Quantum Algorithms and Quantum Entanglement
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1.1 The Emergence of a New Field

The central issue in the emerging field of quantum computation and quantum information theory is the separation of quantum effects from the domain of computation and information processing. Interest in the field has primarily been created by the idea that at the present rate of advancement of our technology, quantum and related technologies will reach their limits around the year 2010. Any information technology that allows the manipulation of quantum bits is expected to outperform classical computers. The field of quantum computation is gaining increasing interest, but the experimental realization of a quantum computer is still some time away. Research is being carried out on a variety of technologies, both practical and theoretical, to make the transition toward a quantum computer. In this field, experimental results are important but theoretical innovations are even more critical. A special technique in quantum computation is the use of quantum information for the manipulation of quantum states, which is the subject of this book. It is important to consider the relationship between these two approaches, for they are complementary.

For example, if a quantum computer is made up of a large number of qubits, it can easily perform operations that are intractable for classical computers, such as factoring large numbers. This is because a quantum computer can manipulate all possible states of a system simultaneously, a phenomenon known as superposition. The field of quantum computation is rapidly expanding, and there is much interest in building a quantum computer. The challenge is to find a way to build a quantum computer that can perform operations on quantum states efficiently, and this is known as the problem of quantum error correction.

In this book, we will discuss the fundamentals of quantum computation, including the basic operations and algorithms that are used to manipulate quantum states. We will also explore the current state of the art in building a quantum computer and discuss the challenges that must be overcome to make this technology a reality.
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