History of dark matter

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(published 15 October 2018)

Although dark matter is a central element of modern cosmology, the history of how it became accepted as part of the dominant paradigm is often ignored or condensed into an anecdotal account focused around the work of a few pioneering scientists. The aim of this review is to provide a broader historical perspective on the observational discoveries and the theoretical arguments that led the scientific community to adopt dark matter as an essential part of the standard cosmological model.

DOI: 10.1103/RevModPhys.90.045002

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I. PREFACE

Dark matter plays a central role in our understanding of modern cosmology. But despite its significance, many of the scientists active in this area of research know relatively little about its interesting history and how it came to be accepted as the standard explanation for a wide variety of astrophysical observations. Most publications and presentations on this topic—whether at a technical or a popular level—either ignore the long history of this field or condense it into a brief anecdotal account, typically centered around the work on galaxy clusters by Fritz Zwicky in the 1930s and on galactic rotation curves by Vera Rubin in the 1970s. Only a small number of scientists, and an even smaller number of historians, have endeavored to systematically analyze the development of the dark matter problem from an historical perspective, and it is surprisingly hard to find articles and books that do justice to the fascinating history of dark matter.

The aim of this article is to provide a review of the theoretical arguments and observations that led to the establishment of dark matter among the pillars of modern cosmology, as well as of the theories that have been proposed to explain its nature. Although we briefly discuss some early ideas and recent developments, the focus of this review is the 20th century, beginning with the first dynamical estimates of dark matter’s abundance in the Universe, and to its role in the current standard cosmological model, and the strategies that have been pursued to reveal its particle nature.

The first part of this review is largely based on the analysis of primary sources, mainly scanned versions of scientific articles and books published in the 19th and 20th centuries, freely accessible via NASA ADS and the Internet Archive Project. We study the emergence of the concept of dark matter in the late 19th century and identify a series of articles and other sources that describe the first dynamical estimates for its...
abundance in the known Universe (Sec. II). We then discuss the pioneering work of Zwicky within the context of the scientific developments of the early 20th century. And although his work clearly stands out in terms of methodology and significance, we find that his use of the term “dark matter” was in continuity with the contemporary scientific literature. We then go on to follow the subsequent development of the virial discrepancy that he discovered, with particular emphasis on the debate that took place around this issue in the 1960s (Sec. III).

The second part of this review focuses on more recent developments, which gave us the opportunity to complement the analysis of the primary sources with extensive discussions with some of the pioneering scientists who contributed to the advancement of this field of research. We discuss the history of galactic rotation curves, from the early work in the 1920s and 1930s to the establishment of flat rotation curves in the 1970s, placing the famous work of Bosma and Rubin and collaborators in 1978 within the broader context of the theories and observations that were available at that time (Sec. IV). We then discuss the theories that have been put forward to explain the nature of dark matter in terms of fundamental particles (Sec. VII), astrophysical objects (Sec. V), or manifestations of non-Newtonian gravity or dynamics (Sec. VI).

Finally, we discuss how the emergence of cosmology as a science in the 1960s and 1970s, the advent on numerical simulations in the 1980s, and the convergence between particle physics and cosmology led most of the scientific community to accept the idea that dark matter was made of nonbaryonic particles (Sec. VIII) and prompted the development of new ideas and techniques to search for dark matter candidates, many of which are still being pursued today (Sec. IX).

One of the main difficulties in reconstructing the history of dark matter is that the key developments took place in a continuously changing landscape of cosmology and particle physics, in which scientists were repeatedly forced to revise their theories and beliefs. The authors of this review are not professional historians, but scientists writing for other scientists. And although we have taken great care in reconstructing the contributions of individuals and groups of scientists, we have little doubt that our work falls short of the standards of the historical profession. We nevertheless hope that this review will contribute to a better understanding and appreciation of the history of dark matter among our fellow astronomers and physicists, and that it will foster an interest among professional historians in this rich and fascinating field of research.

II. PREHISTORY

A. From Epicurus to Galileo

Throughout history, natural philosophers have speculated about the nature of matter and even have contemplated the possibility that there may be forms of matter that are imperceptible, because they were either too far away, too dim, or intrinsically invisible. And although many of the earliest scientific inquiries were less than rigorous and often inseparable from philosophy and theology, they reveal to us the longevity of our species’ desire to understand the world and its contents.

Although many early civilizations imagined their own cosmological systems, it was arguably the ancient Greeks who were the first to attempt the construction of such a model based on reason and experience. The atomists, most famously Leucippus and Democritus who lived in the 5th century BCE, were convinced that all matter was made of the same fundamental and indivisible building blocks, called atoms, and that these atoms were infinite in number, as was the infinite space that contained them. Epicurus (341 BCE–270 BCE) further suggested in his “Letter to Herodotus” that an infinite number of other worlds existed as well, “some like this world, others unlike it.”\(^1\) Others speculated about unobservable matter that might be found within our own Universe. For example, the Pythagorean Philolaus conjectured the existence of the celestial body Antichthon, or counter-earth, which revolves on the opposite side of the “central fire” with respect to the Earth (Kragh, 2006).

The cosmological model of Aristotle, which would dominate discourse throughout the Middle Ages, provided an elegant construction, in which the location of the Earth was fixed to the center of an immutable universe. This model offered what seemed to many to be strong arguments against the existence of invisible or unknown forms of matter. Even the striking appearance of comets, which had no place in Aristotle’s highly organized hierarchy of celestial spheres, was dismissed as an atmospheric phenomenon, a belief that continued to be held until Tycho Brahe measured the absence of parallax for a comet in 1577.

Although many offered challenges to the orthodoxy of Aristotelian cosmology, these attempts were not met without resistance. The statue of Giordano Bruno in Campo de’ Fiori in downtown Rome serves as a reminder of the dangers that were inherent in such departures from the strict Aristotelian world view embraced by the Catholic Church. It was at the location of that statue that Bruno was burned at the stake in 1600 by the Roman Inquisition, after being convicted on charges that included the holding of a heretical belief in the existence of infinite other worlds.

It was arguably Galileo, who himself had his share of trouble with the inquisition, who did the most to break the hold of Aristotelian cosmology. By pointing his telescope toward the sky, Galileo saw much that had been previously imperceptible. Among his many other discoveries, he learned that the faint glow of the Milky Way is produced by a myriad of individual stars, and that at least four satellites, invisible to the naked eye, are in orbit around Jupiter. Each of these observations encapsulate two lessons that remain relevant to dark matter today. First, the Universe may contain matter that cannot be perceived by ordinary means. And second, the introduction of new technology can reveal to us forms of matter that had previously been invisible.

\(^1\)Epicurus, Letter to Herodotus (c. 305 BCE), Extracted from Diogenes Laertius, Lives of Eminent Philosophers, trans. R. D. Hicks, Vol. 2 (1925).
B. Dark stars, dark planets, dark clouds

The course of science, and of astronomy, in particular, was transformed in 1687 when Isaac Newton published his treatise *Philosophiae Naturalis Principia Mathematica*. Newton’s laws of motion and universal gravitation provided scientists with new and formidable tools which, among many other things, enabled them to determine the gravitational mass of astronomical bodies by measuring their dynamical properties.

In 1783, John Michell, also famous for inventing the torsion balance for the measurement of the force of gravity, realized that if light is affected by the laws of gravity, as he reasoned it should, given the universal nature of gravity, then potentially objects could exist that are so massive that even light would not be able to escape their gravitational pull (Michell, 1784). This proposal, also famously discussed a decade later by Pierre Simon Laplace, is often considered to be the first mention of what have become known as black holes. We mention it here, however, as an explicit example of a discussion of a class of invisible astrophysical objects that populate the Universe while residing beyond the reach of astronomical observations.

The mathematician Friederich Bessel was perhaps the first to predict the existence of a specific undiscovered astronomical object, based only on its gravitational influence. In a letter published in 1844 (Bessel, 1844), he argued that the observed proper motion of the stars Sirius and Procyon could be explained only by the presence of faint companion stars, influencing the observed stars through their gravitational pull:

“If we were to regard Procyon and Sirius as double stars, their change of motion would not surprise us.”

Bessel further argued in favor of the existence of many stars, possibly an infinite number of them, also anticipating the modern concept of the mass-to-light ratio:

“But light is no real property of mass. The existence of numberless visible stars can prove nothing against the evidence of numberless invisible ones.”

Only two years later, in 1846, the French astronomer Urbain Le Verrier and the English astronomer John Couch Adams, in order to explain some persistent anomalies in the motion of Uranus, proposed the existence of a new planet. Le Verrier’s calculations were so precise that the German astronomer John Galle (assisted by Heinrich D’Arrest) identified the new planet, which was later named Neptune, at the Berlin observatory the same evening he received the letter from Le Verrier, within 1° of the predicted position.

Interestingly, it was Le Verrier himself who also later noticed the anomalous precession of the perihelion of Mercury and proposed the existence of a perturbing planet to explain it. As it is well known, this “dark planet”—called *Vulcan*—was never observed, and the solution to this problem would have to await the advent of Einstein’s theory of general relativity.

Besides dark stars\(^3\) and planets, astronomers in the 19th century also discussed dark matter in the form of dark clouds, or dark “nebulae.” One of the earliest traces of this discussion can be found in a memoir written in 1877 by father Angelo Secchi, then Director of the Roman College Observatory, describing research on nebulae that had been carried out 20 years earlier (Secchi, 1877):

“Among these studies there is the interesting probable discovery of dark masses scattered in space, whose existence was revealed thanks to the bright background on which they are projected. Until now they were classified as black cavities, but this explanation is highly improbable, especially after the discovery of the gaseous nature of the nebular masses.”

Around the end of the 19th century, an interesting discussion began to take place within the astronomical community. As soon as astronomical photography was invented, scientists started to notice that stars were not distributed evenly in the sky. Dark regions were observed in dense stellar fields, and the question arose of whether they were dark because of a paucity of stars, or due to the presence of absorbing matter along the line of sight. The astronomer Arthur Ranyard, who was among the main proponents of the latter hypothesis, wrote in 1894 (Ranyard, 1894):

“The dark vacant areas or channels running north and south, in the neighborhood of \([\theta\ Ophiuchi]\) at the center … seem to me to be undoubtedly dark structures, or absorbing masses in space, which cut out the light from the nebulous or stellar region behind them.”

This debate went on for quite some time, and it sparked some interesting ideas. W. H. Wesley, who acted for 47 years as the assistant secretary of the Royal Astronomical Society, proposed a curious way to settle the question, involving a rudimentary simulation of the arrangement of stars in the Milky Way (Wesley, 1894):

“It is better to solve the question experimentally. For this purpose [the author] repeated many times the experiment of sprinkling small splashes of Indian ink upon paper with a brush, revolving the paper between each sprinkling, so to avoid the chance of showing any artificial grouping in lines due to the direction in which the spots of ink were thrown from the hairs of the brush.”

C. Dynamical evidence

Lord Kelvin was among the first to attempt a dynamical estimate of the amount of dark matter in the Milky Way. His argument was simple yet powerful: if stars in the Milky Way

\(^2\)This is already implicit in *Query I* of Newton’s *Opticks*: “Do not Bodies act upon Light at a distance, and by their action bend its Rays; and is not this action \((\text{ceteris paribus})\) strongest at the least distance?”

\(^3\)Not to be confused with stars that are powered by dark matter annihilation as have been considered more recently (*Freese et al.*, 2016).
can be described as a gas of particles, acting under the influence of gravity, then one can establish a relationship between the size of the system and the velocity dispersion of the stars (Kelvin, 1904):

“It is nevertheless probable that there may be as many as $10^9$ stars [within a sphere of radius $3.09 \times 10^{16}$ kilometres] but many of them may be extinct and dark, and nine-tenths of them though not all dark may be not bright enough to be seen by us at their actual distances. [...] Many of our stars, perhaps a great majority of them, may be dark bodies.”

Kelvin also obtained an upper limit on the density of matter within such a volume, arguing that larger densities would be in conflict with the observed velocities of stars. Henri Poincaré was impressed by Lord Kelvin’s idea of applying the “theory of gases” to the stellar system of the Milky Way. In 1906 he explicitly mentioned dark matter (“matière obscure” in the original French), and argued that since the velocity dispersion predicted in Kelvin’s estimate is of the same order of magnitude as that observed, the amount of dark matter was likely to be less than or similar to that of visible matter (Poincaré, 1906a) [for an English translation, see Poincaré (1906b)]. See also Poincaré and Vergne (1911) for a more complete discussion:

“There are the stars which we see because they shine; but might there not be obscure stars which circulate in the interstellar space and whose existence might long remain unknown? Very well then, that which Lord Kelvin’s method would give us would be the total number of stars including the dark ones; since his number is comparable to that which the telescope gives, then there is no dark matter, or at least not so much as there is of shining matter.”

Along similar lines, in 1915, the Estonian astronomer Ernst Öpik built a model (published in Russian) of the motion of stars in the Galaxy, also concluding that the presence of large amounts of unseen matter was unlikely (Ópik, 1915).

An important step forward in the understanding of the structure of the Milky Way was made by the Dutch astronomer Jacobus Kapteyn. In his most important publication, which appeared shortly before his death in 1922, Kapteyn attempted “a general theory of the distribution of masses, forces and velocities in the sidereal system”—that is, in the Milky Way.

Kapteyn was among the first to offer a quantitative model for the shape and size of the Galaxy, describing it as a flattened distribution of stars, rotating around an axis that points toward the galactic pole. He argued that the Sun was located close to the center of the Galaxy, and that the motion of stars could be described as that of a gas in a quiescent atmosphere. He then proceeded to establish a relationship between the motion of stars and their velocity dispersion, similar to what Öpik had done a few years earlier.

Kapteyn expressed the local density in terms of an effective stellar mass, by dividing the total gravitational mass by the number of observed stars—including faint ones, through an extrapolation of the luminosity function—and he explicitly addressed the possible existence of dark matter in the Galaxy:

“We therefore have the means of estimating the mass of the dark matter in the universe. As matters stand at present, it appears at once that this mass cannot be excessive. If it were otherwise, the average mass as derived from binary stars would have been very much lower than what has been found for the effective mass.”

In 1932, Kapteyn’s pupil, Jan Oort, published an analysis of the vertical kinematics of stars in the solar neighborhood (Oort, 1932). In this work, Oort added to the list of estimates for the local dark matter density, including those by James Jeans (Jeans, 1922) and Bertil Lindblad (Lindblad, 1926). In his analysis, Oort made a number of improvements on Kapteyn’s seminal work, relaxing for instance the assumption of the “isothermality” of the gas of stars.

Oort derived a most probable value for the total density of matter near the Sun of $0.092 M_\odot /pc^3$, corresponding to $6.3 \times 10^{-24} \text{ g/cm}^3$. He compared this number to the value obtained by Kapteyn, $0.099 M_\odot /pc^3$, and noticed that the agreement was “unexpectedly good,” given the differences in treatment and the data used. The numbers obtained by Jeans and Lindblad were each somewhat higher, $0.143 M_\odot /pc^3$ and $0.217 M_\odot /pc^3$, respectively.

In order to estimate the amount of dark matter, Oort then proceeded with an estimate for the contribution from stars to the local density, arguing that an extrapolation of the stellar mass function based on the observed stars seemed to be able to account for a substantial fraction of the inferred total density. It is interesting to recall the words used by Oort to illustrate the constraint on the amount of dark matter:

“We may conclude that the total mass of nebulous or meteoric matter near the sun is less than $0.05 M_\odot \text{ pc}^{-3}$, or $3 \times 10^{-24} \text{ g cm}^{-3}$; it is probably less than the total mass of visible stars, possibly much less.”

We learn from this quote that the maximum allowed amount of dark matter was about half of the total local density, but also that astronomers thought that the dark matter was likely to consist of faint stars that could be accounted for through a suitable extrapolation of the stellar mass function, along with “nebulous” and “meteoric” matter.

As we shall see in Sec. IV, the pioneering work of Kapteyn, Jeans, Lindblad, Öpik, and Oort opened the path toward modern determinations of the local dark matter density, a subject that remains of importance today, especially for experiments that seek to detect dark matter particles through their scattering with nuclei.

III. GALAXY CLUSTERS

A. Zwicky and Smith

The Swiss-American astronomer Fritz Zwicky is arguably the most famous and widely cited pioneer in the field of dark matter.
matter. In 1933, he studied the redshifts of various galaxy clusters, as published by Edwin Hubble and Milton Humason (Hubble and Humason, 1931), and noticed a large scatter in the apparent velocities of eight galaxies within the Coma Cluster, with differences that exceeded 2000 km/s (Zwicky, 1933a). The fact that Coma exhibited a large velocity dispersion with respect to other clusters had already been noticed by Hubble and Humason, but Zwicky went a step further, applying the virial theorem to the cluster in order to estimate its mass.

This was not the first time that the virial theorem, borrowed from thermodynamics, was applied to astronomy; Poincaré had done so more than 20 years earlier in his *Leçons sur les hypothèses cosmogoniques professées à la Sorbonne* (Poincaré and Vergne, 1911). But, to the best of our knowledge, Zwicky was the first to use the virial theorem to determine the mass of a galaxy cluster.

Zwicky started by estimating the total mass of Coma to be the product of the number of observed galaxies, 800, and the average mass of a galaxy, which he took to be $10^9$ solar masses, as suggested by Hubble. He then adopted an estimate for the physical size of the system, which he took to be around $10^6$ light years, in order to determine the potential energy of the system. From there, he calculated the average kinetic energy and finally a velocity dispersion. He found that 800 galaxies of $10^9$ solar masses in a sphere of $10^6$ light years should exhibit a velocity dispersion of 80 km/s. In contrast, the observed average velocity dispersion along the line of sight was approximately 1000 km/s. From this comparison, he concluded (Zwicky, 1933a) the following:

“If this would be confirmed, we would get the surprising result that dark matter is present in much greater amount than luminous matter.”

This sentence is sometimes cited in the literature as the first usage of the phrase dark matter. It is not, as we have seen in the previous section, and it is not even the first time that Zwicky used it in a publication. He had, in fact, used the same phrase in an article published earlier the same year, pertaining to the sources of cosmic rays (Zwicky, 1933b):

“According to the present estimates the average density of dark matter in our galaxy ($\rho_g$) and throughout the rest of the universe ($\rho_u$) are in the ratio $\rho_g/\rho_u > 100 000$.”

Although he does not explicitly cite any article, it is obvious from this sentence that he was well aware of the work of Kapteyn, Oort, and Jeans discussed in the previous section. His use of the term dark matter is, therefore, in continuity with the community of astronomers that had been studying the dynamics of stars in the local Milky Way.

In 1937, Zwicky published a new article, this time in English, in the *Astrophysical Journal* (Zwicky, 1937), in which he refined and extended his analysis of the Coma Cluster. The purpose of this paper was to determine the mass of galaxies, and he proposed a variety of methods to attack this problem. In particular, he returned to the virial theorem approach that he had proposed in 1933, this time assuming that Coma contained 1000 galaxies within a radius of $2 \times 10^6$ light years, and solving for the average galaxy’s mass. From the observed velocity dispersion of 700 km/s, he obtained a conservative lower limit of $4.5 \times 10^{13} M_\odot$ on the mass of the cluster (to be conservative, he excluded a galaxy with a recession velocity of 5100 km/s as a possible outlier), corresponding to an average mass per galaxy of $4.5 \times 10^{10} M_\odot$. Assuming then an average absolute luminosity for cluster galaxies of $8.5 \times 10^7$ times that of the Sun, Zwicky showed that this led to a surprisingly high mass-to-light ratio of about 500.

Zwicky’s work relied on Hubble’s relationship between redshift and distance, and in the 1937 paper he used the results of Hubble and Humason (1931), which pointed to a Hubble constant of $H_0 = 558$ km/s/Mpc, with an estimated uncertainty of 10%–20%. If we rescale these results adopting the modern value of $H_0 = 67.27 \pm 0.66$ (Ade et al., 2015), we see that Zwicky overestimated the mass-to-light ratio by a factor of $\sim 558/67.27 = 8.3$. Despite this substantial correction, Coma’s velocity dispersion still implies a very high mass-to-light ratio and points to the existence of dark matter in some form.

What did Zwicky think that the dark matter in Coma and other galaxy clusters might be? An illuminating sentence in his 1937 paper provides a rather clear answer to this question:

“[In order to derive the mass of galaxies from their luminosity] we must know how much dark matter is incorporated in nebulae in the form of cool and cold stars, macroscopic and microscopic solid bodies, and gases.”

Meanwhile, another estimate for the mass of a cluster of galaxies had appeared in 1936, this time from Sinclair Smith, who had studied the Virgo Cluster. Assuming that the outer galaxies were in circular motion around Virgo, Smith calculated a total mass for the cluster of $10^{14} M_\odot$. When divided by the number of observed galaxies 500, he found an average mass per galaxy of $2 \times 10^{11} M_\odot$, which he pointed out was much higher than Hubble’s estimate of $10^9 M_\odot$.

Much like Zwicky, whose 1933 work he cites, Smith considered this high value for the mass per galaxy implied by his calculations to be a problem, in particular, in light of its incompatibility with Hubble’s estimate. He also acknowledged, however, that both could be correct, and that:

“the difference represents internebular material, either uniformly distributed or in the form of great clouds of low luminosity surrounding the [galaxies].”

In his famous book *The Realm of Nebulae*, Hubble cites the work of Smith (and not that of Zwicky) and states that he considers the discrepancy between the masses of galaxies inferred from the dynamics of clusters and those from the rotation of galaxies to be “real and important.” And although he argued that this problem might be solved, or at least diminished, by observing that the former were likely upper limits, while the latter lower limits, he acknowledged that this
argued that these galaxies were probably "temporary" members of the cluster, i.e., galaxies on hyperbolic orbits that had fallen into the gravitational potential of the cluster, but were not bound to it. In 1954, Martin Schwarzschild (Schwarzschild, 1954)—son of the famous Karl Schwarzschild who had made important contributions to general relativity—attempted to get rid of "interlopers" and inferred a smaller radial velocity dispersion of \(5 \times 10^8 \text{L}_\odot\), he obtained the "bewildering high" mass-to-light ratio of 800. The distance, mass, luminosity, and mass-to-light ratio of the galaxies and clusters of galaxies compiled by Schwarzschild are shown in Fig. 1.

By the late 1950s, a number of other articles had been published on the mass-to-light ratios of galaxy clusters. Victor Ambartsumian rejected the possibility that dark matter existed in clusters and argued instead that they are unstable and rapidly expanding systems, to which the virial theorem cannot be applied. It was soon realized, however (Burbidge and Burbidge, 1959; Limber, 1962) that this interpretation was in tension with the estimated age of the galaxies (requiring clusters that were younger than the galaxies they contained), and with that of the Universe (the clusters should have evaporated long ago). In August 1961, a conference on the instability of systems of galaxies was held in Santa Barbara and included as participants some of the most important astrophysicists active in that field of research. Jerzy Neyman, Thornton Page, and Elizabeth Scott summarized the discussions that took place around the mass discrepancy in galaxy clusters as follows (Neyman, Page, and Scott, 1961):

"Several possible explanations of this mass discrepancy were discussed at the Conference [...]. Many of those present consider that it might be real and due to invisible inter-galactic material in the clusters, totalling 90 to 99% of their mass. If these possibilities are excluded, however, the discrepancy in mass indicates positive total energy and instability of the system involved."

The overall situation was that of a community that was struggling to find a unified solution to a variety of problems. The dark matter hypothesis was not commonly accepted, nor was it disregarded. Instead, there was a consensus that more information would be needed in order to understand these systems.

In addition to the question of whether the dynamics of galaxy clusters required the presence of dark matter, astronomers around this time began to be increasingly willing to contemplate what this dark matter might be made of. Herbert Rood (Rood, 1965) [later confirmed by Simon White (White, 1977)] studied the relaxation process of galaxy clusters and argued that the mass responsible for their high mass-to-light ratios must be found within the intergalactic space and not in the galaxies themselves. Arno Penzias searched for free hydrogen in the Pegasus I cluster and set an upper limit of one-tenth of its virial mass (Penzias, 1961). Neville Woolf suggested in 1967 that the gas could be ionized, and used radio, visible, and x-ray observations to set limits on it (Woolf, 1967). Turnrose and Rood (1970) discussed the problems of this hypothesis, and Meekins et al. (1971) obtained observational evidence for x-ray emission that limited the amount of hot intracluster gas to be less than 2% of that required for gravitational binding.

With gas ruled out as an explanation for the "missing mass" in galaxy clusters, scientists began to explore more or less exotic possibilities, including massive collapsed objects (van den Bergh, 1969), HI snowballs (Peebles, 1971), and
M8 dwarf stars (Tarter and Silk, 1974). As we discuss in Sec. V, these possibilities, and others like them, were eventually ruled out by measurements of the primordial light element abundances, which instead favor a nonbaryonic nature for the dark matter.

IV. GALACTIC ROTATION CURVES

A. The beginnings

The rotation curves of galaxies, i.e., the circular velocity profile of the stars and gas in a galaxy, as a function of their distance from the Galactic center, played a particularly important role in the discovery of dark matter. Under some reasonable simplifying assumptions, it is possible to infer the mass distribution of galaxies from their rotation curves. Historically, it was the observation of approximately “flat” rotation curves at very large galactocentric distances that did the most to convince the scientific community that large amounts of dark matter are present in the outer regions of galaxies.

In 1914, ten years before Hubble convincingly demonstrated that Andromeda (M31) was a galaxy and located outside of the Milky Way, Wolf (1914) and Slipher (1914) noticed that the spectral lines from this system were inclined when the slit of the spectrogram was aligned with the galaxy’s major axis and straight when it was aligned with the minor axis, allowing them to conclude that Andromeda rotates. Based on 79 h of observation in 1917 with the Mount Wilson Observatory’s 60-in. reflector, Francis Pease measured the rotation of the central region of Andromeda out to an angular radius of 2.5 arc min, finding that it rotates with an approximately constant angular velocity. Several used Andromeda’s observed rotational velocity to calculate its mass and discuss its mass-to-light ratio in comparison with the measured value for the solar neighborhood (see Sec. II), finding values that were in reasonable agreement (Hubble, 1926; Oort, 1932).

In a paper published in 1930, Knut Lundmark made estimates for the mass-to-light ratios of five galaxies based on a comparison of their absolute luminosity, as estimated using novae as distance indicators, and their mass as inferred from spectroscopic observations (Lundmark, 1930). These mass-to-light ratios varied, quite unrealistically, from 100 for M81 to 6 for M33—much larger than those found for the solar neighborhood [Lundmark also made early estimates for the mass of the Milky Way (Lundmark, 1925)]. This demonstrates that astronomers at the time were open to the possibility that large amounts of dark matter might be present in astrophysical systems, in the form of “extinguished stars, dark clouds, meteors, comets, and so on,” as Lundmark writes in 1930.4 Holmberg argued in 1937 that the large spread in mass-to-light ratios found by Lundmark was a consequence of the absorption of light “produced by the dark matter,” and that once this was correctly taken into account, all of the galaxies studied by Lundmark, including the Milky Way, would have mass-to-light ratios between 6 and 7 (Holmberg, 1937).

Zwicky, in his famous 1937 article on galaxy clusters, discussed the possibility of using the rotation curves of galaxies to infer their mass distribution, concluding that (Zwicky, 1937):

“It is not possible to derive the masses of [galaxies] from observed rotations, without the use of additional information.”

Besides the lack of information on the ellipticity of orbits, one of Zwicky’s main concerns was the possible internal “viscosity” resulting from the mutual interactions of stars. Only four years later, Chandrasekhar demonstrated in his classic paper, “The Time of Relaxation of Stellar Systems” (Chandrasekhar, 1941), that these interactions are completely negligible, allowing one to reliably describe galaxies as systems of noninteracting stars.

Meanwhile, in his 1939 Ph.D. dissertation, Horace Babcock presented the rotation curve of M31 out to 100 arc min (i.e., about 20 kpc) away from its center (Babcock, 1939). Interestingly, he found very high values for the circular velocity at large radii, so high, in fact, that they are at odds with modern measurements. Approximating M31 as a sphere surrounded by a flattened ellipsoid, he calculated the mass distribution of the galaxy, recognizing that the observed rising rotation curve at large radii implied the existence of large amounts of mass in the outer parts of the Galaxy. But when interpreting this result, he conservatively argued that:

“The calculated ratio of mass to luminosity in proceeding outward from the nucleus suggests that absorption plays a very important role in the outer portion of the spiral, or, perhaps, that new dynamical considerations are required, which will permit of a smaller mass in the outer parts.”

More than a decade later, observations made by Nicholas Mayall in 1951 at Mount Wilson were used by Martin Schwarzschild to further study the dynamics of M31. In doing so, Schwarzschild showed that a model with a constant mass-to-light ratio was able to explain the rotational velocities measured by Mayall out to 115 arc min (Schwarzschild, 1954).

The German invasion of Poland in 1939 marked the official start of World War II. Hostilities brought death and destruction, but also unexpected benefits for science, as after the war ended military radars began to be used for radio astronomical observations. The Netherlands was particularly active in this field, under the push of the visionary astronomer Jan Oort, who was not only a great scientist, but also a great organizer. A chain of so-called Würtzburg antennas—7.5 m parabolic radars used at 54 cm wavelengths for aircraft tracking—had been left behind in the Netherlands by occupying German forces at the end of the war, and since the reflective surface and tracking precision were also suitable for shorter wavelengths, and, in particular, for the 21 cm line predicted by Oort’s student Hendrik van de Hulst, one was mounted in Kootwijk for the purpose of radio astronomy (van Woerden and Strom, 2006).
When Harold Ewen and Edward Purcell from Harvard detected the 21 cm line in 1951, van de Hulst was visiting Harvard, and so was F.J. Kerr from the Radiophysics Laboratory in Sydney. The Dutch and Australian groups were soon able to confirm the detection: the reports of the American and Dutch groups appeared in the same issue of Nature, together with a confirming telegram from the Australian group. This success provided an important boost to the young field of radio astronomy and had a dramatic impact on the history of astrophysics and cosmology.

Back in the Netherlands, the construction of a new 25 m radio telescope was completed in Dwingeloo in 1955. Only two years later, van de Hulst, Jean Jacques Raimond, and Hugo van Woerden published the first radio rotation curve of M31, extending observations to 2 deg away from its center (van de Hulst, Raimond, and van Woerden, 1957). Although the data seemed at first to be at odds with the rotation curve calculated by Schwarzschild, Maartin Schmidt argued in a paper accompanying the publication of van de Hulst et al. that a constant mass-to-light ratio provided a satisfactory explanation of the data, although also noting that “nothing as yet can be stated about the ratio in the innermost and outermost parts” of M31 (Schmidt, 1957).

In 1959, Franz Kahn and Lodewijk Woltjer proposed an ingenious method to determine the combined mass of M31 and the Milky Way, also known as the “timing argument.” Since 21 cm observations of M31 indicated that it was approaching the Milky Way at a speed of 125 km/s, they derived a lower bound on the reduced mass of the M31-Milky Way system, assuming that the two galaxies are part of a bound system and that the orbital period is smaller than the age of the Universe. That lower bound was, however, 6 times larger than the currently accepted value of the reduced mass of the system (Kahn and Woltjer, 1959). They argued at the time that this provided evidence for intergalactic material in the form of gas stabilizing the local group. In retrospect, this simple argument is one of the earliest clear indications of dark matter halos around galaxies.

In his detailed historical account, Sanders (2010) argued that despite these developments there was no sense of crisis in the field of astrophysics at the end of the 1950s, or at least that there was no consensus that the observed rotation curves were in conflict with the current understanding of galaxies. A decade later, things began to dramatically change.

B. The 1970s revolution

In the 1960s, Kent Ford developed an image tube spectrograph that Vera Rubin and he used to perform spectroscopic observations of the Andromeda Galaxy. The observations of the M31 rotation curve Rubin and Ford published in 1970 represented a step forward in terms of quality (Rubin and Ford, 1970). Their optical data extended out to 110 arc min away from the galaxy’s center and were compatible with the radio measurements obtained previously by Roberts (1966).

It was also in 1970 that the first explicit statements began to appear arguing that additional mass was needed in the outer parts of some galaxies, based on comparisons of the rotation curves predicted from photometry and those measured from 21 cm observations. In the appendix of his seminal paper Freeman (1970) compared the radius at which the rotation curve was predicted to peak, under the assumption of an exponential disk with a scale length fit to photometric observations to the observed 21 cm rotation curve. This combination of theoretical modeling and radio observations extending beyond the optical disk allowed Freeman to reach a striking conclusion. He found that for M33 [based on data summarized by Brandt (1965)] and NGC 300 [based on data from Shobbrook and Robinson (1967)] the observed rotation curves peaked at larger radii than predicted, and, prompted by discussions with Roberts5, concluded the following:

“If [the data] are correct, then there must be in these galaxies additional matter which is undetected, either optically or at 21 cm. Its mass must be at least as large as the mass of the detected galaxy, and its distribution must be quite different from the exponential distribution which holds for the optical galaxy.”

This is perhaps the first convincing (or at least convinced) claim of a mass discrepancy in galaxies. Rogstad and Shostak (1972) performed a similar analysis by analyzing the rotation curves of five galaxies—M33, NGC 2403, IC 342, M101, and NGC 6946—they had themselves obtained using the radio telescope at the Owens Valley Radio Observatory. They found that these rotation curves remained flat out to the largest radii observed (see Fig. 2) and, following the method of Freeman, they derived mass-to-light ratios as high as 20 at large radii. As explicitly stated in their paper, they

“confirm[ed] the requirement of low-luminosity material in the outer regions of these galaxies.”

Morton Roberts was among the first to recognize the implications of the observed flatness of galactic rotation curves. Together with R. Whitehurst, in 1972 he published a rotation curve of M31 that extended to 120 arc min from its center (Whitehurst and Roberts, 1972). In 1973, together with Arnold Rots, he extended the analysis to M81 and M101 and argued that these spiral galaxies each exhibited flat rotation curves in their outer parts (Roberts and Rots, 1973) (see Fig. 3). Their interpretation of these data was unambiguous:

“The three galaxies rotation curves decline slowly, if at all, at large radii, implying a significant mass density at these large distances. It is unreasonable to expect the last measured point to refer to the ‘edge’ of the galaxy, and we must conclude that spiral galaxies must be larger than indicated by the usual photometric measurements […]. The present data also require that the mass to luminosity ratio vary with radius increasing in distance from the center.”

In the Proceedings of the IAU Symposium No. 69, held in Besançon, France in September of 1974, Roberts (1975b) reviewed the status of galactic rotation curves, highlighting

5K. Freeman, private communication.
the importance of radio observations, which extended well beyond the optical radius of the galaxies (see Fig. 4). When discussing the implications of the high mass-to-light ratios implied by these observations, he argued that the excess mass might take the form of intermediate and late dwarf M stars. He further tried to reassure his colleagues by arguing that the required radius-dependent luminosity function need not be alarming, since there was evidence of a dependence on the height above the galactic plane that exhibited a similar trend.

As discussed in Sec. VIII, two influential papers in 1974 brought together the observed mass discrepancies observed in clusters and in galaxies (Einasto, Kaasik, and Saar, 1974; Ostriker, Peebles, and Yahil, 1974). Both of these papers clearly state in their first paragraphs that the mass of galaxies had been until then underestimated by about a factor of 10. In support of this, Ostriker, Peebles, and Yahil (1974) cited Rogstad and Shostak (1972) and Roberts and Rots (1973). Einasto and collaborators (1974) for the observed flat rotation curves. For the same purpose, Einasto, Kaasik, and Saar (1974) cited a review written in 1975 by Roberts for a book edited by A. and M. Sandage together with J. Kristian (Sandage, Sandage, and Kristian, 1975). In a separate paper that appeared in the same year, focusing on the “morphological evidence” of missing mass around galaxies, Einasto and collaborators cited Roberts and Rots (1973). Interestingly, Einasto and collaborators excluded the possibility that this missing mass was in the form of stars and argued that the most likely explanation was the presence of large amounts of gas in the outer parts of galaxies, which they referred to as “coronas” (Einasto et al., 1974).

By 1974, the flat rotation curves obtained by radio astronomers had done much to establish the existence of large amounts of mass in the outer parts of galaxies at least in the eyes of Einasto, Kaasik, and Saar (1974) and Ostriker,
Peebles, and Yahil (1974). Portions of the astronomical community, however, were still not convinced of this conclusion (Rubin, 2004). In the late 1970s, this evidence was strengthened and corroborated by a series of new studies. In 1977, Nathan Krumm and Edwin Salpeter observed six spiral galaxies with the Arecibo Observatory and showed that they each exhibited a flat rotation curve out to radii larger than their optical extent (Krumm and Salpeter, 1977), but these data turned out to be unreliable due to beam smearing [see the discussion at the end of Salpeter (1978)].

In 1978, Albert Bosma published the results of his Ph.D. thesis (Bosma, 1978), including the radio observation of the velocity fields and corresponding rotation curves of 25 galaxies. This work convincingly proved that most of these objects had flat rotation curves out to the largest observed radius, which again exceeded the optical size of the galaxies, therefore demonstrating that their mass continued to grow beyond the region occupied by the stars and gas (see also Fig. 5).

A few months later, Rubin, Thonnard, and Ford (1978) published optical rotation curves for ten high-luminosity spiral galaxies and found that they were flat out to the outermost measured radius. This work has become one of the most well known and widely cited in the literature, despite the fact that the optical measurements did not extend to radii as large as those probed by radio observations, thus leaving open the possibility that galaxies may not have dark matter halos, as pointed out, for example, by Agris J. Kahns in 1983 [see the discussion at the end of Haud and Einasto (1983)] and Stephen Kent (Kent, 1986). Rubin, Ford, and Thonnard themselves acknowledged the credit that was due to the preceding analyses (Rubin, Thonnard, and Ford, 1978):

“Roberts and his collaborators deserve credit for first calling attention to flat rotation curves. […] These results take on added importance in conjunction with the suggestion of Einasto, Kaasik, and Saar (1974) and Ostriker, Peebles and Yahil (1974) that galaxies contain massive halos extending to large r.”

A lucid and timely review of the status of galaxy masses and mass-to-light ratios appeared by Faber and Gallagher (1979). We refer the interested reader to this article for an overview of the various ideas that had been put forward in an effort to understand the complex and diverse observational data that were available at the time. The abstract of that article provides a clear indication of its contents:

“The current status of the ‘missing mass’ problem is reviewed on the basis of standardized mass-to-light (M/L) ratios of galaxies. The stellar mass density in the immediate vicinity of the sun is examined, along with the mass of the Milky Way and the M/L ratios of spiral galaxies, E and S0 galaxies, and binary galaxies. The dynamics of small groups of galaxies is investigated, and mass derivations for cluster galaxies are discussed. It is concluded that the case for invisible mass in the universe is very strong and becoming stronger.”

C. Local measurements

We conclude this section with a brief overview of the efforts to determine the local dark matter density, i.e., the density of dark matter in the solar neighborhood. This quantity was historically important, as it provided the first, albeit rather weak, dynamical evidence for matter in the local Universe beyond visible stars. It is also important today, as the prospects for detecting dark matter particles in underground and astrophysical experiments strongly depend on this quantity.

As we have seen in the previous section, Kapteyn, Lindblad, Jeans, and Oort studied the dynamics of nearby stars and compared the inferred gravitational mass with that of the visible stellar density. After decades of steady improvements (Oort, 1932, 1960; Bahcall, 1984a, 1984b), Konrad Kuijken and Gerry Gilmore published a series of papers based on a refined method and a volume complete sample of K-dwarf data to derive a much more precise value of the local density (Kuijken and Gilmore, 1989). The advent of the Hipparcos, Sloan Digital Sky Survey (SDSS), and Radial Velocity Experiment (RAVE) surveys has more recently triggered many new analyses. We refer the interested reader to the review by Read (2014) for further details and references.

Alternatively, the local dark matter density can be constrained using measurements of the Milky Way’s rotation curve (Fich, Blitz, and Stark, 1989; Merrifield, 1992; Dehnen and Binney, 1998; Sofue, Honma, and Omodaka, 2009; Catena and Ullio, 2010; Salucci et al., 2010; Weber and de Boer, 2010; Iocco et al., 2011; Pato, Iocco, and Bertone, 2015). Although rather precise determinations can be made...
A. Gravitational microlensing

The possibility that light could be deflected by gravity has a long history extending back as far as Newton. In 1915, Einstein made the correct prediction for this phenomenon using the framework of general relativity (which predicts twice the degree of deflection as Newtonian gravity). An early test of general relativity was famously conducted during the solar eclipse of 1919, which provided an opportunity to measure the bending of light around the Sun. Although the measurements obtained by Arthur Eddington favored the relativistic prediction, other simultaneous observations appeared to agree with the Newtonian expectation. Despite this apparent ambiguity, Eddington’s results were seen as persuasive by many astronomers and served to elevate the status of Einstein’s theory.

In 1924, the Russian physicist Orest Chwolson returned to the topic of gravitational lensing, pointing out that a massive body could deflect the light from a more distant source in such a way that would lead to the appearance of multiple images, or of a ring (Chwolson, 1924). In 1936, Einstein himself published a paper on this topic (Einstein, 1936), but concluded that due to the very precise alignment required, “there is no great chance of observing this phenomenon.”

The modern theory of gravitational lensing was developed in the 1960s, with contributions from Klimov (1963a, 1963b, 1963c), Liebes (1964), and Refsdal (1964a, 1964b), followed by the first observation of a lensed quasar by Walsh, Carswell, and Weymann (1979). In the same year, Chang and Refsdal (1979) showed that individual stars could also act as lenses, leading to potentially observable variations over time scales of months. Paczynski (1986) proposed that this phenomenon of gravitational microlensing could be used to search for compact objects in the “dark halo” of the Milky Way, followed by more detailed predictions for the probability and light curves of such events, described in the Ph.D. thesis of Robert Nemiroff (Nemiroff, 1987).6

The strategy proposed was to simultaneously monitor large numbers of stars in a nearby galaxy (such as in the Large Magellanic Cloud), in an effort to detect variations in their brightness. If the halo consisted entirely of MACHOs, approximately 1 out of $2 \times 10^6$ stars should be magnified at a given time, a ratio known as the microlensing optical depth. Furthermore, as the duration of a microlensing event is predicted to be $t \sim 130$ days $\times (M/M_\odot)^{0.5}$, such a program would be best suited to detect objects with masses in the range of $\sim 10^{-7} M_\odot$ to $\sim 10^2 M_\odot$, corresponding to variations over time scales of hours to a year. These factors motivated the approaches taken by the MACHO, EROS (Experience pour la Recherche d’Objets Sombres), and OGLE (Optical Gravitational Lensing Experiment) Collaborations, who each set out to conduct large microlensing surveys in order to test the hypothesis that the Milky Way’s dark halo consisted of MACHOs.

Although the first claim of a microlensing event was reported by Irwin et al. (1989), the implications of microlensing surveys for dark matter only began to take shape a few

6The possibility that objects in the Milky Way’s dark halo could be detected through gravitational lensing was also discussed earlier, in the 1981 Ph.D. thesis of Maria Petrou. On the advice of her supervisor, Petrou did not otherwise attempt to publish this work (Valls-Gabaud, 2006).
But alas, it was not to be.

Eddington and others argued that the fusion of hydrogen into helium nuclei could be capable of providing the primary source of energy in stars and suggested that it might also be possible to generate heavier elements in stellar interiors (Eddington, 1920a, 1920b). In 1939, Hans Bethe expanded significantly upon this idea, describing the processes of the proton-proton chain and the carbon-nitrogen-oxygen cycle that are now understood to dominate the energy production in main sequence stars (Bethe, 1939). Fred Hoyle, in papers that are now understood to dominate the energy production in the proton-proton chain and the carbon-nitrogen-oxygen cycle significantly upon this idea, describing the processes of the

“After the discovery of MACHOs in 1993, some thought that the dark matter puzzle had been solved.”

But alas, it was not to be.

Over a period of 5.7 years, the MACHO Collaboration measured the light curves of $40 \times 10^6$ individual stars, identifying between 14 and 17 candidate microlensing events. This was well above their expected background rate and led them to conclude that between 8% and 50% of the Milky Way’s halo mass consisted of compact objects, most of which had masses in the range of $0.15M_\odot$ to $0.9M_\odot$ (Alcock et al., 2000). After collecting data for 6.7 years, however, the EROS Collaboration identified only one microlensing candidate event, allowing them to place an upper limit of 8% on the halo mass fraction in MACHOs (Lasserre et al., 2000; Tisserand et al., 2007). Compact objects, at least within the mass range probed by microlensing surveys, do not appear to dominate the missing mass in the Milky Way’s halo.

B. The Universe’s baryon budget

Throughout much of the mid-twentieth century, the origin of the various nuclear species remained a subject of considerable mystery and speculation. As early as 1920, Arthur Eddington and others argued that the fusion of hydrogen into helium nuclei could be capable of providing the primary source of energy in stars and suggested that it might also be possible to generate heavier elements in stellar interiors (Eddington, 1920a, 1920b). In 1939, Hans Bethe expanded significantly upon this idea, describing the processes of the proton-proton chain and the carbon-nitrogen-oxygen cycle that are now understood to dominate the energy production in main sequence stars (Bethe, 1939). Fred Hoyle, in papers in 1946 and 1954, calculated that nuclei as heavy as iron could be synthesized in massive stars (Hoyle, 1946), and that even heavier nuclear species could be produced by supernovae (Hoyle, 1954).

An alternative to stellar nucleosynthesis was proposed by Gamow (1946) and followed up upon two years later by Alpher, Bethe, and Gamow (1948). The author list of this later paper also famously included Hans Bethe (who reportedly did not contribute to the research) in order to facilitate the pun that enabled it to become known as the “alpha-beta-gamma” paper. In this pair of papers, it was proposed that all nuclear species (both light and heavy) may have been produced in the early Universe through the process of neutron capture. While of historic significance, there were considerable technical problems with the calculations presented in these early papers, some of which were pointed out by Enrico Fermi, Chushiro Hayashi, and Anthony Turkevich in the years to follow. Among other flaws, Alpher and Gamow did not correctly account for Coulomb barriers in estimating the rates for nuclear fusion. Perhaps more importantly, they did not appreciate that the lack of stable nuclei with atomic numbers in the range of 5–8 would effectively prevent any significant nucleosynthesis from occurring beyond $^4$He. After accounting for these issues, Alpher, along with Robert Herman and James Follin, correctly predicted the abundance of helium produced in the early Universe and reported in 1953 that the heavier elements could not be accounted for by this mechanism (Alpher, Follin, and Herman, 1953). For these and other reasons, stellar nucleosynthesis remained the predominant theory throughout the 1950s and into the 1960s. That being said, by the late 1950s, it was becoming increasingly clear that stellar nucleosynthesis could not generate enough helium to accommodate the observed abundance as summarized in the classic 1957 review paper by Margaret Burbidge, Geoffrey Burbidge, William Fowler, and Fred Hoyle (Burbidge et al., 1957).

The discovery of the cosmic microwave background in 1965 (Penzias and Wilson, 1965) led to increased interest in big bang nucleosynthesis and made it possible to further refine the predictions for the light element abundances. In particular, the temperature of this newly detected background favored a primordial helium fraction in the range of 26%–28% (Peebles, 1966; Wagoner, Fowler, and Hoyle, 1967), consistent with observations. In 1973, a paper by Hubert Reeves, Jean Audouze, William Fowler, and David Schramm focused on the production of deuterium in the early Universe (Reeves et al., 1973). As deuterium had been detected in the interstellar medium, but is not generated in stars, they argued that big bang nucleosynthesis offered the most plausible origin for the observed deuterium. In the same paper, they also used the measured light element abundances to derive an upper limit on the cosmological baryon density that was about one-tenth of the critical density $\Omega_b \lesssim 0.1\Omega_{\text{crit}}$.

Constraints on the cosmological baryon density became increasingly stringent over the decades to follow. Of particular importance were the first high-precision measurements of the primordial deuterium abundance, which were carried out in the late 1990s by Scott Burles, David Tytler, and others (Burles and Tytler, 1998a, 1998b; O'Meara et al., 2001). These measurements were used to determine the baryonic abundance with roughly 10% precision, $\Omega_b h^2 = 0.020 \pm 0.002$ (95% C.L.) (Burles, Nollett, and Turner, 2001), leaving little room for baryonic MACHOs (Fukugita, Hogan, and Peebles, 1998). At around the same time, measurements of the angular power spectrum of the cosmic microwave background were also becoming sensitive

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$^7$One of the two microlensing candidate events originally identified by the EROS Collaboration (Aubourg et al., 1993) was later rejected based on subsequent behavior of its light curve (Lasserre et al., 2000).
to this quantity. In particular, the ratio of the heights of the odd and even peaks in this power spectrum is primarily set by the baryonic density. Although limited measurements of the second peak were made by ground-and balloon-based experiments in the late 1990s, it was not until the satellite-based Wilkinson Microwave Anisotropy Probe (WMAP) experiment that these determinations became competitive with (and superior to) those based on the measured light element abundances. WMAP ultimately achieved a measurement of $\Omega \Delta h^2 = 0.02264 \pm 0.00050$ (68% C.L.) (Hinshaw et al., 2013), while the most recent analysis from the Planck Collaboration arrived at a constraint of $\Omega \Delta h^2 = 0.02225 \pm 0.00016$, corresponding to a fractional uncertainty of less than 1% (Ade et al., 2015). When this is compared to the total matter density as inferred by these and other experiments, one is forced to the conclusion that less than 20% of the matter in the Universe is baryonic.

C. Primordial black holes

By the late 1990s, it had become clear that baryonic dark matter does not constitute a large fraction of the Universe’s dark matter. Although these results seem to imply that the dark matter must consist of one or more new particle species, there remains a caveat to this conclusion: the dark matter might instead consist of black holes that formed before the epoch of big bang nucleosynthesis and with masses below the sensitivity range of microlensing surveys.

The possibility that black holes may have formed in the early Universe was discussed by Barnard Carr and Stephen Hawking as early as 1974 (Carr and Hawking, 1974). Such primordial black holes exhibit a characteristic mass that is on the order of the mass contained within the horizon at the time of formation $M_{\text{horizon}} \sim 10^{15} \text{ kg} \times (10^7 \text{ GeV}/T)^2$, allowing for a very large range of possible masses. A lower limit on this mass range can be placed, however, from the lack of Hawking-radiated gamma rays from a primordial black hole population (Page and Hawking, 1976; MacGibbon, 1987). Combining gamma-ray constraints (Yokoyama, 1998; Kim, Lee, and MacGibbon, 1999) with the null results of microlensing surveys yields an acceptable mass range of $10^{14}$ to $10^{23} \text{ kg}$ for dark matter in the form of primordial black holes.

A major factor that has tempered the enthusiasm for primordial black hole dark matter pertains to the number of such objects that are expected to have formed in the early Universe. If one assumes an approximately scale-invariant spectrum of density fluctuations (normalized to that observed at large scales), the predicted formation rate is cosmologically negligible. To generate a relevant abundance of such black holes, one must postulate a large degree of non-Gaussianity or other such features in the primordial power spectrum.

VI. MODIFIED GRAVITY

In February 1982, Mordehai Milgrom submitted a trio of papers to the Astrophysical Journal (Milgrom, 1983a, 1983b, 1983c). These papers, which Milgrom developed at the Weizmann Institute in Israel and while on sabbatical at Princeton’s Institute of Advanced Study, provided the foundation for what would become the leading alternative to dark matter. This proposal, known as modified Newtonian dynamics (MOND), was a seemingly simple one, but with extremely far reaching consequences. At the heart of MOND is the recognition that if instead of obeying Newton’s second law $F = ma$, the force due to gravity scaled as $F = maG^2/a_0$ in the limit of very low accelerations ($a \ll a_0 \sim 1.2 \times 10^{-10} \text{ m/s}^2$), then it would be possible to account for the observed motions of stars and gas within galaxies without postulating the presence of any dark or otherwise hidden matter. In Milgrom’s proposal, there was no dark matter. Instead, what astronomers had discovered was evidence of a new framework for gravity and dynamics beyond that described by Newtonian physics or even by general relativity.

A. Toward a realistic theory of MOND

Milgrom’s initial proposal was not intended to represent a realistic theory, but rather was presented as the approximate weak-field limit of some unknown, but more complete framework. In its original form, it was not even clear whether MOND was merely a modification of the behavior of gravity or was instead a more general correction to Newton’s second law, applicable to all forces. Within the context of either interpretation, however, it has proven challenging to embed MOND-like behavior within a realistic theoretical framework. First of all, in its original formalism, MOND does not conserve momentum, angular momentum, or energy. Furthermore, Milgrom did not initially propose any means by which MOND could be embedded within a theory consistent with general relativity. Before MOND could be considered a viable alternative to dark matter, a more realistic version of this theory would have to be developed. And while significant progress has been made toward this goal over the past three decades, this progress has often been accomplished at great expense in terms of economy and simplicity.

The first step in this direction was made in 1984 through the collaboration of Milgrom with Jacob Bekenstein, and their proposal of the quadragrade Lagrangian theory (AQUAL) (Bekenstein and Milgrom, 1984). In AQUAL, Bekenstein and Milgrom began with a modification of the Lagrangian of Newtonian gravity, rather than with a modification of Newton’s second law. As a result, this approach automatically preserves the conservation of momentum, angular momentum, and energy and respects the weak equivalence principle. And while the predictions of AQUAL are identical to those of MOND only in special and highly symmetric cases, the differences between the predictions of these two theories are typically modest (at the ~10% level) (Milgrom, 1986).

Despite its advantages over the original version of MOND, AQUAL was still a modification of Newtonian dynamics and is not compatible with the general theory of relativity. In order for any variation of MOND to be taken seriously, it would need to be able to account for the many varieties of relativistic phenomena that have been observed, including those of gravitational lensing and cosmological expansion. The first attempts to embed MOND into a relativistic framework involved theories with more than one metric. In relativistic AQUAL (RAQUAL) (Bekenstein and Milgrom, 1984), for example, the dynamics of matter and radiation are dictated by a metric that is distinct from the standard spacetime metric that...
applies to the gravitational field. The difference between these two metrics is the result of the presence of an additional scalar field which, along with matter, contributes to the gravitational potential. In this respect, RAQUAL shares some of the features of much earlier scalar-tensor theories of gravity (Jordan, 1955; Brans and Dicke, 1961). To avoid causal problems resulting from the superluminal motion of the scalar field, however, RAQUAL had to be further modified (Bekenstein, 1988), and these changes were to the detriment of the theory’s consistency with precision solar system tests. Even more problematic was the fact that these early attempts at a relativistic theory of MOND failed to adequately describe the phenomena of gravitational lensing.

Although the deflection of light is predicted to occur in RAQUAL and other relativistic formulations of MOND, the magnitude of such lensing is generally expected to be proportional to the amount of baryonic mass that is present in the deflecting system. In contrast, the degree of lensing that is observed around galaxy clusters is much larger than can be accounted for by the mass of the baryons alone. In this respect, RAQUAL cannot address the dark matter problem on cluster scales.

Although other efforts to resolve this issue were attempted (Bekenstein and Sanders, 1994; Sanders, 1997), it was not until 2004 that Bekenstein proposed the first realistic solution to the problem of gravitational lensing in relativistic theories of MOND (Bekenstein, 2004). Since its proposal, Bekenstein’s TeVeS theory, short for tensor-vector-scalar gravity, has become the leading theory of MOND and has received a great deal of attention. Beyond those of general relativity, TeVeS contains two additional fields, three free parameters, and one free function. On the one hand, this freedom makes TeVeS somewhat limited in its predictive power. On the other hand, it provides TeVeS with enough flexibility to potentially be consistent with gravitational lensing observations and other cosmological considerations, such as those pertaining to structure formation and the cosmic microwave background.

B. Observational successes and failures

Early in its history, it was appreciated that MOND was capable of explaining the observed dynamics of many spiral and elliptical galaxies. MOND also, however, made predic-
tions for the behavior of low surface brightness galaxies, whose dynamics had not yet been well measured. The fact that such systems were later found to be compatible with MOND (Casertano and van Gorkom, 1991; McGaugh and de Blok, 1998) served to bolster interest in the theory. Today, MOND appears to be compatible with the observed rotation curves of hundreds of spiral galaxies (Begeman, Broeils, and Sanders, 1991; Sanders, 1996; Milgrom and Sanders, 2005, 2007).

In addition to galactic rotation curves, MOND also provides an explanation for the empirical Tully-Fisher formula (Tully and Fisher, 1977), which relates the intrinsic luminosities and rotational velocities of spiral galaxies \( L \propto V_{\text{rot}}^\alpha \), where \( \alpha \approx 4 \). If one assumes a common mass-to-light ratio for all galaxies, MOND precisely predicts this relationship, with a value of \( \alpha = 4 \), which is consistent with observations (McGaugh et al., 2000).

On the scale of galaxy clusters, MOND has not been nearly as successful. While MOND does reduce the need for additional mass in clusters, significant quantities of dark matter are still required. If the three known species of neutrinos were as heavy as \( m_\nu \sim 1–2\ eV \) (near the upper limits from beta decay experiments), it has been suggested that they might be able to account for this discrepancy, essentially acting as dark matter in clusters (Sanders, 2003, 2007; Angus et al., 2007). Massive neutrinos can also help to reduce to some degree the discrepancy between measurements of the cosmic microwave background and the predictions of TeVeS (Skordis et al., 2006).

In recent years, the debate over MOND has been focused on the use of gravitational lensing to measure the mass profiles of galaxy clusters. The idea that lensing could be used to determine the mass of a galaxy or a galaxy cluster was first proposed by Fritz Zwicky in his famous paper of 1937 (see Sec. IIIA). It was more than 40 years later that the first gravitational lens was observed (Walsh, Carswell, and Weymann, 1979), two mirror images of a quasar, and another decade after that before the first observations were made of lensing by a galaxy cluster (Lynds and Petrosian, 1986; Soucail et al., 1987). Today, gravitational lensing is frequently used to study the properties of clusters [see, e.g., Mellier (2010) and Hoekstra et al. (2013) for recent reviews].

In 2006, a group of astronomers including Douglas Clowe transformed the debate between dark matter and MOND with the publication of an article entitled, “A direct empirical proof of the existence of dark matter.” In this paper, they described the observations of a pair of merging clusters collectively known as the “bullet cluster” (and also known as 1E0657-558) (Clowe et al., 2006). As a result of the clusters’ recent collision, the distribution of stars and galaxies is spatially separated from the hot x-ray emitting gas (which constitutes the majority of the baryonic mass in this system). A comparison of the weak lensing and x-ray maps of the bullet cluster reveals that the mass in this system does not trace the distribution of baryons (see Fig. 7). Another source of gravitational potential, such as that provided by dark matter, must instead dominate the mass of this system.

Following these observations of the bullet cluster (and of other similar systems), many researchers expected that this would effectively bring the MOND hypothesis to an end. In the years since, however, anything but has taken place. Since the introduction of TeVeS, MOND has continued to attract a great deal of attention, despite its failure to address the dynamics of galaxy clusters, and, in particular, the bullet cluster. In addition to massive neutrinos, some considered the possibility that TeVeS’s vector field might source the gravitational potential of the bullet cluster, itself acting much like dark matter on cluster scales. Similarly, the failure of TeVeS to predict the observed ratio of the second and third peaks of the cosmic microwave background’s angular power spectrum might be plausibly averted if some of TeVeS’s additional degrees of freedom behaved much like cold dark matter during the early history of the Universe. And although this possibility goes somewhat against the original spirit of MOND, it is hard to rule out at this time. Taken together, the bullet cluster and other increasingly precise cosmological
measurements have been difficult to reconcile with all proposed versions of MOND, and it remains unclear whether TeVeS, in some form, might be compatible with these observations (Dodelson and Liguori, 2006; Skordis et al., 2006; Skordis, 2008). For reviews, see Skordis (2009) and Famaey and McGaugh (2012).

VII. DARK MATTER PARTICLES

Over the past few decades, the very meaning of the phrase dark matter has evolved considerably. Today, this phrase is most frequently used as the name, a proper noun, of whatever particle species accounts for the bulk of our Universe’s matter density. When a modern paper discusses the distribution of dark matter, or the impact of dark matter on structure formation, or the prospects for detecting dark matter with a gamma-ray telescope, one does not have to ask themselves whether the authors might have in mind white dwarfs, neutron stars, or cold clouds of gas—they do not. This is in stark contrast to the earlier usage of the phrase, in which the word “dark” was a mere adjective, and dark matter included all varieties of astrophysical material that happened to be too faint to be detected with available telescopes.

This linguistic transition reflects a larger change that has taken place over the past several decades within the astrophysics and particle physics communities. And although this transformation was driven and initiated by new scientific results and understanding, it also reflects a sociological change in the underlying scientific culture. Half a century ago, cosmology was something of a fringe science, perceived by many astronomers and particle physicists alike to have little predictive power or testability. This can be easy to forget from our modern vantage point in the age of precision cosmology. Furthermore, prior to the last few decades, particle physicists did not often study or pursue research in astrophysics, and most astrophysicists learned and knew little about particle physics. As a result, these scientists did not frequently contribute to each other’s fields of research. When Fermilab founded its theoretical astrophysics group in 1983, for example, the decision to do so was seen by many as a radical departure from the lab’s particle physics mission. From the perspective of many particle physicists in the early 1980s, it was not obvious what astrophysics had to do with the questions being asked by particle physics. This view is shared by few today. As an illustration, we need only to note that the 2014 report of the U.S. Particle Physics Project Prioritization Panel (P5) describes the “cosmic frontier,” along with the energy and intensity frontiers, as coequal areas of inquiry within the larger field of particle physics.

From our contemporary perspective, it can be easy to imagine that Fritz Zwicky, Vera Rubin, and the other early dark matter pioneers had halos of weakly interacting particles in mind when they discussed dark matter. In reality, however, they did not. But over time, an increasing number of particle physicists became interested in cosmology and eventually in the problem of dark matter. By the late 1980s, the hypothesis that the missing mass consists of one or more yet-unknown subatomic particle species had gained enough support to become established as the leading paradigm for dark matter. As alternatives were ruled out one by one (see Secs. V and VI), this view came to be held almost universally among both particle physicists and astrophysicists as well as among their new and now increasingly common hybrids—the particle astrophysicists.

A. Neutrinos

When one considers the dark matter problem from the perspective of the standard model of particle physics, the three neutrinos clearly stand out. Unlike all other known particle species, the neutrinos are stable, or at least very long lived, and do not experience electromagnetic or strong interactions. These are essential characteristics for almost any viable dark matter candidate. And although we know today that dark matter in the form of standard model neutrinos would be unable to account for our Universe’s observed large scale structure, these particles provided an important template for the class of hypothetical particles that would later be known as WIMPs. In this way, standard model neutrinos served as an important gateway particle, leading astrophysicists and particle physicists alike to begin their experimentation with a variety of other, more viable, particle dark matter candidates. And although the first scientists to consider the role of neutrinos in cosmology did not have the dark matter problem in mind, many being unaware that there was any such problem to solve, their work helped to establish the foundations that the field of particle dark matter would later be built upon.

The earliest discussion of the role of neutrinos in cosmology appeared in a paper by Gershtein and Zeldovich (1966). To many scientists working in the fields of cosmology and particle astrophysics, it will be no surprise to see Zeldovich’s name attributed to this pioneering work. Yakov Borisovich Zeldovich was an utterly prolific and versatile physicist, making major contributions to the fields of material science, nuclear physics (including the Soviet weapons program), particle physics, relativity, astrophysics, and cosmology. In terms of research at the interface between particle physics and
cosmology, it can sometimes seem like Zeldovich did almost everything first.

In the early 1960s, Zeldovich was one of only a handful of particle physicists who were also thinking about problems in cosmology. During this period, he made early contributions to black hole thermodynamics, recognized that accretion disks around black holes could power quasars, discussed the possibility of primordial black holes, and studied the problem of how the large scale structure of the Universe formed. He is probably most famous for his paper with Rashid Sunyaev, which predicted that the cosmic microwave background would be distorted by its inverse Compton scattering with high-energy electrons in galaxy clusters (Sunyaev and Zeldovich, 1970). This so-called “Sunyaev-Zeldovich effect” was observed for the first time in 1983 and continues to be of considerable importance in modern cosmology. So sweeping were Zeldovich’s contributions to cosmology that upon being introduced, Stephen Hawking is said to have expressed to him, “Before I met you, I believed you to be a collective author, like Bourbaki.”

In their 1966 paper, Zeldovich and Gershtein considered the production of neutrinos under the conditions that existed shortly after the big bang. Making use of the knowledge of the newly discovered 3° cosmic microwave background (Penzias and Wilson, 1965), they predicted how many electron and muon neutrinos would have existed in thermal equilibrium in the early Universe, and at what temperature those particles would have ceased to efficiently self-annihilate, leading to a population of neutrinos that survived as a thermal relic. Considering how the density of those neutrinos would impact the expansion history of the Universe, and comparing that to the dynamical evidence for dark matter, it was an indication that weakly interacting particles could be very abundant in our Universe and may have had an observable impact on its evolution. The connection between particle physics and the missing mass problem did gradually become more appreciated over the years to come. In 1976, A. S. Szalay and G. Marx published a paper that not only derived an upper limit on neutrino masses from cosmology, but also discussed the possibility that ~10 eV neutrinos might make up the missing mass in the Universe and in galaxy clusters (Szalay and Marx, 1976). Then, a few years later, a sequence of related papers appeared in rapid succession. In a paper received in April 1977, Piet Hut presented a limit on the neutrino mass from cosmological considerations, ruling out masses in the range of 120 eV to 3 GeV (Hut, 1977). In contrast to the authors of the preceding papers, Hut pointed out that quite heavy neutrinos ($m_\nu > 3$ GeV) would be produced in the big bang with an abundance that would not overclose the Universe. Only about a week later, Ben Lee and Steven Weinberg submitted a paper that included a very similar lower bound ($m_\nu > 2$ GeV) (Lee and Weinberg, 1977). In the same month, a paper by Sato and Kobayashi (1977) presented similar conclusions, and another by Duane Dicus, Edward “Rocky” Kolb, and Vigdor Teplitz pointed out that such bounds could be evaded if neutrinos were unstable (Dicus, Kolb, and Teplitz, 1977). A month later, a new paper by Zeldovich (with M. I. Vysotsky and A. D. Dolgov) appeared, updating their own cosmological constraints on neutrino mass (Vysotsky, Dolgov, and Zeldovich, 1977).

Despite the very interesting and important results of these papers, it is evident that most of them did not attempt to address, or even acknowledge, the possibility that neutrinos could account for the missing mass observed by astronomers on galactic and cluster scales. Exceptions to this include the 1976 paper of Szalay and Marx and the 1977 paper of Lee and Weinberg, whose final sentence reads as follows (Lee and Weinberg, 1977):

“Of course, if a stable heavy neutral lepton were discovered with a mass of order 1–15 GeV, the gravitational field of these heavy neutrinos would provide a plausible mechanism for closing the universe.”

While this is still a long way from acknowledging the dynamical evidence for dark matter, it was an indication that physicists were beginning to realize that weakly interacting particles could be very abundant in our Universe and may have had an observable impact on its evolution. The connection between particle physics and the missing mass problem did gradually become more appreciated over the years to come. In 1978, for example, a paper by James Gunn, Ben Lee, Ian Lerche, David Schramm, and Gary Steigman...
included the following statement in their abstract (Gunn et al., 1978):

“… such a lepton is an excellent candidate for the material in galactic halos and for the mass required to bind the great clusters of galaxies.”

By the end of the decade, a number of scientists, including Zeldovich and his Moscow group (Doroshkevich et al., 1980a, 1980b; Zeldovich et al., 1980), had begun to argue in favor of neutrinos as dark matter. Interest in this possibility grew considerably in and after 1980, when a group studying tritium beta decay reported that they had measured the mass of the electron antineutrino (and presumably also the electron neutrino) to be approximately 30 eV (Lubimov et al., 1980). With a mass of this value, neutrinos would be expected to have played a significant role in cosmology. And although this “discovery” was eventually refuted, it motivated many particle physicists to further investigate the cosmological implications of their research and encouraged many astrophysicists to consider the possibility that the dark matter halos surrounding galaxies and galaxy clusters might not be made up of faint stars or other astrophysical objects, but instead might consist of a gas of nonbaryonic particles.

By the middle of the 1980s, a new tool had come into use that would put neutrino dark matter to the test. This tool, numerical simulations, could be used to predict how large numbers of dark matter particles would evolve under the force of gravity in an expanding universe and thus was able to assess the cosmological role and impact of dark matter particles on the formation of large scale structure. Importantly, such tests could be used to discriminate between different dark matter candidates, at least in some cases.

The primary characteristic of a given particle dark matter candidate that can be probed by numerical simulations is whether it was relativistic (hot) or nonrelativistic (cold) during the epoch of structure formation. Standard model neutrinos, being very light thermal relics, are predicted to emerge from the early Universe with a highly relativistic velocity distribution and thus represent an example of hot dark matter (Schramm and Steigman, 1981; Peebles, 1982b). Simulations have shown that the distribution of hot dark matter particles would tend to collapse and form very large structures first and only later go on to form smaller (i.e., galaxy-sized) halos through the fragmentation of larger halos. In contrast to this “top-down” sequence of structure formation, cold dark matter particles form structures through a “bottom-up” sequence, beginning with the smallest halos, which go on to form larger halos through a succession of mergers.

From these early simulations, it quickly became clear that hot and cold dark matter lead to very different patterns of large scale structure. By comparing the results of these simulations with those of galaxy surveys (in particular, the CfA survey, which was the first extensive 3D survey of galaxies in the local universe) (Davis et al., 1982), it was determined that standard model neutrinos, or any other examples of hot dark matter, could not account for most of the dark matter in the Universe (White, Frenk, and Davis, 1983). In their 1983 paper, Simon White, Carlos Frenk, and Marc Davis made the following statement about a neutrino-dominated universe (White, Frenk, and Davis, 1983):

“We find [the coherence length] to be too large to be consistent with the observed clustering scale of galaxies… The conventional neutrino-dominated picture appears to be ruled out.”

We will discuss numerical simulations, and their role in the history of dark matter, in greater detail in Sec. VIII.C.

As it became accepted that standard model neutrinos could not make up most of the Universe’s dark matter, it also became clear that there must exist at least one currently unknown particle species that makes up the missing mass. But although standard model neutrinos were far too light and hot to make up the dark matter, this new information did not preclude the possibility that other types of neutrino-like particles might make up this elusive substance (see, for example, Olive and Turner (1982)). In 1993, Scott Dodelson and Lawrence Widrow proposed a simple scenario in which an additional neutrino species, without the electroweak interactions experienced by standard model neutrinos, could be produced in the early Universe and realistically make up the dark matter (Dodelson and Widrow, 1994). Other than through gravity, the particles envisioned by Dodelson and Widrow interact only through a small degree of mixing with the standard model neutrinos. With such feeble interactions, such particles would have never been in thermal equilibrium in the early Universe, but instead would have been produced through the oscillations of the other neutrino species. Depending on their mass, such sterile neutrinos could be produced with a wide range of temperatures, and thus could constitute either a warm ($m_{\nu_s} \sim$ keV) or a cold ($m_{\nu_s} \gg$ keV) candidate for dark matter. Although dark matter in the form of sterile neutrinos generated strictly through the mechanism originally described by Dodelson and Widrow is now ruled out, variants of this scenario remain viable.

B. Supersymmetry

Among the particle species contained within the standard model, neutrinos are the only examples that are stable, electrically neutral, and not strongly interacting, and therefore are the only known particles that were viewed as potentially viable candidates for dark matter. Physicists’ imagination, however, would not remain confined to the standard model for long, but instead would turn to the contemplation of many speculative and yet undiscovered candidates for the dark matter of our Universe. In particular, beginning in the early 1970s, many physicists began to consider the possibility that nature may contain a spacetime symmetry relating fermions to bosons, dubbed “supersymmetry” (Gervais and Sakita, 1971;11A possible exception being the tau neutrino, whose mass would not be measured for another two decades and thus could not at the time be ruled out as a cold dark matter candidate.

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10The terms “hot” and “cold” dark matter were coined in 1983 by Joel Primack and Dick Bond (J. Primack, private communication).
Golfand and Likhtman, 1971; Volkov and Akulov, 1972; Wess and Zumino, 1974). Supersymmetry requires that for every fermion, a boson must exist with the same quantum numbers, and vice versa. Supersymmetry, therefore, predicts the existence of several new electrically neutral and nonstrONGLY interacting particles, including the superpartners of the neutralinos, photon, Z boson, Higgs boson, and graviton. If any of these superpartners were stable, they could be cosmologically abundant and may have played an important role in the history and evolution of our Universe.

The cosmological implications of supersymmetry began to be discussed as early as the late 1970s. In Piet Hut’s 1977 paper on the cosmological constraints on the masses of neutralinos (as previously described), the discussion was not entirely limited to neutrinos or even to weakly interacting particles. Even the abstract of that paper mentions another possibility (Hut, 1977):

“Similar, but much more severe, restrictions follow for particles that interact only gravitationally. This seems of importance with respect to supersymmetric theories.”

The paper goes on to close with the first cosmological bounds on the mass of the supersymmetric partner of the graviton, the spin 3/2 gravitino:

“Assuming the standard big bang model to be relevant in the context of supergravity theories, one can make the following remark. If there exist light massive spin 3/2 particles interacting only gravitationally, having four spin degrees of freedom, their mass must be less than 15 eV if they are their own antiparticles, otherwise their mass is less than 1.5 eV. Also, they may exist with masses very much larger than 1 TeV.”

Although such bounds would be revised in the decades to follow, in particular, being shown to depend on the temperature to which the Universe was reheated following inflation, this result is essentially the basis of what is known today as the “cosmological gravitino problem.”

In their 1982 paper, Heinz Pagels and Joel Primack also considered the cosmological implications of gravitinos (Pagels and Primack, 1982). But unlike Hut’s paper, or the other preceding papers that had discussed neutralinos as a cosmological relic, Pagels and Primack were clearly aware of the dark matter problem and explicitly proposed that gravitinos could provide the solution by making up the missing mass (Pagels and Primack, 1982):

“Gravitinos could also provide the dark matter required in galactic halos and small clusters of galaxies.”

In many ways, Pagel and Primack’s letter reads like a modern paper on supersymmetric dark matter, motivating supersymmetry by its various theoretical successes and attractive features, and going on to discuss not only the missing mass in galaxies and clusters, but also the role that dark matter could play in the formation of large scale structure. At the time of Pagel and Primack’s submission, however, supersymmetry itself had not yet taken its modern form, and no truly realistic supersymmetric models had been proposed (although many important steps had been made in this direction) (Fayet, 1975, 1976, 1977a, 1977b). This changed in December 1981, when a paper by Savas Dimopoulos and Howard Georgi described a model that would become known as the minimal supersymmetric standard model (MSSM) (Dimopoulos and Georgi, 1981).

The advent of the MSSM opened the door to considering superpartners other than the gravitino as cosmological relics. In particular, in the MSSM, the superpartners of the photon, the Z, and two neutral scalar Higgs bosons mix to form four particles that would become known as neutralinos. Over the past three and a half decades, neutralinos have been the single most studied candidate for dark matter, having been discussed in many thousands of scientific publications. In order to be the dark matter, however, something must stabilize the lightest neutralino, preventing these particles from decaying shortly after being created.

In supersymmetric extensions of the standard model, there exist interactions that violate the conservation of baryon and lepton number. Unless the relevant couplings are highly suppressed, such interactions are expected to cause the proton to decay on unacceptably short time scales, on the order of a year or less. It was recognized early in supersymmetry’s development, however, that the proton’s lifetime could be made to safely exceed observational limits if an additional, and well-motivated, symmetry known as R parity (Fayet, 1975, 1977b; Salam and Strathdee, 1975; Farrar and Fayet, 1978) is imposed. The R parity of a given particle is defined as follows:

\[ P_R = (-1)^{2s+3B+L}, \]

where \( s \) is the spin of the particle, and \( B \) and \( L \) are the particle’s baryon number and lepton number, respectively. Under this definition, all of the standard model particles have positive R parity, \( P_R = +1 \), while all of their superpartners have \( P_R = -1 \). As a consequence, this parity ensures that superpartners can be created or destroyed only in pairs. A heavy superpartner can decay into a lighter superpartner, along with any number of standard model particles, but the lightest of the superpartners cannot decay. Thus if the lightest superpartner of the MSSM is either a neutralino or a sneutrino (the superpartner of a standard model neutrino), R parity will stabilize it, allowing it to be a potentially viable dark matter candidate. As far as we are aware, it was Pagels and Primack who were the first to invoke R parity in order to stabilize a dark matter candidate (Pagels and Primack, 1982).

Papers discussing the cosmological implications of stable neutralinos began to appear in 1983.\footnote{Unstable but long-lived photinos had been considered earlier by Cabibbo, Farrar, and Maiani (1981).} In the first two of these papers, Weinberg (1983) and Goldberg (1983) independently discussed the case of a photino—a neutralino whose composition is dominated by the superpartner of the photon—and
derived a lower bound of 1.8 GeV on its mass by requiring that the density of such particles does not overclose the Universe. A few months later, a longer paper by John Ellis, John Hagelin, Dimitri Nanopoulos, Keith Olive, and Mark Srednicki considered a wider range of neutralinos as cosmological relics (Ellis et al., 1984). In Goldberg’s paper, there is no mention of the phrase dark matter or of any missing mass problem, and Ellis et al. took a largely similar approach, simply requiring that the cosmological abundance of neutralinos not be so large as to overly slow or reverse the Universe’s expansion rate. Ellis et al., however, did mention the possibility that neutralinos could make up the dark matter, although only in a single sentence (Ellis et al., 1984):

“A more restrictive constraint follows from the plausible assumption that a non-relativistic [supersymmetric] fermion would participate in galaxy formation, in which case the limits on “dark matter” in galaxies allow one to deduce that $\rho_x \leq 2 \times 10^{-30}(\Omega h^2)$ gm/cm$^3$.”

and in a passing footnote of (Ellis et al., 1983)

“This bound comes from the overall density of the universe and is very conservative. One can argue that massive neutral fermions probably condense into galaxies in which case a more stringent limit coming from missing galactic matter could be applied.”

Although far from a full embrace of a particle physics solution to the dark matter problem, these sentences [along with those expressed by Pagels and Primack (1982), and Peebles (1982b) within the context of massive neutrinos] reflected the emergence of a new perspective. Throughout the decades to follow, a countless number of particle physicists would motivate their proposals for physics beyond the standard model by showing that their theories could account for the Universe’s dark matter. Despite any other attractive features that a given theory might possess, if it cannot provide a dark matter candidate, it would come to be viewed as incomplete.

That supersymmetric particles, and the lightest neutralino, in particular, have received so much attention as dark matter candidates, it would come to be viewed as incomplete. The cosmological density of matter to be consistent with the observed density of stars, gas, and other baryons, particle physicists in that universe may have been just as interested in supersymmetry as they are in ours. In this respect, supersymmetry’s ability to provide a viable dark matter candidate is seen by many particle physicists as something of a bonus, rather than as the primary motivation to study such theories.

Supersymmetry, however, is not the only particle physics framework that is both strongly motivated in its own right and able to provide a viable candidate for the dark matter of our Universe. In the next section, we will turn our attention to perhaps the second most studied candidate for dark matter, the axion.

C. Axions

By all measures, quantum chromodynamics (QCD) has been an incredibly successful theory and describes the strong force and the quarks and gluons which experience it with remarkable precision. That being said, QCD does suffer from one troubling issue, known as the strong-CP problem. This problem comes down to the fact that the QCD Lagrangian contains the following term:

$$\mathcal{L}_{\text{QCD}} \supset \tilde{\Theta} \frac{g^2}{32\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu},$$

where $G^{\mu\nu}$ is the gluon field strength tensor and $\tilde{\Theta}$ is a quantity closely related to the phase of the QCD vacuum. If $\tilde{\Theta}$ were of order unity, as would naively be expected, this term would introduce large charge-parity (CP) violating effects, causing the electric dipole moment of the neutron to be $\sim 10^{10}$ times larger than experimental upper bounds permit. Therefore, to be consistent with observations, the quantity $\tilde{\Theta}$ must be smaller than $\sim 10^{-10}$. While this could be nothing more than a highly unlikely coincidence, it has been interpreted by many as an indication that some new physics comes into play to explain why $\tilde{\Theta}$ is so small. This is the essence of the strong-CP problem.

What is perhaps the most promising solution to this problem was proposed by Peccei and Quinn (1977a, 1977b). They showed that by introducing a new global $U(1)$ symmetry that is spontaneously broken, the quantity $\tilde{\Theta}$ can be dynamically driven toward zero, naturally explaining the small observed value. Later in the same year, Wilczek (1978) and Weinberg (1978) each independently pointed out that such a broken global symmetry also implies the existence of a Nambu-Goldstone boson called the axion. The axion acquires a small mass as a result of the $U(1)$ symmetry’s chiral anomaly, on the order of $m_a \sim \lambda_{\text{QCD}}^2/f_{\text{PQ}}$, where $f_{\text{PQ}}$ is the scale at which the symmetry is broken and $\lambda_{\text{QCD}} \sim 200$ MeV is the scale of QCD.

In its original conception, $f_{\text{PQ}}$ was taken to be near the weak scale, leading to an MeV-scale axion mass. This scenario was quickly ruled out, however, by a combination of laboratory and astrophysical constraints. In particular, in contradiction with observation, axions heavier than $\sim 10$ keV are predicted to induce sizable rates for a number of exotic meson decays, such as $K^+ \rightarrow \pi^+ + a$ and $J/\psi \rightarrow \gamma + a$. Similarly,
axions heavier than $\sim 1$ eV would lead to the very rapid cooling of red giant stars, again in contradiction with observations. Some years later, after the occurrence and observation of Supernova 1987A, even stronger constraints were placed on the axion mass $m_a \lesssim 10^{-3}$ eV. For a review of constraints on axions, see Chap. 10 of Kolb and Turner (1990).

In order to evade these constraints, axions must be much lighter, and much more feebly interacting (Kim, 1979; Shifman, Vainshtein, and Zakharov, 1980; Dine, Fischler, and Srednicki, 1981), than had been originally envisioned by Wilczek and Weinberg. Such light and “invisible” axions, however, can have interesting consequences for cosmology. Being stable over cosmological time scales, any such axions produced in the early Universe will survive and, if sufficiently plentiful, could constitute the dark matter.

A number of mechanisms have been considered for the production of axions in the early Universe. As with other particle species, axions can be produced thermally (Kephart and Weiler, 1987; Turner, 1987). For axions light enough to avoid the constraints, however, the thermal relic abundance is predicted to be very small and would be able to account only for a small fraction of the dark matter density. There is, however, another production mechanism, related to the misalignment of the Peccei-Quinn field, that is likely to be more important in the mass range of interest (Abbott and Sikivie, 1983; Dine and Fischler, 1983; Preskill, Wise, and Wilczek, 1983). Although the quantity $\Theta$ is dynamically driven to zero by the mechanism proposed by Peccei and Quinn, its initial value was likely to be some much larger value, presumably determined through some random process. As the temperature of the Universe dropped below $T \sim \lambda_{\text{QCD}}$, and the value of $\Theta$ was driven toward zero, the energy that had been stored in the Peccei-Quinn field gets transferred into the production of a nonthermal axion population. For typical initial conditions, this process of misalignment production is predicted to generate a density of axions that is comparable to the dark matter density for masses on the order of $m_a \sim 10^{-5}$ eV. Alternatively, it was pointed out that as a consequence of $\Theta$ taking on different initial values in different locations throughout space, a network of topological defects (axionic strings and domain walls) may be expected to form. The subsequent decay of these defects is predicted to generate a quantity of axions that is comparable to that resulting from misalignment production (Davis, 1986). Inflation will erase this network of topological defects, however, unless it occurs prior to the breaking of the Peccei-Quinn symmetry.

In light of these considerations, axions with masses in the range of $m_a \sim 10^{-6} - 10^{-4}$ eV, and generated largely via misalignment production, have become one of the most popular and well-studied candidates for dark matter. Alternatively, it was also pointed out that if inflation occurs after the breaking of the Peccei-Quinn symmetry, then there may also be a viable anthropic scenario in which the axion mass could be much lighter (Linde, 1991; Wilczek, 2004; Tegmark et al., 2006). In this scenario, the initial value of $\Theta$ is of order unity in most regions, leading to very high axion densities and to the rapid contraction of space. In a small fraction of the overall cosmic volume, however, the initial value of $\Theta$ will be much lower, leading to far less axion production. If we speculate that life is only able to emerge in those regions in which the Universe is allowed to expand for millions or billions of years or more, we should expect to find ourselves in a region with a density of axions that is similar to the observed density of dark matter, even if the axion is much lighter than nonanthropic estimates would lead us to expect.

D. The WIMP paradigm

By the end of the 1980s, the conclusion that most of the mass in the Universe consists of cold and nonbaryonic particles had become widely accepted, among many astrophysicists and particle physicists alike. And while alternatives continued to be discussed (see the following two sections), cold dark matter in the form of some unknown species of elementary particle had become the leading paradigm. In addition to massive neutrinos (sterile or otherwise), supersymmetric particles (neutralinos, gravitinos, sneutrinos, axinos) and axions were each widely discussed as prospective dark matter candidates. And as the evidence in favor of nonbaryonic dark matter became increasingly compelling, an ever greater number of particle physicists began to openly speculate about the nature of this invisible substance. The result of this was a long and diverse list of exotic possibilities, ranging from topological defects produced through spontaneous symmetry breaking in the early Universe (monopoles, cosmic strings) (Kibble, 1976) to macroscopic configurations of quark matter (centimeter-scale “nuggets” with nuclear-scale densities) (Witten, 1984), and even “pyrgons” (Kaluza-Klein excitations) that could appear within the context of models with extra spatial dimensions (Kolb and Slansky, 1984).

While this proliferation of dark matter candidates was taking place, however, a commonality among many of the proposed particles was becoming increasingly appreciated. In order for a particle species to freeze-out of thermal equilibrium in the early Universe to become a cold relic, it must not be too light (roughly heavier than $\sim 1-100$ keV). Furthermore, for the predicted thermal relic abundance of such a species to match the observed dark matter density, the dark matter particles must self-annihilate with a cross section on the order of $\sigma v \sim 10^{-26}$ cm$^3$/s (where $v$ is the relative velocity between the annihilating particles). This number is strikingly similar to the cross section that arises from the weak force. For example, a stable neutrino with a mass of several GeV, annihilating through the exchange of a $Z^0$ boson, would freeze-out with a relic abundance that is roughly equal to the measured density of dark matter. Furthermore, such conclusions are not limited to neutrinos, but apply to a broad range of electroweak-scale dark matter candidates—including any number of stable particles with MeV–TeV masses and interactions that are mediated by the exchange of electroweak-scale particles. This observation, combined with theoretical arguments in favor of the existence of new physics at or around the electroweak scale, has elevated WIMPs (Steigman and Turner, 1985) to
the leading class of candidates for dark matter.\textsuperscript{14} WIMPs have been the subject of thousands of theoretical studies, leading to the refinement of many calculations, including that of the dark matter’s thermal relic abundance (Srednicki, Watkins, and Olive, 1988; Gondolo and Gelmini, 1991; Griest and Seckel, 1991). Furthermore, WIMPs (and to a somewhat lesser degree, axions) have motivated an expansive experimental program that continues to this day. With the advent of the Large Hadron Collider at CERN, and ever more sensitive astrophysical experiments, many believe that the moment of truth has come for WIMPs: either we will discover them soon or we will begin to witness the decline of the WIMP paradigm (Bertone, 2010).

\section*{VIII. PIECING THE PUZZLE}

\subsection*{A. Discrepancies at all scales}

When Fritz Zwicky proposed in 1933 that dark matter might be responsible for the high velocity dispersion of galaxies in the Coma Cluster (see Sec. III.A), he was familiar with the concept of dark matter, and with earlier attempts to dynamically measure the density of dark matter in the Galaxy. Over the decades that followed, however, the presence of dark matter in clusters and in galaxies was discussed largely independently of each other. It was not until the 1960s that mass discrepancies on multiple scales began again to be considered within a common context.

In his pioneering paper of 1963, Arigo Finzi cited Zwicky’s 1933 work on galaxy clusters, the 1957 observation of M31’s rotation curve from van de Hulst \textit{et al.}, as well as more recent determinations of the mass of the Milky Way, and argued in favor of a common interpretation for these phenomena (Finzi, 1963). He then went on to consider various possible forms of what we today call “baryonic” dark matter (see Sec. V), ruling them out one by one. He even went as far as to suggest that these phenomena might be explained by modifying Newton’s gravitational force law, so that it scaled as $r^{-3/2}$ at large distances.

Despite the highly original and prescient nature of Finzi’s work, it was largely ignored by the scientific community (Sanders, 2010), attracting only 50 citations over the past 50 years. Although it is impossible to unambiguously identify the precise reasons for this, the bold nature of Finzi’s conclusions may have been difficult for many of his colleagues to accept or even seriously consider. In any case, this work had little impact, and it would be another decade before other scientists began to pursue similar lines of inquiry.

As mentioned in Sec. IV, two independent groups published groundbreaking papers in 1974, each presenting a strong case for the existence of large amounts of mass in the outer parts of galaxies. The first of these papers, by the Estonian astronomers Jaan Einasto, Ants Kaasik, and Enn Saar, was submitted on April 10 and was entitled “Dynamic evidence on massive coronas of galaxies” (Einasto, Kaasik, and Saar, 1974). These authors began with a discussion of galactic rotation curves, citing the work of Roberts (1975a) and presented an analysis of rotation curve data that included estimates for the contributions from stars for five galaxies of different masses. They argued that the discrepancy between the total mass and the stellar mass implied the existence of a corona, consisting of a “previously unrecognized, massive population.” They then used 105 pairs of galaxies to estimate the total mass and dimensions of their galactic coronas, concluding that the total mass of galaxies exceeded that in stars by an order of magnitude. Finally, the authors argued that these new mass estimates could also explain the mass discrepancy that had been observed in clusters. Similar arguments had also been sketched earlier by Einasto, including at the 1972 IAU meeting in Athens (Einasto, 1972).

On May 28—about six weeks after Einasto \textit{et al.}—Jerry Ostriker, Jim Peebles, and Amos Yahil submitted a paper of similar content and scope, entitled “The size and mass of galaxies, and the mass of the universe” (Ostriker, Peebles, and Yahil, 1974). This paper did not present any new observations, but instead compiled existing estimates for the masses of mostly giant spiral galaxies. They began with galactic rotation curves, citing Roberts and Rots (1973) and Rogstad and Shostak (1972) as evidence for their flatness in the outer parts of galaxies. They then went on to build a case for the existence of large amounts of dark matter in the outer parts of galaxies, based on mass estimates from galaxy pairs, the dynamics of dwarf galaxies, and the so-called timing argument for the local group (see Sec. IV). And although the observations presented in this paper were not new, and were subject to large uncertainties, they appear to have been confident in their conclusions, stating that the trend of increasing mass with increasing radius is “almost certainly real” and arguing that this trend was in line with the “virial discrepancy” that had been observed in clusters and groups of galaxies (Rood, Rothman, and Turnrose, 1970; Field and Saslaw, 1971; Rood, 1974). The first sentences of this paper’s body summarizes well their sentiment:

“There are reasons, increasing in number and quality, to believe that the masses of ordinary galaxies may have been underestimated by a factor of 10 or more. Since the mean density of the Universe is computed by multiplying the observed number density of galaxies by the typical mass per galaxy, the mean density of the Universe would have been underestimated by the same factor.”

In 1979, Sandra Faber and John Gallagher published an influential review, “Masses and mass-to-light ratios of galaxies” (Faber and Gallagher, 1979), which played an important role in crystallizing the opinion among cosmologists and astronomers that dark matter was indeed abundant in the Universe. Interestingly, they chose not to use the terms

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\textsuperscript{14}Although the term WIMP, as coined by Gary Steigman and Michael Turner in 1984, was originally intended to include all particle dark matter candidates, including axions, gravitinos, etc., the definition of this term has since evolved to more often denote only those particles that interact through the weak force.
“corona” or “halo” as suggested by the two previous papers, but instead adopted the phrase “massive envelope” to describe the distribution of dark matter in astrophysical systems.\footnote{In the discussions that took place as part of our research for this historical review, we encountered a considerable range of opinions regarding the relative importance of galactic rotation curves in establishing the existence of dark matter. This supports a picture in which different groups of scientists found quite different lines of evidence to be compelling during this period of time. Despite these disagreements regarding the strengths and weaknesses of the various observations and arguments, a consensus nonetheless began to emerge in favor of dark matter’s existence.}

B. Cosmology

As astronomers continued to gather information on the masses of galaxies, and on other observables of cosmological relevance, cosmologists began to increasingly reflect upon the implications of those findings for the formation of structure and the evolution of the Universe. In 1974, the same year as the two key papers previously described (Einasto, Kaasik, and Saar, 1974; Ostriker, Peebles, and Yahil, 1974), Richard Gott, James Gunn, David Schramm, and Beatrice Tinsley published a paper that provides us with an illuminating snapshot of the status of cosmology at that time (Gott et al., 1974). The conclusions of this paper, entitled “An unbound universe,” appear within the original abstract:

“A variety of arguments strongly suggest that the density of the universe is no more than a tenth of the value required for closure. Loopholes in this reasoning may exist, but if so, they are primordial and invisible, or perhaps just black.”

In this paper, they argued that the body of astronomical data indicated that there was simply not enough matter in the Universe—even accounting for the large mass-to-light ratios observed among galaxies—to equal or exceed the critical density of the Universe. Among other caveats to this conclusion, they considered possible contributions from low-mass neutrinos as had been suggested by Cowsik and McClelland (1972), but ultimately ruled out this possibility as well.

In the early 1980s, the introduction of the theory of inflation profoundly changed the thinking of the cosmological community and allowed one for the first time to make specific predictions for the total cosmological density and for the spectrum of density perturbations (Guth, 1981; Guth and Pi, 1982; Hawking, 1982; Linde, 1982; Starobinsky, 1982; Bardeen, Steinhardt, and Turner, 1983). This began a decade-long struggle to reconcile models of structure formation with the evolution of the Universe in 1998 (Riess et al., 1998; Perlmutter et al., 1999), and the contribution to the total energy density arising from a cosmological constant or “dark energy.”

Meanwhile, Jim Peebles had pointed out that the absence of fluctuations in the cosmic microwave background at a level of $\sim 10^{-4}$ was incompatible with a universe that was composed of only baryonic matter and argued that this problem would be relieved if the Universe were instead dominated by massive, weakly interacting particles, whose density fluctuations could begin to grow prior to decoupling (Peebles, 1982a) [see also Chemin (1981)]. This and other papers that will be discussed in the next section received enormous attention from the scientific community and rapidly led to the establishment of cold dark matter as the leading paradigm to describe the structure and evolution of the Universe at all scales.

C. Numerical simulations

Much of our current understanding of the structure and evolution of dark matter halos in the Universe is based on the results of computer simulations. Such explorations have a longer history than one might expect. Working in the 1940s, the ingenious Swedish scientist Erik Holmberg exploited the fact that light follows the same inverse square law as the gravitational force and performed the first simulation of the interaction between two galaxies on an analog computer that consisted of 74 light bulbs, photocells, and galvanometers. He then calculated the amount of light received by each cell and manually moved the light bulbs in the direction that received the most light (Holmberg, 1941).

Holmberg published his paper in November 1941, shortly before the United States entered World War II. In the following years, the work of many research institutes ground to a halt, but science meanwhile continued to make progress, thanks in large part to the enormous resources made available to military research, especially at the Los Alamos National Laboratory, at which computers and advanced numerical techniques were developed within the context of the Manhattan Project. The first application of such computers to gravitational systems was arguably performed by John Pasta and Stanislaw Ulam in 1953. Their numerical experiments were performed on the Los Alamos computer, which by then had been applied to a variety of other problems, including early attempts to decode DNA sequences and the first chess-playing program. A number of other pioneering studies exploring the evolution of a system of gravitationally interacting massive particles appeared throughout the 1960s and 1970s, gradually increasing the number of simulated particles from $N \sim 100$ in the early works to $N \sim 1000$ (von Hoerner, 1960, 1963; Aarseth, 1963; Peebles, 1970; White, 1976).

By the early 1970s, it had become possible to numerically simulate the dynamics of galaxies. Simulations carried out by Miller, Prendergast, and Quirk (1970) as well as by Hohl (1971) each found rotationally supported galaxies consisting of a stellar disk to be unstable, in contradiction with observations. Instead of reaching an equilibrium configuration, such systems were found to change rapidly, forming bars and evolving toward a more elliptical and pressure supported configuration. The solution to this problem was proposed in 1973 by Jerry Ostriker and Jim Peebles, who recognized that a
rotational support of a stellar disk could be stable if embedded within a massive spherical halo (Ostriker and Peebles, 1973).

The first attempt to numerically solve the formation and evolution of cosmological structures in an expanding universe was presented by Press and Schechter, (1974). This was followed by a number of developments in the late 1970s and early 1980s that significantly advanced the power of such endeavors (Aarseth, Turner, and Gott, 1979; Gott, Turner, and Aarseth, 1979; Turner et al., 1979; Efstathiou and Eastwood, 1981; Centrella and Melott, 1983; Klypin and Shandarin, 1983). First, a combination of improvements in processor speed and in numerical techniques made it possible for the first time to simultaneously simulate millions of particles. Second, the newly proposed theory of inflation (Guth, 1981; Linde, 1982) offered a physical means by which initial density perturbations could be generated, providing the initial conditions for cosmological simulations. And, third, the results of the first large 3D survey of galaxies, the CfA redshift survey, were published in 1982, providing a distribution that could be directly compared with the output of simulations (Davis et al., 1982).

In some ways, the results of cosmological simulations do not depend much on what the dark matter consists of. In particular, they are largely insensitive to the electroweak or other non-gravitational interactions that may (or may not) be experienced by dark matter particles—for the purposes of structure formation, such particles are effectively “collisionless.” What does impact the results of such simulations, however, is the initial velocity distribution of the dark matter particles (Bond, Efstathiou, and Silk, 1980; Doroshkevich et al., 1980a; Bond and Szalay, 1983). Importantly, this provides cosmologists with a way to discriminate between different classes of dark matter candidates. Standard model neutrinos, for example, decoupled from thermal equilibrium in the early Universe at a temperature that is much greater than their mass and thus remained highly relativistic throughout the epoch of structure formation. In contrast, supersymmetric neutralinos are predicted to freeze-out of thermal equilibrium at a temperature below their mass, and are thus non-relativistic throughout cosmic history. Axions generated through misalignment production are also predicted to be produced with non-relativistic velocities.

At the largest scales, those associated with galaxy clusters and superclusters, cosmological simulations predict a pattern of structure that is largely insensitive to the initial velocities of the dark matter. At smaller scales, however, density fluctuations can be washed out by the random thermal motion of individual dark matter particles. As a result, the growth of small scale structure is predicted to be suppressed if the dark matter is relativistic, or hot (Schramm and Steigman, 1981; Peebles, 1982b). Nonrelativistic, or cold dark matter particles undergo a very different sequence of structure formation. The much shorter free-streaming length of such particles allows them to form very low-mass halos; roughly in the range of $10^{-3}M_\odot$ to $10^{-9}M_\odot$ for a typical neutralino, for example. These very small halos form very early in the Universe’s history and then go on to merge with one another, gradually building up larger and larger dark matter structures. This bottom-up, or hierarchical, process of structure formation is in stark contrast to the top-down sequence predicted for hot dark matter.

Simulations of large scale structure are, of course, useful only if their results can be compared to the actual patterns of structure found in the Universe. This was made possible with the CfA survey, which was the first extensive 3D survey of galaxies in the local universe (Davis et al., 1982). Among other features, CfA revealed the first indications of the “cosmic web,” which described the distribution of matter on the largest scales. This survey also identified the presence of significant structure on subcluster scales, in conflict with the predictions of hot dark matter simulations (White, Frenk, and Davis, 1983).

In the wake of the failures of hot dark matter, it was quickly becoming appreciated that cold dark matter could do a much better job of accounting for the observed patterns of large scale structure. To quote the 1984 paper by George Blumenthal, Sandra Faber, Joel Primack, and Martin Rees (Blumenthal et al., 1984):

>[“We have shown that a universe with ~10 times as much cold dark matter as baryonic matter provides a remarkably good fit to the observed universe. This model predicts roughly the observed mass range of galaxies, the dissipational nature of galaxy collapse, and the observed Faber-Jackson and Tully-Fisher relations. It also gives dissipationless galactic halos and clusters. In addition, it may also provide natural explanations for galaxy-environment correlations and for the differences in angular momenta between ellipticals and spiral galaxies.”]

The first simulations of cold dark matter were carried out by Marc Davis, George Efstathiou, Carlos Frenk, and Simon White, who published their results in 1985 (Davis et al., 1985). The resemblance of their simulated distribution of dark matter halos to that of the galaxies in the CfA survey was clear, serving to further elevate the status of cold dark matter within the cosmological community.

By the middle of the 1980s, the paradigm of cold dark matter was well on its way to becoming firmly established. And although scenarios involving mixed dark matter (containing significant quantities of both cold and hot dark matter) and warm dark matter (suppressing structure only on the scale of dwarf galaxies and below) would each continue to be discussed in the literature, the possibility that the dark matter was dominated by neutrinos or other relativistic particles was quickly abandoned.

A decade later, the predictions of cosmological simulations had shifted in focus from the distribution of cold dark matter halos to the shapes of those halos. In 1996, Julio Navarro, Carlos Frenk, and Simon White published a remarkable result, based on an analysis of the halos generated in their cold dark matter simulations (Navarro, Frenk, and White, 1996):

>[“The spherically averaged density profiles of all our halos can be fit over two decades in radius by scaling a simple universal profile. The characteristic overdensity of a halo, or equivalently its..."
concentration, correlates strongly with halo mass in a way which reflects the mass dependence of the epoch of halo formation.”

The simple fitting formula derived became known as the Navarro-Frenk-White profile. This parametrization is still widely used today and represents the primary benchmark for most indirect dark matter detection studies, despite the fact that it is expected to be inaccurate in the innermost regions of galaxies, where baryons dominate the gravitational potential (Pato, Iocco, and Bertone, 2015).

In more recent years, the frontier for cosmological simulations has focused on the implementation of baryonic physics, including the hydrodynamical evolution of gas in astrophysical structures, stellar formation, and feedback from supernova explosions and black holes. Current simulations are not yet able to resolve all relevant scales, which range from subparsec distances for stellar formation to Gpc scales for cosmological structures, but implement baryonic physics through the introduction of suitable “subgrid” parameters which attempt to encode the collective behavior of large amounts of gas and stars. Such parameters are generally tuned to match observable quantities, such as the galaxy mass function and the galaxy-central black hole mass relation, as in, e.g., the recent suite of Eagle simulations (Schaye et al., 2015).

IX. THE HUNT FOR DARK MATTER PARTICLES

As particle physicists became increasingly interested in the problem of the missing matter of the Universe, some began to turn their attention toward ways that individual particles of dark matter might be detected, either directly or indirectly. Although many of the leading techniques were first conceived of in the 1980s, dark matter searches have continued with vigor ever since, occupying the attention of generations of experimental particle astrophysicists.

A. Scattering with nuclei

In 1984, an article by Andrzej Drukier and Leo Stodolsky at the Max Planck Institute in Munich appeared in Physical Review D, discussing techniques that might be used to detect neutrinos scattering elastically off nuclei (Drukier and Stodolsky, 1984). Among other possibilities, the article proposed the use of a superconducting coil detector, consisting of micron-scale superconducting grains maintained at a temperature just below their superconducting transition. Even a very small quantity of energy deposited by the recoil of an incident neutrino could cause a superconducting grain to flip into the normal state, collapsing the magnetic field and producing a potentially measurable electromagnetic signal. In January 1985, Mark Goodman and Ed Witten submitted a paper to the same journal, arguing that this technology could also be used to detect some types of dark matter particles (Goodman and Witten, 1985).

Although Drukier and Stodolsky’s original detector concept was never employed at a scale sensitive to dark matter, the broader notion of experiments capable of detecting \( \sim 1–100 \text{ keV} \) nuclear recoils provided a path through which it appeared possible to test the WIMP hypothesis.

In their original paper, Goodman and Witten considered three classes of dark matter candidates: (1) those that undergo coherent scattering with nuclei (also known as spin-independent scattering), (2) those that scatter with nuclei through spin-dependent couplings, and (3) those with strong interactions. The first two of these three categories provide the basis for how most direct dark matter detection results have since been presented. If mediated by unsuppressed couplings to the Z boson (an important early benchmark), coherent scattering was predicted to lead to large scattering rates, typically hundreds or thousands of events per day per kilogram of target material. With such high rates, the prospects for detecting dark matter in the form of a heavy neutrino or sneutrino appeared very encouraging. Dark matter candidates that scatter with nuclei only through spin-dependent couplings, in contrast, were generally predicted to yield significantly lower rates and would require larger and more sensitive detectors to test. Even as early as in this first paper, Goodman and Witten pointed out that such experiments would have difficulty detecting dark matter particles lighter than \( \sim 1–2 \text{ GeV} \), due to the modest quantity of momentum that would be transferred in the collisions.

The first experiment to place constraints on the scattering cross section of dark matter with nuclei was carried out in 1986 at the Homestake Mine in South Dakota by a collaboration of scientists at the Pacific Northwest National Laboratory, the University of South Carolina, Boston University, and Harvard (Ahlen et al., 1987). Using a low-background germanium ionization detector (originally designed to search for neutrinoless double beta decay), they accumulated an exposure of 33 kg day, yielding a limit that significantly constrained dark matter candidates with unsuppressed spin-independent scattering cross sections with nuclei (such as heavy neutrinos or sneutrinos) (Ahlen et al., 1987). Shortly thereafter, similar results were obtained by an independent collaboration of scientists from the Universities of California at Santa Barbara and Berkeley (Caldwell et al., 1988).

Despite the importance of these first dark matter scattering limits, the reach of such detectors quickly became limited by their backgrounds, making it difficult to achieve significant improvements in sensitivity. One possible solution to this problem, first suggested by Drukier, Freese, and Spergel (1986), was to search for an annual variation in the rate of dark matter induced events in such an experiment as was predicted to result from the combination of the Earth’s motion around the Sun and the Sun’s motion through the dark matter halo. Such a technique could, in principle, be used to identify a signal of dark matter scattering over a large rate of otherwise indistinguishable background events. The most well-known group to employ this technique was the DAMA/NaI Collaboration (and later DAMA/LIBRA). The original DAMA/NaI experiment consisted of nine 9.70 kg scintillating thallium-doped sodium iodide crystals, located in Italy’s deep underground Gran Sasso Laboratory. In 1998, they published their first results, reporting the observation of an annually

\[ \text{[14]} \] A similar paper by Ira Wasserman was submitted shortly after Goodman and Witten’s (Wasserman, 1986).
modulating rate consistent with dark matter scattering (Bernabei et al., 1998). Over the past nearly two decades, DAMA’s signal has persisted and become increasingly statistically significant as more data were collected (Bernabei et al., 2003), including with the more recent DAMA/LIBRA detector (Bernabei et al., 2008, 2010). At this point in time, it seems hard to reconcile dark matter interpretations of the DAMA/LIBRA signal with the null results of other direct detection experiments, featuring much lower backgrounds. On the other hand, no convincing alternative explanation for this signal has been so far identified.

During the period of time that DAMA/NaI was being developed and collecting its first data, experimental techniques were being pursued that could discriminate dark matterlike nuclear recoil events from various backgrounds. These efforts ultimately led to the technologies employed by the Cryogenic Dark Matter Search (CDMS), the Experience pour DEetecter Les Wimps En Site Souterrain (EDELWEISS), and the Cryogenic Rare Event Search with Superconducting Thermometers (CRESST) Collaborations. These experimental collaborations make use of two-channel detectors, capable of measuring both ionization and heat (CDMS, EDELWEISS) or scintillation and heat (CRESST), the ratio of which could be used to discriminate nuclear recoil events from electron recoils generated by gamma and beta backgrounds. They employ crystalline target materials, maintained at cryogenic temperatures, consisting of germanium and silicon, germanium, and calcium tungstate (CaWO₄), respectively. Throughout most of the first decade of the 21st century, the CDMS and EDELWEISS experiments led the field of direct detection, providing the most stringent constraints and improving in sensitivity by more than 2 orders of magnitude over that period of time (see Fig. 8).

In order to continue to increase the sensitivity of direct dark matter experiments, it was necessary for experiments to employ ever larger targets, gradually transitioning from the kilograms of detector material used by EDELWEISS and CDMS (9.3 kg in the case of SuperCDMS) to the ton scale and beyond. Cryogenic solid state detectors, however, have proven to be costly to scale up to ton-scale experiments. In the late 1990s, Pio Picchi, Hanguo Wang, and David Cline pioneered an alternative technique that exploited liquid noble gas targets (most notably liquid xenon). Like solid state detectors, such experiments discriminate nuclear recoils from electron recoils by measuring two quantities of deposited energy. In the case of dual-phase xenon detectors [first proposed by Dolgoshein, Lebedenko, and Rodionov (1970), and pioneered by the Zeplin-II Collaboration in Alner et al. (2007)], these two quantities are scintillation and ionization. Between 2010 and 2015, the XENON100 and LUX experiments (each of which utilize a liquid xenon target) have improved upon the limits placed by CDMS by approximately 2 orders of magnitude, thanks to their larger target mass and lower background. It is generally anticipated that future experiments employing liquid xenon targets (XENON1T, LZ, XENONnT, PandaX) will continue along this trajectory for years to come.

As CDMS, EDELWEISS, XENON100, LUX, and other direct detection experiments have increased in sensitivity over the past decades, they have tested and ruled out an impressive range of particle dark matter models. And although results from the CoGeNT (Aalseth et al., 2011a, 2011b), CRESST (Angloher et al., 2012), and CDMS (Agnese et al., 2013) experiments were briefly interpreted as possible dark matter signals, they now appear to be the consequences of poorly understood backgrounds (Angloher et al., 2014; Kelso et al., 2014) and/or statistical fluctuations. While many viable WIMP models remain beyond the current reach of this experimental program, a sizable fraction of the otherwise most attractive candidates has been excluded. Of particular note is the fact that these experiments now strongly constrain dark matter particles that scatter coherently with nuclei through Higgs exchange, representing an important theoretical benchmark.

B. Annihilation and decay

In the 14 February 1978 issue of Physical Review Letters, there appeared two articles that discussed, for the first time, the possibility that the annihilations of pairs of dark matter particles might produce an observable flux of gamma rays. And although each of these papers [by Jim Gunn, Ben Lee,17 Ian Lerche, David Schramm, and Gary Steigman (Gunn et al., 1978), and by Floyd Stecker (Stecker, 1978)] focused on dark matter in the form of a heavy stable lepton (i.e., a heavy neutrino), similar calculations would later be applied to a wide range of dark matter candidates. On that day, many hopeless romantics became destined to a lifetime of searching for signals of dark matter in the gamma-ray sky.

At the time, the most detailed measurement of the astrophysical gamma-ray background was that made using data from the Small Astronomy Satellite (SAS) 2 (Fichtel, Simpson, and Thompson, 1978). Although the intensity of

17With regards to Ben Lee, who died in a traffic accident in 1977, this article was published posthumously.
35–100 MeV gamma rays measured by this telescope (∼6 × 10⁻⁵ cm⁻² sr⁻¹) was several orders of magnitude higher than that predicted from annihilating dark matter particles smoothly distributed throughout the Universe, it was recognized that inhomogeneities in the dark matter distribution could increase this prediction considerably. In particular, annihilations taking place within high-density dark matter halos, such as that of the Milky Way, could plausibly produce a flux of gamma rays that was not much fainter than that observed at high galactic latitudes and with a distinctive gradient on the sky (Gunn et al., 1978; Stecker, 1978). Focusing on GeV-scale dark matter particles, Gunn et al. went as far as to state that such a signal “may be discoverable in future γ-ray observations.”

Several years later, in 1984, Joe Silk and Mark Srednicki built upon this strategy, considering not only gamma rays as signals of annihilating dark matter particles, but also cosmic-ray antiprotons and positrons (Silk and Srednicki, 1984) [see also Stecker, Rudaz, and Walsh (1985), Ellis et al. (1988), and Kamionkowski and Turner (1991)]. They argued that the observed flux of ∼0.6–1.2 GeV antiprotons (Buffington, Schindler, and Pennypacker, 1981) provided the greatest sensitivity to annihilating dark matter and noted that ∼10 GeV WIMPs would be predicted to produce a quantity of cosmic-ray antiprotons that was comparable to the observed flux.

In 1985, Lawrence Krauss, Katherine Freese, David Spergel, and William Press published a paper suggesting that neutrinos might be detected from dark matter annihilating in the core of the Sun (L. M. Krauss et al., 1985) [see also Press and Spergel (1985)]. Shortly thereafter, Silk, Olive, and Srednicki pointed out that not only could elastic scattering cause dark matter particles to become gravitationally bound to and captured within the Sun, but that the number of WIMPs captured over the age of the Solar System could be sufficiently high to attain equilibrium between the processes of capture and annihilation (Silk, Olive, and Srednicki, 1985). Observations over the subsequent few years by the proton decay experiments IMB, FREJUS, and Kamioka capitalized on this strategy, strongly constraining some classes of dark matter candidates, most notably including light electron or muon neutrinos. Similar approaches using dark matter capture by the Earth were also proposed around the same time (Freese, 1986; Krauss, Srednicki, and Wilczek, 1986).

In the decades that followed, measurements of astrophysical gamma ray, antimatter, and neutrino fluxes improved dramatically. In parallel, the scientific community’s understanding of the astrophysical sources and propagation of such particles also matured considerably. Information from successive gamma-ray satellite missions, including COS-B (Hermes, Blitz, and Bloemen, 1984), energetic gamma ray experiment telescope (EGRET) (Sreekumar et al., 1998), and the Fermi gamma-ray space telescope, gradually led to the conclusion that most of the observed gamma-ray emission could be attributed to known gamma-ray source classes (such as active galactic nuclei), although it remains possible that a non-negligible component of the high-latitude background could originate from dark matter (Ackermann et al., 2015a).

Motivated by their high densities of dark matter and low levels of baryonic activity, dwarf spheroidal galaxies, satellites of the Milky Way, have in recent years become a prime target of gamma-ray telescopes searching for evidence of dark matter annihilations. Fermi’s study of dwarf galaxies has provided the strongest limits on the dark matter annihilation cross section to date, strongly constraining WIMPs lighter than ∼100 GeV or so in mass (Ackermann et al., 2015b). Ground based gamma-ray telescopes have also used observations of dwarf galaxies to constrain the annihilations of heavier dark matter candidates. Although complicated by imperfectly understood backgrounds, gamma-ray observations of the Milky Way’s Galactic center are also highly sensitive to annihilating WIMPs. A significant excess of GeV-scale gamma rays has been identified from this region, consistent with arising from the annihilations of ∼50 GeV particles (Goodenough and Hooper, 2009; Daylan et al., 2014). An active debate is currently taking place regarding the interpretation of these observations. Alternative targets for indirect searches have also been proposed, including Galactic dark matter subhalos not associated with dwarf galaxies (Diemand, Kuhlen, and Madau, 2007; Pieri, Bertone, and Branchini, 2008), and density “spikes” of dark matter around black holes (Gondolo and Silk, 1999; Bertone, Zentner, and Silk, 2005; Zhao and Silk, 2005).

Over approximately the same period of time, great progress has also been made in the measurement of the cosmic-ray antiproton spectrum, including successive advances by the CAPRICE (Boezio et al., 1997, 2001), BESS (Asaoka et al., 2002; Abe et al., 2008), AMS-01 (Aguilar et al., 2002), and PAMELA (Adriani et al., 2010) experiments. When these measurements are combined with our current understanding of cosmic-ray production and propagation, they appear to indicate that the observed cosmic-ray antiproton spectrum originates largely from conventional secondary production (cosmic-ray interactions with gas), although a significant contribution from dark matter remains a possibility. These measurements generally yield constraints on annihilating dark matter that are not much less stringent than those derived from gamma-ray observations.

Compared to antiprotons, measurements of the cosmic-ray positron spectrum have been more difficult to interpret. Building upon earlier measurements (Mueller and Tang, 1987; Golden et al., 1996; Boezio et al., 2000), the balloon-borne HEAT experiment observed in 1994, 1995, and 2000 indications of an excess of cosmic-ray positrons at energies above ∼10 GeV, relative to the rate predicted from standard secondary production (Barwick et al., 1997). This was later confirmed and measured in much greater detail by a series of space-based experiments: AMS (Aguilar et al., 2007), PAMELA (Adriani et al., 2009), and AMS-02 (Aguilar et al., 2013). Although this positron excess received much attention as a possible signal of annihilating dark matter, this possibility is now strongly constrained by a variety of arguments (Bertone et al., 2009; Galli et al., 2009; Slatyer, Padmanabhan, and Finkbeiner, 2009), and plausible astrophysical explanations have also been proposed (Hooper, Blasi, and Serpico, 2009).

As large volume neutrino telescopes began to be deployed, such experiments became increasingly sensitive to dark matter annihilating in the interiors of the Sun and Earth. The AMANDA detector at the South Pole (Ahrens et al., 2002)
along with Super-Kamiokande in Japan (Desai et al., 2004) each significantly improved upon previous limits to be followed most notably by IceCube (Aartsen et al., 2013) and ANTARES (Adrian-Martinez et al., 2013). Constraints from neutrino telescopes are currently competitive with those derived from direct detection experiments for the case of WIMPs with spin-dependent interactions with protons.

Many of the strategies employed to search for annihilating dark matter have also been used to constrain the rate at which dark matter particles might decay. In addition to constraints on gravitinos and other potentially unstable particles, such searches are particularly interesting within the context of sterile neutrino dark matter. Sterile neutrinos with masses in the range of \( \sim 1–100 \text{ keV} \) are predicted to decay (into an active neutrino and a photon) at a rate that could generate a potentially observable x-ray line (Abazajian, Fuller, and Tucker, 2001). In fact, considering the standard case of Dodelson-Widrow production (as discussed in Sec. VII), the combination of constraints from x-ray observations and measurements of the Lyman-\( \alpha \) forest (Viel et al., 2005; Seljak, Slosar, and McDonald, 2006) disfavor sterile neutrino dark matter over this entire mass range. Models with enhanced production in the early Universe (Shi and Fuller, 1999) can evade such constraints, however, and continue to receive considerable interest. In particular, reports of a 3.55 keV line observed from a collection of galaxy clusters (Boyarsky et al., 2014; Bulbul et al., 2014) have recently received a great deal of attention within the context of a decaying sterile neutrino (Abazajian, 2017).

### C. Axion experiments

For some time, there has been an active experimental program searching for dark matter axions, most notably in the form of the axion dark matter experiment (ADMX). The idea behind this effort is to make use of the photon-photon-axion coupling, generically present in axion models, to convert dark matter axions in a strong and static magnetic field into a signal of nearly monochromatic microwave photons. This possibility was first suggested by Sikivie (1983), and was later expanded upon (Sikivie, 1985) along with L. Krauss et al. (1985). As the signal in such an experiment is maximized for a specific cavity frequency (corresponding to a specific axion mass), it is necessary that the resonant frequency of the cavity be tunable, making it possible to scan over a range of axion masses.

The first laboratory constraints on dark matter axions were presented in the late 1980s by a number of groups (De Panfilis et al., 1987; Wuensch et al., 1989; Hagmann et al., 1990). While the frequency range covered by these experiments was well suited to axion masses favored by dark matter abundance considerations (covering \( m_a \sim 4.5–16.3 \mu \text{eV} \)), their sensitivity was orders of magnitude below that required to test realistic axion models. In 2003, however, the ADMX Collaboration reported results that constrained realistic axion dark matter models, although only for a relatively narrow range of masses 1.9–3.3 \( \mu \text{eV} \) (Asztalos et al., 2004). With anticipated upgrades (Asztalos et al., 2010), ADMX is expected to be sensitive to a much larger range of axion masses and couplings, significantly constraining the axion dark matter parameter space in the coming years.

Other experimental techniques, such as the so-called helioscopes SUMICO, CAST, and IAXO, have recently allowed one to set complementary constraints on axions, and axionlike particles (Irastorza et al., 2011).

### ACKNOWLEDGMENTS

We are grateful to the many pioneering physicists and astronomers who helped us to reconstruct the methods, ideas, and circumstances that led to the establishment of the dark matter paradigm. In particular, we thank Lars Bergström, Andrea Biviano, Albert Bosma, Shantanu Desai, Jaco de Swart, Jaan Einasto, John Ellis, Pierre Fayet, Ken Freeman, Katherine Freese, Steve Kent, Rocky Kolb, Keith Olive, Jerry Ostriker, Sergio Palomares-Ruiz, Jim Peebles, Joel Primack, Morton Roberts, Bernard Sadoulet, Bob Sanders, Gary Steigman, Alar Toomre, Juri Toomre, Scott Tremaine, Virginia Trimble, Michael Turner, and Simon White. We also acknowledge many useful discussions with Jaco de Swart and Jeroen van Dongen. This work would not have been possible without the vast collection of articles and books, dating back to the 19th century, made freely available by NASA ADS and the Internet Archive Project. G. B. acknowledges support from the European Research Council through the ERC starting grant WIMPs Kairos. D. H. is supported by the U.S. Department of Energy.

### APPENDIX: FURTHER READING

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