Price dynamics in equilibrium models

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Chapter 7

A Model of Political Business Cycles

7.1 Introduction

In this thesis we have studied a number of models that feature cyclic behaviour. These cycles can be perfect foresight cycles that arise due to the structure of the aggregate excess demand functions, as the cycles in the overlapping generations models studied in Chapter 3. Alternatively, they can arise from some typical learning algorithm as was the case with the cycles encountered in the models of Chapters 2, 4 and 6. Finally, we have seen that cycles might arise due to a competition between different behavioural rules as in the Cournot model in Chapter 5. All these models have in common that markets are allowed to operate freely. In reality, however, markets are often subject to regulation by governmental authorities, who try to influence the income distribution. In this chapter we introduce the possibility of price regulations by the government and study the implications for the price dynamics.

There is a close interaction between politics and economics. Political decisions influence the functioning of the economy and the performance of politicians is evaluated according to the state the economy is in. This interaction between politics and economics may result in a situation of economic inefficiency. It may even cause the emergence of so-called political business cycles, that is fluctuations in prices, employment and output arising from the competition between political parties.

In this chapter we study a simple general equilibrium model with endogenous political behaviour and show that economically inefficient equilibria and political business cycles may emerge endogenously. The mechanism driving these results differs from the mechanisms identified in the earlier literature on political business cycles.

1On the subject of economic efficiency versus political efficiency see Magee, Brock and Young (1989) who show that it may be political efficient to introduce some economic inefficiency, such as tariffs.
In voting theory it is usually assumed that political parties select platforms in a way that maximizes the probability of being elected. If there are two political parties, the policy space is one-dimensional and voting behaviour is deterministic, that is, an agent votes for the political party closest to his ideal point. This leads to the well-known median voter result: both political parties choose a platform coinciding with the ideal point of the median voter.\(^2\) Nordhaus (1975) shows that in such a model an opportunistic political business cycle might emerge. The incumbent party stimulates the economy prior to an election, for example, by trading off a higher inflation rate and a lower unemployment level, in order to be re-elected. After the election, the government lowers the high inflation rate, which leads to a reduction of employment again. This model therefore predicts an upswing of the economy just before each election and a downswing after an election. This political business cycle only emerges if agents form expectations adaptively\(^3\), or if there is some information asymmetry between the government and the voters. A number of contributions have criticized the assumption that political parties only care about winning elections (for example Alesina (1987), Alesina and Rosenthal (1995) and Wittman (1977)). Rather, it has been argued that political parties have certain preferences and different political parties execute different policies when elected.\(^4\) A “left-wing” political party may choose policies to reduce unemployment and a “right-wing” party may choose policies focusing on lowering inflation. A business cycle may arise since, if a left-wing party is elected, inflation will rise and, if a right-wing party is elected, inflation will decrease. Business cycles emerging in this way are called partisan business cycles (Alesina (1987)). A necessary condition for these partisan business cycles to exist is incomplete information about voter preferences (see Alesina and Rosenthal (1995)).

In these partisan models party preferences are given exogenously. In this chapter we study the behaviour of a political-economic model with two parties where party preferences differ because parties represent different groups in the economy. An important aspect of our model is that the sizes of these groups are determined endogenously and depend upon government policies. In particular, the size of the group might decrease when the political party representing this group is in office. This occurs because a policy that is beneficial

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2 However, if the policy space is multidimensional an equilibrium only can be shown to exist under very restrictive assumptions (Kramer (1973)).

3 Rogoff and Sibert (1988) show that these opportunistic business cycles can also arise when there is some information asymmetry between the government and the voters (this can, however, only explain cycles in macroeconomic policy and not cycles in unemployment).

4 Wittman (1977) writes: “The extensive literature concerned with formal models of political candidate strategies has almost without exception viewed policy as a means to winning.....I suggest that the reverse is true—that candidates view winning as a means to policy.”
7.2. MINIMUM WAGES FOR HIGH-SKILLED LABOUR

To an average individual member of this group may cause a number of members of this group to transfer to the other group.

As a very simple example consider an economy where agents can be divided in two groups, the "rich" and the "poor", for lack of better terminology. If the political party representing the poor is in office, it will devise policies such that the average poor agent will be better off, resulting in some poor people becoming rich, which decreases the number of poor people and increases the number of rich people. If, on the other hand, the party representing rich people is in office and if we make the realistic assumption that income for a rich agent is a decreasing function of the number of rich people, then this political party will execute policies that will at least not increase, and possibly decrease the number of rich people. In this way, an endogenous political business cycle emerges.

We study these ideas in a simple general equilibrium model with two sectors, one of which uses high-skilled labour and the other low-skilled labour, where the number of people working in each sector is determined endogenously. A political party in office can impose a minimum wage to redistribute income in favour of the group it represents. We study the existence of a political economic equilibrium. Several situations may occur: i) the Walrasian equilibrium may be a political economic equilibrium, ii) there may be a continuum of political economic equilibria, possibly including the Walrasian equilibrium and iii) a political-economic equilibrium may not exist, in which case, an endogenous political business cycle emerges.

Our model is similar to that of Herings (1997), who shows that price rigidities may emerge endogenously in a general equilibrium model through competition for votes between two political parties.

The outline of the rest of the chapter is as follows. In Section 7.2 we describe the model and show what can happen if a government can set a minimum wage for high-skilled labour. In Section 7.3 we study what happens if the government can set a minimum wage for low-skilled labour. Section 7.4 discusses the results.

7.2 Minimum wages for high-skilled labour

In this section we investigate circumstances where a government is able to impose a minimum wage on high-skilled labour. In subsection 7.2.1 we briefly describe the general equilibrium model and give a definition of the fixed price equilibria we want to study. Subsection 7.2.2 gives the structure of political institutions and derives the optimal policy functions for the two different political parties. These optimal policy functions allow a classification into different types of equilibria can be made. Finally, subsection 7.2.3
briefly considers the outcome if a government can set a maximum wage for high-skilled labour.

### 7.2.1 The economic sphere

#### The firms

There are two sectors producing the same homogenous nonstorable commodity, sector $h$ and sector $l$. Each sector consists of a large number of firms with mass 1 having the following production technologies respectively

\[ y_h = f(x), \quad y_l = g(x), \]  

(7.1)

where $y_i$ denotes production by a firm from sector $i$ and $x$ denotes labour input. There is perfect competition on the goods market. There are two types of labour, labour of type $h$ and labour of type $l$. We interpret type $h$ labour as high-skilled labour and type $l$ labour as low-skilled labour. We make the following assumptions about the production technologies $f(\cdot)$ and $g(\cdot)$.

- Firms in sector $h$ only use labour of type $h$, firms in sector $l$ can use labour of type $h$ and labour of type $l$.

- $f$ and $g$ are twice differentiable, strictly monotonically increasing and strictly concave: $f'(x) > 0$, $f''(x) < 0$, $g'(x) > 0$ and $g''(x) < 0$.

- Firms in sector $h$ are more productive than firms in sector $l$: $f(x) > g(x)$ for all $x > 0$.

Firms take prices as given and solve the following problem

\[ \max_{x} y_i - w_i x, \quad i = h, l, \]  

(7.2)

where we take the price of consumption to be equal to one and where $w_i$ is the real wage rate of labour of type $i$. This gives the following labour demand functions, consumption supply functions and profit functions

\[ x_h^d(w_h) = (f')^{-1}(w_h) = x_h(w_h), \]

\[ y_h^s(w_h) = f(x_h(w_h)), \]

\[ \pi_h(w_h) = y_h^s(w_h) - w_h x_h^d(w_h), \]  

(7.3)
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and

\[ x_i^d (w_i) = (g')^{-1} (w_i) = x_i (w_i), \]
\[ y_i^d (w_i) = g (x_i (w_i)), \]
\[ \pi_i (w_i) = y_i^d (w_i) - w_i x_i^d (w_i). \]  

(7.4)

We assume that total profits made by both sectors are equally distributed over all consumers. Since the number of consumers and the number of firms is large, practically all stockholders of the firm work in other firms. Therefore profit maximization is the proper objective for each firm.

The consumers

There are two types of consumers in this economy, those possessing labour of type \( h \) and those possessing labour of type \( l \). Each consumer is endowed with one unit of labour. There is a large number of consumers with mass 1 and the fraction of consumers endowed with labour of type \( h \) is exogenously given and equal to \( \beta \in (0, 1) \). The fraction of consumers endowed with labour of type \( l \) is \( 1 - \beta \). We assume that all labour is supplied inelastically. First consider an agent of type \( l \) (that is, an agent endowed with one unit of labour of type \( l \)). He solves

\[
\max_{\{c_l, x_l\}} c_l \quad \text{s.t.} \quad c_l \leq w_l x_l + \pi_l + \pi_h, \\
x_l \leq 1.
\]

(7.5)

This gives

\[ c_l^d = w_l + \pi_l + \pi_h, \quad x_l^d = 1. \]  

(7.6)

An agent of type \( h \) can supply labour to both sectors. Under the condition that wages in sector \( h \) are always at least as high as wages in sector \( l \), we find

\[ c_h^d = w_h + \pi_l + \pi_h, \quad x_h^d = 1. \]

Consumption of agents working in the two different sectors can be expressed as

\[ c_h (b) = f (b) + g (1 - b) + (1 - b) (f' (b) - g' (1 - b)), \]
\[ c_l (b) = f (b) + g (1 - b) + b (g' (1 - b) - f' (b)), \]  

(7.7)

where \( b \) is the fraction of agents working in sector \( h \). Since marginal production in sector \( h (l) \) is decreasing (increasing) in \( b \), a sufficient condition for an agent of type \( h \) to supply labour to sector \( h \) is

\[ f' (\beta) \geq g' (1 - \beta). \]  

(7.8)
Under this condition, wages (and therefore consumption) for agents working in sector $h$ will always be higher than wages for agents working in sector $l$.

Throughout this section, we illustrate our results with the following simple example. Production technologies are given by

$$g(x) = x - \frac{1}{2}x^2, \quad f(x) = \alpha g(x),$$

where $\alpha > 1$ measures the difference in productiveness between labour of type $h$ and labour of type $l$. Notice that these production technologies indeed satisfy the assumptions for $x < 1$. Condition (7.8) requires that $\alpha \geq \beta/(1 - \beta)$. The wage rates in the two different sectors are

$$w_h = \alpha (1 - x_h), \quad w_l = 1 - x_l$$

and the profit made by the two sectors are

$$\pi_h (x_h) = \frac{1}{2} \alpha x_h^2 \quad \text{and} \quad \pi_l (x_l) = \frac{1}{2} x_l^2.$$ 

Given that employment in sector $h$ is $b$, consumption of agents working in sector $h$ and agents working in sector $l$ then becomes

$$c_h (b) = \alpha (1 - b) + \frac{1}{2} \alpha b^2 + \frac{1}{2} (1 - b)^2 \quad \text{and} \quad c_l (b) = b + \frac{1}{2} \alpha b^2 + \frac{1}{2} (1 - b)^2.$$ 

**Fixed price equilibria**

If condition (7.8) is satisfied, all high-skilled workers supply their labour to sector $h$. Total supply of high-skilled labour is $\beta$ and total supply of low-skilled labour is $1 - \beta$.

The Walrasian equilibrium wage rates are easily seen to be

$$\hat{w}_h = f'(\beta) \quad \text{and} \quad \hat{w}_l = g'(1 - \beta). \quad (7.9)$$

If wage rates are flexible and adjusted according to market pressure, these might be the wage rates prevailing in this economy. However, in this chapter we assume that wage rates do not adjust flexibly, but are partly determined by the government. In particular, in this section we assume that the government can impose a minimum wage on high-skilled labour. Given this minimum wage rate, the wage rate for low-skilled labour adjusts competitively.

Consider a minimum wage rate for high-skilled labour above the equilibrium wage rate $\hat{w}_h$. Then labour demand of firm $h$ decreases and a number of high-skilled agents is

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5It is by now well known that price adjustment processes such as the tâtonnement process need not converge to the Walrasian equilibrium price vector (see for example Chapter 2).
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Figure 7.1: The Walrasian equilibrium \((\bar{w}_h, \bar{w}_l)\) and a fixed price equilibrium \((\bar{w}_h, \bar{w}_l)\).

Rationed on the labour market. These agents then supply their labour to firms in sector \(l\). Total supply of labour to firms in sector \(l\) then increases and the wage rate for low-skilled labour will adjust until demand equals supply. Figure 7.1 illustrates the consequences of a minimum wage for high-skilled labour.

We can now define a fixed price equilibrium as follows:

**Definition 7.1** A fixed price equilibrium \(W(\bar{w}_h)\) induced by a price regulation \(\bar{w}_h \geq \hat{w}_h\), is a six-tuple of quantities and prices

\[
W(\bar{w}_h) = \{(b, c_h, w_h), \{1 - b, c_l, w_l\}\}
\]

where the first triple corresponds to employment in sector \(h\), consumption of an agent working in sector \(h\) and the wage rate for labour of type \(h\), respectively and the second triple corresponds to employment in sector \(l\), consumption of an agent working in sector \(l\) and the wage rate for labour of type \(l\). These quantities are determined by the price regulation \(\bar{w}_h\) in the following way

\[
b = x_h(\bar{w}_h), \quad c_h = \bar{w}_h + \pi_h(\bar{w}_h) + \pi_l(\bar{w}_l), \quad \bar{w}_l = g'(1 - b), \quad c_l = \bar{w}_l + \pi_h(\bar{w}_h) + \pi_l(\bar{w}_l).
\]

6 We assume part-time work does not exist. This implies that a fraction of \(b\) high-skilled workers are not rationed and a fraction of \(\beta - b\) is completely rationed. If part-time work were available all high-skilled workers might be rationed such that they supplied \(\beta\) of their labour to sector \(h\) and \(\beta - b\) to sector \(l\). This possibility is excluded.

7 For an analysis of fixed price equilibria in general equilibrium models, see for example Benassy (1982) and Drèze (1975).
We can also express the fixed price equilibria in terms of employment in sector \( h \) as \( \mathcal{W}(b) \). The Walrasian equilibrium then corresponds to \( \mathcal{W}(\beta) \).

Due to the decreasing returns to scale technology, we have from (7.7)

\[
\frac{\partial c_h(b)}{\partial b} = (1 - b) (f''(b) + g''(1 - b)) < 0, \\
\frac{\partial q(b)}{\partial b} = -b (g''(1 - b) + f''(b)) > 0.
\]

(7.11)

This is an important feature of the model: agents working in sector \( i \) benefit from a decrease of the number of agents working in sector \( i \).

### 7.2.2 The political sphere

**Political institutions**

There are two clearly distinguishable groups in this economy, agents working in sector \( h \) and agents working in sector \( l \). The political system consists of two political parties, one representing agents working in sector \( h \) (political party \( H \)) and one representing agents working in sector \( l \) (political party \( L \)). There is an election at the beginning of each period \( t \) (\( t = 0, 1, 2, \ldots \)). Elections are decided by majority rule and if both parties receive the same number of votes the incumbent party remains in office. If political party \( H \) (\( L \)) is elected in period \( t \), it chooses a minimum wage \( \bar{w}_h \) in order to maximize average expected consumption for period \( t \) for people working in sector \( h \) (\( l \)) at time \( t - 1 \). We assume that agents and political parties have complete knowledge of the economic model. Agents vote for the political party giving them highest expected consumption in period \( t \) if elected.

The **optimal policy problem** of political party \( H \) (\( L \)), given that employment at time \( t \) in sector \( h \) is \( b_{t-1} \), is to maximize average expected consumption in period \( t \) of an agent working in sector \( h \) (\( l \)) in period \( t - 1 \), that is, it solves

\[
\max_b V_i(b, b_{t-1}), \quad i = H, L,
\]

(7.12)

where \( V_i(b, b_{t-1}) \) is the average expected consumption for period \( t \) of a consumer working in sector \( i \) in period \( t - 1 \), when employment in sector \( h \) is \( b_{t-1} \) in period \( t - 1 \) and \( b \) in period \( t \). So the political party in office in period \( t \) determines the fixed price equilibrium \( \mathcal{W}(b) \) that will obtain in period \( t \), by choosing the relevant minimum wage \( \bar{w}_h \) for high-skilled labour. On the other hand, the optimal policies of the two different political parties determine which political party will win the election at the beginning of period \( t \). We can then define a **political economic equilibrium** as follows.
Definition 7.2 A fixed price equilibrium $W(b^*)$ is a political economic equilibrium if one of the following conditions holds:

1. Given $W(b^*)$, political party $H$ is elected and

   \[ b^* = \arg \max_b V_H(b, b^*) \]

2. Given $W(b^*)$, political party $L$ is elected and

   \[ b^* = \arg \max_b V_L(b, b^*) \]

Given this definition of a political economic equilibrium, we want to know if such an equilibrium exists and if it is unique. Furthermore, we are interested in the question whether the Walrasian equilibrium $W(\beta)$ is a political economic equilibrium. We will now study the optimal policies of political party $H$ and $L$ respectively, and then study the existence of a political economic equilibrium.

Optimal policies for political party $H$

Expected consumption for an agent of type $h$ working in sector $h$ in period $t - 1$ is

\[ V_H(b, b_{t-1}) = (1 - P(b, b_{t-1})) c_h(b) + P(b, b_{t-1}) c_l(b), \]

(7.13)

where $P(b, b_{t-1})$ is the probability for an agent working in sector $h$ in period $t - 1$ to get rationed on the high-skilled labour market in period $t$. It is important to be clear about the rationing scheme here. We assume that a high-skilled agent working in sector $h$ in period $t - 1$ is never replaced by a high-skilled agent working in sector $l$ in period $t - 1$. The probability of being rationed therefore becomes

\[ P(b, b_{t-1}) = \begin{cases} \frac{b_{t-1} - b}{b_{t-1}} & b < b_{t-1} \\ 0 & b \geq b_{t-1} \end{cases} \]

(7.14)

hence expected consumption of an agent employed in sector $h$ in period $t - 1$ is

\[ V_H(b, b_{t-1}) = \begin{cases} \frac{b_{t-1} - b}{b_{t-1}} c_h(b) + \frac{b_{t-1} - b}{b_{t-1}} c_l(b) & b < b_{t-1} \\ c_h(b) & b \geq b_{t-1} \end{cases} \]

(7.15)

Notice that, for a high-skilled agent working in sector $h$ at time $t - 1$, there are two, partially offsetting, effects of a decrease in the employment level $b$. Firstly, if this agent can keep working in sector $h$, his consumption level increases. Secondly, there is a positive probability that this agent will be rationed on the market for high-skilled labour and then
he has to work in sector $l$, which will make him worse off, since wages in sector $l$ are lower than wages in sector $h$.

To be able to derive some conclusions about the optimal policies of political party $L$, we want to ensure that the function $V_H(b, b_{t-1})$ is strict concave to the left of $b_{t-1}$.

**Proposition 7.1** The function $V_H(b, b_{t-1})$ is concave in $b$ on the interval $[0, b_{t-1}]$ for every $b_{t-1}$, if for all $b \in [0, \beta]$ we have

$$2 \left( f''(b) + g''(1 - b) \right) + b \left( f'''(b) - g'''(1 - b) \right) < 0. \quad (7.16)$$

**Proof.** Let

$$H(b, b_{t-1}) = b c_h(b) + (b_{t-1} - b) c_l(b) = b_{t-1} \left( f(b) + g(1 - b) \right) + b(1 - b_{t-1}) \left( f'(b) - g'(1 - b) \right).$$

Observe that $H$ is proportional to $V_H$ for $b < b_{t-1}$ (compare (7.15)). Then we have

$$F(b, b_{t-1}) = \frac{\partial H(b, b_{t-1})}{\partial b} = \left( f'(b) - g'(1 - b) \right) + b(1 - b_{t-1}) \left( f''(b) + g''(1 - b) \right). \quad (7.17)$$

Notice that $F(b, b_{t-1}) = 0$ corresponds to the first order condition for an optimum. The second order derivative of $H(b, b_{t-1})$ with respect to $b$ is

$$\frac{\partial^2 H(b, b_{t-1})}{\partial b^2} = (2 - b_{t-1}) \left( f''(b) + g''(1 - b) \right) + b(1 - b_{t-1}) \left( f'''(b) - g'''(1 - b) \right)$$

and since $(2 - b_{t-1})/(1 - b_{t-1}) \geq 2$ a sufficient condition for strict concavity of $H$ and therefore for strict concavity of $V_H$ for $b \leq b_{t-1}$ is (7.16). \[\blacksquare\]

Condition (7.16) is a technical condition without a straightforward economic interpretation. However, it is satisfied for a number of standard production technologies, such as the quadratic ones in our examples and production technologies of the form $h(l) = Al^\gamma$, with $A > 0$ and $0 < \gamma < 1$.

Let the solution to the optimal policy problem be described by the following optimal policy function

$$\Phi_H(b_{t-1}) = \arg \max_b V_H(b, b_{t-1}). \quad (7.18)$$

Since by (7.11) and (7.15) $V_H(b, b_{t-1})$ is downward sloping to the right of $b_{t-1}$ we know that $\Phi_H(b_{t-1}) \leq b_{t-1}$, so when party $H$ is in office and finds itself not at a steady state of $\Phi_H$ it will lower employment in sector $h$ and approach a steady state of $\Phi_H$. We therefore want to find the steady states of the function $\Phi_H$. The first question we want to answer is whether the Walrasian equilibrium is a steady state or not.
Proposition 7.2 The Walrasian equilibrium \( \beta \) is not a steady state of \( \Phi_H (b) \) if and only if

\[
\bar{c}_h - \bar{c}_l < -\beta \frac{\partial \bar{c}_h}{\partial b} \quad (7.19)
\]

Proof. The Walrasian equilibrium is not a steady state of \( \Phi_H \) if and only if

\[
F(\beta, \beta) = (f'(\beta) - g'(1 - \beta)) + \beta (1 - \beta) (f''(\beta) + g''(1 - \beta)) < 0 \quad (7.20)
\]

since then expected consumption for agents working in sector \( h \) can be increased by a decrease in employment in sector \( h \). Now by (7.11) and \( f'(\beta) - g'(1 - \beta) = \bar{w}_h - \bar{w}_l = \bar{c}_h - \bar{c}_l \), condition (7.20) is equivalent with condition (7.19). \( \blacksquare \)

If the optimal policy function is concave to the left of \( b_{t-1} \), we can say more about the steady states of \( \Phi_H \).

Proposition 7.3 Suppose condition (7.16) is satisfied. Then \( \Phi_H \) has a continuum of steady states, of the form \( B_H = \{ b \mid b \in [0, b^*] \} \), where \( b^* = \min \{ \beta, b^0 \} \) and \( b^0 \) is implicitly determined by

\[
c_h(b) - c_l(b) = -b \frac{\partial c_h(b)}{\partial b}. \quad (7.21)
\]

Furthermore \( \frac{\partial \Phi_H}{\partial b_{t-1}} > 0 \).

Proof. \( b^* \) is a steady state of \( \Phi_H \) if it is the global maximum of \( H(b, b^*) \). Due to the concavity of \( H(., b^*) \), \( b^* \) is the global maximum of \( H \) if \( J(b^*) = F'(b^*, b^*) \geq 0 \). We therefore need

\[
J(b^*) = (f'(b^*) - g'(1 - b^*)) + b^*(1 - b^*) (f''(b^*) + g''(1 - b^*)) \geq 0.
\]

This can be rewritten as condition (7.21). Furthermore, we have \( J(0) = F'(0) - g'(1) > 0 \) and

\[
J'(b) = 2(1 - b) (f''(b) + g''(1 - b)) + b(1 - b) (f'''(b) + g'''(1 - b)) \leq 0
\]

where the inequality follows from condition (7.16). Hence \( J(b) \) has a unique zero between 0 and \( \beta \), and if \( J(b^*) \geq 0 \), then \( J(b) \geq 0 \) for all \( b \leq b^* \). Finally to proof that \( \Phi_H \) is upward sloping on \( [b^*, \beta] \) notice that

\[
\frac{\partial^2 H(b, b_{t-1})}{\partial b_{t-1} \partial b} = -b(f''(b) + g''(1 - b)) > 0 \quad \text{and} \quad \frac{\partial^2 H(b, b_{t-1})}{\partial b^2} \leq 0
\]
so an increase in $b_{t-1}$ has to be accompanied by an increase in $b$ to keep $\frac{\partial H(b, b_{t-1})}{\partial b}$ equal to 0.

The interpretation of condition (7.21) is straightforward. The left-hand side gives the loss in utility resulting from a lay-off in the high-skill sector, the right-hand side gives a measure of the increase in expected consumption. Furthermore, notice that there always is a continuum of steady states, which may or may not include the Walrasian equilibrium. Figure 7.2 gives a picture of the expected consumption function $V_H$ and the corresponding optimal policy function $\Phi_H$.

![Figure 7.2: Expected consumption for an agent working in sector $h$ and the optimal policy function for political party $H$.](image)

For our example, we find that the optimization problem for political party $H$ can be written as

$$\max_b V_H(b, b_{t-1}) = \max_b \frac{1}{b_{t-1}} \left[ \frac{1}{2} b_{t-1} + \alpha b - \left( 1 - \frac{1}{2} b_{t-1} \right) (1 + \alpha) b^2 \right],$$

which is clearly a concave function. We then obtain

$$\Phi_H(b_{t-1}) = \arg\max_b V_H(b, b_{t-1}) = \min \left\{ b_{t-1}, \frac{\alpha}{1 + \alpha \cdot 2 - b_{t-1}} \right\}.$$

The steady states of this optimal policy function are $b \in [0, b^*]$, where

$$b^* = \min \left\{ 1 - \sqrt{\frac{1}{1 + \alpha}}, \beta \right\}.$$
Optimal policies for political party $L$

There can be two types of agents working in sector $l$ at time $t-1$, agents of type $l$ and agents of type $h$ who are rationed on the market for labour of type $h$. Expected consumption of an agent of type $l$, given that employment in sector $h$ will be $b$, is

$$W_l(b, b_{t-1}) = c_l(b).$$  \hfill (7.22)

Expected utility for an agent of type $h$ working in sector $l$ at time $t-1$ is

$$W_h(b, b_{t-1}) = Q(b, b_{t-1}) c_h(b) + (1 - Q(b, b_{t-1})) c_l(b)$$  \hfill (7.23)

$$= \frac{b - b_{t-1}}{\beta - b_{t-1}} c_h(b) + \frac{\beta - b}{\beta - b_{t-1}} c_l(b), \quad \text{if } b > b_{t-1},$$

where $Q(b, b_{t-1})$ is the probability for an agent of type $h$ working in sector $l$ in period $t-1$ to find employment in sector $h$ in period $t$. The fraction of agents of type $l$ in sector $l$ in period $t-1$ is $\frac{1 - \beta}{1 - b_{t-1}}$ and the fraction of agents of type $h$ working in sector $l$ in period $t-1$ is $\frac{\beta - b_{t-1}}{1 - b_{t-1}}$. Average expected utility then is

$$V_L(b, b_{t-1}) = \frac{1 - \beta}{1 - b_{t-1}} W_l(b, b_{t-1}) + \frac{\beta - b_{t-1}}{1 - b_{t-1}} W_h(b, b_{t-1})$$  \hfill (7.24)

$$= \begin{cases} \frac{b - b_{t-1}}{1 - b_{t-1}} c_h(b) + \frac{1 - b}{1 - b_{t-1}} c_l(b) & b \geq b_{t-1}, \\ c_l(b) & b < b_{t-1}. \end{cases}$$

Given $b_{t-1}$, political party $L$ chooses $b$ to maximize $V_L(b, b_{t-1})$.

**Proposition 7.4** We have

$$\Phi_L(b_{t-1}) = \arg \max_b V_L(b, b_{t-1}) = \beta,$$  \hfill (7.25)

for all $b_{t-1} \in [0, \beta]$.

**Proof.** First notice that $\arg \max_b V_L(b, b_{t-1}) \geq b_{t-1}$ since $V_L(b, b_{t-1})$ is a monotonically increasing function on the interval $[0, b_{t-1})$. So we have to focus on the interval $[b_{t-1}, \beta]$. On this interval we have

$$V_L(b, b_{t-1}) = \frac{b - b_{t-1}}{1 - b_{t-1}} c_h(b) + \frac{1 - b}{1 - b_{t-1}} c_l(b)$$

$$= (f(b) + g(1-b)) - (1-b) \frac{b_{t-1}}{1 - b_{t-1}} (f'(b) - g'(1-b))$$

and we have

$$\frac{\partial V_L(b, b_{t-1})}{\partial b} = \frac{1}{1 - b_{t-1}} (f'(b) - g'(1-b)) - (1-b) \frac{b_{t-1}}{1 - b_{t-1}} (f''(b) + g''(1-b)) > 0$$
by \( f'(b) - g'(1 - b) = c_h(b) - c_l(b) > 0 \) and decreasing returns to scale of \( f \) and \( g \). Clearly \( V_L(b, b_{t-1}) \) is monotonically increasing from 0 to \( \beta \) and hence political party \( L \) will always return immediately to the Walrasian equilibrium \( \beta \). 

The reason that the behaviour of political party \( L \) differs so much from the behaviour of political party \( H \) lies in the fact that, for political party \( H \) a decrease in the employment in the high-skill sector has two opposing effects. Agents still employed in sector \( h \) are better off, but agents who transfer to firm \( I \) are worse off. Party \( L \) always wants to increase employment in sector \( h \) since all agents presently working in sector \( l \) benefit from this.

### Political economic equilibria and political business cycles

In the previous two subsections, we determined the policies the two political parties choose when elected. Now we close our political-economical system by determining the voting behaviour of agents. Assuming that agents vote for the political party that will present their interest in the best way, the following result follows intuitively.

**Proposition 7.5** It is an optimal strategy for an agent working in sector \( h \) (\( l \)) in period \( t - 1 \) to vote for political party \( H \) (\( L \)).

**Proof.** First consider agents working in sector \( h \) in period \( t - 1 \). Since party \( H \) maximizes expected consumption of an agent working in sector \( h \) and all agents working in sector \( h \) are the same, by definition each agent working in sector \( h \) is at least as good off with party \( H \) as with party \( L \). Now consider agents working in sector \( l \) in period \( t - 1 \). Consumption for an agent working in sector \( l \) becomes \( c_l \) or \( c_l \) if political party \( L \) is elected and since \( \min \{ \hat{c}_h, \hat{c}_l \} = \hat{c}_l \geq c_l(b_{t-1}) \geq c_l(\Phi_H(b_{t-1})) \) an agent working in sector \( l \) in period \( t - 1 \) is at least as well off with party \( L \) as with party \( H \).

This leads to the following characterization of aggregate voting behaviour:

If \( b_{t-1} > \frac{1}{2} \), party \( H \) will be in office in period \( t \), if \( b_{t-1} < \frac{1}{2} \) party \( L \) will be in office in period \( t \), and if \( b_{t-1} = \frac{1}{2} \), the incumbent party will remain in office.

Our model is now completely determined and we can study the existence of political economic equilibria. Government policies are given by

\[
b_t = \Psi(b_{t-1}) = \begin{cases} 
\Phi_H(b_{t-1}) & \text{if } b_{t-1} > \frac{1}{2} \\
\beta & \text{if } b_{t-1} < \frac{1}{2}, 
\end{cases}
\]

(7.26)

where we ignore the case \( b_{t-1} = \frac{1}{2} \) for the moment. Political-economic equilibria of our model correspond to steady states of (7.26). The most interesting case arises when \( \beta > \frac{1}{2} \).
The behaviour of the economy then is determined by the iterations of the Walrasian equilibrium \( \beta \) under \( \Phi_H : \Phi_H(\beta), \Phi_H^2(\beta) = \Phi_H(\Phi_H(\beta)) \), etc. Clearly this is a decreasing sequence. Starting at the Walrasian equilibrium and given that \( \beta \geq \frac{1}{2} \), we have three possibilities

1. \( \Phi_H(\beta) = \beta \)

2. \( \lim_{k \to \infty} \Phi_H^k(\beta) = b^* \geq \frac{1}{2} \)

3. \( \exists k^0 \geq 1 \) such that \( \Phi_H^{k^0-1}(\beta) \geq \frac{1}{2} \text{ and } \Phi_H^{k^0}(\beta) < \frac{1}{2} \).

In the first case we find that the Walrasian equilibrium is a political-economic equilibrium. In the second case the Walrasian equilibrium is not a political-economic equilibrium but there is another equilibrium. In the third case we find that there is a political business cycle of the form

\[
\{\beta, \Phi_H(\beta), \Phi_H^2(\beta), \ldots, \Phi_H^{k^0}(\beta)\}.
\]

If we make the assumption that \( V_H(b, b_{t-1}) \) is a concave function in \( b \) on \([0, b_{t-1}]\) we can derive more explicit properties of the political economic system. An important role is played by the number \( b^* \) from proposition 7.3.

**Theorem 7.1** Assume condition (7.16) holds. For \( \beta \leq \frac{1}{2} \), the Walrasian equilibrium \( W(\beta) \) is the unique political-economic equilibrium. If \( \beta > \frac{1}{2} \) we can distinguish two cases. If \( b^* \geq \frac{1}{2} \) we have a continuum of equilibria of the form \( \{W(b) \mid b \in \left[\frac{1}{2}, b^*\right]\} \), the Walrasian equilibrium belongs to this continuum if and only if \( b^* = \beta \). If \( b^* < \frac{1}{2} \) no political-economic equilibrium exists. In this case a political business cycle exists.

**Proof.** First consider \( \beta < \frac{1}{2} \). Then since \( b_t \leq \beta < \frac{1}{2} \), \( b_t \) evolves according to \( b_t = \Phi_L(b_{t-1}) = \beta \) and therefore \( W(\beta) \) is the unique political-economic equilibrium. Now consider \( \beta = \frac{1}{2} \). If party \( L \) is in office once, it will be in office forever and \( b_t \) will be equal to \( \frac{1}{2} \) forever. If party \( H \) is in office (implying \( b = \beta = \frac{1}{2} \) and \( \Phi_H(\frac{1}{2}) < \frac{1}{2} \) party \( L \) will be elected in the next period. If party \( H \) is in office and \( \Phi_H(\frac{1}{2}) = \frac{1}{2} \), it will be in office forever and stay at the Walrasian equilibrium. This concludes the case \( \beta \leq \frac{1}{2} \). Now consider the case \( \beta > \frac{1}{2} \). First take \( b^* \geq \frac{1}{2} \). Political party \( H \) will be in office at a certain time \( t_0 \) and employment will then be \( b_0 \geq \frac{1}{2} \). If \( b_0 \leq b^* \) then \( b_0 \) will be an equilibrium since \( \Phi_H(b_0) = b_0 \). If \( b_0 > b^* \) then \( b_t \) will converge to \( b^* \). Now consider the case \( b^* < \frac{1}{2} \). In this case \( b_t \) will fall below \( \frac{1}{2} \) at a certain time \( t_1 \). At time \( t_1 + 1 \) government \( L \) is elected and \( b_t \) goes directly to \( \beta \), after which government \( H \) is elected again and \( b_t \) decreases. \( b_t \) will never converge and a political business cycle emerges.
Figure 7.3 gives an example of such a business cycle. For our example we have $b^* = \min \left\{ 1 - \sqrt{\frac{1}{1+\alpha}}, \beta \right\}$. So for $\beta \leq 1 - \sqrt{\frac{1}{1+\alpha}}$ we have a continuum of steady states of $\Phi_H$, including the Walrasian equilibrium. Furthermore $b^* \geq \frac{1}{2}$ if $\alpha \geq 3$ and $b^* < \frac{1}{2}$ if $\alpha < 3$. Hence we can conclude the following:

1. If $\beta \leq \frac{1}{2}$, the Walrasian equilibrium is the unique political-economic equilibrium,

2. if $\frac{1}{2} < \beta < 1 - \sqrt{\frac{1}{1+\alpha}}$ there is a continuum of political-economic equilibria including the Walrasian equilibrium,

3. if $\beta > 1 - \sqrt{\frac{1}{1+\alpha}}$ and $\alpha \geq 3$, there is a continuum of political-economic equilibria not including the Walrasian equilibrium, and finally,

4. if $\beta > \frac{1}{2}$ and $\alpha < 3$, there exists a political business cycle and no political-economic equilibria.

Figure 7.4 shows the different areas in $(\alpha, \beta)$ space for which the different regimes occur for this example. Notice that the existence of political business cycles does not require extreme values of $\alpha$ and $\beta$.

**7.2.3 An extension to maximum wages for high-skilled labour**

In this section we have considered a government setting a minimum wage for high-skilled labour. We will now briefly discuss what happens if we also allow the government to set
7.2. MINIMUM WAGES FOR HIGH-SKILLED LABOUR

Figure 7.4: Different political economic regimes for the example where the government can set a minimum wage for high-skilled labour. 1: Walrasian equilibrium is the unique political economic equilibrium. 2: Continuum of political economic equilibria, including the Walrasian equilibrium. 3: Continuum of political economic equilibria, not including the Walrasian equilibrium. 4: Existence of political business cycles.

...maximum wage for high-skilled labour. Consider the implications of such a restriction on the high-skilled labour wage below the Walrasian equilibrium wage $\tilde{w}_h$. As long as this maximum wage rate is higher than the wage for low-skilled labour, all high-skilled workers will supply their labour to sector $h$. Firms from sector $h$ will demand more labour than is available and therefore will be rationed on the high-skilled labour market. Sector $h$ then uses the same amount of labour as before and total production remains the same. However, the division of the revenues from production into wage and profit income changes. In particular, part of the wage income of the high-skilled workers is transferred as profit income to the low-skilled workers. This provides an incentive for political party $L$ to impose a maximum wage on high-skilled labour when it is in office. In fact, consumption for low-skilled workers is

$$c_l = w_l + \pi_h (w_h) + \pi_l (w_l) = f(\beta) - w_h \beta + g(1 - \beta) + \beta g'(1 - \beta),$$

which clearly increases with a decrease in $w_h$. The wage for high-skilled labour will never be decreased below $\tilde{w}_l = g'(1 - \beta)$. This would attract high-skilled workers to sector $l$ and lead to a decrease in the wage for low-skilled labour (as well as a decrease in total production and aggregate profits). It is therefore optimal for political party $L$ to
set \( w_h \) equal to \( \hat{w}_h = g'(1 - \beta) \), leading to the same consumption level for all workers
\( (c_l = c_h = f(\beta) + g(1 - \beta)) \). Notice that this allocation is Pareto efficient and the maximum wage on high-skilled labour in fact corresponds to a nondistortionary lump-sum tax transferring income from high-skilled to low-skilled workers.

How would the dynamics of the political-economic system be affected if the government could set a minimum or a maximum wage for high-skilled labour? Clearly, for political party \( H \) the optimal policy is unchanged. If \( \beta < \frac{1}{2} \) political party \( L \) will always be in office and will equalize wages across sectors. The Walrasian equilibrium then no longer corresponds to a political economic equilibrium. If \( \beta > \frac{1}{2} \) there are two possibilities. If a continuum of political-economic equilibria exists \( (\beta^* > \frac{1}{2}) \), its structure remains the same as before. If no political-economic equilibrium exists \( (\beta^* < \frac{1}{2}) \) a political business cycle emerges again. This political business cycle differs from the one discussed above, since in periods in which party \( L \) is in office, it will have an incentive to equalize wages across sectors.

7.3 Minimum wages for low-skilled labour

In the previous section, the government was able to influence the allocation of income by setting a minimum wage for high-skilled labour. Minimum wages for low-skilled labour, however, seem to be of more empirical relevance. This case is analyzed in this section and we will see that similar phenomena occur. Since the analysis is very similar to that of the previous section, we will skip most of the details.

7.3.1 The economic sphere

We saw that rationing high-skilled workers on the high-skill labour market leads a number of high-skilled workers to seek employment in sector \( l \). An important difference from the model presented in this section is that here low-skilled workers who are rationed on the market for low-skilled labour become unemployed since they are, by assumption, not suited for working in sector \( h \). This introduces a third class of agents, unemployed low-skilled workers. Furthermore, it follows that in any period all high-skilled workers will be working in the high-skilled sector. A priori, minimum wages in sector \( l \) may be increased beyond the wage for high-skilled labour, but this would have to be accompanied by a decrease in employment in the low-skill sector. Hence, even if high-skilled workers would want to work in the low-skill sector, there would be excess supply and they would be rationed on this market (assuming that the rationing scheme is such that no low-skilled
worker is replaced by a high-skilled worker). Employment in the high-skill sector therefore will always be $\beta$ with the corresponding wage rate $\hat{w}_h = f'(\beta)$.

Let $d \in [0, 1 - \beta]$ be employment in sector 1. The government can manipulate $d$ by imposing a minimum wage $\bar{w}_l$ for low-skilled labour, equal to $\bar{w}_l = g'(d)$. We can then define a fixed price equilibrium induced by such a minimum wage $\bar{w}_l$ by the following eight-tuple of prices and quantities

$$\mathcal{W}(\bar{w}_l) = \{\beta, \bar{c}_h, \bar{w}_h\}, \{d, \bar{c}_l, \bar{w}_l\}, \{1 - \beta - d, \bar{c}_u\},$$

where

$$c_h = w_h + \pi_h(w_h) + \pi_l(w_l) = f(\beta) + g(d) + (1 - \beta) f'(\beta) - dg'(d),$$

$$c_l = w_l + \pi_h(w_h) + \pi_l(w_l) = f(\beta) + g(d) + (1 - d) g'(d) - \beta f'(\beta) - dg'(d) + (\beta - d) g'(d),$$

$$c_u = \pi_h(w_h) + \pi_l(w_l) = f(\beta) + g(d) - \beta f'(\beta) - dg'(d).$$

Notice that aggregate profits are once more redistributed over all agents, including those that are unemployed.

**7.3.2 The political sphere**

A political party can use the minimum wage for low-skilled labour in order to arrive at a fixed price equilibrium that is beneficial to the people it represents. First consider political party $H$, representing people working in sector $h$. As observed above employment and the wage rate in sector $h$ are the same for any value of $d$. All agents working in this sector in period $t - 1$ will also be working in this sector in period $t$. Consumption of an agent working in sector $h$ will therefore be

$$V_H(d, d_{t-1}) = c_l = f(\beta) + g(d) + (1 - \beta) f'(\beta) - dg'(d).$$

Notice that the above expression does not depend upon $d_{t-1}$. We have $\partial V_H/d_{t-1} = -dg''(d) > 0$ due to the concavity of $g(.)$. Therefore political party $H$ always wants to maximize employment in the low-skill sector, that is, the optimal policy function is

$$\Phi_H(d_{t-1}) = 1 - \beta.$$

The intuition is clear: since profits of sector $l$ are an increasing function of employment in that sector, people working in sector $h$ benefit from an increase in employment in sector $l$ through higher profit income.
CHAPTER 7. A MODEL OF POLITICAL BUSINESS CYCLES

The optimal policy problem of the party representing low-skilled workers, political party $L$, is more complicated. Expected consumption for an agent working in sector $I$ in period $t - 1$, given that employment in period $t$ will be $d$ is

$$
V_L(d, d_{t-1}) = \begin{cases} 
\frac{d}{d_{t-1}} c_l(d) + \frac{d_{t-1} - d}{d_{t-1}} c_u(d) & d \leq d_{t-1} \\
c_l(d) & d > d_{t-1},
\end{cases}
$$

where $\frac{d}{d_{t-1}}$ is the probability of being employed in sector $I$, if employment at time $t - 1$ is $d_{t-1}$ and employment at time $t$ is $d \leq d_{t-1}$, where again it is assumed that no agent working in sector $I$ is replaced by an unemployed low-skilled worker. The following result describes the structure of the optimal policy function of political party $L$.

**Proposition 7.6** Assume that

$$2g''(d) + dg'''(d) < 0 \quad (7.28)$$

holds for all $d \in [0, 1 - \beta]$. Then there is a unique solution $\Phi_L(d_{t-1})$ to the problem of maximizing $V_L(d, d_{t-1})$. Furthermore $\Phi_L(d_{t-1})$ is increasing in $d_{t-1}$ and $\Phi_L(d_{t-1})$ has a continuum of steady states of the form $D^L = \{d|d \in [0, d^*]\}$, where $d^* = \min\{1 - \beta, d^0\}$ and $d^0$ is implicitly defined by

$$c_l - c_u = -d \frac{\partial c_l}{\partial d}. \quad (7.29)$$

**Proof.** First observe that $\frac{\partial c_l}{\partial d} = (1 - d) g''(d) < 0$. Therefore political party $L$ will never increase employment in sector $I$. We therefore concentrate on $d \leq d_{t-1}$. Let

$$L(d, d_{t-1}) = dc_l(d) + (d_{t-1} - d) c_u = d_{t-1} (f(\beta) - \beta f'(\beta) + g(d) - dg'(d)) + dg'(d).$$

The first order condition for an optimum becomes

$$\frac{\partial L(d, d_{t-1})}{\partial d} = g'(d) + (1 - d_{t-1}) dg''(d) = 0.$$ 

The second order derivative is

$$\frac{\partial^2 L(d, d_{t-1})}{\partial d^2} = (2 - d_{t-1}) g''(d) + (1 - d_{t-1}) g'''(d).$$

Since $(2 - d_{t-1}) / (1 - d_{t-1}) \geq 2$, a sufficient condition for the strict concavity of $L(d, d_{t-1})$ in $d$ is $(7.28)$. Hence, $\tilde{\Phi}_L(d_{t-1}) = \arg\max L(d, d_{t-1})$ exists and is unique. Furthermore we have $\frac{\partial L(d, d_{t-1})}{\partial d_{t-1} \partial d} = -dg''(d) > 0$, which implies that $\tilde{\Phi}_L(d_{t-1})$ is upward sloping. This
implies that $\Phi_L (d_{t-1}) = \min \{d_{t-1}, \tilde{\Phi}_L (d_{t-1})\} = \arg\max V_H (d, d_{t-1})$ exists, is unique and is upward sloping.

Now consider the set of fixed points of $\Phi_L (d_{t-1})$. Let $d^0$ be a fixed point of $\tilde{\Phi}_L (d_{t-1})$, then $d^0$ is implicitly defined as the solution to
\[
g'(d) + (1 - d) g''(d) = 0.
\]
Since by (7.27) we have $g'(d) = w_t = c_t - c_u$ and $\frac{\partial g}{\partial d} = (1 - d) g''(d)$, this condition is equivalent to condition (7.29). Furthermore we have
\[
\frac{\partial \tilde{\Phi}_L (d_{t-1})}{\partial d_{t-1}} = - \frac{\partial^2 L (d, d_{t-1}) / \partial d_{t-1} \partial d}{\partial^2 L (d, d_{t-1}) / \partial d^2} = \frac{dg''(d)}{(2 - d_{t-1}) g''(d) + (1 - d_{t-1}) g'''(d)},
\]
and this expression is smaller than 1 in any fixed point of $\tilde{\Phi}_L$ (unless $d = d_{t-1} = 1$), by virtue of condition (7.28). This implies that $d^0$ is unique and $\tilde{\Phi}_L (d_{t-1})$ lies below $d_{t-1}$ to the right of $d^0$ and lies above $d_{t-1}$ to the left of $d^0$. Therefore $\Phi_L (d_{t-1}) = \min \{d_{t-1}, \tilde{\Phi}_L (d_{t-1})\}$ has a continuum of fixed points of the form $D^L = \{d | d \in [0, d^*]\}$ where $d^* = \min \{1 - \beta, d^0\}$.

Notice that (7.29) has an interpretation similar to (7.21). The left-hand side corresponds to the loss in consumption for a low-skilled worker who becomes unemployed, the right-hand side corresponds to the expected marginal increase in utility corresponding to a decrease in employment in sector $l$. From proposition 7.6 it follows that the form of $\Phi_L (d_{t-1})$ is similar to the form of $\Phi_H (b_{t-1})$ as shown in Figure 7.2. We can now turn to the political-economic system. It can easily be seen that, for the low-skilled agents working in sector $l$, it is optimal to vote for political party $L$ and for the high-skilled workers and unemployed low-skilled workers it is optimal to vote for political party $H$.

Similar to the approach in the previous section, we can then show that the evolution of employment in the sector $l$ can be described by
\[
d_t = \Psi (d_{t-1}) = \begin{cases} 
\Phi_L (d_{t-1}) & \text{if } d_{t-1} > \frac{1}{2} \\
1 - \beta & \text{if } d_{t-1} \leq \frac{1}{2}.
\end{cases}
\]
Political-economic equilibria again correspond to fixed points of this mapping. As in the previous sections, the Walrasian equilibrium can be the unique political-economic equilibrium (if $\beta > \frac{1}{2}$), there can be a continuum of equilibria that may or may not include the Walrasian equilibrium and we may have political business cycles. Our main result then becomes

**Theorem 7.2** Assume condition (7.28) holds. For $\beta \geq \frac{1}{2}$, the Walrasian equilibrium $W(\beta)$ is the unique political economic equilibrium. If $\beta < \frac{1}{2}$ we can distinguish between
two cases. If \( d^* > \frac{1}{2} \) we have a continuum of equilibria of the form \( \{ W(d) \mid d \in \left[ \frac{1}{2}, d^* \right] \} \), the Walrasian equilibrium belongs to this continuum if and only if \( d^* = 1 - \beta \). If \( d^* < \frac{1}{2} \) no political-economic equilibrium exists. In this case a political business cycle exists.

Notice that we would have obtained precisely the same results had we replaced political party \( H \) by a political party that represents the unemployed, or one that would have as its main objective reduced unemployment. The model presented in this section shows that unemployment can arise as an equilibrium phenomenon in a model where political behaviour is modelled explicitly.

We again illustrate our main results by a simple numerical example. Consider the following specification of the production function of sector \( I \):

\[
g(x) = \begin{cases} 
\frac{1}{2\gamma} x^2 & x < \gamma \\
\frac{1}{2\gamma} & x \geq \gamma 
\end{cases}
\]

where \( \gamma \in (1 - \beta, 1] \). Notice that this production function does not everywhere satisfy the assumptions we imposed on production functions but does satisfy them in the relevant part since \( 1 - \beta < \gamma \). Assume furthermore that \( f \) does satisfy the assumptions of the model. The optimal policy for political party \( L \) can then be obtained as

\[
\Phi_L(d_{t-1}) = \min \left\{ d_{t-1}, \frac{\gamma}{2 - d_{t-1}} \right\}
\]

It can be easily be checked that this is monotonically increasing in \( d_{t-1} \) and that the fixed point is given by

\[
d^* = 1 - \sqrt{1 - \gamma}.
\]

Now for \( \gamma < \frac{3}{4} \), \( d^* \) will be below \( \frac{1}{2} \) and for \( \beta > \sqrt{1 - \gamma} \) we will have \( d^* < \beta \). We can again identify four different regimes. They are (remember that for all these regimes we assume \( \beta > 1 - \gamma \)):

1. If \( \beta \geq \frac{1}{2} \) the Walrasian equilibrium is the unique political-economic equilibrium,
2. if \( \sqrt{1 - \gamma} < \beta < \frac{1}{2}, \) there is a continuum of political-economic equilibria, including the Walrasian equilibrium,
3. if \( 1 - \gamma < \beta < \sqrt{1 - \gamma} \) and \( \gamma \geq \frac{3}{4} \) there is a continuum of political-economic equilibria, not including the Walrasian equilibrium, and
4. if \( 1 - \gamma < \beta < \frac{1}{2} \) and \( \gamma < \frac{3}{4} \), no political equilibrium exists and political business cycles will occur.

Figure 7.5 shows these different regimes. Notice the similarity with Figure 7.4.
Figure 7.5: Different political economic regimes for the example where the government can set a minimum wage for low skilled labour. 1: Walrasian equilibrium is the unique political economic equilibrium. 2: Continuum of political economic equilibria, including the Walrasian equilibrium. 3: Continuum of political economic equilibria, not including the Walrasian equilibrium. 4: Existence of political business cycles.

7.4 Discussion

In this chapter we have studied a simple general equilibrium model with two groups of agents, each represented by a distinct political party. We have shown that the Walrasian equilibrium might be the only political equilibrium. However, there might also be a continuum of political-economic equilibria that do not necessarily include the Walrasian equilibrium. If no political-equilibrium exists a political business cycle emerges. The interaction between politics and economics therefore can result in economically inefficient allocations of consumption and labour. The mechanism driving the results is that, for at least one of the parties choosing a policy that is beneficial to an “average” member of its platform the size of its constituency is decreased. In alternative models of political business cycles, it is assumed that political parties choose policies in order to maximize the number of votes (as is the case with opportunistic political business cycles as discussed in Nordhaus (1975) and Rogoff and Sibert (1989)) or that political parties themselves have preferences about policies (as is the case with partisan political business cycles, discussed for example in Alesina (1988) and Alesina and Rosenthal (1995)). An important characteristic of our model is that government behaviour is endogenized. This is driven
by the preferences of economic agents and not by the (exogenously given) preferences of "politicians", as in the opportunistic and partisan models.

Of course, the general equilibrium model that is used in this chapter has been very simple. We believe, however, that the mechanism driving the results remains valid in more complicated models. Notice that agents and parties in our framework maximize (expected) consumption and have complete knowledge of the political-economic system. The possibility of political business cycles is therefore not due to incomplete information of voters, as is the case with alternative models of political business cycles. However, political parties and agents are myopic, in the sense that they are only interested in the current period's expected consumption level, rather than maximizing lifetime utility.

Does this model help us to explain empirical facts? Casual observation suggests that political parties influence the economy and that, as a consequence, the platforms for political parties change over time. Our model explains how platforms depend on the policies executed by political parties without needing to resort to some kind of incomplete information of voters. Minimum wages and the resulting unemployment are features of most contemporary democratic economies. This chapter shows that minimum wages and unemployment can be an equilibrium phenomenon if we take the political system into account. With respect to the existence of political business cycles empirical evidence has been provided by, amongst others, Alesina and Roubini (1992). They find evidence that supports the conjecture that for most OECD countries political business cycles exist and that these correspond to partisan business cycles. Another interesting model with predictions similar to ours is that of Alesina and Rosenthal (1995). They develop a partisan model with two political parties. In their model partisan business cycles can emerge because voters want to moderate the government, that is, most voters prefer government behaviour that lies somewhere between the optimal policies of the political parties. In this way a political business cycle can emerge. Alesina and Rosenthal (1995) provide empirical evidence that supports this conjecture. A similar idea lies behind the political business cycles model presented in this chapter. Here a political business cycle emerged because the political party that is in office alienates some of its voters with its policy. Our model predicts that the number of agents voting for the political party that is not in office is nondecreasing. Again, this seems to be in accordance with casual empirical observations.