Appendix: Not for publication. Available upon request.

A The Household Problem

The objective of the household is to maximize (1) subject to (3). As the household budget constraint will hold with equality in every period, we can form the Lagrangian as

$$\mathcal{L} = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right] + \beta^t \lambda_t^c \left(W_t N_t + B_t + \Upsilon_t - T_t - P_t C_t - \mathbb{E}_t Q_{t,t+1} B_{t+1} \right) \right\}.$$

where λ_t^c is the Lagrange multiplier associated with household budget constraint in period t. Intuitively, it is the gain in utility by one extra unit of nominal income. The first-order conditions (FOCs) with respect to C_t, N_t and B_{t+1} are given as

$$C_t^{-\sigma} = \lambda_t^c P_t, \quad N_t^{\phi} = \lambda_t^c W_t, \quad \mathbb{E}_t Q_{t,t+1} = \beta \mathbb{E}_t \left(\frac{\lambda_{t+1}^c}{\lambda_t^c} \right).$$

Substituting $\mathbb{E}_t Q_{t,t+1} = \frac{1}{R_t}$, $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$, we can write the FOCs as

$$\frac{N_t^{\phi}}{C_t^{-\sigma}} = \frac{W_t}{P_t}, \qquad 1 = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{R_t}{\Pi_{t+1}} \right].$$

The stochastic discount factor is defined as

$$Q_{t,\tau} = \beta^{\tau-t} \frac{\lambda_\tau^c}{\lambda_t^c} = \beta^{\tau-t} \frac{C_\tau^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_\tau}.$$

B The Firm Problem

We work from top to bottom, first solving the allocation over the two intermediates sectors, then solving the allocation over the individual intermediates.

B.1 Final Good Producer Problem - second stage

The second stage problem of the final good producer is to minimize the cost of production given the demand for final output and sector-level price indices. Formally, we can state the problem as

$$\min_{Y_{s,t},Y_{f,t}} P_{s,t}Y_{s,t} + P_{f,t}Y_{f,t} \qquad s.t. \qquad Y_t = \frac{Y_{s,t}^{\eta}Y_{f,t}^{1-\eta}}{\eta^{\eta}(1-\eta)^{1-\eta}}.$$

We can form the Lagrangian as

$$\mathcal{L} = P_{s,t}Y_{s,t} + P_{f,t}Y_{f,t} + \lambda_t^{P2}\left[Y_t - \frac{Y_{s,t}^{\eta}Y_{f,t}^{1-\eta}}{\eta^{\eta}(1-\eta)^{1-\eta}}\right],$$

where λ_t^{P2} is the marginal cost of production in period t. The first-order conditions with respect to $Y_{s,t}$ and $Y_{f,t}$ are given as

$$\lambda_t^{P2}\ \eta\ \frac{Y_{s,t}^{\eta-1}Y_{f,t}^{1-\eta}}{\eta^{\eta}(1-\eta)^{1-\eta}} = P_{s,t}, \qquad \lambda_t^{P2}\ (1-\eta)\ \frac{Y_{s,t}^{\eta}Y_{f,t}^{-\eta}}{\eta^{\eta}(1-\eta)^{1-\eta}} = P_{f,t}. \tag{26}$$

Taking the ratio of these two first-order conditions we get

$$\frac{P_{f,t}}{P_{s,t}} = \left(\frac{1-\eta}{\eta}\right)\frac{Y_{s,t}}{Y_{f,t}}. \tag{27}$$

Substituting the ratio $\frac{Y_{s,t}}{Y_{f,t}}$ into the production function yields the demands for output from the sectors s and f as a function of final demand Y_t and the relative price level $\frac{P_{f,t}}{P_{s,t}}$. Using (27) to eliminate $\frac{Y_{s,t}}{Y_{f,t}}$ from (26) we obtain,

$$\lambda_t^{P2} = P_{s,t}^{\eta}P_{f,t}^{1-\eta}.$$

Define $\lambda_t^{P2} = P_t$, where P_t is the aggregate price index.

B.2 Final Goods Producer Problem - first stage

Here, we will solve for the demand by final goods producers for intermediates from sector s only, as the derivation for sector f is analogous. The problem is to minimize the cost, given the aggregate demand for output from sector s and the prices charged by the intermediate producers. Formally, the problem is to:

$$\min_{Y_{s,t}(i), i\in[0,n)} \int_0^n P_{s,t}(i) Y_{s,t}(i) di, \qquad s.t. \qquad Y_{s,t} = \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n Y_{s,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}.$$

Forming the Lagrangian we get

$$\mathcal{L} = \int_0^n P_{s,t}(i) Y_{s,t}(i) di + \lambda_t^{P1} \left(Y_{s,t} - \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n Y_{s,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \right).$$

where λ_t^{P1} is the sector marginal cost of production in period t. The first-order conditions with respect to $Y_{s,t}(i)$ and $Y_{s,t}(j)$, $i, j \in [0, n)$ are given as

$$\lambda_t^{P1} \left(\frac{Y_{s,t}}{n}\right)^{1/\epsilon} Y_{s,t}(i)^{-1/\epsilon} = P_{s,t}(i), \quad \lambda_t^{P1} \left(\frac{Y_{s,t}}{n}\right)^{1/\epsilon} Y_{s,t}(j)^{-1/\epsilon} = P_{s,t}(j).$$

Taking the ratio of these two first-order conditions we get

$$\frac{Y_{s,t}(i)}{Y_{s,t}(j)} = \left(\frac{P_{s,t}(i)}{P_{s,t}(j)}\right)^{-\epsilon}.$$

From the first FOC we get,

$$Y_{s,t}(i) = \left(\frac{P_{s,t}(i)}{\lambda_t^{P1}}\right)^{-\epsilon} \frac{Y_{s,t}}{n}.$$

Substituting this into the production technology, and solving for λ_t^{P1} we will get

$$\lambda_t^{P1} = \left[\frac{1}{n} \int_0^n P_{s,t}(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}.$$

where $\lambda_t^{P1} = P_{s,t}$ is the sector s price index. Now substituting

$$Y_{s,t}(i) = \left(\frac{P_{s,t}(i)}{P_{s,t}(j)}\right)^{-\epsilon} Y_{s,t}(j),$$

into the production function and solving for $Y_{s,t}(j)$ we obtain

$$Y_{s,t}(j) = \frac{1}{n}\left(\frac{P_{s,t}(j)}{P_{s,t}}\right)^{-\epsilon} Y_{s,t}.$$

B.3 Intermediate Producer Problem and Derivation of the New-Keynesian Phillips curves

The intermediate producer also solves its problem in two stages. First, it minimizes the cost of production given the demand for its product, the factor prices and the technology. Second, subject to the Calvo (1983) price-setting mechanism, when it gets the chance to change the price of its product, it does so in order to maximize expected future nominal profits, while taking into account the effect of its price choice on the demand for its product.

Formally, we write the first problem of the sector s intermediate producer as

$$\min_{X_{s,t}(i),N_{s,t}(i)} P_t X_{s,t}(i) + W_t N_{s,t}(i) \qquad s.t. \qquad Y_{s,t}(i) = Z_{s,t} X_{s,t}(i)^{\chi} N_{s,t}(i)^{1-\chi}.$$

Forming the Lagrangian

$$\mathcal{L} = P_t X_{s,t}(i) + W_t N_{s,t}(i) + \lambda^m_{s,t}[Y_{s,t}(i) - Z_{s,t} X_{s,t}(i)^{\chi} N_{s,t}(i)^{1-\chi}].$$

where $\lambda^m_{s,t}$ is the marginal cost of production for producer i in sector s. The point to notice here is that the marginal cost does not varying from producer to producer. This is because of the assumption of the constant-returns-to-scale production technology. The first-order conditions with respect to $X_{s,t}(i)$ and $N_{s,t}(i)$ are given as

$$\lambda^m_{s,t}\chi Z_{s,t} X_{s,t}(i)^{\chi-1} N_{s,t}(i)^{1-\chi} = P_t, \qquad \lambda^m_{s,t}(1-\chi) Z_{s,t} X_{s,t}(i)^{\chi} N_{s,t}(i)^{-\chi} = W_t.$$

The ratio of the two first-order conditions is given as

$$\frac{X_{s,t}(i)}{N_{s,t}(i)} = \left(\frac{\chi}{1-\chi}\right)\frac{W_t}{P_t}.$$

Rewriting the first first-order condition and combining with this expression we obtain

$$P_t = \lambda^m_{s,t} Z_{s,t} \chi \left(\frac{N_{s,t}(i)}{X_{s,t}(i)} \right)^{1-\chi} \Rightarrow \frac{\lambda^m_{s,t}}{P_t} = \frac{\Gamma^{1-\chi}}{\chi} \left(\frac{W_t}{P_t} \right)^{1-\chi} \frac{1}{Z_{s,t}},$$

where $\Gamma = \frac{\chi}{1-\chi}$ and $\frac{\lambda^m_{s,t}}{P_t}$ is the real marginal cost of production. We define $MC_{s,t} \equiv \frac{\lambda^m_{s,t}}{P_t}$. The production technology for the intermediate producer i in sector s can be written as

$$Y_{s,t}(i) = Z_{s,t} \left(\frac{X_{s,t}(i)}{N_{s,t}(i)} \right)^{\chi} N_{s,t}(i).$$

Substituting for $\frac{X_{s,t}(i)}{N_{s,t}(i)}$ we get

$$N_{s,t}(i) = \Gamma^{-\chi} \left(\frac{W_t}{P_t} \right)^{-\chi} \frac{Y_{s,t}(i)}{Z_{s,t}},$$

and hence $X_{s,t}(i) = \Gamma^{1-\chi} \left(\frac{W_t}{P_t} \right)^{1-\chi} \frac{Y_{s,t}(i)}{Z_{s,t}}$.

The second problem of the intermediate producer can be formalized as

$$\max_{P_{s,t}(i)} \mathbb{E}_t \sum_{\tau=t}^{\infty} \theta_s^{\tau-t} Q_{t,\tau} \left[P_{s,t}(i)(1+\tau_s) - MC_{s,\tau} P_\tau \right] Y_{s,t,\tau}(i).$$

where $Y_{s,t,\tau}(i) \equiv \frac{1}{n} \left(\frac{P_{s,t}(i)}{P_{s,\tau}} \right)^{-\epsilon} Y_{s,\tau}$, τ_s is the subsidy given by the government in order to remove the monopolistic power which accrues to the intermediate firms because of monopolistic competition and $Q_{t,\tau}$ is the stochastic discount factor defined earlier. Substituting for $Y_{s,t,\tau}(i)$ and $Q_{t,\tau}$, and taking the first-order condition we get

$$P_{s,t}(i) \mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_s \beta)^{\tau-t} C_\tau^{-\sigma} P_{s,\tau}^{\epsilon-1} Y_\tau = \frac{\epsilon}{(\epsilon-1)(1+\tau_s)} \mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_s \beta)^{\tau-t} C_\tau^{-\sigma} MC_{s,\tau} P_{s,\tau}^{\epsilon-1} P_\tau Y_\tau.$$

The government sets $\tau_s = \frac{1}{\epsilon-1}$, thereby removing the price markup, hence eliminating the distortion in prices due to monopolistic competition. Hence, if producer i in sector s gets the chance to change his price, he sets it optimally at:

$$P^{opt}_{s,t} = \frac{\mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_s \beta)^{\tau-t} C_\tau^{-\sigma} MC_{s,\tau} P_{s,\tau}^{\epsilon-1} P_\tau Y_\tau}{\mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_s \beta)^{\tau-t} C_\tau^{-\sigma} P_{s,\tau}^{\epsilon-1} Y_\tau}.$$

Next, we log-linearize the optimal price around the steady state with constant prices. The optimal price can be written as

$$P_{s,t}^{opt}\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C_\tau^{-\sigma}P_{s,\tau}^{\epsilon-1}Y_\tau=\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C_\tau^{-\sigma}MC_{s,\tau}P_{s,\tau}^{\epsilon-1}P_\tau Y_\tau.$$

First, consider the left hand side. Define for some generic variable $\widehat{x}_t=\frac{X_t-X}{X}$, where X is variable X_t's steady state. Using this notation we can write

$$P\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C^{-\sigma}P^{\epsilon-1}Y+P\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C^{-\sigma}P^{\epsilon-1}Y\widehat{p}_{s,t}^{opt}+P\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C^{-\sigma}P^{\epsilon-1}Y(-\sigma\widehat{c}_\tau)$$
$$P\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C^{-\sigma}P^{\epsilon-1}Y((\epsilon-1)\widehat{p}_{s,\tau})+P\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C^{-\sigma}P^{\epsilon-1}Y\widehat{y}_\tau,$$

which reduces to

$$PC^{-\sigma}P^{\epsilon-1}Y\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}(1+\widehat{p}_{s,t}^{opt}-\sigma\widehat{c}_\tau+(\epsilon-1)\widehat{p}_{s,\tau}+\widehat{y}_\tau).$$

Now log-linearizing the right hand side of the equation,

$$\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C^{-\sigma}MC_sP^{\epsilon-1}PY+\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C^{-\sigma}MC_sP^{\epsilon-1}PY(-\sigma\widehat{c}_\tau)+$$
$$\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C^{-\sigma}MC_sP^{\epsilon-1}PY\widehat{mc}_{s,\tau}+\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C^{-\sigma}MC_sP^{\epsilon-1}PY((\epsilon-1)\widehat{p}_{s,\tau})+$$
$$\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C^{-\sigma}MC_sP^{\epsilon-1}PY\widehat{p}_\tau+\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}C^{-\sigma}MC_sP^{\epsilon-1}PY\widehat{y}_\tau.$$

Equating the left-hand and the right-hand side of the log-linearized equation and making repeated use of the steady-state version of the initial equation, we obtain[16]

$$\widehat{p}_{s,t}^{opt}=(1-\theta_s\beta)\mathbb{E}_t\sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s,\tau}+\widehat{p}_\tau\}.$$

Combining the loglinearized versions of the relative price index, $T_t=\frac{P_{s,t}}{P_{f,t}}$, and the aggregate price

[16] In an efficient steady state, $\bar{T}=1$. As the aggregate price level is given by $P_t=P_{s,t}^{\eta}P_{f,t}^{1-\eta}=\left(\frac{P_{s,t}}{P_{f,t}}\right)^{\eta}P_{f,t}=T_t^{\eta}P_{f,t}$, in an efficient steady state, $P=P_s=P_f$.

index (5), we obtain

$$\widehat{p}_t = \widehat{p}_{s,t} - (1-\eta)\widehat{T}_t = \widehat{p}_{f,t} + \eta\widehat{T}_t.$$

Combining this expression with the preceding expression, we can write

$$\widehat{p}^{opt}_{s,t} = (1-\theta_s\beta)\mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s,\tau} + \widehat{p}_{s,\tau} - (1-\eta)\widehat{T}_\tau\}.$$

Similarly, for sector f we have

$$\widehat{p}^{opt}_{f,t} = (1-\theta_f\beta)\mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_f\beta)^{\tau-t}\{\widehat{mc}_{f,\tau} + \widehat{p}_{f,\tau} + \eta\widehat{T}_\tau\}.$$

Now, we will derive the New Keynesian Phillips Curve (NKPC) for sector s. The derivation for sector f is similar. The expression for $\widehat{p}^{opt}_{s,t}$ just derived can be written as:

$$\widehat{p}^{opt}_{s,t} - \widehat{p}_{s,t-1} = (1-\theta_s\beta)\mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s,\tau} + \widehat{p}_{s,\tau} - \widehat{p}_{s,t-1} - (1-\eta)\widehat{T}_\tau\},$$

which can be written further as

$$\widehat{p}^{opt}_{s,t} - \widehat{p}_{s,t-1} = (1-\theta_s\beta)\mathbb{E}_t \sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s,\tau} - (1-\eta)\widehat{T}_\tau\} + (1-\theta_s\beta)\mathbb{E}_t[(\theta_s\beta)^0(\widehat{p}_{s,t} - \widehat{p}_{s,t-1}) +$$
$$(\theta_s\beta)^1(\widehat{p}_{s,t+1} - \widehat{p}_{s,t} + \widehat{p}_{s,t} - \widehat{p}_{s,t-1}) + (\theta_s\beta)^2(\widehat{p}_{s,t+2} - \widehat{p}_{s,t+1} + \widehat{p}_{s,t+1} - \widehat{p}_{s,t} + \widehat{p}_{s,t} - \widehat{p}_{s,t-1}) + \cdots],$$

which, in turn, can be further written as

$$\widehat{p}^{opt}_{s,t} - \widehat{p}_{s,t-1} = (1-\theta_s\beta)\mathbb{E}_t \sum_{\tau=t}^{\infty}(\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s,\tau} - (1-\eta)\widehat{T}_\tau\} + (1-\theta_s\beta)\mathbb{E}_t[(\theta_s\beta)^0\pi_{s,t} +$$
$$(\theta_s\beta)^1(\pi_{s,t+1} + \pi_{s,t}) + (\theta_s\beta)^2(\pi_{s,t+2} + \pi_{s,t+1} + \pi_{s,t})\cdots].$$

Manipulating this further, we obtain

$$\widehat{p}^{opt}_{s,t} - \widehat{p}_{s,t-1} = (1-\theta_s\beta)\mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s,\tau} - (1-\eta)\widehat{T}_\tau\} + \mathbb{E}_t[(\theta_s\beta)^0\pi_{s,t} + (\theta_s\beta)^1(\pi_{s,t+1}+\pi_{s,t}) + (\theta_s\beta)^2(\pi_{s,t+2}+\pi_{s,t+1}+\pi_{s,t})\cdots] - [(\theta_s\beta)^1\pi_{s,t} + (\theta_s\beta)^2(\pi_{s,t+1}+\pi_{s,t}) + (\theta_s\beta)^3(\pi_{s,t+2}+\pi_{s,t+1}+\pi_{s,t})\cdots],$$

which results into

$$\begin{aligned}
\widehat{p}^{opt}_{s,t} - \widehat{p}_{s,t-1} &= (1-\theta_s\beta)\mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s,\tau} - (1-\eta)\widehat{T}_\tau\} + \mathbb{E}_t \sum_{\tau=t}^{\infty} (\theta_s\beta)^{\tau-t}\pi_{s,\tau} \\
&= (1-\theta_s\beta)\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + \pi_{s,t} + \\
&\quad (1-\theta_s\beta)\mathbb{E}_t \sum_{\tau=t+1}^{\infty} (\theta_s\beta)^{\tau-t}\{\widehat{mc}_{s,\tau} - (1-\eta)\widehat{T}_\tau\} + \mathbb{E}_t \sum_{\tau=t+1}^{\infty} (\theta_s\beta)^{\tau-t}\pi_{s,\tau} \\
&= (1-\theta_s\beta)\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + \pi_{s,t} + \theta_s\beta\mathbb{E}_t\{\widehat{p}^{opt}_{s,t+1} - \widehat{p}_{s,t}\}.
\end{aligned}$$

Under the Calvo price setting mechanism we know that the price index in sector s evolves as

$$P^{1-\epsilon}_{s,t} = \theta_s P^{1-\epsilon}_{s,t-1} + (1-\theta_s)(P^{opt}_{s,t})^{1-\epsilon}.$$

Hence, log-linearizing this equation yields

$$\pi_{s,t} = (1-\theta_s)(\widehat{p}^{opt}_{s,t} - \widehat{p}_{s,t-1}).$$

Hence, we can write $\widehat{p}^{opt}_{s,t} - \widehat{p}_{s,t-1}$ as

$$\widehat{p}^{opt}_{s,t} - \widehat{p}_{s,t-1} = (1-\theta_s\beta)\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + (1-\theta_s)(\widehat{p}^{opt}_{s,t} - \widehat{p}_{s,t-1}) + \theta_s\beta\mathbb{E}_t\{\widehat{p}^{opt}_{s,t+1} - \widehat{p}_{s,t}\},$$

or

$$\widehat{p}_{s,t}^{opt} - \widehat{p}_{s,t-1} = \frac{(1-\theta_s\beta)}{\theta_s}\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + \beta\mathbb{E}_t\{\widehat{p}_{s,t+1}^{opt} - \widehat{p}_{s,t}\}.$$

Multiplying both sides by $(1-\theta_s)$ and using the fact that $\pi_{s,t} = (1-\theta_s)(\widehat{p}_{s,t}^{opt} - \widehat{p}_{s,t-1})$, we get the NKPC for sector s, which is given as

$$\pi_{s,t} = \kappa_s\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1},$$

where $\kappa_s = \frac{(1-\theta_s\beta)(1-\theta_s)}{\theta_s}$. Similarly, the NKPC for sector f is given as

$$\pi_{f,t} = \kappa_f\{\widehat{mc}_{f,t} + \eta\widehat{T}_t\} + \beta\mathbb{E}_t\pi_{f,t+1},$$

where $\kappa_f = \frac{(1-\theta_f\beta)(1-\theta_f)}{\theta_f}$.

C Aggregation

Demand faced by intermediate producer i in sector s is given by (7). Using the production technology of the intermediate producer we can write

$$Z_{s,t}X_{s,t}(i)^{\chi}N_{s,t}(i)^{1-\chi} = \frac{1}{n}\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon}Y_{s,t}.$$

The constant-returns-to-scale of production implies

$$\frac{X_{s,t}(i)}{N_{s,t}(i)} = \frac{X_{s,t}}{N_{s,t}},$$

where $X_{s,t}$ and $N_{s,t}$ are the aggregate output and labor demand from sector s, respectively. Hence,

$$Z_{s,t}\left(\frac{X_{s,t}}{N_{s,t}}\right)^{\chi}N_{s,t}(i)=\frac{1}{n}\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon}Y_{s,t}.$$

Hence,

$$Z_{s,t}\left(\frac{X_{s,t}}{N_{s,t}}\right)^{\chi}\int_0^n N_{s,t}(i)di=\int_0^n \frac{1}{n}\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon}Y_{s,t}.$$

Hence,

$$Z_{s,t}X_{s,t}^{\chi}N_{s,t}^{1-\chi}=Y_{s,t}D_{s,t},$$

where $D_{s,t}=\frac{1}{n}\int_0^n\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon}$ is the measure of price dispersion in sector s. Similarly, we get

$$Z_{f,t}X_{f,t}^{\chi}N_{f,t}^{1-\chi}=Y_{f,t}D_{f,t}.$$

Using Jensen's inequality it can be proved that $D_{s,t},D_{f,t}\geq 1$. Hence, the presence of relative price dispersion will push equilibrium output below its flexible-price equilibrium level. Now, labour demand from firm i in sector s is given by (9). Aggregate labour demand by sector s is given by

$$\begin{aligned}\int_0^n N_{s,t}(i)&=\Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi}\frac{1}{Z_{s,t}}\int_0^n Y_{s,t}(i)\\&=\Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi}\frac{Y_{s,t}}{Z_{s,t}}\frac{1}{n}\int_0^n\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon},\end{aligned}$$

which yields (and analogously for sector f)

$$N_{s,t}=\Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi}\frac{Y_{s,t}}{Z_{s,t}}D_{s,t},\qquad N_{f,t}=\Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi}\frac{Y_{f,t}}{Z_{f,t}}D_{f,t}.\tag{28}$$

Similarly,

$$X_{s,t} = \Gamma^{1-\chi} \left(\frac{W_t}{P_t}\right)^{1-\chi} \frac{Y_{s,t}}{Z_{s,t}} D_{s,t}, \qquad X_{f,t} = \Gamma^{1-\chi} \left(\frac{W_t}{P_t}\right)^{1-\chi} \frac{Y_{f,t}}{Z_{f,t}} D_{f,t}. \tag{29}$$

Now, the objective is to represent $D_{s,t}$ and $D_{f,t}$ in a recursive form. We will only focus on sector s, because the derivation is analogous for sector f:

$$\begin{aligned}
D_{s,t} &= \frac{1}{n} \int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon} di \\
&= \frac{1}{n} \left[n(1-\theta_s) \left(\frac{P^*_{s,t}}{P_{s,t}}\right)^{-\epsilon} + n\theta_s \int_0^n \left(\frac{P_{s,t-1}(i)}{P_{s,t}}\right)^{-\epsilon} di \right] \\
&= (1-\theta_s) \left(\frac{P^*_{s,t}}{P_{s,t}}\right)^{-\epsilon} + \theta_s(1-\theta_s) \left(\frac{P^*_{s,t-1}}{P_{s,t}}\right)^{-\epsilon} + \theta_s^2(1-\theta_s) \left(\frac{P^*_{s,t-2}}{P_{s,t}}\right)^{-\epsilon} + \cdots \\
&= (1-\theta_s) \left(\frac{P^*_{s,t}}{P_{s,t}}\right)^{-\epsilon} + (1-\theta_s) \sum_{j=1}^{\infty} \theta_s \left(\frac{P^*_{s,t-j}}{P_{s,t}}\right)^{-\epsilon}.
\end{aligned}$$

Now, use the transformation $j = j^* + 1$:

$$\begin{aligned}
D_{s,t} &= (1-\theta_s) \left(\frac{P^*_{s,t}}{P_{s,t}}\right)^{-\epsilon} + (1-\theta_s)\theta_s \sum_{j^*=0}^{\infty} \theta_s^{j^*} \left(\frac{P^*_{s,t-1-j^*}}{P_{s,t-1}}\right)^{-\epsilon} \left(\frac{P_{s,t-1}}{P_{s,t}}\right)^{-\epsilon} \\
&= (1-\theta_s) \left(\frac{P^*_{s,t}}{P_{s,t}}\right)^{-\epsilon} + \theta_s \Pi^{\epsilon}_{s,t} D_{s,t-1}.
\end{aligned}$$

Now, we want to substitute for $\left(\frac{P^*_{s,t}}{P_{s,t}}\right)^{-\epsilon}$. We know that under Calvo pricing, the price index in sector s evolves as

$$P^{1-\epsilon}_{s,t} = (1-\theta_s) P^{*1-\epsilon}_{s,t} + \theta_s P^{1-\epsilon}_{s,t-1}.$$

Hence,

$$1 = (1-\theta_s) \left(\frac{P^*_{s,t}}{P_{s,t}}\right)^{1-\epsilon} + \theta_s \Pi^{\epsilon-1}_{s,t}.$$

Hence,

$$\left(\frac{P^*_{s,t}}{P_{s,t}}\right) = \left[\frac{1}{1-\theta_s} - \left(\frac{\theta_s}{1-\theta_s}\right)\Pi_{s,t}^{\epsilon-1}\right]^{\frac{1}{1-\epsilon}}.$$

Substituting for $\left(\frac{P^*_{s,t}}{P_{s,t}}\right)$ gives

$$D_{s,t} = (1-\theta_s)\left[\frac{1}{1-\theta_s} - \left(\frac{\theta_s}{1-\theta_s}\right)\Pi_{s,t}^{\epsilon-1}\right]^{\frac{\epsilon}{\epsilon-1}} + \theta_s \Pi_{s,t}^{\epsilon} D_{s,t-1}. \tag{30}$$

D The Steady State

We derive the steady state of the endogenous variables. We now denote the steady state of some generic variable X_t by $\bar{X}$. In the steady state, all endogenous variables are constant, and the technology shocks $Z_{s,t}$ and $Z_{f,t}$ have the respective values $\bar{Z}_s = \bar{Z}_f = 1$. Conjecture that in the steady state the gross rate of aggregate inflation is 1, i.e. $\bar{\Pi} = 1$. Using the relative price identity, $\bar{\Pi}_s = \bar{\Pi}_f$. Hence, using the conjecture about aggregate inflation, $\bar{\Pi} = \bar{\Pi}_s = \bar{\Pi}_f = 1$. Then, using (30) we get $\bar{D}_s = \bar{D}_f = 1$. From the steady-state versions of (13) and (14) we have

$$\frac{\bar{H}_s/\bar{K}_s}{\bar{H}_f/\bar{K}_f} = 1.$$

From the steady-state versions of (12) we have

$$\mu_s \frac{\bar{H}_s}{\bar{K}_s} = \bar{T}^{1-\eta}, \qquad \mu_f \frac{\bar{H}_f}{\bar{K}_f} = \bar{T}^{-\eta}.$$

Hence,

$$\bar{T} = \frac{\mu_s}{\mu_f}\frac{\bar{H}_s/\bar{K}_s}{\bar{H}_f/\bar{K}_f},$$

and, hence, $\bar{T} = \frac{\mu_s}{\mu_f}$. Defining $\mu = \mu_s^{\eta}\mu_f^{1-\eta}$, we can write $\frac{\bar{H}_s}{\bar{K}_s} = \frac{\bar{H}_f}{\bar{K}_f} = \frac{1}{\mu}$. The steady-state versions of (13) and (14) imply

$$\frac{\bar{H}_s}{\bar{K}_s} = \frac{\Gamma^{1-\chi}}{\chi}(\bar{C}^{\sigma}\bar{N}^{\phi})^{1-\chi}.$$

Hence,

$$\bar{C}^{\sigma}\bar{N}^{\phi} = \left(\frac{\chi}{\mu}\right)^{\frac{1}{1-\chi}} \frac{1}{\Gamma}.$$

From (29) we can write:[17]

$$\begin{aligned}\bar{X}_s + \bar{X}_f &= \Gamma^{1-\chi}(\bar{C}^{\sigma}\bar{N}^{\phi})^{1-\chi}(\eta\bar{T}^{\eta-1} + (1-\eta)\bar{T}^{\eta})\bar{Y} \\ &= \left(\frac{\chi}{\mu}\right)\left[\eta\bar{T}^{\eta-1} + (1-\eta)\bar{T}^{\eta}\right]\bar{Y},\end{aligned}$$

where we also used (6). Now, using the goods market clearing condition, $\bar{C} + \bar{X}_s + \bar{X}_f = \bar{Y}$, and labor market clearing condition $\bar{N} = \bar{N}_s + \bar{N}_f$, we can further write $\bar{C} = k_{CY}\bar{Y}$ and $\bar{N} = k_{NY}\bar{Y}$, where k_{CY} and k_{NY} are given as[18]

$$k_{CY} = 1 - \left(\frac{\chi}{\mu}\right)\left[\eta\bar{T}^{\eta-1} + (1-\eta)\bar{T}^{\eta}\right], \qquad k_{NY} = \left(\frac{\chi}{\mu}\right)^{-\frac{\chi}{1-\chi}}(\eta\bar{T}^{\eta-1} + (1-\eta)\bar{T}^{\eta}).$$

We know that $\frac{\bar{H}_s}{\bar{K}_s} = \frac{1}{\mu} = \frac{\Gamma^{1-\chi}}{\chi}(\bar{C}^{\sigma}\bar{N}^{\phi})^{1-\chi}$. Substituting for $\bar{C}$ and $\bar{N}$ we get $\bar{Y}$, which is given as

$$\bar{Y} = \left[\left(\frac{\chi}{\mu}\right)^{\frac{1}{1-\chi}} \frac{1}{\Gamma}\frac{1}{k_{CY}^{\sigma}k_{NY}^{\phi}}\right]^{\frac{1}{\sigma+\phi}}.$$

Since all the remaining steady-state values of the endogenous variables depend on $\bar{Y}$, they can now all be determined. Using (4) the steady-state value of short term nominal interest rate is given by $\bar{R} = \beta^{-1}$.

E The Flexible-Price Equilibrium

The flexible-price equilibrium refers to the equilibrium when prices in both sectors are fully flexible. In other words, the degree of price stickiness is zero in both sectors. Under the flexible-price equilibrium and the assumption that the government subsidy eliminates the monopolistic distortions,

[17] From equation (9), in the steady state one has $\bar{X}_s = \Gamma^{1-\chi}\left(\frac{\bar{W}}{\bar{P}}\right)^{1-\chi}\bar{Y}_s$. Further, use $\frac{\bar{W}}{\bar{P}} = \bar{C}^{\sigma}\bar{N}^{\phi}$ and $\bar{Y}_s = \eta T^{\eta-1}\bar{Y}$. Similarly, for $\bar{X}_f$.

[18] We make use of $\frac{\bar{W}}{\bar{P}} = \bar{C}^{\sigma}\bar{N}^{\phi}$ and $\bar{C}^{\sigma}\bar{N}^{\phi} = \left(\frac{\chi}{\mu}\right)^{\frac{1}{1-\chi}}\frac{1}{\Gamma}$.

(11) implies

$$P_{f,t} = P_t MC_{f,t}, \qquad P_{s,t} = P_t MC_{s,t}.$$

Using (5), we can substitute for $P_{s,t}$ and $P_{f,t}$,

$$MC_{s,t}^{\eta} MC_{f,t}^{1-\eta} = 1.$$

Substituting for $MC_{s,t}$ and $MC_{f,t}$ using (10) we get

$$\left(\frac{W_t}{P_t}\right)^{1-\chi} = \frac{\chi}{\Gamma^{1-\chi}} Z_{s,t}^{\eta} Z_{f,t}^{1-\eta}.$$

Log-linearizing around the steady state, and denoting log-linearized flexible-price variables by $*$, we get

$$w_t^* - p_t^* = \frac{1}{1-\chi}(\eta z_{s,t} + (1-\eta) z_{f,t}).$$

Log-linearizing the marginal costs, using (10) we get

$$\begin{aligned} mc_{s,t}^* &= (1-\chi)(w_t^* - p_t^*) - z_{s,t} \\ &= (1-\eta)(z_{f,t} - z_{s,t}), \\ mc_{f,t}^* &= (1-\chi)(w_t^* - p_t^*) - z_{f,t} \\ &= \eta(z_{s,t} - z_{f,t}). \end{aligned}$$

Using the definition of the relative price, i.e. $T_t = \frac{P_{s,t}}{P_{f,t}}$, and the log-linearized versions of $P_{a,t} = P_t MC_{a,t}$ for $a \in \{s, f\}$, we get the log-linearized flexible-price version as

$$\begin{aligned} T_t^* &= p_{s,t}^* - p_{f,t}^* \\ &= mc_{s,t}^* - mc_{f,t}^* \\ &= z_{f,t} - z_{s,t}. \end{aligned}$$

Using (6) we get

$$\begin{aligned} y_{s,t}^* &= (\eta - 1)T_t^* + y_t^*, \\ y_{f,t}^* &= \eta T_t^* + y_t^*. \end{aligned}$$

The aggregate demand for the final good as an input by sector s is given by (note that $D_{s,t} = 1$ under flexible prices):

$$X_{s,t} = \Gamma^{1-\chi} \left(\frac{W_t}{P_t} \right)^{1-\chi} \frac{Y_{s,t}}{Z_{s,t}}.$$

Log-linearizing this equation yields

$$x_{s,t}^* = (1 - \chi)(w_t^* - p_t^*) + y_{s,t}^* - z_{s,t}.$$

Using the preceding expressions for $(w_t^* - p_t^*)$, $y_{s,t}^*$ and T_t^* gives $x_{s,t}^* = y_t^*$. Similarly, $x_{f,t}^* = y_t^*$. The final goods market clearing condition implies $\bar{Y} y_t^* = \bar{C} c_t^* + \bar{X}_s x_{s,t}^* + \bar{X}_f x_{f,t}^*$. Hence, substituting, and using the steady-state composition of final output, we get $y_t^* = c_t^*$. Aggregate labor demand from sector s is given as

$$N_{s,t} = \Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi}\frac{Y_{s,t}}{Z_{s,t}}.$$

The log-linearized version of this equation is given as

$$n^*_{s,t} = -\chi(w^*_t - p^*_t) + y^*_{s,t} - z_{s,t}.$$

Substituting for $(w^*_t - p^*_t)$ and $y^*_{s,t}$, after some manipulations we get

$$n^*_{s,t} = -(w^*_t - p^*_t) + y^*_t.$$

Similarly we get $n^*_{f,t} = -(w^*_t - p^*_t) + y^*_t$. Using the labor market clearing condition, i.e. $N_t = N_{s,t} + N_{f,t}$, we get

$$n^*_t = -(w^*_t - p^*_t) + y^*_t,$$

where we have used the fact that $\frac{\bar{N}_s}{\bar{N}} = \eta$ and $\frac{\bar{N}_s}{\bar{N}} = 1 - \eta$. Using (4), i.e. the labor supply equation derived from the household's optimization problem, we get

$$\frac{N^\phi_t}{C^{-\sigma}_t} = \frac{W_t}{P_t},$$

which upon log-linearization yields

$$\phi n^*_t + \sigma c^*_t = w^*_t - p^*_t.$$

Substituting $n_t^* = -(w_t^* - p_t^*) + y_t^*$, $w_t^* - p_t^* = \frac{1}{1-\chi}(\eta z_{s,t} + (1-\eta)z_{f,t})$ and $c_t^* = y_t^*$, after some manipulations we get

$$y_t^* = c_t^* = x_{s,t}^* = x_{f,t}^* = \frac{(1+\phi)}{(\phi+\sigma)(1-\chi)}(\eta z_{s,t} + (1-\eta)z_{f,t}).$$

Again using $n_t^* = -(w_t^* - p_t^*) + y_t^*$, we obtain

$$n_t = n_{s,t}^* = n_{f,t}^* = \left(\frac{1-\sigma}{(\phi+\sigma)(1-\chi)}\right)(\eta z_{s,t} + (1-\eta)z_{f,t}).$$

We also obtain the sector level output, $y_{s,t}^*$ and $y_{f,t}^*$

$$\begin{aligned} y_{s,t}^* &= \left(\frac{(1-\sigma) + (\phi+\sigma)\chi}{(\phi+\sigma)(1-\chi)}\right)(\eta z_{s,t} + (1-\eta)z_{f,t}) + z_{s,t}, \\ y_{f,t}^* &= \left(\frac{(1-\sigma) + (\phi+\sigma)\chi}{(\phi+\sigma)(1-\chi)}\right)(\eta z_{s,t} + (1-\eta)z_{f,t}) + z_{f,t}. \end{aligned}$$

Log-linearizing the Euler equation, (4), yields

$$c_t^* = \mathbb{E}_t c_{t+1}^* - \sigma^{-1}(r_t^* - \mathbb{E}_t \pi_{t+1}^*).$$

Define the ex-ante natural real interest rate as $r_t^{n*} = r_t^* - \mathbb{E}_t \pi_{t+1}^*$. Hence,

$$\begin{aligned} r_t^{n*} &= \sigma(\mathbb{E}_t(c_{t+1}^* - c_t^*)) \\ &= \frac{\sigma(1+\phi)}{(\phi+\sigma)(1-\chi)}\mathbb{E}_t(\eta \Delta z_{s,t+1} + (1-\eta)\Delta z_{f,t+1}) \\ &= \frac{\sigma(1+\phi)}{(\phi+\sigma)(1-\chi)}\left[\eta(\rho_{z,s} - 1)z_{s,t} + (1-\eta)(\rho_{z,f} - 1)z_{f,t}\right]. \end{aligned}$$

F The Sticky-Price Equilibrium

For a generic endogenous variable X, we define $\widetilde{x} = \widehat{x} - x^*$, where $\widehat{x}$ is the log-linearized value of X in the sticky-price equilibrium, while x^* is the log-linearized value of X in the flexible-price equilibrium.[19] Log-linearizing expression (10) for the marginal cost in sector s in the sticky-price equilibrium yields

$$\begin{aligned}
\widehat{mc}_{s,t} &= (1-\chi)(\widehat{w}_t - \widehat{p}_t) - z_{s,t} \\
&= (1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + (1-\chi)(w_t^* - p_t^*) - z_{s,t} \\
&= (1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + (1-\eta)T_t^*,
\end{aligned}$$

where the last equality follows from the definition of flexible-price equilibrium. Substituting this expression into the NKPC of sector s, we get

$$\begin{aligned}
&\pi_{s,t} = \kappa_s\{\widehat{mc}_{s,t} - (1-\eta)\widehat{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1} \\
\Rightarrow\ &\pi_{s,t} = \kappa_s\{(1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + (1-\eta)T_t^* - (1-\eta)\widehat{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1} \\
\Rightarrow\ &\pi_{s,t} = \kappa_s\{(1-\chi)(\widetilde{w}_t - \widetilde{p}_t) - (1-\eta)\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1}
\end{aligned}$$

The log-linearized version of the marginal cost in sector f is given by

$$\begin{aligned}
\widehat{mc}_{f,t} &= (1-\chi)(\widehat{w}_t - \widehat{p}_t) - z_{f,t} \\
&= (1-\chi)(\widetilde{w}_t - \widetilde{p}_t) - \eta T_t^*.
\end{aligned}$$

Substituting this into the NKPC of sector f we get

$$\pi_{f,t} = \kappa_f\{(1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + \eta\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{f,t+1}.$$

[19] For log-linearized values we use small letters, except, in order to avoid notational confusion, in the case of the relative price, where $\widetilde{T} = \widehat{T} - T^*$ and where $\widehat{T}$ is the log-linearized value of T in sticky-price equilibrium, while T^* is the log-linearized value of T in the flexible-price equilibrium.

Now our aim is to substitute out $(\widetilde{w}_t - \widetilde{p}_t)$ and express NKPC's in terms of the output gap, i.e. $(\widehat{y}_t - y_t^*)$, and relative price gap, $(\widehat{T}_t - T_t^*)$. The log-linearized versions of the household labor supply equation, (4) are given as

$$\phi\widehat{n}_t + \sigma\widehat{c}_t = \widehat{w}_t - \widehat{p}_t,$$

$$\phi\widetilde{n}_t + \sigma\widetilde{c}_t = \widetilde{w}_t - \widetilde{p}_t$$

So, if we can express $\widetilde{n}_t$ and $\widetilde{c}_t$ in terms of $\widetilde{y}_t$, then we can express the NKPCs in terms of the output gap. From the labor-market clearing condition, equation (10), in sector s we get

$$\widehat{n}_{s,t} = -\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t} - z_{s,t}.$$

Hence,

$$\widetilde{n}_{s,t} + n_{s,t}^* = -\chi(\widetilde{w}_t - \widetilde{p}_t) - \chi(w_t^* - p_t^*) + \widetilde{y}_{s,t} + y_{s,t}^* - z_{s,t}.$$

From the flexible price equilibrium we know,

$$n_{s,t}^* = -\chi(w_t^* - p_t^*) + y_{s,t}^* - z_{s,t}.$$

Hence we get,

$$\widetilde{n}_{s,t} = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_{s,t}.$$

Similarly, from the sector f labor-market clearing condition we get

$$\widetilde{n}_{f,t} = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_{f,t}.$$

The sector-level goods market clearing condition is given by (6), which in log-linearized form is given by

$$\begin{aligned}
\widehat{y}_{s,t} &= (\eta-1)\widehat{T}_t + \widehat{y}_t, \qquad \widehat{y}_{f,t} = \eta\widehat{T}_t + \widehat{y}_t \\
\Rightarrow \widetilde{y}_{s,t} &= (\eta-1)\widetilde{T}_t + \widetilde{y}_t, \qquad \widetilde{y}_{f,t} = \eta\widetilde{T}_t + \widetilde{y}_t
\end{aligned}$$

Substituting $\widetilde{y}_{s,t}$ and $\widetilde{y}_{f,t}$ into the sector-level labor-market clearing conditions yields

$$\widetilde{n}_{s,t} = -\chi(\widetilde{w}_t - \widetilde{p}_t) + (\eta-1)\widetilde{T}_t + \widetilde{y}_t, \qquad \widetilde{n}_{f,t} = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \eta\widetilde{T}_t + \widetilde{y}_t.$$

The "overall" labor-market clearing condition implies

$$\widehat{n}_t = \frac{\bar{N}_s}{\bar{N}}\widehat{n}_{s,t} + \frac{\bar{N}_f}{\bar{N}}\widehat{n}_{f,t},$$

where $\frac{\bar{N}_s}{\bar{N}} = \eta$ and $\frac{\bar{N}_f}{\bar{N}} = 1-\eta$. Substituting this into the preceding equation, we get

$$\begin{aligned}
\widehat{n}_t &= \eta\widehat{n}_{s,t} + (1-\eta)\widehat{n}_{f,t}, \\
\widetilde{n}_t + n^*_t &= \eta\widetilde{n}_{s,t} + (1-\eta)\widetilde{n}_{f,t} + \eta n^*_{s,t} + (1-\eta)n^*_{f,t}.
\end{aligned}$$

From the flexible price equilibrium we know that $n^*_t = n^*_{s,t} = n^*_{f,t}$. Hence we can write

$$\widetilde{n}_t = \eta\widetilde{n}_{s,t} + (1-\eta)\widetilde{n}_{f,t}.$$

Substituting for $\widetilde{n}_{s,t}$ and $\widetilde{n}_{f,t}$ we get

$$\widetilde{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_t.$$

Substituting for $(\widetilde{w}_t - \widetilde{p}_t) = \phi \widetilde{n}_t + \sigma \widetilde{c}_t$ we get

$$\widetilde{n}_t = -\frac{\chi\sigma}{1+\chi\phi}\widetilde{c}_t + \frac{1}{1+\chi\phi}\widetilde{y}_t. \tag{31}$$

The final goods market clearing condition implies

$$Y\widehat{y}_t = C\widehat{c}_t + X_s\widehat{x}_{s,t} + X_f\widehat{x}_{f,t} \Rightarrow Y\widetilde{y}_t = C\widetilde{c}_t + X_s\widetilde{x}_{s,t} + X_f\widetilde{x}_{f,t}.$$

The steady-state values of $X_{s,t}$, $X_{f,t}$ and C_t are, respectively, $\bar{X}_s = \eta\chi\bar{Y}$, $\bar{X}_f = (1-\eta)\chi\bar{Y}$ and $\bar{C} = (1-\chi)\bar{Y}$.[20] Our aim now is to express $\widetilde{x}_{s,t}$ and $\widetilde{x}_{f,t}$ in terms of $\widetilde{y}_t$. We know from the aggregation,

$$X_{f,t} = \Gamma^{1-\chi}\left(\frac{W_t}{P_t}\right)^{1-\chi}\frac{Y_{f,t}}{Z_{f,t}}D_{f,t}, \qquad X_{s,t} = \Gamma^{1-\chi}\left(\frac{W_t}{P_t}\right)^{1-\chi}\frac{Y_{s,t}}{Z_{s,t}}D_{s,t}.$$

Log-linearization of these equations gives us

$$\widehat{x}_{s,t} = (1-\chi)(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t} - z_{s,t}, \qquad \widehat{x}_{f,t} = (1-\chi)(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{f,t} - z_{f,t}$$
$$\Rightarrow \widetilde{x}_{s,t} = (1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_{s,t}, \qquad \widetilde{x}_{f,t} = (1-\chi)(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_{f,t}.$$

Substituting for $\widetilde{y}_{s,t}$ and $\widetilde{y}_{f,t}$ and the above steady-state values of $X_{s,t}$ and $X_{f,t}$, we get

$$\bar{X}_s\widetilde{x}_{s,t} + \bar{X}_f\widetilde{x}_{f,t} = \chi(1-\chi)\bar{Y}(\widetilde{w}_t - \widetilde{p}_t) + \chi\bar{Y}\widetilde{y}_t.$$

Substituting $\widetilde{w}_t - \widetilde{p}_t = \phi\widetilde{n}_t + \sigma\widetilde{c}_t$, we get

$$\bar{X}_s\widetilde{x}_{s,t} + \bar{X}_f\widetilde{x}_{f,t} = \chi(1-\chi)\bar{Y}(\phi\widetilde{n}_t + \sigma\widetilde{c}_t) + \chi\bar{Y}\widetilde{y}_t.$$

[20] To obtain the expressions for the steady state values we used $\mu = 1$ and $\bar{T} = 1$.

Substituting this into the final goods market clearing condition implies

$$
\begin{aligned}
\bar{Y}\widetilde{y}_t &= \bar{C}\widetilde{c}_t + \bar{X}_s\widetilde{x}_{s,t} + \bar{X}_f\widetilde{x}_{f,t} \\
\Rightarrow \bar{Y}\widetilde{y}_t &= \bar{C}\widetilde{c}_t + \chi(1-\chi)\bar{Y}(\phi\widetilde{n}_t + \sigma\widetilde{c}_t) + \chi\bar{Y}\widetilde{y}_t \\
\Rightarrow \bar{Y}(1-\chi)\widetilde{y}_t &= \bar{C}\widetilde{c}_t + \bar{Y}(1-\chi)\chi(\phi\widetilde{n}_t + \sigma\widetilde{c}_t) \\
\Rightarrow \widetilde{y}_t &= (1+\chi\sigma)\widetilde{c}_t + \chi\phi\widetilde{n}_t,
\end{aligned}
$$

where in the last equality we used the fact that $\bar{C} = (1-\chi)\bar{Y}$. Substituting for $\widetilde{n}_t$ from (31) and solving further we get

$$
\widetilde{y}_t = (1 + \chi(\phi + \sigma))\widetilde{c}_t.
$$

Hence, $\widetilde{n}_t$ can be expressed as

$$
\begin{aligned}
\widetilde{n}_t &= -\frac{\chi\sigma}{1+\chi\phi}\widetilde{c}_t + \frac{1}{1+\chi\phi}\widetilde{y}_t \\
&= -\frac{\chi\sigma}{1+\chi\phi}\widetilde{c}_t + \frac{1+\chi(\phi+\sigma)}{1+\chi\phi}\widetilde{c}_t \\
&= \widetilde{c}_t
\end{aligned}
$$

So $\widetilde{w}_t - \widetilde{p}_t$ can be expressed as

$$
\begin{aligned}
\widetilde{w}_t - \widetilde{p}_t &= (\phi+\sigma)\widetilde{c}_t \\
&= \frac{\phi+\sigma}{1+\chi(\phi+\sigma)}\widetilde{y}_t.
\end{aligned}
$$

Define $\Theta \equiv \frac{(1-\chi)(\phi+\sigma)}{1+\chi(\phi+\sigma)}$. Then, we can write the NKPC for sector s as

$$
\begin{aligned}
\pi_{s,t} &= \kappa_s\{(1-\chi)(\widetilde{w}_t - \widetilde{p}_t) - (1-\eta)\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1} \\
&= \kappa_s\{\Theta\widetilde{y}_t - (1-\eta)\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1}.
\end{aligned}
$$

Similarly, the NKPC for sector f is given as

$$\pi_{f,t} = \kappa_f \{\Theta \widetilde{y}_t + \eta \widetilde{T}_t\} + \beta \mathbb{E}_t \pi_{f,t+1}.$$

The log-linearized household Euler equation is given by

$$\widehat{c}_t = \mathbb{E}_t \widehat{c}_{t+1} - \sigma^{-1}(r_t - \mathbb{E}_t \pi_{t+1}).$$

Combining with the log-linearized version under flexible prices we get

$$\widetilde{c}_t = \mathbb{E}_t \widetilde{c}_{t+1} - \sigma^{-1}(r_t - \mathbb{E}_t \pi_{t+1} - r_t^{n*}).$$

Using the aggregate price index (5) we can write

$$\pi_{t+1} = \eta \pi_{s,t+1} + (1-\eta)\pi_{f,t+1}.$$

Hence, the Euler equation becomes

$$\widetilde{c}_t = \mathbb{E}_t \widetilde{c}_{t+1} - \sigma^{-1}(r_t - \mathbb{E}_t(\eta \pi_{s,t+1} + (1-\eta)\pi_{f,t+1}) - r_t^{n*}).$$

Substituting away $\widetilde{c}_t$ using $\widetilde{y}_t$ and defining $\sigma^{-1}(1 + \chi(\sigma + \phi)) = \Xi$, we get

$$\widetilde{y}_t = \mathbb{E}_t \widetilde{y}_{t+1} - \Xi(r_t - \mathbb{E}_t(\eta \pi_{s,t+1} + (1-\eta)\pi_{f,t+1}) - r_t^{n*}).$$

Recall the expression for the relative price:

$$T_t = \frac{P_{s,t}}{P_{f,t}}.$$

It can be rewritten as

$$T_t = \frac{P_{s,t}}{P_{s,t-1}} \frac{P_{s,t-1}}{P_{f,t-1}} \frac{P_{f,t-1}}{P_{f,t}} \qquad = \frac{\Pi_{s,t} T_{t-1}}{\Pi_{f,t}}.$$

The log-linearized version is

$$
\begin{aligned}
\widehat{T}_t &= \pi_{s,t} - \pi_{f,t} + \widehat{T}_{t-1} \\
\Rightarrow \widehat{T}_t - T^*_t &= \pi_{s,t} - \pi_{f,t} + \widehat{T}_{t-1} - T^*_{t-1} + T^*_{t-1} - T^*_t \\
\Rightarrow \widetilde{T}_t &= \pi_{s,t} - \pi_{f,t} + \widetilde{T}_{t-1} - (T^*_t - T^*_{t-1}) \\
\Rightarrow \Delta\widetilde{T}_t &= \pi_{s,t} - \pi_{f,t} - \Delta T^*_t.
\end{aligned}
$$

Note that, because $T^*_t = z_{f,t} - z_{s,t}$, we have $\Delta T^*_t = \Delta z_{f,t} - \Delta z_{s,t}$.

Summarizing, the sticky price equilibrium is given by:

$$\widetilde{y}_t = \mathbb{E}_t\widetilde{y}_{t+1} - \Xi(r_t - \mathbb{E}_t(\eta\pi_{s,t+1} + (1-\eta)\pi_{f,t+1}) - r^{n*}_t), \tag{32}$$

$$\pi_{s,t} = \kappa_s\{\Theta\widetilde{y}_t - (1-\eta)\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{s,t+1}, \tag{33}$$

$$\pi_{f,t} = \kappa_f\{\Theta\widetilde{y}_t + \eta\widetilde{T}_t\} + \beta\mathbb{E}_t\pi_{f,t+1}, \tag{34}$$

$$\Delta\widetilde{T}_t = \pi_{s,t} - \pi_{f,t} - \Delta T^*_t. \tag{35}$$

G Derivation of the Utility-Based Loss Function

Linearizations are done around the steady state with flexible prices and in which shocks are absent. The government sets subsidies to eliminate the monopolistic competition distortions, hence the gross markups in the two sectors are equal to one.

Repeating the expression for period t utility:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}. \tag{L1}$$

We can write C_t as $\exp(\log C_t)$. Taking a second-order Taylor approximation yields:[21]

$$
\begin{aligned}
C_t &\approx \exp(\log C_t) = \exp(\log \bar{C}) + \exp(\log \bar{C})(\log C_t - \log \bar{C}) + \frac{1}{2}\exp(\log \bar{C})(\log C_t - \log \bar{C})^2 \\
\Rightarrow C_t &\approx \bar{C} + \bar{C}\widehat{c}_t + \frac{\bar{C}}{2}\widehat{c}_t^2 \\
&= \bar{C}\left[1 + \widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right]
\end{aligned}
$$

Hence,

$$\frac{C_t - \bar{C}}{\bar{C}} = \widehat{c}_t + \frac{1}{2}\widehat{c}_t^2.$$

Similarly for N_t we get

$$\frac{N_t - \bar{N}}{\bar{N}} = \widehat{n}_t + \frac{1}{2}\widehat{n}_t^2.$$

For notational simplicity, we denote $U(C_t, N_t) = U_t$, $U_C(C_t, N_t) = U_{Ct}$, $U_N(C_t, N_t) = U_{Nt}$ and so on. $\bar{U}, \bar{U}_C, \bar{U}_N, \cdots$ have analogous interpretations, but are evaluated at the steady state. Taking a second-order approximation of period t utility, and evaluating at the steady state, we get:

$$
\begin{aligned}
U_t &= \bar{U} + \bar{U}_C(C_t - \bar{C}) + \frac{1}{2}\bar{U}_{CC}(C_t - \bar{C})^2 + \bar{U}_N(N_t - \bar{N}) + \frac{1}{2}\bar{U}_{NN}(N_t - \bar{N})^2 + \mathcal{O}(||z^3||) \\
&= \bar{U} + \bar{U}_C\bar{C}\left(\frac{C_t - \bar{C}}{\bar{C}}\right) + \frac{1}{2}\bar{U}_{CC}\bar{C}^2\left(\frac{C_t - \bar{C}}{\bar{C}}\right)^2 + \bar{U}_N\bar{N}\left(\frac{N_t - \bar{N}}{\bar{N}}\right) + \frac{1}{2}\bar{U}_{NN}\bar{N}^2\left(\frac{N_t - \bar{N}}{\bar{N}}\right)^2 + \mathcal{O}(||z^3||),
\end{aligned}
$$

where $\mathcal{O}(||z^3||)$ represents the terms of third- or higher order. Substituting for $\frac{C_t - \bar{C}}{\bar{C}}$ and $\frac{N_t - \bar{N}}{\bar{N}}$, we get

[21] The approximation is taken of the function $\exp(X_t)$ and evaluated at $\log \bar{C}$.

$$U_t = \bar{U} + \bar{U}_C \bar{C}\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) + \frac{1}{2}\bar{U}_{CC}\bar{C}^2\left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right)^2 + \bar{U}_N \bar{N}\left(\widehat{n}_t + \frac{1}{2}\widehat{n}_t^2\right) + \frac{1}{2}\bar{U}_{NN}\bar{N}^2\left(\widehat{n}_t + \frac{1}{2}\widehat{n}_t^2\right)^2 + \mathcal{O}(||z^3||).$$

Substituting $\bar{U}_C\bar{C} = \bar{C}^{1-\sigma}$, $\bar{U}_{CC}\bar{C}^2 = -\sigma\bar{C}^{1-\sigma}$, $\bar{U}_N\bar{N} = -\bar{N}^{1+\phi}$ and $\bar{U}_{NN}\bar{N}^2 = -\phi\bar{N}^{1+\phi}$, and neglecting the third- and higher-order terms, we get

$$U_t - \bar{U} = \bar{C}^{1-\sigma}\left(\widehat{c}_t + \frac{1-\sigma}{2}\widehat{c}_t^2\right) - \bar{N}^{1+\phi}\left(\widehat{n}_t + \frac{1+\phi}{2}\widehat{n}_t^2\right) + \mathcal{O}(||z^3||). \tag{L2}$$

Our objective is to write this expression in terms of the output gap $\tilde{y}_t$ and inflation. First, we eliminate $\widehat{n}_t$. Recall the economy-wide labor-market clearing condition

$$N_t = N_{s,t} + N_{f,t},$$

as well as the sectoral labor-market clearing conditions (28),

$$N_{s,t} = \Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi}\frac{Y_{s,t}}{Z_{s,t}}D_{s,t}, \qquad N_{f,t} = \Gamma^{-\chi}\left(\frac{W_t}{P_t}\right)^{-\chi}\frac{Y_{f,t}}{Z_{f,t}}D_{f,t}.$$

For a generic variable X_t, we will use x_t as short-hand notation for $\log X_t$. Taking logarithms on both sides of the expression for sector s we get

$$n_{s,t} = -\chi(w_t - p_t) + y_{s,t} - z_{s,t} + d_{s,t} + \log(\Gamma^{-\chi}).$$

Subtract the steady-state counterpart

$$n_{s,t} - \bar{n}_s = -\chi(w_t - p_t) + \chi(\bar{w} - \bar{p}) + y_{s,t} - \bar{y}_s - z_{s,t} + d_{s,t}.$$

Here, we have used that the steady-state value $\bar{D}_s$ of $D_{s,t}$ is one, as all the prices in sector s are constant and equal to the sector-level price index. Hence, $\log(\bar{D}_s) = 0$. We can further write

$$\widehat{n}_{s,t} = -\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t} - z_{s,t} + d_{s,t}. \tag{L3}$$

Although in the above log-linearizations we neglected $d_{s,t}$, when taking our second approximation to utility we cannot do this as $d_{s,t}$ is of second order. Before continuing with (L3) we focus on $d_{s,t}$, which is given by

$$d_{s,t} = \log\left[\frac{1}{n}\int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon} di\right].$$

From the definition of the sector s price index we have

$$\begin{aligned} P_{s,t}^{1-\epsilon} &= \left[\frac{1}{n}\int_0^n P_{s,t}(i)^{1-\epsilon} di\right] \\ \Rightarrow 1 &= \left[\frac{1}{n}\int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon} di\right]. \end{aligned}$$

Hence, $\mathbb{E}_i \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon} = 1$. Now, consider $\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon}$, which can be written as

$$\begin{aligned} \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon} &= \exp\left((1-\epsilon)\log\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)\right) \\ &= \exp\left((1-\epsilon)(p_{s,t}(i) - p_{s,t})\right). \end{aligned}$$

Define $\hat{p}_{s,t}(i) = p_{s,t}(i) - p_{s,t}$. Note that $\hat{p}_{s,t}(i)$ is *not* the deviation of $\log(P_{s,t}(i))$ from its steady state counter part. Hence, we can write

$$\begin{aligned}
\left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon} &= \exp\left((1-\epsilon)\hat{p}_{s,t}(i)\right) \\
&\approx \exp(0) + \exp(0)(1-\epsilon)\hat{p}_{s,t}(i) + \exp(0)\frac{(1-\epsilon)^2}{2}\hat{p}_{s,t}(i)^2 \\
&= 1 + (1-\epsilon)\hat{p}_{s,t}(i) + \frac{(1-\epsilon)^2}{2}\hat{p}_{s,t}(i)^2
\end{aligned}$$

Hence, we can write $1 = \frac{1}{n}\int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon} di = \mathbb{E}_i \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon}$ as

$$\begin{aligned}
\mathbb{E}_i \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{1-\epsilon} &= \mathbb{E}_i(1 + (1-\epsilon)\hat{p}_{s,t}(i) + \frac{(1-\epsilon)^2}{2}\hat{p}_{s,t}(i)^2) \\
\Rightarrow 1 &= 1 + (1-\epsilon)\mathbb{E}_i\hat{p}_{s,t}(i) + \frac{(1-\epsilon)^2}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2)
\end{aligned}$$

Therefore, we can write $\mathbb{E}_i\hat{p}_{s,t}(i)$ as

$$\mathbb{E}_i\hat{p}_{s,t}(i) = -\frac{(1-\epsilon)}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2).$$

We can substitute this into our rewritten measure of price dispersion in sector s:

$$\begin{aligned}
D_{s,t} &= \frac{1}{n}\int_0^n \left(\frac{P_{s,t}(i)}{P_{s,t}}\right)^{-\epsilon} di \\
&= \frac{1}{n}\int_0^n \exp(-\epsilon(\hat{p}_{s,t}(i)))di \\
&\approx \frac{1}{n}\int_0^n [1 - \epsilon\hat{p}_{s,t}(i) + \frac{\epsilon^2}{2}\hat{p}_{s,t}(i)^2]di \\
&= 1 - \epsilon\ \mathbb{E}_i\hat{p}_{s,t}(i) + \frac{\epsilon^2}{2}\ \mathbb{E}_i(\hat{p}_{s,t}(i)^2),
\end{aligned}$$

to give

$$
\begin{aligned}
D_{s,t} &= 1 + \frac{\epsilon(1-\epsilon)}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) + \frac{\epsilon^2}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) \\
&= 1 + \frac{\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2).
\end{aligned}
$$

Since $d_{s,t} = \log(D_{s,t})$, we can write

$$
\begin{aligned}
d_{s,t} &= \log(D_{s,t}) \\
&= \log(1 + \frac{\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2)) \\
&\approx \frac{\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2).
\end{aligned}
$$

Hence, we can write (L3) as

$$
\begin{aligned}
\widehat{n}_{s,t} &= -\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t} - z_{s,t} + d_{s,t} \\
&= -\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t} - z_{s,t} + \frac{\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2).
\end{aligned}
$$

Similarly, for sector f we have

$$
\widehat{n}_{f,t} = -\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{f,t} - z_{f,t} + \frac{\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2).
$$

Recall the labor-market clearing condition

$$
\widehat{n}_t = \frac{\bar{N}_s}{\bar{N}}\widehat{n}_{s,t} + \frac{\bar{N}_f}{\bar{N}}\widehat{n}_{f,t}.
$$

Using $\frac{\bar{N}_s}{\bar{N}} = \eta$ and $\frac{\bar{N}_f}{\bar{N}} = 1 - \eta$, we can write

$$\begin{aligned}\widehat{n}_t &= \eta\left(-\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{s,t} - z_{s,t} + \frac{\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2)\right) + (1-\eta)\left(-\chi(\widehat{w}_t - \widehat{p}_t) + \widehat{y}_{f,t} - z_{f,t} + \frac{\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2)\right) \\ &= -\chi(\widehat{w}_t - \widehat{p}_t) + \eta\widehat{y}_{s,t} + (1-\eta)\widehat{y}_{f,t} - (\eta z_{s,t} + (1-\eta)z_{f,t}) + \frac{\eta\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2).\end{aligned}$$

Recall that $\eta z_{s,t} + (1-\eta)z_{f,t} = (1-\chi)(w_t^* - p_t^*)$. Substituting this into the equation above and using the fact that $\widetilde{w}_t = \widehat{w}_t - w_t^*$, we can write

$$\widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \eta\widehat{y}_{s,t} + (1-\eta)\widehat{y}_{f,t} - (w_t^* - p_t^*) + \frac{\eta\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2).$$

Recall furthermore that $\widehat{y}_{s,t} = (\eta-1)\widehat{T}_t + \widehat{y}_t$ and $\widehat{y}_{f,t} = \eta\widehat{T}_t + \widehat{y}_t$. Substituting this into the above equation we get

$$\widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widehat{y}_t - (w_t^* - p_t^*) + \frac{\eta\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2).$$

We also recall that $y_t^* = \frac{1+\phi}{(\phi+\sigma)(1-\chi)}(\eta z_{s,t} + (1-\eta)z_{f,t}) = \frac{(1+\phi)(1-\chi)}{(\phi+\sigma)(1-\chi)}(w_t^* - p_t^*)$. So, we can further rewrite the above equation as

$$\widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widehat{y}_t - \frac{\phi+\sigma}{1+\phi}y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2).$$

Using $\widehat{y}_t = \widetilde{y}_t + y_t^*$, this can be further rewritten as

$$
\widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_t + \left(1 - \frac{\phi+\sigma}{1+\phi}\right) y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2)
$$
$$
\Rightarrow \widehat{n}_t = -\chi(\widetilde{w}_t - \widetilde{p}_t) + \widetilde{y}_t + \left(\frac{1-\sigma}{1+\phi}\right) y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2).
$$

Finally, recall that $\widetilde{w}_t - \widetilde{p}_t = \left(\frac{\phi+\sigma}{1+\chi(\phi+\sigma)}\right)\widetilde{y}_t$. Substituting this in the above equation we get

$$
\widehat{n}_t = \left(\frac{1}{1+\chi(\phi+\sigma)}\right)\widetilde{y}_t + \left(\frac{1-\sigma}{1+\phi}\right) y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2).
$$

Let us now turn to the consumption part of period utility. First notice that $\bar{C}^{1-\sigma} = \bar{N}^{1+\phi}$. We know that $\bar{C} = k_{CY}\bar{Y}$ and $\bar{N} = k_{NY}\bar{Y}$, where $k_{CY} = 1-\chi$ and $k_{NY} = \chi^{-\frac{\chi}{1-\chi}}$. Using this and the value of $\bar{Y} = \left[\chi^{\frac{1}{1-\chi}}\left(\frac{1-\chi}{\chi}\right)\frac{1}{k_{CY}^{\sigma}k_{NY}^{\phi}}\right]^{\frac{1}{\sigma+\phi}}$, we can show that $\bar{C}^{1-\sigma} = \bar{N}^{1+\phi}$. To see this, write

$$
\begin{aligned}
\frac{\bar{N}^{1+\phi}}{\bar{C}^{1-\sigma}} &= \frac{K_{NY}^{1+\phi}}{k_{CY}^{1-\sigma}}\bar{Y}^{\sigma+\phi} \\
&= \frac{k_{NY}}{k_{CY}}\chi^{\frac{1}{1-\chi}}\left(\frac{1-\chi}{\chi}\right), \\
&= 1
\end{aligned}
$$

where the second equality uses the value of $\bar{Y}$ and last equality uses the values of k_{CY} and k_{NY}. Now, we can write $\widehat{c}_t + \frac{1-\sigma}{2}\,\widehat{c}_t^2$, using $\widetilde{c}_t = \widehat{c}_t - c_t^*$, as

$$
\widetilde{c}_t + \frac{1-\sigma}{2}\widetilde{c}_t^2 + (1-\sigma)\widetilde{c}_t c_t^* + t.i.p,
$$

where $t.i.p$ is short-hand for "terms independent of policy". Then, using the relationship $\widetilde{c}_t = \kappa_1\widetilde{y}_t$, where $\kappa_1 \equiv \frac{1}{1+\chi(\phi+\sigma)}$, and $y_t^* = c_t^*$, we can write this expression as

$$
\kappa_1\widetilde{y}_t + \frac{1-\sigma}{2}\kappa_1^2\widetilde{y}_t^2 + (1-\sigma)\kappa_1\widetilde{y}_t y_t^* + t.i.p + \mathcal{O}(||z^3||).
$$

Now, coming onto simplifying the notation of $\widehat{n}_t + \frac{1+\phi}{2}\widehat{n}_t^2$. As we had derived above, $\widehat{n}_t$ is given as

$$\widehat{n}_t = \left(\frac{1}{1+\chi(\phi+\sigma)}\right)\widetilde{y}_t + \left(\frac{1-\sigma}{1+\phi}\right)y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2).$$

Defining $\kappa_2 \equiv \left(\frac{1-\sigma}{1+\phi}\right)$, and using the definition of κ_1, we can simplify our expression for $\widehat{n}_t$ to

$$\widehat{n}_t = \kappa_1\widetilde{y}_t + \kappa_2 y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2).$$

Hence, $\widehat{n}_t + \frac{1+\phi}{2}\widehat{n}_t^2$ becomes

$$\widehat{n}_t + \frac{1+\phi}{2}\widehat{n}_t^2 = \kappa_1\widetilde{y}_t + \left(\frac{1+\phi}{2}\right)\kappa_1^2\widetilde{y}_t^2 + (1+\phi)\kappa_1\kappa_2\widetilde{y}_t y_t^* + \frac{\eta\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2) + t.i.p + \mathcal{O}(||z^3||).$$

Now, using $\bar{C}^{1-\sigma} = \bar{N}^{1+\phi}$ and substituting $\widehat{n}_t + \frac{1+\phi}{2}\widehat{n}_t^2$ and $\widetilde{c}_t + \frac{1-\sigma}{2}\widetilde{c}_t^2 + (1-\sigma)\widetilde{c}_t c_t^*$ into (L2) we get

$$U_t - \bar{U} = -\bar{C}^{1-\sigma}\left[\left(\frac{\sigma+\phi}{2}\right)\kappa_1^2\widetilde{y}_t^2 + \frac{\eta\epsilon}{2}\mathbb{E}_i(\hat{p}_{s,t}(i)^2) + \frac{(1-\eta)\epsilon}{2}\mathbb{E}_i(\hat{p}_{f,t}(i)^2)\right] + t.i.p + \mathcal{O}(||z^3||). \quad (36)$$

We still need to express the price dispersions $\frac{\epsilon}{2}\mathbb{E}_i(\widehat{p}_{s,t}(i)^2)$ and $\frac{\epsilon}{2}\mathbb{E}_i(\widehat{p}_{f,t}(i)^2)$ in terms of the inflation rates. Because the derivation for sector f is analogous, we focus on sector s. We can write $\mathbb{E}_i(\widehat{p}_{s,t}(i)^2)$ as

$$\begin{aligned}\mathbb{E}_i\left(\widehat{p}_{s,t}(i)^2\right) &= \mathbb{E}_i\left(p_{s,t}(i) - p_{s,t}\right)^2 \\ &\approx \mathbb{E}_i\left(p_{s,t}(i) - \mathbb{E}_i p_{s,t}(i)\right)^2 \\ &= var\ (p_{s,t}(i)),\end{aligned}$$

which is the variance calculated over index i. We denote $var\ (p_{s,t}(i))$ by $\mathfrak{D}_{s,t}$. We first prove that

$\mathfrak{D}_{s,t}$ can be written in a recursive form as

$$\mathfrak{D}_{s,t} = \theta_s \mathfrak{D}_{s,t-1} + \left(\frac{\theta_s}{1-\theta_s}\right)\pi_{s,t}^2.$$

To this end, define $\bar{P}_{s,t} = \mathbb{E}_i p_{s,t}$. (Note that variables with upperbars and subscripts are NOT used to denote steady state values, unlike variables with upperbars and no subscripts.) Hence, we can write

$$\begin{aligned}
\bar{P}_{s,t} &= \mathbb{E}_i p_{s,t}(i) \\
\Rightarrow \bar{P}_{s,t} - \bar{P}_{s,t-1} &= \mathbb{E}_i (p_{s,t}(i) - \bar{P}_{s,t-1}) \\
&= \theta_s \mathbb{E}_i \left(p_{s,t-1}(i) - \bar{P}_{s,t-1}\right) + (1-\theta_s)(p_{s,t}^* - \bar{P}_{s,t-1}) \\
&= (1-\theta_s)(p_{s,t}^* - \bar{P}_{s,t-1}),
\end{aligned}$$

where second line is an identity and the third line uses the Calvo (1983) pricing assumption that a fraction $1-\theta_s$ of the firms change prices in period t. The last line uses the fact that $\mathbb{E}_i(p_{s,t-1}(i)) = \bar{P}_{s,t-1}$. Furthermore, we can write

$$\begin{aligned}
var_i(p_{s,t}(i)) &= var_i(p_{s,t}(i) - \bar{P}_{s,t-1}) \\
&= \mathbb{E}_i \left[(p_{s,t}(i) - \bar{P}_{s,t-1})^2\right] - \left(\mathbb{E}_i(p_{s,t}(i) - \bar{P}_{s,t-1})\right)^2 \\
&= \theta_s \mathbb{E}_i \left[(p_{s,t-1}(i) - \bar{P}_{s,t-1})^2\right] + (1-\theta_s)(p_{s,t}^* - \bar{P}_{s,t-1})^2 - (\bar{P}_{s,t} - \bar{P}_{s,t-1})^2 \\
&= \theta_s var_i\ (p_{s,t-1}(i)) + \left(\frac{\theta_s}{1-\theta_s}\right)\left(\bar{P}_{s,t} - \bar{P}_{s,t-1}\right)^2
\end{aligned}$$

where for the third equality we have used the fact that a fraction θ_s of the firms do not change prices and a fraction $(1-\theta_s)$ of the firms do change prices. For the last equality we have used

$$\begin{aligned}
\bar{P}_{s,t} &= \mathbb{E}_i[p_{s,t}(i)] \\
&= \theta_s E_i[p_{s,t-1}(i)] + (1-\theta_s)\mathbb{E}_i[p_{s,t}^*] \\
&= \theta_s \bar{P}_{s,t-1} + (1-\theta_s)p_{s,t}^*
\end{aligned}$$

where we used the fact that $E_i[p^*_{s,t}] = p^*_{s,t}$, because the firms that get the chance to re-set their price choose the same price. Then we get $p^*_{s,t} - \bar{P}_{s,t-1} = \left(\frac{1}{1-\theta_s}\right)[\bar{P}_{s,t} - \bar{P}_{s,t-1}]$, which we used to obtain the last expression for $var_i(p_{s,t}(i))$.

Following Woodford (2003) we can write $\bar{P}_{s,t} \approx \log P_{s,t}$. Hence, we get the final expression

$$\mathfrak{D}_{s,t} = \theta_s \mathfrak{D}_{s,t-1} + \left(\frac{\theta_s}{1-\theta_s}\right)\pi^2_{s,t}.$$

By one period iteration we get,

$$\mathfrak{D}_{s,t} = \theta_s^2 \mathfrak{D}_{s,t-2} + \theta_s \left(\frac{\theta_s}{1-\theta_s}\right)\pi^2_{s,t-1} + \left(\frac{\theta_s}{1-\theta_s}\right)\pi^2_{s,t}.$$

Following the same logic we get,

$$\mathfrak{D}_{s,t} = \theta_s^{t+1} \mathfrak{D}_{-1} + \sum_{k=0}^{t} \theta_s^{t-k} \left(\frac{\theta_s}{1-\theta_s}\right)\pi^2_{s,k}.$$

Hence, we can write, assuming that $\mathfrak{D}_{-1} = 0$,

$$\sum_{t=0}^{\infty} \beta^t \mathfrak{D}_{s,t} = \frac{\theta_s}{(1-\theta_s)(1-\theta_s\beta)} \sum_{t=0}^{\infty} \beta^t \pi^2_{s,t}.$$

Substitute into the second-order approximation (36) of utility at time t:

$$\frac{1}{\bar{C}^{1-\sigma}}(U_t - \bar{U}) \approx -\left(\left(\frac{\sigma+\phi}{2}\right)\kappa_1^2 \widetilde{y}_t^2 + \frac{\epsilon}{2}(\eta\mathfrak{D}_{s,t} + (1-\eta)\mathfrak{D}_{f,t})\right).$$

Hence, we get

$$\sum_{t=0}^{\infty} \beta^t \frac{1}{\bar{C}^{1-\sigma}}(U_t - \bar{U}) = -\sum_{t=0}^{\infty} \beta^t L_t + t.i.p + \mathcal{O}(||z^3||), \tag{37}$$

where L_t is given as

$$\left(\frac{\sigma+\phi}{2}\right)\kappa_1^2\widetilde{y}_t^2+\frac{\epsilon\eta}{2}\frac{\theta_s}{(1-\theta_s)(1-\theta_s\beta)}\pi_{s,t}^2+\frac{\epsilon(1-\eta)}{2}\frac{\theta_f}{(1-\theta_f)(1-\theta_f\beta)}\pi_{f,t}^2.$$

H Derivation of the Optimal Interest Rate Rule

The four first-order conditions corresponding to the minimization of loss function are given as

$$\begin{aligned}
&\lambda_{\pi,s}\pi_{s,t}+\Lambda_{s,t}-\Lambda_{s,t-1}-\Lambda_{T,t}=0,\\
&\lambda_{\pi,f}\pi_{f,t}+\Lambda_{f,t}-\Lambda_{f,t-1}+\Lambda_{T,t}=0,\\
&\lambda_y\widetilde{y}_t-\kappa_s\Theta\Lambda_{s,t}-\kappa_f\Theta\Lambda_{f,t}=0,\\
&\kappa_s(1-\eta)\Lambda_{s,t}-\kappa_f\eta\Lambda_{f,t}+\Lambda_{T,t}-\beta\mathbb{E}_t\Lambda_{T,t+1}=0.
\end{aligned}$$

Using the third first-order condition we can write

$$\kappa_s\Lambda_{s,t}+\kappa_f\Lambda_{f,t}=\frac{\lambda_y}{\Theta}\widetilde{y}_t.$$

Multiplying the first first-order condition by κ_s, the second by κ_f, adding the resulting two equations and solving for $\Lambda_{T,t}$, gives,

$$\Lambda_{T,t}=\frac{\kappa_s\lambda_{\pi,s}}{\kappa_s-\kappa_f}\pi_{s,t}+\frac{\kappa_f\lambda_{\pi,f}}{\kappa_s-\kappa_f}\pi_{f,t}+\frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)}\Delta\widetilde{y}_t.$$

Hence,

$$\beta\mathbb{E}_t\Lambda_{T,t+1}=\frac{\kappa_s\lambda_{\pi,s}}{\kappa_s-\kappa_f}\beta\mathbb{E}_t\pi_{s,t+1}+\frac{\kappa_f\lambda_{\pi,f}}{\kappa_s-\kappa_f}\beta\mathbb{E}_t\pi_{f,t+1}+\frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)}\beta\mathbb{E}_t\Delta\widetilde{y}_{t+1}.$$

Rewriting the NKPCs of sectors s and f, (19) and (20) respectively, we get

$$\begin{aligned}
\beta\mathbb{E}_t\pi_{s,t+1}&=\pi_{s,t}-\kappa_s\Theta\widetilde{y}_t+\kappa_s(1-\eta)\widetilde{T}_t,\\
\beta\mathbb{E}_t\pi_{f,t+1}&=\pi_{f,t}-\kappa_f\Theta\widetilde{y}_t-\kappa_f\eta\widetilde{T}_t.
\end{aligned}$$

Similarly, using the Euler equation, (18), we can write,

$$
\begin{aligned}
\beta\mathbb{E}_t\Delta\widetilde{y}_{t+1} &= \beta\Xi(r_t - \eta\mathbb{E}_t\pi_{s,t+1} - (1-\eta)\mathbb{E}_t\pi_{f,t+1} - r_t^{n*}) \\
&= \beta\Xi(r_t - r_t^{n*}) - \eta\Xi\pi_{s,t} - (1-\eta)\Xi\pi_{f,t} + \Xi\Theta(\eta\kappa_s + (1-\eta)\kappa_f)\widetilde{y}_t - \eta(1-\eta)\Xi(\kappa_s - \kappa_f)\widetilde{T}_t.
\end{aligned}
$$

Hence, we can write $\beta\mathbb{E}_t\Lambda_{T,t+1}$ as

$$
\begin{aligned}
\beta\mathbb{E}_t\Lambda_{T,t+1} = &\left[\frac{\kappa_s\lambda_{\pi,s}}{\kappa_s-\kappa_f} - \frac{\lambda_y\eta\Xi}{\Theta(\kappa_s-\kappa_f)}\right]\pi_{s,t} + \left[\frac{\kappa_f\lambda_{\pi,f}}{\kappa_s-\kappa_f} - \frac{\lambda_y(1-\eta)\Xi}{\Theta(\kappa_s-\kappa_f)}\right]\pi_{f,t} + \\
&\frac{\lambda_y\beta\Xi}{\Theta(\kappa_s-\kappa_f)}(r_t - r_t^{n*}) + \\
&\left[-\frac{\kappa_s^2\Theta\lambda_{\pi,s}}{\kappa_s-\kappa_f} - \frac{\kappa_f^2\lambda_{\pi,f}\Theta}{\kappa_s-\kappa_f} + \frac{\Xi\lambda_y(\eta\kappa_s + (1-\eta)\kappa_f)}{\kappa_s-\kappa_f}\right]\widetilde{y}_t + \\
&\left[\frac{\kappa_s^2\lambda_{\pi,s}(1-\eta)}{\kappa_s-\kappa_f} - \frac{\kappa_f^2\lambda_{\pi,f}\eta}{\kappa_s-\kappa_f} - \frac{\eta(1-\eta)\Xi\lambda_y}{\Theta}\right]\widetilde{T}_t.
\end{aligned}
$$

Hence, $\Lambda_{T,t} - \beta\mathbb{E}_t\Lambda_{T,t+1}$ is given as

$$
\begin{aligned}
\Lambda_{T,t} - \beta\mathbb{E}_t\Lambda_{T,t+1} = &\frac{\lambda_y\eta\Xi}{\Theta(\kappa_s-\kappa_f)}\pi_{s,t} + \frac{\lambda_y(1-\eta)\Xi}{\Theta(\kappa_s-\kappa_f)}\pi_{f,t} + \\
&\left[\frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)} + \frac{\kappa_s^2\Theta\lambda_{\pi,s}}{\kappa_s-\kappa_f} + \frac{\kappa_f^2\lambda_{\pi,f}\Theta}{\kappa_s-\kappa_f} - \frac{(\eta\kappa_s + (1-\eta)\kappa_f)\Xi\lambda_y}{\kappa_s-\kappa_f}\right]\widetilde{y}_t \\
&- \frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)}\widetilde{y}_{t-1} - \left[\frac{\kappa_s^2\lambda_{\pi,s}(1-\eta)}{\kappa_s-\kappa_f} - \frac{\kappa_f^2\lambda_{\pi,f}\eta}{\kappa_s-\kappa_f} - \frac{\eta(1-\eta)\Xi\lambda_y}{\Theta}\right]\widetilde{T}_t \\
&- \frac{\lambda_y\beta\Xi}{\Theta(\kappa_s-\kappa_f)}(r_t - r_t^{n*}),
\end{aligned}
$$

which can be further written as

$$
\Lambda_{T,t} - \beta\mathbb{E}_t\Lambda_{T,t+1} = \rho_s\pi_{s,t} + \rho_f\pi_{f,t} + \rho_y\widetilde{y}_t - \frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)}\widetilde{y}_{t-1} - \rho_T\widetilde{T}_t - \rho_r(r_t - r_t^{n*}),
$$

where

$$\rho_s = \frac{\lambda_y \eta \Xi}{\Theta(\kappa_s - \kappa_f)}, \qquad \rho_f = \frac{\lambda_y(1-\eta)\Xi}{\Theta(\kappa_s - \kappa_f)},$$
$$\rho_y = \frac{\lambda_y}{\Theta(\kappa_s - \kappa_f)} + \frac{\kappa_s^2 \Theta \lambda_{\pi,s}}{\kappa_s - \kappa_f} + \frac{\kappa_f^2 \lambda_{\pi,f} \Theta}{\kappa_s - \kappa_f} - \frac{(\eta\kappa_s + (1-\eta)\kappa_f)\Xi\lambda_y}{\kappa_s - \kappa_f},$$
$$\rho_T = \frac{\kappa_s^2 \lambda_{\pi,s}(1-\eta)}{\kappa_s - \kappa_f} - \frac{\kappa_f^2 \lambda_{\pi,f}\eta}{\kappa_s - \kappa_f} - \frac{\eta(1-\eta)\Xi\lambda_y}{\Theta}, \qquad \rho_r = \frac{\lambda_y \beta \Xi}{\Theta(\kappa_s - \kappa_f)}.$$

Defining $F_t = \Lambda_{T,t} - \beta \mathbb{E}_t \Lambda_{T,t+1}$, we can write the last of the above first-order conditions as

$$-\kappa_s(1-\eta)\Lambda_{s,t} + \kappa_f \eta \Lambda_{f,t} = F_t.$$

Additionally, using $\kappa_s \Lambda_{s,t} + \kappa_f \Lambda_{f,t} = \frac{\lambda_y}{\Theta}\widetilde{y}_t$, we can solve for $\Lambda_{f,t}$, which is given as

$$\Lambda_{f,t} = \frac{F_t}{\kappa_f} + \frac{\lambda_y(1-\eta)}{\Theta\kappa_f}\widetilde{y}_t.$$

Substituting $\Lambda_{f,t}$ into the second of the above first-order conditions we get

$$\lambda_{\pi,f}\pi_{f,t} + \frac{\Delta F_t}{\kappa_f} + \frac{\lambda_y(1-\eta)}{\Theta\kappa_f}\Delta\widetilde{y}_t + \Lambda_{T,t} = 0,$$

where ΔF_t is given as

$$\Delta F_t = \rho_s \Delta\pi_{s,t} + \rho_f \Delta\pi_{f,t} + \rho_y \Delta\widetilde{y}_t - \frac{\lambda_y}{\Theta(\kappa_s - \kappa_f)}\Delta\widetilde{y}_{t-1} - \rho_T \Delta\widetilde{T}_t - \rho_r(\Delta r_t - \Delta r_t^{n*}).$$

Using the actual form of ΔF_t, we can write

$$\lambda_{\pi,f}\pi_{f,t} + \frac{\rho_s}{\kappa_f}\Delta\pi_{s,t} + \frac{\rho_f}{\kappa_f}\Delta\pi_{f,t} + \frac{\rho_y}{\kappa_f}\Delta\widetilde{y}_t - \frac{\lambda_y}{\Theta\kappa_f(\kappa_s - \kappa_f)}\Delta\widetilde{y}_{t-1} - \frac{\rho_T}{\kappa_f}\Delta\widetilde{T}_t - \frac{\rho_r}{\kappa_f}(\Delta r_t - \Delta r_t^{n*})$$
$$+ \frac{\lambda_y(1-\eta)}{\Theta\kappa_f}\Delta\widetilde{y}_t + \Lambda_{T,t} = 0.$$

Now, substituting $\Lambda_{T,t} = \frac{\kappa_s \lambda_{\pi,s}}{\kappa_s - \kappa_f}\pi_{s,t} + \frac{\kappa_f \lambda_{\pi,f}}{\kappa_s - \kappa_f}\pi_{f,t} + \frac{\lambda_y}{\Theta(\kappa_s - \kappa_f)}\Delta\widetilde{y}_t$ gives

$$\frac{\rho_r}{\kappa_f}(\Delta r_t - \Delta r_t^{n*}) = \frac{\kappa_s \lambda_{\pi,s}}{\kappa_s - \kappa_f}\pi_{s,t} + \frac{\kappa_s \lambda_{\pi,f}}{\kappa_s - \kappa_f}\pi_{f,t} + \frac{\rho_s}{\kappa_f}\Delta\pi_{s,t} + \frac{\rho_f}{\kappa_f}\Delta\pi_{f,t} +$$
$$\left[\frac{\rho_y}{\kappa_f} + \frac{\lambda_y(1-\eta)}{\Theta\kappa_f} + \frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)}\right]\Delta\widetilde{y}_t - \frac{\lambda_y}{\Theta\kappa_f(\kappa_s-\kappa_f)}\Delta\widetilde{y}_{t-1} - \frac{\rho_T}{\kappa_f}\Delta\widetilde{T}_t,$$

and solving for r_t we get

$$r_t = r_{t-1} + \frac{\kappa_s\kappa_f}{\kappa_s-\kappa_f}\frac{\lambda_{\pi,f}}{\rho_r}\pi_{f,t} + \frac{\kappa_s\kappa_f}{\kappa_s-\kappa_f}\frac{\lambda_{\pi,s}}{\rho_r}\pi_{s,t} + \frac{\rho_s}{\rho_r}\Delta\pi_{s,t} + \frac{\rho_f}{\rho_r}\Delta\pi_{f,t} +$$
$$\frac{\kappa_f}{\rho_r}\left[\frac{\rho_y}{\kappa_f} + \frac{\lambda_y(1-\eta)}{\Theta\kappa_f} + \frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)}\right]\Delta\widetilde{y}_t - \frac{\lambda_y}{\Theta\rho_r(\kappa_s-\kappa_f)}\Delta\widetilde{y}_{t-1} - \frac{\rho_T}{\rho_r}\Delta\widetilde{T}_t + \Delta r_t^{n*}.$$

Now, we can simplify each coefficient by making appropriate substitutions. For example, the coefficient associated with $\pi_{s,t}$ we can write as

$$\begin{aligned}
\frac{\kappa_s\kappa_f}{\kappa_s-\kappa_f}\frac{\lambda_{\pi,s}}{\rho_r} &= \frac{\kappa_s\kappa_f}{\kappa_s-\kappa_f}\frac{\eta\epsilon}{2\kappa_s}\frac{\Theta(\kappa_s-\kappa_f)}{\lambda_y\eta\Xi} \\
&= \frac{\kappa_s\kappa_f}{\kappa_s-\kappa_f}\frac{\eta\epsilon}{2\kappa_s}\frac{(1-\chi)(\sigma+\phi)}{1+\chi(\phi+\sigma)}\frac{1}{\beta}\frac{\sigma}{1+\chi(\phi+\sigma)}\frac{2[1+\chi(\phi+\sigma)]^2}{\sigma+\phi} \\
&= \frac{\sigma\epsilon\eta\kappa_f(1-\chi)}{\beta},
\end{aligned}$$

where for the first equality we substituted $\lambda_{\pi_s} = \frac{\eta\epsilon}{2\kappa_s}$ and for the second equality we substitute $\Theta = \frac{(1-\chi)(\sigma+\phi)}{1+\chi(\phi+\sigma)}$, $\Xi = \frac{(1+\chi(\phi+\sigma))}{\sigma}$ and $\lambda_y = \frac{(\sigma+\phi)}{2(1+\chi(\phi+\sigma))^2}$. The coefficient associated with $\Delta\widetilde{y}_t$ (denoted by $\mathcal{A}_{\Delta\widetilde{y}_0}$) can be written as

$$\begin{aligned}
\mathcal{A}_{\Delta\widetilde{y}_0} &= \frac{\kappa_f}{\rho_r}\left[\frac{\rho_y}{\kappa_f} + \frac{\lambda_y(1-\eta)}{\Theta\kappa_f} + \frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)}\right] \\
&= \frac{\rho_y}{\rho_r} + \frac{\lambda_y(1-\eta)}{\Theta\rho_r} + \frac{\lambda_y\kappa_f}{\Theta(\kappa_s-\kappa_f)\rho_r}.
\end{aligned}$$

Substituting ρ_y and ρ_r gives

$$\mathcal{A}_{\Delta\widetilde{y}_0} = \frac{\Theta(\kappa_s-\kappa_f)}{\lambda_y\beta\Xi}\left[\frac{\lambda_y}{\Theta(\kappa_s-\kappa_f)} + \left(\frac{\Theta}{\kappa_s-\kappa_f}\right)(\kappa_s^2\lambda_{\pi,s}+\kappa_f^2\lambda_{\pi,f}) - \frac{[\eta\kappa_s+(1-\eta)\kappa_f]\Xi\lambda_y}{\kappa_s-\kappa_f}\right] + \frac{\lambda_y}{\Theta\rho_r}\left[1-\eta+\frac{\kappa_f}{\kappa_s-\kappa_f}\right].$$

By substituting $\lambda_{\pi,s} = \frac{\eta\epsilon}{2\kappa_s}$ and $\lambda_{\pi,f} = \frac{(1-\eta)\epsilon}{2\kappa_f}$ we get

$$\mathcal{A}_{\Delta\widetilde{y}_0} \equiv \frac{\Theta}{\lambda_y\beta\Xi}\left[\frac{\lambda_y}{\Theta} + \frac{\Theta\eta\epsilon\kappa_s}{2} + \frac{\Theta(1-\eta)\epsilon\kappa_f}{2} - (\eta\kappa_s+(1-\eta)\kappa_f)\Xi\lambda_y\right] + \frac{1}{\beta\Xi}(\eta\kappa_f+(1-\eta)\kappa_s).$$

In a similar way we can derive the other coefficients.

I Proof of Claim in Footnote 7

The four first-order conditions from the minimization of the authority's loss function subject to the constraints (19), (20) and (21), are given as

$$\lambda_{\pi,s}\pi_{s,t} + \Lambda_{s,t} - \Lambda_{s,t-1} - \Lambda_{T,t} = 0, \tag{OMP, π_s}$$

$$\lambda_{\pi,f}\pi_{f,t} + \Lambda_{f,t} - \Lambda_{f,t-1} + \Lambda_{T,t} = 0, \tag{OMP, π_f}$$

$$\lambda_y\widetilde{y}_t - \kappa_s\Theta\Lambda_{s,t} - \kappa_f\Theta\Lambda_{f,t} = 0, \tag{OMP,y}$$

$$\kappa_s(1-\eta)\Lambda_{s,t} - \kappa_f\eta\Lambda_{f,t} + \Lambda_{T,t} - \beta\mathbb{E}_t\Lambda_{T,t+1} = 0. \tag{OMP,t}$$

From (OMP,y) we can write $\widetilde{y}_t$ as

$$\widetilde{y}_t = \left(\frac{\Theta}{\lambda_y}\right)(\kappa_s\Lambda_{s,t} + \kappa_f\Lambda_{f,t}).$$

Substituting this into (19), (20), (21), (OMP, π_s), (OMP, π_f) and (OMP,t), and defining $S_t = \left(\Lambda_{s,t-1}, \Lambda_{f,t-1}, \widetilde{T}_{t-1}, \pi_{s,t}, \pi_{f,t}, \Lambda_{T,t}\right)'$, hence $\mathbb{E}_t S_{t+1} = \left(\Lambda_{s,t}, \Lambda_{f,t}, \widetilde{T}_t, \mathbb{E}_t \pi_{s,t+1}, \mathbb{E}_t \pi_{f,t+1}, \mathbb{E}_t \Lambda_{T,t+1}\right)'$, we can write the above system of equations as

$$A\mathbb{E}_t S_{t+1} = BS_t + V\Delta T_t^*,$$

where A and B are 6×6 matrices of coefficients and V is a 6×1 matrix of coefficients. A, B and V are given as

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ \kappa_s(1-\eta) & -\kappa_f\eta & 0 & 0 & 0 & -\beta \\ \frac{\kappa_s^2\Theta^2}{\lambda_y} & \frac{\kappa_s\kappa_f\Theta^2}{\lambda_y} & -\kappa_s(1-\eta) & \beta & 0 & 0 \\ \frac{\kappa_s\kappa_f\Theta^2}{\lambda_y} & \frac{\kappa_f^2\Theta^2}{\lambda_y} & \kappa_f\eta & 0 & \beta & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & -\lambda_{\pi,s} & 0 & 1 \\ 0 & 1 & 0 & 0 & -\lambda_{\pi,f} & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 0 \end{bmatrix},$$

$$V = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}'.$$

The determinant of A is β^3 and, hence, under the accepted parametrization, matrix A is always invertible. Hence we can write $\mathbb{E}_t S_{t+1}$ as

$$\mathbb{E}_t S_{t+1} = A^{-1}BS_t + A^{-1}V\Delta T_t^*.$$

There are three predetermined variables and three non-predetermined variables. In order to have a unique bounded solution, there should be exactly three eigenvalues of the matrix $A^{-1}B$ with modulus greater than 1. It can be shown computationally that for all the combinations of $\theta_s, \theta_f, \eta, \chi$ and ϵ that we consider, there are always exactly three eigen values with modulus greater than one. Hence, using Blanchard Kahn (1980) conditions, we can conclude that the solution for $\mathbb{E}_t S_{t+1}$ is unique and bounded. Some algebra shows that A^{-1}, $A^{-1}B$ and $A^{-1}V$ are

$$
A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{\Theta^2\kappa_s^2}{\beta\lambda_y} & -\frac{\Theta^2\kappa_s\kappa_f}{\beta\lambda_y} & 0 & \frac{1}{\beta} & 0 & \frac{\kappa_s(1-\eta)}{\beta} \\ -\frac{\Theta^2\kappa_s\kappa_f}{\beta\lambda_y} & -\frac{\Theta^2\kappa_f^2}{\beta\lambda_y} & 0 & 0 & \frac{1}{\beta} & -\frac{\kappa_f\eta}{\beta} \\ -\frac{\kappa_s(1-\eta)}{\beta} & \frac{\kappa_f\eta}{\beta} & -\frac{1}{\beta} & 0 & 0 & 0 \end{bmatrix},
$$

$$
A^{-1}B = \begin{bmatrix} 1 & 0 & 0 & -\lambda_{\pi,s} & 0 & 1 \\ 0 & 1 & 0 & 0 & -\lambda_{\pi,f} & -1 \\ 0 & 0 & 1 & 1 & -1 & 0 \\ -\frac{\Theta^2\kappa_s^2}{\beta\lambda_y} & -\frac{\Theta^2\kappa_s\kappa_f}{\beta\lambda_y} & \frac{\kappa_s(1-\eta)}{\beta} & \frac{\lambda_y+\kappa_s\lambda_y+\Theta^2\kappa_s^2\lambda_{\pi,s}-\eta\kappa_s\lambda_y}{\beta\lambda_y} & \left(\frac{\kappa_s}{\beta\lambda_y}\right)(\kappa_s\lambda_{\pi,f}\Theta^2-\lambda_y(1-\eta)) & \frac{\kappa_s\Theta^2(\kappa_f-\kappa_s)}{\beta\lambda_y} \\ -\frac{\Theta^2\kappa_s\kappa_f}{\beta\lambda_y} & -\frac{\Theta^2\kappa_f^2}{\beta\lambda_y} & -\frac{\eta\kappa_f}{\beta} & \left(\frac{\kappa_f}{\beta\lambda_y}\right)(\kappa_s\lambda_{\pi,s}\Theta^2-\eta\lambda_y) & \frac{\lambda_{\pi,f}\Theta^2\kappa_f^2+\eta\lambda_y\kappa_f+\lambda_y}{\beta\lambda_y} & \frac{\Theta^2\kappa_f^2-\kappa_s\kappa_f\Theta^2}{\beta\lambda_y} \\ \frac{\kappa_s(1-\eta)}{\beta} & -\frac{\kappa_f\eta}{\beta} & 0 & -\frac{\kappa_s(1-\eta)\lambda_{\pi,s}}{\beta} & \frac{\eta\kappa_f\lambda_{\pi,f}}{\beta} & \frac{\kappa_s(1-\eta)+\kappa_f\eta+1}{\beta} \end{bmatrix},
$$

$$
A^{-1}V = \begin{bmatrix} 0 & 0 & -1 & \frac{(1-\eta)\kappa_s}{\beta} & \frac{\eta\kappa_f}{\beta} & 0 \end{bmatrix}'.
$$

J Derivation of (25)

Recall that period t utility is $U_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}$, while its steady-state is $\bar{U} = \frac{\bar{C}^{1-\sigma}}{1-\sigma} - \frac{\bar{N}^{1+\phi}}{1+\phi}$. Then, from the equation (37) we know that

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t - \frac{1}{1-\beta}\bar{U} = -\bar{C}^{1-\sigma} \ \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t L_t,
$$

where the first term on the left-hand side is actual lifetime utility and the second term is (minus) the lifetime utility if the individual is always in the steady state. Denoting the right-hand side as $-\bar{C}^{1-\sigma}L$ and using that in the steady state $\bar{C}^{1-\sigma} = \bar{N}^{1+\phi}$, we can write

$$
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t - \mathbb{A}\bar{C}^{1-\sigma} = -\bar{C}^{1-\sigma} L, \tag{38}
$$

where $\mathbb{A} \equiv \frac{\sigma+\phi}{(1-\beta)(1-\sigma)(1+\phi)}$. Now, define c as the permanent decrease in the steady-state consumption level, such that the following equality holds

$$\begin{aligned}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t = \sum_{t=0}^{\infty} \beta^t \left[\frac{(C-c)^{1-\sigma}}{1-\sigma} - \frac{N^{1+\phi}}{1+\phi}\right] &= \frac{1}{1-\beta}\left[\frac{(C-c)^{1-\sigma}}{1-\sigma} - \frac{N^{1+\phi}}{1+\phi}\right] \\ &= \frac{1}{1-\beta}\left[\frac{C^{1-\sigma}\left(1-\frac{c}{C}\right)^{1-\sigma}}{1-\sigma} - \frac{N^{1+\phi}}{1+\phi}\right].\end{aligned}$$

Using $\bar{C}^{1-\sigma} = \bar{N}^{1+\phi}$, this equation reduces to

$$\left(1-\frac{c}{C}\right)^{1-\sigma} = (1-\sigma)\left(\frac{(1-\beta)\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t}{C^{1-\sigma}} + \frac{1}{1+\phi}\right).$$

Now, using (38) we can write

$$\left(1-\frac{c}{C}\right)^{1-\sigma} = (1-\sigma)\left(\frac{(1-\beta)(\mathbb{A}C^{1-\sigma} - C^{1-\sigma}L)}{C^{1-\sigma}} + \frac{1}{1+\phi}\right),$$

which reduces to equation (25):

$$\frac{c}{C} = 1 - \left[(1-\sigma)\left((1-\beta)(\mathbb{A}-L) + \frac{1}{1+\phi}\right)\right]^{\frac{1}{1-\sigma}}.$$