements that measure destructive interference between forward scattered light and the direct beam. In analogy to standard transmittance measurements probing extinction, we here define the extinction \(E\) as \(E \equiv 1 - |r|^2\), with the normalized reflectance \(|r|^2\) given by \(|r|^2 \equiv |r'|^2/|r_{\text{glass}}|^2\). The use of \(|r|^2\) has the advantage that results obtained at different excitation angles (leading to different values of \(r_{\text{glass}}\)) are more easily compared. Moreover, the introduction of the variable \(E\) simplifies the interpretation of our experiment: a decrease in antenna-extinction (increasing \(|r|^2\)) is mapped to decreasing values for \(E\). Inspired by the case of a single scatterer at resonance with a cavity discussed e.g. in Section 2.4.2, our prediction is that the polarizability will show a reduction over a narrow spectral region [118, 137] once the antennas are subject to back-action through the cavity mode, i.e., once they are offered the additional possibility of radiation damping due to the Purcell factor associated with the cavity mode. This will then also appear as a minimum in \(E\) (see Fig. 8.4b).

8.3 Experimental results

8.3.1 Observation of cavity-mediated back-action

Figure 8.4c displays the response of the antenna array in absence (orange points) and presence (blue points) of the cavity for an incident beam with \(k_\parallel/k_0 = 0.84\). The narrow frequency window displayed in Fig. 8.4c is close to the plasmon resonance, evident from the fact that \(E\) is close to unity, meaning that \(|r|^2\) is close to zero. Comparing the trace without cavity and with the cavity approached to several microns distance away (antennas weakly couple to the cavity) shows a small back-action effect of the cavity on the array, visible as a \(\sim 1\%\) dip in \(E\). This dip is tantamount to a reduction in the extinction that the antennas cause when they are offered the cavity as an additional channel to radiate into. Expressed in polarizability, our measurement implies a decrease in \(\text{Im}[\alpha]\) due to back-action, occurring over a narrow bandwidth that is commensurate with the linewidth (\(\sim 30\text{MHz}\)) of the high-Q cavity mode. In Fig. 8.4c the cavity-array distance was several microns, limiting the back-action experienced by the antennas. Moving the cavity closer to the array results in much larger effects. For instance, Fig. 8.4d shows a > 25% change in polarizability when approaching the cavity to within approximately 1 micron (about 6.5 times the evanescent decay length of the squared mode field) from the antennas. This is direct evidence that the magnitude of polarizability can be substantially controlled by the photonic environment.
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Figure 8.4: Cavity-induced back-action. (a) Sketch of a typical reflectance signal $|r'|^2$ as measured in the experiment. The plasmon feature introduces a broadband dip, with respect to the non-resonant $|r_{\text{glass}}|^2$ value. (b) Sketch of the extinction $E$, obtained from (a). We expect the cavity mode to reduce $\text{Im}\{\alpha\}$ (and thus $E$) in a narrow frequency band. (c-d) Experiment. (c) With the cavity present (blue points), the extinction $E$ decreases by 1% at the cavity frequency, a feature that is absent without cavity (orange points). (d) At smaller cavity-array distance the dip increases to 25%, indicating a strong suppression of antenna extinction. The cavity linewidth increases compared to (c) as a result of increased cavity losses.

8.3.2 Tuning back-action strength through phase matching

While our experiment probes several antennas, it was previously realized that for single antennas the polarizability modification must be directly linked to the cavity Purcell factor at the location of the antenna [118]. In other words, one viewpoint on our experiment is that it evidences that the polarizability of a nano-antenna is modified, which is mathematically expressed as $\alpha^{-1} = \alpha_0^{-1} - G$, with $\text{Im}[G]$ the LDOS and $\text{Re}[G]$ the Lamb shift [305] provided by the cavity mode. As such, an antenna is analogous to a quantum emitter in the sense that it probes the LDOS of the cavity. The effect of an LDOS peak, however, is distinctly different: the antenna emission is quenched on resonance rather than, as would be the case for an emitter, enhanced. The fact that in our experiment the mode density provided by the cavity results from a single Lorentzian mode offers an alternative viewpoint. In essence, the reduction of polarizability over the cavity bandwidth can be viewed as a ‘transparency’ feature in direct analogy to electromagnetically/plasmon/optomechanically induced transparency [296, 306–309]. In these systems, a broad resonator
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(here: plasmonic scatterer) is rendered ‘transparent’ in its susceptibility to driving over a narrow frequency band due to coupling to a narrow resonator (here: WGM resonator), even though that narrow resonator is not directly driven. Beyond purely Lorentzian transparency dips, one can obtain Fano-type [310] lineshapes depending on the phase of the coupling constants that connect the broad and narrow resonance. Inspired by this analogy we explore the shape of the back-action feature by varying the angle of incidence of the incoming drive field. As shown in Fig. 8.5a, this effectively sweeps the (−2) diffraction order over the finite k-space width of the cavity mode, thus varying the degree to which the array and the cavity mode are coupled through phase-matching. From the resulting spectra (Fig. 8.5b) we qualitatively observe a dependence of the back-action strength and lineshape on the incoming angle, which is expressed as a varying depth and asymmetry of the cavity-induced dip. In line with the phase-matching argument, visual inspection of Fig. 8.5a and Fig. 8.5b shows that cavity-mediated back-action is most prominent when the cavity mode profile and the (−2) diffraction order of the array experience better overlap. In Section 8.4.2, this behavior is verified using analytical coupled dipole calculations.

The antenna-cavity cooperativity

Full quantification of the back-action is not straightforward, as it requires analysis of the Fano lineshapes. A detailed multiple scattering analysis particular for our system, which will be discussed in Section 8.4.2, shows that the plasmon antennas in our experiment are simultaneously subject to the resonant back-action of the cavity and a nonresonant back-action term from the interface on which the antennas are placed (glass-air) [12, 311]. The nonresonant back-action is governed by the complex Fresnel coefficient associated with the interface, which exhibits a phase change for the (evanescent) (−2) diffraction order upon sweeping k∥/k0. In our experiment we measure the scatterers’ response in the presence of all back-action, which is a coherent sum of the broadband interface-induced back-action plus the resonant cavity-mediated back-action. Sweeping k∥ thus directly affects the Fano lineshape that we observe. We develop a simple model based on coupled-mode theory [127] that can disentangle the resonant back-action from the nonresonant background. Treating the array and cavity as resonators, coupled at rate g, both described by a Lorentzian response with complex field amplitudes a and c, respectively, we solve the driven system

\[
\begin{pmatrix}
\Delta_a + i\gamma/2 & g \\
g & \Delta_c + i\kappa/2
\end{pmatrix}
\begin{pmatrix}
a \\
c
\end{pmatrix}
= \begin{pmatrix}
i\sqrt{\gamma_{ex}s_{in}} \\
0
\end{pmatrix}
\tag{8.5}
\]

for a. Here we defined \(\Delta_a \equiv \omega - \omega_a\) and \(\Delta_c \equiv \omega - \omega_c\), where \(\omega\) is the frequency of the incident field \(s_{in}\) driving the array and \(\gamma_{ex}\) the rate at which the array and input/output channel are coupled. Antenna and cavity resonance
8.3 Experimental results

Figure 8.5: Tuning back-action through angle of incidence. (a) Fourier-image overlay that shows the position of the diffraction orders at the start (blue dot, \(k_\parallel = 0.69 k_0\)) and stop (pink dot, \(k_\parallel = 0.88 k_0\)) values of the \(k_\parallel\) sweep displayed in panel (b). (b) The strength and lineshape of the back-action strongly depend on the incoming angle. The black dashed lines are fits using our coupled-mode model (Eq. (8.7)). (c) Values for the cooperativity obtained from fitting our coupled-mode model to the spectra in panel (b). Black line: fit with a Gaussian lineshape. (d) Horizontal cross-cut through the cavity radiation pattern shown in (a). A 2-Gaussian fit (dashed black line) yields cavity modes with peak position and width in good agreement with results from the cooperativity profile in (c).

frequencies are \(\omega_a\) and \(\omega_c\), and their damping rates are \(\gamma\) and \(\kappa\), respectively. Next, we use the input-output relation \(s_{\text{out}} = s_{\text{in}} - \sqrt{\gamma_{\text{ex}}} a\) [127] (such that \(r = s_{\text{out}}/s_{\text{in}}\)) and parametrize the coupling via the cooperativity \(C = 4g^2/(\gamma\kappa)\), the determining quantity for the strength of the sharp spectral feature observed in electromagnetically/optomechanically induced transparency [23, 258]. We obtain

\[
r = \frac{s_{\text{out}}}{s_{\text{in}}} = 1 - \frac{2i\gamma_{\text{ex}}}{2\Delta_a + i\gamma(1 + C\frac{\kappa/2}{-i\Delta_c + \kappa/2})}.
\]  

(8.6)

Introducing an arbitrary phase pickup \(\phi\) in the direct reflection channel (first term in Eq. (8.6)) and assuming \(\omega \approx \omega_a\), we find

\[
|r|^2 = \left| \exp(i\phi) - \frac{2\eta}{1 + C\frac{\kappa/2}{-i(\Delta_c - \Delta) + \kappa/2}} \right|^2,
\]  

(8.7)
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where $\eta \equiv \gamma_{ex}/\gamma$ and $\Delta$ is an additional small detuning that captures small fluctuations in $\omega_c$ due to e.g. thermal drift. We use Eq. (8.7) to fit our experimental data in Fig. 8.5b, yielding values for $C$ as a function of $k_\parallel$ (Fig. 8.5c, blue points). A Gaussian lineshape (black line) is fit (center(width): $k_\parallel/k_0 \approx 0.78(0.14)$) to the blue data points, giving a maximum cooperativity of $C \approx 0.5$. Notably, the width and center of the Gaussian agree with expected values based on a fit to the cross-cut of the cavity mode profile (shown in Fig. 8.5d), which reveals modes with linewidths of $\sim 0.15k_0$. The cavity mode to which we couple via second order diffraction is centered at $k_\parallel/k_0 = -1.23$. Considering the incident free-space wavelength of 1540 nm and a pitch of 1500 nm, one would expect maximum coupling between the array and cavity (via the 2nd order diffraction) for an incident wavevector of $-[1.23 + 2 \times (1540/1500)] = 0.82k_0$, which matches with the experimentally observed incident wavevector $0.78k_0$ for which we observe our maximum in cooperativity.

The relation between cooperativity and Purcell factor

The cooperativity quantifies the degree of coupling between the array and the cavity. More than that, in the limit of a single scatterer and single cavity mode, it is directly equivalent to the product of the scatterer albedo ($A$) and the cavity Purcell factor ($F$, see Eq. (3.1)) at the location of the scatterer. This can be seen by rewriting our expression for the hybridized polarizability $\alpha_H$ in such a system (Eq. (2.36)), obtained from the coupled-oscillator model discussed in Chapter 2. If we make the assumption that we are close to both cavity and antenna resonance, such that $\omega_a^2 - \omega^2 \approx 2\omega_a\Delta_a$ and $\omega_c^2 - \omega^2 \approx 2\omega_c\Delta_c$, we find

$$\alpha_H \approx \frac{\beta/\omega_a}{-2\Delta_a - i\gamma_i - i\gamma_r - i\frac{\beta Q}{\epsilon_0\omega_a} - i\Delta_c + \kappa/2} = \frac{\beta/\omega_a}{-2\Delta_a - i\gamma \left(1 + AF\frac{\kappa/2}{-i\Delta_c + \kappa/2}\right)}, \quad (8.8)$$

where $\beta$ is antenna oscillator strength, $V_{eff}$ cavity effective mode volume and $Q = \omega_c/\kappa$. Furthermore, $\gamma_i$ and $\gamma_r$ are antenna Ohmic and radiative damping rates, respectively, with $\gamma = \gamma_i + \gamma_r$ and $A = \gamma_i/\gamma$, and we used the expression $\gamma_r = \beta\omega^2\sqrt{\epsilon}/(6\pi\epsilon^3\epsilon_0)$ for $\gamma_r$ in a homogeneous medium (see Eq. (2.22)). Comparison of Eq. (8.8) with the resonant term in Eq. (8.6) shows that the cooperativity $C$ is equal to the $AF$ product in the case of a single scatterer.

In our experiment the cooperativity can not be directly cast into a Purcell factor, as we probe an array of antennas at specific wavevector, meaning that we probe a lattice-sum dressed polarizability [134] that experiences back-action from a wavevector-resolved mode density. We can, however, make an estimation of the Purcell factor by comparing our measurements to rigorous coupled dipole calculations which will be discussed in Section 8.4.1. This reveals that measured cooperativity of $C = 0.5$ actually corresponds to a value
8.4 Modeling of an antenna array coupled to a microcavity

of $C = 1.7$ as it is felt by a single antenna, without a lattice, located at the lattice origin. Considering that $A < 1$, the back-action feature in our experiment is tantamount to a modest Purcell factor of $F \geq 1.7$. Obviously this effect could be much stronger in experiments where the scatterers are placed right in the mode maximum, as opposed to the arrangement in our setup where scatterers are placed at in the evanescent tail (estimated $|E|^2$ decay length of 145 nm, based on finite-element simulations) of the cavity mode at approximately 1 µm distance. We verified that the obtained value of $F \geq 1.7$ is in reasonable agreement with results from finite element simulations on a microtoroid. These simulations predict $F \approx 0.54$ at 1 µm distance, which however rises quickly with decreasing distance (e.g. to 2.1 at 800 nm distance). It is important to note that the quoted cavity-array distance of 1 µm was only approximately determined by comparing the experimental cavity linewidth-broadening due to the glass substrate to a finite element simulation.

8.4 Modeling of an antenna array coupled to a microcavity

In Section 8.3.2 we use a general coupled-oscillator model to fit our experimental data. This allowed us to disentangle the resonant and non-resonant features in the data and retrieve the apparent cooperativity between the antenna lattice and the cavity. However, these results leave us with two open questions. (1) What is the origin of the non-resonant background in our measurements? (2) How can we relate the measured cooperativity to the Purcell factor of the cavity, given that we do not measure a single antenna but an array? In this section we therefore go beyond the simple coupled-oscillator model and instead employ a full electrodynamic theory to answer these questions. Doing so, we provide a deeper understanding of the rich behaviour that occurs in these complex system.

As our system consists of two very distinct elements, i.e. a quasi-infinite lattice of scatterers and a finite-sized cavity, no obvious choice of theoretical model exists. We therefore consider two extremes, neither of which models the system perfectly, yet each with its own merits. In both cases, the models treat each antenna in the array as a separate dipole and calculate the total response of the array using an analytical point-dipole model (see for example [134, 311]). It is essential to understand that in such a coupled dipole model a dipole is driven not only by the driving field and its own backscattered field, but also by the field scattered by the other dipoles. For a lattice of $N$ identical scatterers, the dipole moment of particle $n$ reads

$$p_n = \alpha_0 \left[ E_{ext}(r_n) + \sum_{m} G(r_n, r_m, \omega) p_m \right], \quad (8.9)$$
with $E_{\text{ext}}(r_n)$ the driving field and $\tilde{\alpha}_0$ the electrostatic particle polarizability. The Green’s function $\tilde{\omega} = \tilde{G}_{\text{bg}} + \tilde{G}_c$ is the total Green’s function, consisting, in our case, of the background and the cavity contributions. Each of the two models solves Eq. (8.9) in a different way.

The first model, discussed in Section 8.4.1, considers a finite array, coupled to the finite-sized cavity. The array is assumed to be in vacuum. This ‘finite’ approach has the benefit that it allows us to assign a position dependent Purcell factor that each individual antenna in the array is subject to. In other words, the strength of this approach is that it can take into account important aspects of the cavity that include the cavity mode profile, $Q$ and effective mode volume $V_{\text{eff}}$, as well as the fact that the toroid curvature means that only a finite set of particles are in its mode. This allows us to answer the second question above by connecting the observed cooperativity to the cavity Purcell factor. While its strength is the description of the finite-sized cavity, its weakness is that it can only deal with a finite number of particles and cannot account for the air-glass interface on which the particles are placed in the experiment.

The second method we discuss (Section 8.4.2) is complementary as it assumes an infinite array of scatterers including all retarded electrodynamic interactions. However, because an infinite array requires Ewald summation in $k$-space, this method approximates the cavity as a translation invariant resonantly reflecting slab. For an infinite periodic array, the polarizability is entirely summarized by the polarizability of a particle at the origin [134]. Importantly, it has recently been shown that the theory that typically describes such infinite arrays in vacuum [134] can be extended to take into account a reflective surface on which the particles are placed [311]. As the resulting theory only requires Fresnel reflection and transmission coefficients, in fact one can even use stacked (resonant) planar layers as an interface [304]. This is also the approach we take in this second, ‘infinite’ model: we essentially lump the response of the glass-air interface and cavity into a single Fresnel coefficient, and calculate the response of the array using the resulting ‘engineered’ metasurface. As our calculation in this second scenario allows us to include interfaces such as the glass-air interface that characterizes our sample surface, we are able to reproduce the angle-dependent Fano lineshapes that are observed in our experiment (Fig. 8.5b).

8.4.1 Finite array and cavity

In our experiment we did not probe the response of a single scatterer, but instead measured on an array of dipoles. Here we use a brute-force coupled dipole model to show that the response of an array qualitatively matches the response of a single scatterer when coupled to a single cavity mode. The spectral lineshapes that we calculate in both scenarios are similar, although lattice effects can lead to a significantly stronger response for some particles
in the array, compared to that of the single scatterer case. We first introduce the coupled dipole model, before deriving the Greens function of a single whispering-gallery mode (WGM) and finally showing the results that we obtain using our model.

**Effective polarizability retrieval in a finite lattice**

For $N$ dipoles, Eq. (8.9) leads to a set of $3N$ coupled equations of motion. To simplify the math, we take the particles to be only polarizable along the $y$-axis, reducing Eq. (8.9) from $3N$ to $N$ equations. Reshuffling the terms, we can now write an equation of the form

$$
\tilde{M}^{-1} \tilde{P} = \tilde{E}_{\text{ext}}
$$

(8.10)

with

$$
\tilde{M}^{-1} = \begin{pmatrix}
\alpha_{yy}^{-1} - G_{yy}(r_0, r_0) & -G_{yy}(r_0, r_1) & \ldots & -G_{yy}(r_0, r_N) \\
-G_{yy}(r_1, r_0) & \alpha_{yy}^{-1} - G_{yy}(r_1, r_1) & \ldots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
-G_{yy}(r_N, r_0) & \ldots & \alpha_{yy}^{-1} - G_{yy}(r_N, r_N)
\end{pmatrix}
$$

(8.11)

and where $\tilde{P}$ and $\tilde{E}_{\text{ext}}$ are column vectors of length $N$ containing the dipole moments of all particles and the driving fields at their positions, respectively.

We can solve this system of equations by setting up $\tilde{M}^{-1}$ and numerically inverting it. One then multiplies it with the driving fields $\tilde{E}_{\text{ext}}$ to obtain $\tilde{P}$. Dividing $\tilde{P}$ element-wise by $\tilde{E}_{\text{ext}}$, one obtains the effective polarizability $\alpha_{\text{eff}}$ of each particle, defined as usual through $p_{n} = \alpha_{\text{eff}}E_{\text{ext}}(r_{n})$. The effective polarizability, averaged over the number of particles in the detection area, determines the response of the lattice to an incident field.

**The cavity Green’s function**

To couple multiple dipoles via the cavity, we require an explicit expression that describes the full cavity Green function $\hat{G}_{c}(r, r', \omega)$. The field of a single cavity mode can be described as (see Section 2.2.2)

$$
E(r, \omega) = a(\omega)e_{c}(r),
$$

(8.12)

where $a(\omega)$ is the frequency-dependent amplitude and $e_{c}(r)$ is the normalized field profile. If a dipole $p'$ at position $r'$ is coupled to the cavity, we find $a(\omega)$ by solving the cavity equation of motion (Eq. (2.20), with $s_{\text{in}} = 0$, taking the small linewidth approximation $\kappa \ll \omega_{c}$ and evaluating near cavity resonance) as

$$
a(\omega) = \frac{i}{4} [e_{c}^{*}(r') \cdot p'] \frac{\omega_{c}}{-i\Delta_{c} + \kappa/2},
$$

(8.13)
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The cavity Green’s function is defined through the cavity fields generated at position \( r \) by a dipole at position \( r' \) as

\[
E(r) = \tilde{\tilde{G}}_{c}(r, r', \omega) p'.
\]  

(8.14)

Plugging in Eqs. (8.12) and (8.13) for \( E(r) \) and reshuffling terms, we get

\[
G_{c}(r', r, \omega) = L(\omega) e_{c}(r) e_{c}^{*}(r'),
\]  

(8.15)

with

\[
L(\omega) = \frac{i}{4} \frac{\omega_{c}}{\Delta + \kappa/2}
\]

the Lorentzian lineshape function.

From Eq. (8.15) it is clear that we require an expression for the spatial mode profile of the cavity. It was shown that the mode profile of a fundamental TE WGM in a microtoroid with major radius \( R_1 \) and minor radius \( R_2 \), outside the glass of the cavity, takes on the shape [302]

\[
e_{c}(r) = e_{c}(y = 0, r = R_1) e^{-y^2/2r_y^2} e^{-\kappa_r(r-R_1)} e^{\pm il\phi},
\]  

(8.16)

where \( e_{c}(y = 0, r = R_1) \) is the field somewhere on the edge of the cavity, in the equatorial plane, \( r_y \) is a Gaussian width along the \( y \)-direction (which depends on \( R_1, R_2 \) and \( l \)), \( \kappa_r \approx \frac{2\pi \sqrt{\epsilon_c - \epsilon}}{\lambda} \) (with \( \epsilon_c \) and \( \epsilon \) the permittivities of the cavity and the surrounding, respectively) is the radial decay length and \( l \) is the azimuthal mode number. \( r \) and \( \phi \) are cylindrical coordinates, where we have taken the origin to lie in the toroid center. See Fig. 8.6 for a sketch of the geometry. A plus or a minus sign in the azimuthal dependence determines whether it is the anticlockwise (ACW) or clockwise (CW) mode in the toroid. Note that the total cavity Green’s function is the sum of the CW and ACW mode contributions. Since we only want to know the field in the plane of the antennas, close to where the toroid approaches the lattice, we can make a
Taylor expansion around $x = 0$ in the Gaussian term. Doing the same in the last term describing the azimuthal dependence, we obtain for the field in the plane of the antennas

$$e_c(r) = e_c(\{x, y\} = 0, z = R_1)e^{-x^2/2r_x^2}e^{-y^2/2r_y^2}e^{-\kappa_r(|z|-R_1)}e^{\pm ik_cx},$$  \hspace{1cm} (8.17)

where $r_x = \sqrt{z/4\kappa_r}$ and $k_c = -l/z$ is the effective wavevector of the cavity mode in the antenna plane. From Eq. (8.15), the cavity Green’s function in the plane of the antennas ($z = z_0$) can now be described as

$$G_c(r', r, \omega) = L(\omega)\hat{e}^{\omega}e^{-(x'^2+x'^2)/2r_x^2}e^{-(y'^2+y'^2)/2r_y^2}e^{\pm ik_c(x-x')}.$$  \hspace{1cm} (8.18)

where $r'$ and $r$ are the source and detection positions, respectively, and

$$\hat{O} = e_c(\{y, x\} = 0, z = R_1)e^*_{c}(\{y, x\} = 0, z = R_1)e^{-2\kappa_r(|z_0|-R_1)} = e_c(r_0)e^*_{c}(r_0)$$  \hspace{1cm} (8.19)

is a matrix with the fields at the origin $r_0$ of the lattice. It is easy to verify that, for a dipole $p$ at position $r$

$$\text{Im}\left\{\hat{p} \cdot \hat{G}_c(r, r, \omega_c) \cdot \hat{p}\right\} = \frac{Q}{V_{\text{eff}}(r)\epsilon_0\epsilon(r)} = \frac{Fk^3}{6\pi\epsilon_0\epsilon},$$  \hspace{1cm} (8.20)

with $V_{\text{eff}}$ defined in Eq. (2.23). This confirms that the dipole experiences additional radiation damping in proportion to the cavity Purcell factor $F$ \cite{118}, as also discussed in Section 8.3.2.

**Results**

For simplicity, we restrict ourselves to a lattice of scatterers in vacuum. The Greens function of the background $\hat{G}_{bg}(r_n, r_m, \omega)$ is then a well-known expression \cite{12}. \(^\dagger\) We ignore its real part for $r_m = r_n$, taking the divergent electrostatic contribution to be included in the static polarizability $\hat{\alpha}_0$. The cavity Greens function is the sum of the CW and ACW contributions described in Eq. (8.18). As we assume that the scatterers are only polarizable along the $y$-axis, we only require the yy-component of $\hat{O}$, which is related to the effective mode volume $V_{\text{eff}}(r_0)$ felt by a y-oriented dipole at the lattice origin through $O_{yy} = 2/(V_{\text{eff}}(r_0)\epsilon_0\epsilon(r_0))$.

We assume particles with a Lorentzian polarizability $\alpha_0$ with resonance frequency $\omega_n = 2\pi\epsilon/\lambda_n$, $\lambda_n = 1.5 \mu m$, oscillator strength corresponding to a

\(^\dagger\)Note that one could, in principle, also include the glass-air interface, since the Green’s function near an interface is also known \cite{12}. However, it involves an integral over reciprocal space which, although solvable \cite{311}, is computationally very intensive to perform. In practice, this limits the lattice size to no more than a few particles.
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sphere of volume of $1.5 \times 10^{-3} \, \mu m^3$ and an ohmic damping rate $\gamma_i$ of $\omega_a/10$, matching literature values of gold [139]. The dipoles are positioned in a square lattice as in the experiment with pitches $d_x = 1.5 \, \mu m$ and $d_y = 0.8 \, \mu m$, containing 465 particles, i.e. 31 (15) particles in the x-(y-)direction. Our cavity is made of glass ($n = 1.5$) and surrounded by air, has a major radius of 18 $\mu m$, $l_{100}$, $\omega_c = \omega_a$, $Q = 3 \cdot 10^6$ and is located at 2 $\mu m$ distance from the lattice. This leads to a cavity in-plane wavevector of $k_c = 1.19 k_0$ (in good correspondence with the experimentally observed $1.23 k_0$) and a Gaussian width $r_x \approx 1.46 \, \mu m$. We take $r_y = r_x/2.6$, corresponding to the measured cavity mode profile. We choose $V_{\text{eff}}(r_0) = 5 \cdot 10^4 \lambda_0^3$ for the CW and ACW modes, which implies that the cavity Purcell factor (including both the CW and ACW mode) at the lattice origin is 9.1. We excite the lattice with a plane wave at an angle with the normal, along the x-axis, i.e. $k_\parallel = k_0 \hat{x}$.

**Figure 8.7:** Effective polarizability $\alpha$ in a finite lattice of dipolar scatterers coupled to a WGM cavity. (a-c) Narrowband spectra of $\text{Im}\{\alpha\}$, for three different incident parallel wavevectors $k_\parallel$ (indicated). We show $\text{Im}\{\alpha\}$ for a single dipole (located at the lattice origin $r_0$) without a lattice (blue) and for dipoles in a lattice, where we show the dipole at the lattice origin (green) and the mean value of the dipoles within a spatial region of 4.5 $\mu m$ diameter around the origin (red). (d) Broadband spectrum of $\text{Im}\{\alpha\}$ for $k_\parallel/k_0 = 0.8$, where we have optimal phase matching to the cavity mode via the (-2,0) diffraction order. Color coding is the same as in (a-c). (e-f) Spatial profiles of $\text{Im}\{\alpha\}$ in the x- and the y-direction, centered at the lattice origin. We show $\text{Im}\{\alpha\}$ for a single scatterer that is moved in the plane of the lattice (blue) and for the particles in the lattice (red). We chose $\omega = \omega_c$ and $k_\parallel/k_0 = 0.8$.

Figure 8.7 shows the effective polarizability of particles in this lattice. From Fig. 8.7 (a-c) we can see firstly that, while the polarizability of a single particle coupled to a cavity does not depend on angle of incidence, that of particles in a lattice does. We see exactly the type of phase-matching condition that
was also observed in our experiment (Fig. 8.5), where the effect of the cavity on polarizability is strongest when we are phase-matched to the cavity mode via the (-2,0) diffraction order. Here this occurs at $k_{\parallel}/k_0 = 0.8$. Moreover, we see that if phase-matching is achieved, the back-action effect introduced by the cavity can be stronger in a lattice than for a single particle: in a lattice, the particle at the origin has a more strongly modified polarizability than the single particle. This is because of the constructive interference of all particles radiating into the cavity, leading to an enhanced back-action field for particles near the origin. Figure 8.7 (d) displays the broadband polarizability of the dipoles, showing that both the single particle and the lattice follow the same Lorentzian lineshape outside the cavity spectral window. In Fig. 8.7 (e-f) we see that the particles close to the origin are more strongly affected by the cavity, i.e., that the effect diminishes for antennas at larger distance from the origin, roughly following the 2D Gaussian profile of the cavity mode. To compare the analytical results with our experiment, we also show in Fig. 8.7 (a-d) the mean polarizability of particles within a spatial region of 4.5 µm diameter around the origin, corresponding to the size of the real-space filter used in our experiments. To first order, the field scattered by the dipoles within the filter area is proportional to their mean polarizability. We see that this shows the same lineshape, but the averaging decreases the effect of the cavity.

These results show that the polarizability in a finite lattice of dipoles is qualitatively similar to the polarizability of a single dipole. We can thus use the ratio between the averaged lattice response and that of the single particle to predict the response we would have obtained in our measurement if we would have measured on a single particle, instead of on an array of dipoles. For this purpose, we fit the curves for a single particle and for the averaged lattice response at optimal phase matching ($k_{\parallel}/k_0 = 0.8$, Fig. 8.7c) with our simple coupled-oscillator model (Eq. (8.7)). We find a cooperativity of 1.4 and 0.41 for the single particle and the averaged lattice response, respectively. The former is in good agreement with the $AF$ product in these calculations, i.e $A = 0.15$, $F = 9.1$, $AF = 1.37$. We therefore expect that the experimentally measured maximum cooperativity of 0.5 implies a cooperativity of 1.7 for a single particle (in absence of other scatterers) located at the lattice origin. Thus the Purcell factor at the lattice origin must be higher than 1.7. This value of $F \geq 1.7$ is in reasonable agreement with results from finite element simulations on a microtoroid, as discussed in Section 8.3.2.

8.4.2 Infinite array and cavity

In this section we will discuss an infinite array of scatterers in front of an interface. With this model we aim to justify a specific claim made in the main text, namely that the Fano lineshapes that we observe in our experiment result from non-trivial background signals originating from the interface. Before we move to the details of our model, we first briefly recall the foundations
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of lattice sum theory, which describes infinite arrays of scatterers (see for example [134, 311]). We then discuss how we implemented the cavity in our model, followed by a discussion of the calculation results.

Lattice sum theory

The dipole moment $\mathbf{p}_n$ of a dipole in a lattice is described by Eq. (8.9). In the case of an infinite lattice of identical particles, one can simplify this equation using the fact that the solutions $\mathbf{p}_n$ will have a Bloch wave form, i.e. $\mathbf{p}_n = e^{i \mathbf{k}_{ext} \cdot \mathbf{r}_n} \mathbf{p}_0$, with $\mathbf{k}_{ext}$ the in-plane wavevector of the incident light and $\mathbf{p}_0$ the dipole moment of the particle at the lattice origin. Equation (8.9) can then be written as [134]

$$\mathbf{p}_n = \left[ \hat{\alpha}_0 - \hat{\mathcal{G}}(\mathbf{k}_{ext}, \mathbf{r}_n, \omega) \right]^{-1} \mathbf{E}_{ext}(\mathbf{r}_n) \quad (8.21)$$

with

$$\hat{\mathcal{G}}(\mathbf{k}_{ext}, \mathbf{r}_n, \omega) = \sum_{n'} \hat{\mathcal{G}}(\mathbf{r}_{n'}, \mathbf{r}_n, \omega) e^{i \mathbf{k}_{ext} \cdot (\mathbf{r}_{n'} - \mathbf{r}_n)} \quad (8.22)$$

the lattice-summed Green’s function, i.e. the field at $\mathbf{r}_n$ generated by all particles in the lattice (including the $n$-th particle itself). To find the response of the lattice, one should solve Eq. (8.22). We will not discuss in detail on how to do this, but instead point to the relevant literature (see e.g. [311, 312] for details). The formalism as it is described by Eqs. (8.21) and (8.22) is well established for 2D lattices in homogeneous space, using Ewald summation for exponential convergence of the lattice sums in the case that $\hat{\mathcal{G}}$ is $\hat{\mathcal{G}}_{hom}$ (with $\hat{\mathcal{G}}_{hom}$ the Green function for homogeneous space) [134]. Recently, Kwadrin et al. [311] showed how to generalize this approach for the case of lattices placed in front of a single reflective interface. In this case, one separates $\hat{\mathcal{G}}$ as the sum of $\hat{\mathcal{G}}_{hom}$ and a reflected Green function $\hat{\mathcal{G}}_{refl}$, where $\hat{\mathcal{G}}_{refl}$ is written in the angular spectrum representation [12], taking the wavevector dependent Fresnel coefficient as an input. After solving for $\hat{\mathcal{G}}$, the subsequently obtained lattice- and interface-corrected polarizability $\hat{\alpha}_{lat}$, defined as [134]

$$\hat{\alpha}_{lat} = \left[ \hat{\alpha}_0 - \hat{\mathcal{G}}(\mathbf{k}_{ext}, 0, \omega) \right]^{-1}, \quad (8.23)$$

can be used to calculate far-field observables such as, for example, reflection and transmission properties [12].

Simple model for cavity interaction

It is important to realize that Eq. (8.21) only holds if the entire system obeys translation invariance (or has a common periodicity). Only in that case is the
8.4 Modeling of an antenna array coupled to a microcavity

$p_n$ guaranteed to have a Bloch waveform. Due to the finite extent of the microtoroid cavity, however, translation invariance is not maintained in our system. To nonetheless approximate the experiment, we propose to mimic the cavity response by a resonant planar structure. This is a feasible approach, because the extension of lattice sum theory to lattices near mirrors, as reported by [311], is not restricted to a single reflective interface. Instead, one can also consider an array of scatterers positioned in a half space in front of an arbitrary multi-layer stack [304]. This approach simply relies on replacing the Fresnel coefficient of the single interface with the multi-layer reflection coefficient $r_{\text{eff}}$.

![Figure 8.8: Geometry used in the infinite lattice calculations. (a) Actual experimental geometry. (b) Geometry assumed in the infinite lattice theory. The finite-sized cavity is replaced by a resonant multistack, and the antennas are placed just inside the glass.](image)

Our model now considers a geometry as shown in Fig. 8.8 — an infinite antenna array placed 50 nm from a glass-air interface, inside the glass. Behind the glass-air interface, there is a multilayer stack which models the cavity. The total lumped reflection coefficient, including interface and cavity, is given by the well-known expression for an etalon [313]

$$r_{\text{eff}} = \frac{r_{\text{int,12}} + r_c e^{2ik_{z,\text{air}}d}}{1 - r_{\text{int,21}}r_c e^{2ik_{z,\text{air}}d}},$$  \hspace{1cm} (8.24)

where $r_{\text{int,12}}$ ($r_{\text{int,21}}$) is the reflection coefficient of the glass-air interface, incident from the glass (air) side, $r_c$ is the cavity reflection coefficient, $k_{z,\text{air}}$ is the component of the $k$-vector in air perpendicular to the lattice plane, and $d$ is the distance between cavity and interface (which, for simplicity, we set to zero). Due to the resonant nature of the single cavity mode (in frequency and wavevector), $r_{\text{eff}}$ is equivalent to $r_{\text{int,12}}$, except for very specific frequencies and wavevectors at which it is possible to excite the cavity. For our calculations, we approximate $r_{\text{cav}}$ as

$$r_{\text{cav}} = \frac{i\kappa_{\text{ex}}}{(-i\Delta_c + \kappa/2)},$$  \hspace{1cm} (8.25)

with

$$\kappa_{\text{ex}} \equiv \kappa/2 \times e(-|k_{\parallel}-k_{||,c}|^2/(2\sigma^2)).$$  \hspace{1cm} (8.26)
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the effective coupling rate to the cavity. We recognize that $r_c$ takes on a similar shape as the cavity Green’s function $G_c$ (Eq. (8.18)). Its frequency-dependency is given by a Lorentzian, and the momentum dependency shows Gaussian peaks centered at $\pm k_{\parallel,c}$ (the wavevector of the cavity mode) with a width given by $\sigma$. It can be easily shown that a spatial Fourier transform of $G_c$ in the detector coordinate $r$ would lead to a similar double Gaussian dependency (matching the measured Fourier pattern shown in Fig. 8.3d). The part of $G_c$ that is not contained in $r_c$, is the dependence on source position $r'$, because this would break translational invariance. As such, the resonant planar structure has an equal interaction with all antennas in the array, in contrast to the microtoroid in the experiment which interacts only with a select number of antennas. We note that the pre-factor $\kappa/2$ in Eq. (8.26) is chosen such that Eq. (8.25) yields unity reflection for perfect phase-matching and $\omega = \omega_c$, maximizing the effect of our resonant structure. A drawback of our model is that we can not easily determine the ‘real’ pre-factor that we should use. In reality, the pre-factor should relate to the cavity-array distance, and determine the strength of the back-action.

Finally, a difference with the experimental situation is the positioning of the antenna array inside the glass environment, which is necessary to lump the interface and cavity as a simple reflective multilayer. This positioning will slightly influence the total field at the position of the array. However, taking into account that s-polarized fields are continuous across the boundary, and that we positioned our antennas at a distance of $\lambda/20$ from the substrate, we estimate the resulting difference originating from this change in position to be relatively small.

Calculation results

Using our model, we plot the specular reflectance spectra for two different illumination conditions in Fig. 8.9a (normal incidence) and Fig. 8.9b ($k_{\parallel}/k_0 = 0.78$). In both scenarios we observe a broadband dip together with a glass-related background reflection signal. This means that the reduction in reflection can be attributed to the plasmon resonance. Our calculations thus validate our claim in Section 8.2.3 that a dip in reflectance is a measure for extinction, and that an increase in reflectance signals a reduction in extinction. The exact shape of the broadband dip varies slightly with incident angle, because the particle-particle interactions, captured in the Green’s function $G$, depend on this angle. The narrowband, angle-dependent resonances in reflection that are typical for lattices in a homogeneous medium and associated to Rayleigh anomalies [134], however, are not observed. This is due to the

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‡We use a cavity with $\omega_c/2\pi = 194.4$ THz, $Q = 3 \cdot 10^6$, $k_{\parallel,c}/k_0 = 1.23$ and $\sigma/k_0 = 0.07$. The antennas have $\omega_a \approx \omega_c$, $\gamma_i = \omega_a/10$ and oscillator strength corresponding to a sphere of volume of $3.4 \cdot 10^{-3} \mu m^3$, and organized in a lattice with pitches as in the experiment. They are only polarizable along the y-axis.
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presence of the interface, which suppresses these narrow features, as was also observed earlier [314].

Figure 8.9: Calculation results for an infinite system. (a) The calculated specular reflectance of the array (for normal incidence) shows a clear broadband dip associated with the plasmon resonance. The dashed line indicates the resonance frequency associated with the cavity mode. (b) For $k_\parallel /k_0 = 0.78$, we observe roughly the same signature. Note that due to the larger angle of incidence, the background signal coming from the glass-air interface is higher than in (a). Importantly, the sharp feature that is visible at $\omega_c$ is directly related to the presence (back-action) of the cavity. (c-d) Narrowband spectra of extinction $E$ (c) and lattice polarizability $\text{Im}[\alpha_{\text{lat}}]$ (d) for different $k_\parallel$ (indicated). Similar to our experiment, the calculation predicts a narrow bandwidth dip in extinction resulting from back-action, that is maximized for a wavevector for which the second diffraction order matches the wavevector of the cavity.

In addition to the broadband dip, we observe a small peak in the reflectance spectrum in Fig. 8.9b that is associated with the presence of the cavity. To investigate this in more detail, Fig. 8.9c displays narrowband extinction spectra for various $k_\parallel$, where extinction is defined as in the experiment ($E \equiv 1 - |r'|^2/|r_{\text{glass}}|^2$). From this figure we directly observe a dip in $E$ similar to that in the experimental spectra in Fig. 8.5. Importantly, the depth of this dip significantly depends on the angle of incidence, showing that the we can reproduce the main feature of our experiment using our model.

In addition, these calculations provide access to the (corrected) polarizability $\alpha_{\text{lat}}$ of the array. Figure 8.9d shows the imaginary part of $\alpha_{\text{lat}}$ as we obtain
it from our model. We observe a rapid change of polarizability around the cavity resonance frequency, indicative of cavity-induced back-action. Similar to the extinction, this plot shows that the back-action and its effect on $\alpha_{\text{lat}}$ can be controlled via the incident angle of the incoming drive field. Interestingly, whereas the extinction (Fig. 8.9c) displays a clear dip, the polarizability has a more asymmetric Fano-like resonant signature. We attribute this somewhat surprising discrepancy in shape between $E$ and $\text{Im}[\alpha_{\text{lat}}]$ to the fact that we do not perform a pure extinction measurement, but instead also partly probe $|\alpha_{\text{lat}}|^2$, which can be related to scattering by the antennas. The interplay between these two contributions, scattering and extinction, most likely gives rise to a more complex behaviour.

Finally, we observe a change in resonance lineshape both in extinction and in polarizability as $k_\parallel$ is varied, similar to what was observed in the experiment. From our model, we can trace the origin of this change as a phase change in $r_{\text{int}}$, the reflection coefficient of the glass-air interface. Back-action from the multilayer, i.e. the contribution of $\hat{G}_{\text{refl}}$ to $\hat{G}$, is mostly determined by its reflection $r_{\text{eff}}$ evaluated at the angles of the diffraction orders (which is also why we see cavity back-action only when a diffraction order overlaps with the cavity mode). At the angle of the $(-2)$ diffraction order, the glass-air interface shows total internal reflection, for which $r_{\text{int}}$ becomes complex and its phase depends strongly on angle. In contrast, the phase of the cavity reflection coefficient $r_c$ does not depend on angle. This explains the shape change of the Fano lineshape. We do, however, observe a difference in Fano lineshapes between calculations (Fig. 8.9c) and experiment (Fig. 8.5b). For example, for large angle of incidence (bottom panel in both figures) the Fano lineshapes have opposite asymmetry, i.e. opposite phase. We attribute this difference to the positioning of the antennas. In the experiment we place the array on the air side of the interface, while in the calculation we put them inside the glass environment. For evanescent waves (i.e. at the angle of the $(-2)$ diffraction order) the phase of $r_{\text{int}}$ when incident from the air side is opposite to that experienced when entering from the glass side. This opposite phase alters the observed shape of the extinction signal.

In conclusion, we have developed a full electrodynamic model for an infinite lattice of point scatterers coupled to a resonant multilayer stack which mimics a WGM cavity. Using this model, we were able to reproduce the main features in our experiment. As in the experiment, a suppression of extinction by the antenna array was observed, with strength tunable through angle of incidence. We showed that the effective antenna polarizability is similarly modified, which links the observed extinction dip to cavity-mediated back-action on the antenna. Furthermore, the lineshape change observed in both theory and experiment was explained as originating from the complex reflection coefficient of the glass-air interface.
8.5 Conclusion and outlook

We have shown that cavity back-action can alter the polarizability of an array of scatterers, and that the strength of the back-action can be controlled via the incoming drive field. Whereas in this work the Purcell enhancement provided by the cavity effectively depolarizes the nano-rods, which is related to the fact that the cavity and array are nearly resonant, it has been predicted in Section 2.4.2 that both an increase and decrease in polarizability can be obtained by controlling the detuning between cavity and scatterers. As the antenna polarizability dictates properties such as scattering and extinction, but also near-field enhancement and local density of states, this result demonstrates the feasibility of antenna-cavity hybrids as a tunable platform for scattering and emission control. We expect that this type of control over antenna polarizability will facilitate the exploration of antenna-cavity hybrids for applications such as single-photon sources, strong coupling to single quantum emitters, as well as classical applications like single-molecule sensing [26, 105, 109, 118].