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Comment on “What is a Gauge Transformation in Quantum Mechanics?”

In a recent Letter [1], Rovelli addresses a technical issue in constrained quantization which is of great potential importance for quantum gravity, Yang-Mills theories, and other fundamental theories. Let \( \mathcal{H} \) be the Hilbert space of unconstrained states of a quantum theory with quantized first-class constraints \( \{C\} \). Rovelli defines two vectors \( c \) and \( w \) in \( \mathcal{H} \) to be related by a “complete gauge transformation” when \( c^2 \langle L \rangle \mathcal{H}^2 \delta^1 \rho_i \), \( \rho_i \in \mathcal{H} \). He then proposes that the physical Hilbert space be \( \mathcal{H}_{\text{ph}} = \mathcal{H} / L \), and he shows that when \( \mathcal{H} \) is finite dimensional, \( \mathcal{H}_{\text{ph}} \) is the same as the physical state space defined in Dirac’s well-known theory of constrained quantization.

Rovelli’s analysis of the situation in which \( \mathcal{H} \) is infinite dimensional is based on his remark that “In infinite dimensions, the orthogonal complement \( L \) of a subspace \( L \) may be trivial even if \( L \) is smaller than \( \mathcal{H} \). But \( \mathcal{H} / L \) exists nevertheless…” This is true when \( L \) is not closed, but it is not clear what the inner product on \( \mathcal{H} / L \) should be in that case, and how it is to be completed so as to become a Hilbert space.

I here wish to point out that the infinite-dimensional case may be handled [2] by modifying the inner product \( \langle | \rangle \) on \( \mathcal{H} \) (which is positive definite) into a positive semidefinite sesquilinear form \( \langle | \rangle_{0} \), which in the infinite-dimensional case is defined only on a suitable dense subspace \( \mathcal{D} \) of \( \mathcal{H} \). This form has a nontrivial null space \( \mathcal{N} = \{ \psi \in \mathcal{H} \mid \langle \psi | \psi \rangle_{0} = 0 \} \), in terms of which the physical state space of the constrained system is the closure of \( \mathcal{D} / \mathcal{N} \) in the inner product inherited from \( \langle | \rangle_{0} \). When the dimension of \( \mathcal{H} \) is finite, the space \( \mathcal{N} \) coincides with Rovelli’s \( L \) (this is immediate if one combines the theorem on p. 4614 of [1] with the analysis in section 1.3 of [3]).

In addition, one would like to specify the action of physical observables on \( \mathcal{H}_{\text{ph}} \); recall that a Hilbert space as such conveys practically no physical information, since all Hilbert spaces of the same dimension are isomorphic. One may proceed by (i) defining a weak physical observable as an operator on \( \mathcal{H} \) satisfying the modified Hermiticity condition \( \langle \psi | A | \varphi \rangle_{0} = \langle \varphi | A | \psi \rangle_{0} \); (ii) noting that this implies that \( A \) maps \( \mathcal{N} \) into itself; (iii) concluding that \( A \) induces a well-defined operator \( A_{\text{ph}} \) on \( \mathcal{H}_{\text{ph}} \). In cases that are well understood, this procedure indeed turns out to yield the correct physical quantum observables [3].

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