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## Energy correlators beyond angles

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**ABSTRACT:** Energy correlators are theoretically simple and physically intuitive observables that bridge experimental and theoretical particle physics. They have for example enabled the most precise jet substructure determination of the strong coupling constant to date, and recent proposals suggest that they may be used to precisely determine the top quark mass with calculable, small theoretical uncertainties. However, existing energy correlators all measure correlations in angles between particles, from which other observables such as mass must be inferred through potentially complicated procedures. In this work, we generalize energy correlators to enable straightforward measurements of non-angular correlations, which we call Energy Weighted Observable Correlations (EWOCs). To enforce collinear safety, EWOCs quantify correlations between *subjets* rather than particles. The subjet radius can be tuned to control both the physical scales probed by EWOCs and their sensitivity to non-perturbative physics. We focus on the phenomenologically relevant example of the mass EWOC, which measures mass correlations between pairs of subjets, in the task of extracting mass scales from jets. In jet substructure determinations of the mass of a hadronically-decaying  $W$  boson, we show that the mass EWOC outperforms the angle-based energy correlator, and performs comparably to the soft-drop groomed jet mass. As a first exploration of the theoretical properties of EWOCs, we also calculate the mass EWOC on light-quark jets and compare to results obtained with `Pythia 8.309`.

**KEYWORDS:** Jets and Jet Substructure, Specific QCD Phenomenology

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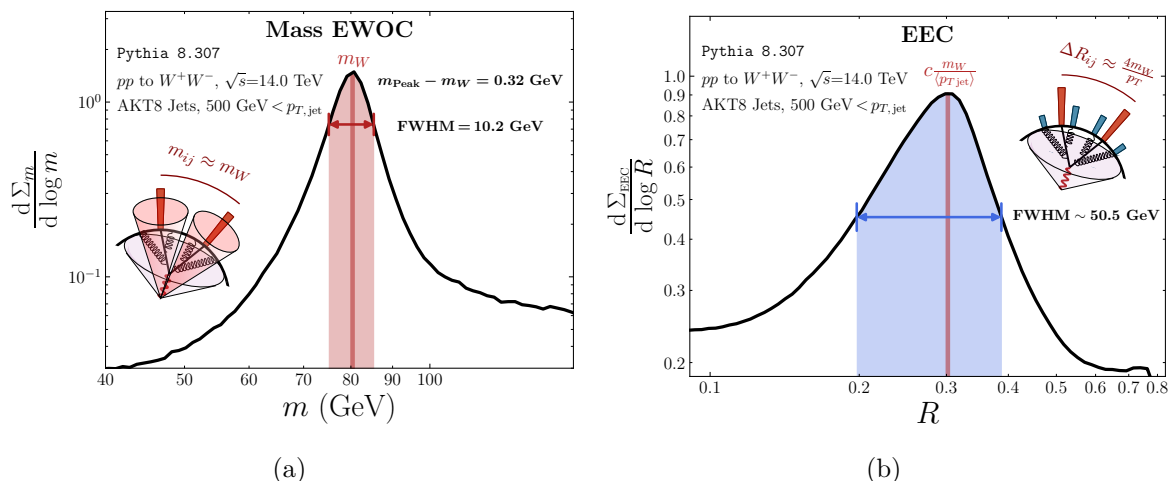
## 1 Introduction

The discovery of hadronic jets in high-energy particle collisions is one of the most important milestones in human understanding of the microscopic universe, and has solidified quantum chromodynamics (QCD) as our fundamental theory of the strong nuclear force. Jets were originally predicted [1–5], and later measured [6–14], as collimated sprays of hadronic radiation produced in high-energy collisions involving the strong interaction. The quantification of the internal structure of jets, enabled by the excellent performance of the current generation of experimental detectors, has been a particularly fruitful area of study. The study of jet substructure has led to the development of a large class of observables that characterize hadronic radiation in particle collisions, facilitating our understanding of the physical properties of the Standard Model of particle physics (SM) and beyond.

Among collider observables with the potential to probe jet substructure, the Energy-Energy Correlator (EEC) [15–17] is particularly noteworthy: it is conceptually simple, insensitive to low-energy radiation, and naturally separates physics at different scales. Physically, the EEC measures the mean correlation in the energy passing through any two detectors separated by an angle  $\Delta$ , averaged over many particle collisions:  $\text{EEC}(\Delta) \propto \langle E_{\text{det.1}} E_{\text{det.2}} \rangle$ .<sup>1</sup> From the birth of the EEC to its use in recent collider studies, the simplicity of the EEC has enabled highly accurate perturbative computations [18–46] and has facilitated measurements of the strong coupling constant  $\alpha_s$  [47–56] including the most precise jet substructure determinations of  $\alpha_s$  to date [57]. The EEC has been investigated as a probe of jets traveling through the quark-gluon plasma (QGP) produced in heavy-ion collisions [58–67] and of correlations in nuclear physics [68–72]. The study of the EEC has also shed light on the structure of

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<sup>1</sup>More concretely, this means that the contribution of each particle in an event to the EEC is weighted by the particle’s energy, see eq. (2.1). This energy weighting mitigates the effects of the low-energy physics of QCD that complicates the theoretical description of other jet substructure observables.



**Figure 1.** Visualizations of the hadronic decay of a  $W$  boson in 14 TeV LHC events simulated with Pythia 8.309. (a) The mass EWOC, introduced in this work, which captures the mass of the  $W$  boson directly from the pairwise mass of subjets (the red cones) of radius  $r_{\text{sub}} = 0.3$ . (b) The EEC evaluated on pairs of particles; we tune the constant  $c \sim 2.5$  to extract  $m_W$  from the peak of the EEC distribution. The full width at half maximum (FWHM) of the mass EWOC (about 10 GeV) is significantly smaller than the mass difference associated with the FWHM of the EEC (corresponding to about 50 GeV), suggesting that the peak of the mass EWOC will provide a more precise estimation of  $m_W$ .

non-perturbative QCD [73–80] and of quantum field theory in general [81–100]. It also been put forward as a way to search for new physics in hadronic decays of electroweak bosons [101]. Finally, while most jet substructure observables are dominated by contributions from two-particle correlations, the EEC can be naturally extended to multi-point energy correlators which can characterize correlations between an arbitrary number of particles [100, 102–105]. Recent theoretical work suggests that multi-point generalizations of the EEC may be used to extract the mass of the top quark at the LHC [106–111].

However, the EEC is limited to probing angular correlations, and angular correlations are not the only correlations of interest in particle collisions. Additional, non-trivial steps are needed when using energy correlators to extract non-angular parameters of interest. For example, in extracting the top quark mass, the conversion of angular scales to mass scales involves the transverse momentum of top quark jets; large experimental uncertainties in the jet  $p_T$  then directly affect the resulting extraction of the top quark mass  $m_t$ .<sup>2</sup> Similarly, heavy-ion collisions and the properties of the QGP are characterized by a wide variety of correlations extending beyond the simple angular correlations probed by energy correlators [58–67].

In this paper, we introduce **Energy Weighted Observable Correlations (EWOCs)** as generalizations of the EEC that preserve the strengths of energy weighting as a paradigm

<sup>2</sup>A solution to this  $p_T$  smearing was proposed by the authors of ref. [108], leveraging the  $W$ -bosons that emerge during top quark decay and the well-measured mass of the  $W$ -boson. In particular, they define an integrated three-point energy correlator with two peaks, at angular scales  $\zeta_{\text{peak}} \sim m^2/p_T^2$  with  $m = m_W$  and  $m = m_t$ . The ratio of the peak locations provides a robust measure of  $m_t/m_W$  which is insensitive to uncertainties in jet  $p_T$ . The approach we develop here does not rely on an existing, precisely determined particle mass.

for extracting fundamental information from measurements, while simultaneously probing correlations in non-angular observables. A pairwise (or 2-point) EWOC takes the form<sup>3</sup>

$$\frac{d\Sigma_{\mathcal{O}}}{d\chi} = \frac{1}{\sigma} \int d\sigma \sum_{\text{subjects } i,j} x_i x_j \delta(\chi - \mathcal{O}_{ij}), \quad (1.1)$$

where  $(d)\sigma$  indicates the (differential) cross section for jet production in the process under consideration and the sum on  $i$  and  $j$  run over the subjects within the jet. The energy-weighting factors  $x_{i,j}$ , adopted from the EEC, ensure that higher-energy subjects contribute more to the EWOC of eq. (1.1).<sup>4</sup>  $\mathcal{O}_{ij}$  is a user-defined observable on pairs of subjects, such as their angular separation or invariant mass. For example, using the two-particle angular separation in eq. (1.1),  $\mathcal{O}_{ij} \rightarrow \theta_{ij}$ , recovers the traditional EEC of  $e^+e^-$  collisions in the limit of zero subjet radius (i.e. where the sum over pairs of subjects is replaced by a sum over pairs of particles). However, the EWOC framework we introduce expands on the strengths of the EEC in two ways:

- We study correlations of generic **pairwise observables**, i.e. any observables that depend on the properties of two particles; this includes not only the relative angle between their momenta, probed by the EEC, but also their total invariant mass, our main focus in this work.
- We study correlations between **subjects** rather than particles within a jet; this isolates correlations between collective degrees of freedom (the subjects), and is essential to ensure that EWOCs are collinear-safe when considering general pairwise observables.

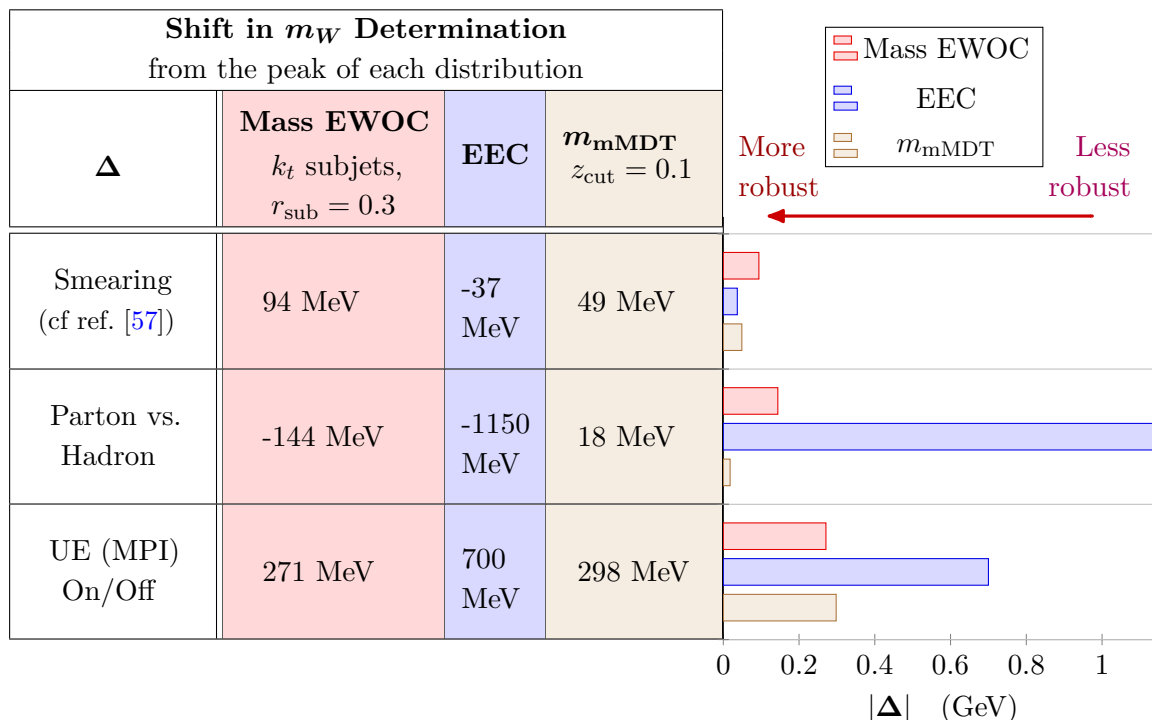
The radius of the subjects controls the sensitivity of EWOCs to non-perturbative effects, though it also limits the angular resolution that EWOCs can probe; therefore, the choice of subjet radius in defining an EWOC for a particular physics context is motivated by balancing these considerations.

In this work, we focus on the mass EWOC, showing its power in determining the mass of a hadronically decaying particle. In figure 1, we demonstrate the use of the mass EWOC in extracting the mass of the  $W$  boson (as a proxy for a generic hadronically-decaying resonance) from simulations of  $W$  boson pair production at the LHC generated with Pythia 8.309 [113]. The EWOC framework may also be used to extract more general and phenomenologically important correlations between subjects or the masses of decaying resonances with a greater number of decay products. For example, we expect that the three-point mass EWOC may be used as an alternative to the three-point, angle-based energy correlator for extracting the mass of top quark from its three-pronged decay [106–111]. While we do not explore these phenomenologically important measurements in this work, we hope that the EWOC framework will be helpful for extracting a wide variety of correlations of physical interest from future experimental measurements.

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<sup>3</sup>We will present this definition in more detail in section 2.2, and a slightly more general definition with arbitrary energy weights in section 2.3. One can also generalize the pairwise EWOCs we present here to higher-point correlators that we leave to future work. Here, the new parametrization proposed in ref. [112] would be particularly convenient to reduce computation time and enhance interpretability.

<sup>4</sup>For proton-proton collisions at the LHC, transverse momentum fractions  $x_i = p_{T,i}/p_{T,\text{jet}}$  are a natural choice, while energy fractions  $x_i = E_i/E_{\text{jet}}$  are more natural in the context of electron-positron collisions.



**Table 1.** Several measures of robustness of determinations of  $m_W$  based only on the peak of the mass EWOC introduced in this work, the EEC, or the mass distribution of jets groomed using the modified mass drop tagger (mMDT). These are all evaluated on Pythia 8.309 samples of a pair of hadronically-decaying  $W$  bosons at the LHC. While each measure of robustness may be addressed by appropriate calibration, smaller values indicate more robust determinations of  $m_W$ .

In table 1, we further evince the phenomenological value of the mass EWOC by comparing shifts in the  $m_W$  determination obtained by using the peaks of the mass EWOC, the EEC, and the mass distribution of jets groomed using the modified mass drop tagger (mMDT, which far outperforms the ungroomed jet mass) [114, 115]. For each distribution,  $m_W$  is estimated by performing a quadratic fit of each distribution near the peak at  $m_W$ . In particular, we show the amount by which each estimation of  $m_W$  varies due to:

- an emulation of experimental detector effects via Gaussian smearing of particle momenta by 3% for photons, 5% for neutral particles, and 1% for charged particles (the values used in CMS-SMP-22-01 [57]);
- turning on/off the effects of hadronization;
- turning on/off the effects of multiple-parton interactions (MPI), with hadronization on, as a proxy for the underlying event (UE).

We note that, in this simplified context, the EEC is more robust to the effects of particle-level momentum smearing than the mass EWOC in the estimation of  $m_W$ , but that the mass EWOC is much more resilient to non-perturbative QCD effects than the EEC. The presented values for the variations in the EEC are commensurate with corresponding values in the estimation of the top quark mass using the three-point energy correlator, as in table

I of ref. [107]. The shifts in the mass EWOC determination of  $m_W$  due to smearing and hadronization are greater in absolute value than those of the determination of  $m_W$  using the mMDT-groomed jet mass. However, when smearing, hadronization, and MPI are considered together, the overall shift in the peak of the mass EWOC — and the resulting shift in the determination of  $m_W$  — is smaller than that of the mMDT-groomed mass distribution.

The rest of the paper proceeds as follows: In section 2, we define EWOCs and discuss their properties and some applications. In section 3, we examine the phenomenology of the mass EWOC in both proton-proton and electron-positron collisions. In our study of LHC collisions in section 3.2, we show that the mass EWOC is a particularly powerful tool for extracting mass scales from collision data, using the  $W$ -boson as an example of a generic hadronically-decaying resonance. In section 3.3 we present fixed-order results for EWOCs of light-quark jets produced in  $e^+e^-$  collisions and compare them to Pythia 8.309 simulations. We give concluding thoughts and discuss avenues for future exploration in section 4.

## 2 Energy Weighted Observable Correlations (EWOCs)

In this section, we provide an introduction to EWOCs and the physics they probe. Sections 2.1 and 2.2 introduce the general considerations and ingredients that go into the construction of EWOCs, presenting EWOCs as flexible, easily interpretable extensions of the more familiar EEC. Section 2.3 discusses how the use of subjets ensures collinear safety and suppresses non-perturbative effects for generic EWOCs involving non-angular correlations.

### 2.1 A review of the Energy-Energy Correlator (EEC)

Qualitatively, the EEC captures information about the angular scales at which energetic pairs of particles tend to strike a detector after a particle collision event. As visualized in the inset of figure 1(b), the EEC at the angular scale  $\chi$  is the average product of the energies of particles separated by an angle  $\chi$ , averaged over an ensemble of jets.

More precisely, the EEC is defined as<sup>5</sup>

$$\frac{d\Sigma_{\text{EEC}}}{d\chi} = \frac{1}{\sigma} \int d\sigma \sum_{\text{particles } i, j} x_i x_j \delta(\chi - \theta_{ij}), \quad (2.1)$$

where  $\chi$  is an angular variable and  $d\sigma$  is a differential cross-section with  $\int d\sigma = \sigma$ . The subscripts  $i$  and  $j$  denote outgoing particles,  $x_i$  and  $x_j$  describe their energy or transverse momentum fraction and  $\theta_{ij}$  denotes an angular metric.<sup>6</sup>

<sup>5</sup>The definition of eq. (2.1) is common in particle phenomenology. When all particles are massless, there is an equivalent definition of the EEC in terms of *light-ray operators*: integrals of the stress energy tensor along light rays travelling from the location of the particle collision; see, for example, refs. [83] and [93]. The light-ray definition of the EEC may even be applied in the context of conformal field theory, when the concept of a single particle no longer makes sense.

<sup>6</sup>In electron-positron collisions, it is common to replace the  $\delta(\chi - \theta_{ij})$  of eq. (2.1) with  $\delta[\chi - (1 - \cos \theta_{ij})/2]$ .  $\theta_{ij}$  denotes the real-space angle between the momenta of particles  $i$  and  $j$ . The sum on  $i, j$  often runs over all particles in an event (in which case it is called an *event shape*), though it can alternatively run over all particles within the set of jets obtained by an appropriate jet definition. When treated as an event shape, the EEC energy weights are  $x_i = E_i/E_{\text{event}}$ , while for the jet-based EEC  $x_i = E_i/E_{\text{jet}}$ . In proton-proton collisions, the sum runs over all particles in a jet,  $x_i = p_{T,i}/p_{T,\text{jet}}$ , and  $\theta_{ij}$  denotes instead the rapidity-azimuth distance between particles  $i$  and  $j$ . When computing EECs and general EWOCs in this work, we will always use jet-based definitions.

Though the EEC is conceptually simple observable, the use of particles in the definition of the EEC presented in eq. (2.1) makes the EEC somewhat vulnerable to the non-perturbative physics of low-energy QCD, which is notoriously difficult to predict and control, through effects such as hadronization and the underlying event. The EEC partially mitigates this issue through the use of energy weighting (the factor  $x_i x_j$  in eq. (2.1)) such that these low-energy effects are suppressed by their small energy fractions. The resulting observable is less sensitive than traditional jet observables to the effects of low-energy QCD, but the EEC still uses particles directly, and therefore has not completely eliminated the potential for complicated, non-perturbative effects.

Furthermore, while the angular scales in a particle collision convey a great deal of information about the underlying fundamental physics, they are not able to efficiently characterize everything about the interactions of our microscopic universe. In order to gather a more rich collection of information about collider physics using energy-weighted correlations, it is tempting to simply change the argument of the delta function in eq. (2.1), which singles out angular scales. However, the resulting observable is collinear unsafe (infrared safety is still ensured by the energy weighting). In the following section, we will introduce EWOCs, which solve this issue by introducing energy-weighted correlations on *subjets*.

## 2.2 Introducing EWOCs

The framework of (subjet-level) energy-weighted observable correlations (EWOCs) that we introduce in this section takes the mitigation of low-energy effects one step further. In addition to using the energy weighting of the EEC, we use subjets rather than particles; subjets are constructed by applying a jet algorithm with radius  $r_{\text{sub}} < R_{\text{jet}}$  to the particles clustered within each jet in an event, and the subjet radius provides an explicit collinear cutoff on contributing radiation.<sup>7</sup> Any collection of particles which lead to the same subjet configurations will have the same EWOCs, regardless of the details of the particles themselves.

EWOCs therefore trade the ability to capture correlations between all particles in a jet for the ability to more easily control the effects of low-energy QCD. As we will see, this tradeoff dramatically increases the richness of the correlations that can be probed by EWOCs. In addition to the angular correlations probed by the EEC, EWOCs can be used to capture many other correlations — such as correlations in mass, formation time, or any other observable on subjets — which are forbidden by collinear safety in the absence of a subjet radius. An EWOC for a generic observable  $\mathcal{O}(\text{subjet 1}, \text{subjet 2})$  on pairs of subjets takes a similar form to eq. (2.1). Its definition was given in eq. (1.1) and is repeated here for convenience:

$$\frac{d\Sigma_{\mathcal{O}}}{d\chi} = \frac{1}{\sigma} \int d\sigma \sum_{\text{subjets } i, j} x_i x_j \delta(\chi - \mathcal{O}_{ij}), \quad (2.2)$$

where the sum over particles has been replaced by a sum over subjets, and the role of  $\theta_{ij}$  has been taken instead by  $\mathcal{O}_{ij} = \mathcal{O}(\text{subjet } i, \text{subjet } j)$ , the value of the observable  $\mathcal{O}$  on subjets  $i$  and  $j$ . The normalization  $\sigma$  is the jet cross section, and ensures that the EWOC is normalized:

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<sup>7</sup>Subjet algorithms are not the only regularization schemes which provide a collinear cutoff on the physics of the EEC. For example, refs. [65] and [116] both compute the EEC on collinear-safe collective degrees of freedom based on the Lund plane or on the clustering history of the jet under study, respectively.

$\int d\chi \frac{d\Sigma}{d\chi} = 1$ .<sup>8</sup> The observable  $\mathcal{O}_{ij}$  can depend on the energies and relative angle of subjects  $i$  and  $j$ , but also on additional information such as the subject masses.

The advantage of EWOCs is that they directly probe correlations in an observable of interest. The choice of subject definition (e.g. radius and recombination scheme) determines the set of subjects that contribute to the EWOC, and therefore may also be tuned to capture different physical effects. For example, the subject radius may be chosen to capture the subjects emerging from the hadronic decays of a boosted particle, as we explore in section 3. In this work, we focus on the concrete example of the mass EWOC as a proof-of-concept, with a subject radius chosen to probe decays of the  $W$  boson, but the use of EWOCs is by no means limited to the measurement of mass correlations. Possible pairwise observables include:

- The opening angle  $\theta_{ij}$  or the rapidity-azimuth distance  $R_{ij}$  between subjects, which probe angular correlations (used in  $e^+e^-$  or  $pp$  collisions, respectively). These observables lead to the EEC, for which the use of subjects does not provide novel results;<sup>9</sup>
- The mass  $m_{ij} = \sqrt{(p_i + p_j)^2}$ , which probes mass scales,<sup>10</sup> applied to the phenomenology of massive decaying particles in section 3;
- The formation time  $\tau_{ij} = \max(E_i, E_j)/m_{ij}^2$ , which probes the Lorentz-dilated time scale at which partons (subjects)  $i$  and  $j$  become “separate”.<sup>11</sup> At times  $t > \tau_{ij}$ , partons  $i$  and  $j$  are expected to behave as separated, distinct color charges, while for  $t < \tau_{ij}$  they are expected to behave “coherently” as a single color charge. We defer the study of formation time EWOCs, especially in the study of heavy-ion collisions, to future work.<sup>12</sup>

Each observable is useful for the study of different physical effects, and each comes with its own behavior under non-perturbative corrections and in analytic computations.

The EEC has traditionally been applied to particle physics in order to extract the strong coupling constant [47–57, 105], and more recently to a broad set of applications including

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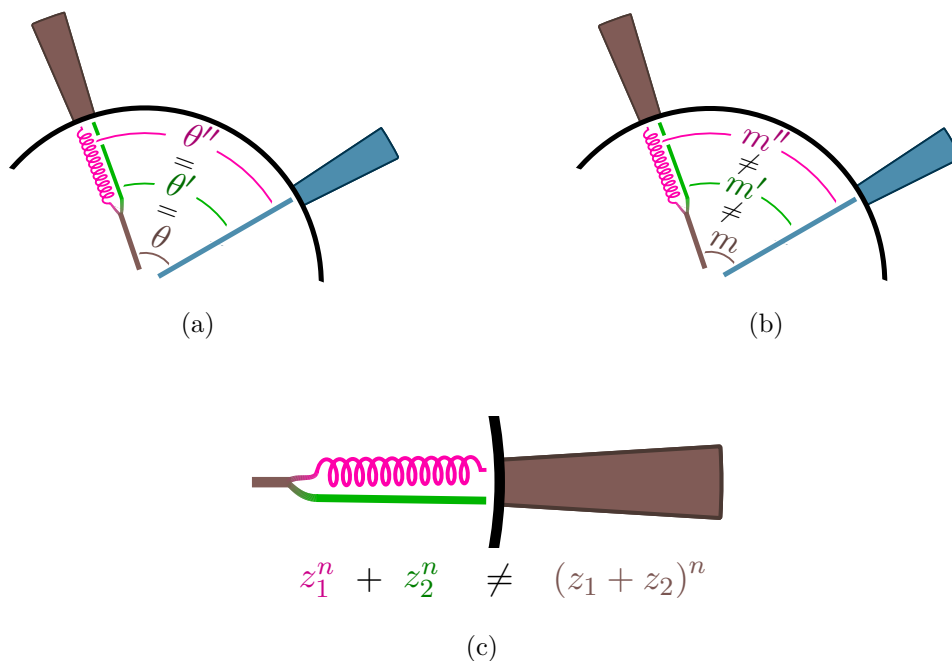
<sup>8</sup>EWOCs can also be defined as event shapes, for which the sum on subjects  $i$  within a particular jet is instead replaced by a sum on jets  $i$  within an entire event. However, in this work, we use the jet-based definition of eq. (2.2) for all EWOC computations.

<sup>9</sup>With a subject radius  $r_{\text{sub}}$ , the value of the subject EEC is nearly identical to the particle-level EEC for  $\chi > r_{\text{sub}}$ , while the collinear cutoff of the subject radius sets the subject EEC to zero for  $0 < \chi < r_{\text{sub}}$ . The subject EEC and the particle-level EEC would be exactly the same for  $\chi > r_{\text{sub}}$  if all particles in each subject were exactly aligned with the subject direction, with small differences suppressed by the subject radius size.

<sup>10</sup>The four-momenta of the subjects depends on the recombination scheme used to define the subjects. In this work, we use winner-take-all schemes which enforce that all subjects are massless. We have found that this choice of recombination scheme does not affect the performance of the mass EWOC in constraining the  $W$ -boson mass.

<sup>11</sup>Since formation time is usually associated with partonic splittings, formation time EWOCs may be more useful if the sum on subject pairs only includes subjects associated with splittings in the branching history of a jet.

<sup>12</sup>The formation time is an especially useful observable in relativistic heavy-ion collisions, where it can probe whether two partons “split” before or after the time scale set by the mean-free path of the QGP medium. Splittings with formation times much smaller than the mean free path of the medium are therefore expected to be roughly governed by the physics of QCD in empty space/without a medium; the formation time at which predictions deviate from QCD without a medium can be used to infer the mean free path of the medium, and therefore how medium effects modify the behavior of QCD splittings [117–135].



**Figure 2.** Cartoons of collinear splittings (colored lines inside the black hemispheres, for which the brown quark line splits into the pink gluon line and the green quark line) producing the same subjects (colored bars outside of the hemispheres) and the collinear safety properties of (a) the EEC (see eq. (2.4)), (b) the mass EWOC (see eq. (2.5)), and (c) the EWOCs with non-unity energy weights of eq. (2.3). (a) The EEC is collinear safe at the level of both particles and subjects: the collinear splitting does not change the set of angles between particles in a jet (see eq. (2.4)). (b) The particle-level mass EWOC is collinear unsafe (as are non-angular EWOCs in general) because the set of masses of particle pairs changes after a collinear splitting:  $m \neq m' \neq m''$  (see eq. (2.5)). The subject-level mass EWOC, however, is collinear safe as long as subjects are unchanged by collinear splittings. For the same reason, (c) generic subject-level EWOCs remain collinear-safe even in the presence of non-unity energy weights.

extracting mass scales of boosted objects in colliders [106–111] and the characterization of relativistic heavy-ion collisions [58–67]. More general EWOCs refine and extend the goals of the EEC in particle physics, and the addition of a subject radius or new pairwise observables may be useful in several contexts.

### 2.3 Infrared/Collinear (IRC) safety and non-perturbative effects

As mentioned in section 2.2, particle-level EWOCs — with  $r_{\text{sub}} = 0$  — utilizing non-angular observables are collinear unsafe. For the special case of the EEC, there is no problem of collinear unsafety: the particles produced in a collinear splitting still contribute at the exact same angle in the EEC. However, the values of a *generic* observable that would enter a *particle-level* EWOC differ before and after one of the particles undergoes a collinear splitting. On the other hand, EWOCs which utilize subjects — which we will simply refer to as EWOCs — are manifestly collinear safe as long as the subject algorithm being used is collinear safe. Furthermore, the use of subjects allows for IRC-safe EWOCs with non-unity energy weights,

$$\frac{d\Sigma_{\mathcal{O}}^{(n,m)}}{d\chi} = \frac{1}{\sigma} \int d\sigma \sum_{\text{subjects } i,j} x_i^n x_j^m \delta(\chi - \mathcal{O}_{ij}), \quad (2.3)$$

which, for  $n, m > 1$ , are even less sensitive to the soft radiation within an event. The benefits of non-unity weights will be discussed below.

The collinear safety of the EEC and the collinear unsafety of the would-be particle-level mass EWOC is illustrated in figure 2, where subjects are visualized as bars on the outside of the circles which are unchanged under a collinear splitting. Figure 2 focuses on the simple example of a two-particle final state and the corresponding three-particle final state after a collinear splitting. Figure 2(a) visualizes the collinear safety of the particle-level EEC, contrasted against the collinear unsafety of generic, particle-level EWOCs, such as the mass EWOC visualized in figure 2(b). Figure 2(c) visualizes the collinear unsafety of generic energy weighting factors in would-be particle-level EWOCs, including the particle-level EEC. While the use of non-unity energy weights is collinear unsafe at particle-level, the subjects used in the computation of EWOC are unchanged under collinear splittings, as long as the subjet algorithm is collinear safe.

To understand the collinear safety of the EEC, and the unsafety of the particle-level mass EWOC, more precisely, let us denote the momentum fraction of the right (blue) particle by  $z_2$ , the left (brown) particle before a collinear splitting by  $z$ , and the particles emerging from the collinear splitting (green and pink) by  $z'$  and  $z''$ . The pairwise angles between the right particle and either the collinear parent and collinear-split children are  $\theta, \theta'$  and  $\theta''$ , respectively, and the associated pairwise masses are  $m, m'$ , and  $m''$ . Since the splitting is collinear, we have  $\theta = \theta' = \theta''$  as well as  $z = z' + z''$ . Already, we find that the associated pairwise correlations captured by the EEC do not change under a collinear splitting.<sup>13</sup>

$$\frac{d\Sigma_{\text{EEC}}}{d\chi} \supset 2 z_2 z \delta(\chi - \theta) = 2 z_2 (z' \delta(\chi - \theta') + z'' \delta(\chi - \theta'')) \subset \left. \frac{d\Sigma_{\text{EEC}}}{d\chi} \right|_{\text{split}}. \quad (2.4)$$

For the would-be particle-level mass EWOC, however, the collinear splitting changes the value of the distribution

$$\frac{d\Sigma_m}{d\chi} \supset 2 z_2 z \delta(\chi - m) \neq 2 z_2 (z' \delta(\chi - m') + z'' \delta(\chi - m'')) \subset \left. \frac{d\Sigma_m}{d\chi} \right|_{\text{split}}. \quad (2.5)$$

Neither  $m'$  nor  $m''$  is equal to  $m$ , and the support of the distribution has changed under a collinear splitting. We conclude that the would-be particle-level EWOC is collinear-unsafe.

The use of energy weights larger than one, as defined in eq. (2.3), has the potential to further reduce the effects of additive contamination such as pileup and the overwhelming underlying event in relativistic heavy-ion collisions [65].<sup>14</sup> In section 3.2, we restrict to the case  $n = m$  and examine how energy weights can be used to mitigate non-perturbative effects and yield more robust results for EWOCs.<sup>15</sup> We note, however, that the additional flexibility

<sup>13</sup>Similar arguments hold for any number of particles, as well as for *contact terms* — the singular contributions at  $\chi = 0$  due to particle self-correlations:  $d\Sigma_{\text{EEC}}/d\chi \supset z^2 \delta(\chi) = (z'^2 + z''^2 + 2z'z'') \delta(\chi) \subset d\Sigma_{\text{EEC}}/d\chi \Big|_{\text{split}}$ .

<sup>14</sup>Ref. [65] provides an earlier discussion of how a different collinear regularization of the EEC facilitates the IRC-safe use of non-unity energy weights, and notes that the use of higher energy weights may provide a way to mitigate the low-energy effects of the formidable underlying event of relativistic heavy-ion collisions.

<sup>15</sup>We do not consider the case  $n \neq m$ , though it may be interesting to do so e.g. to pick out configurations with one energetic and one softer subjet, or when using asymmetric pairwise observables  $\mathcal{O}_{ij} \neq \mathcal{O}_{ji}$ . The latter in particular may be interesting in the study of EWOCs characterizing formation times.

and infrared stability granted by the energy weights come at the expense of losing a simple sum rule: the integral over  $\chi$  of  $d\Sigma_{\mathcal{O}}^{(n,m)}/d\chi$  will no longer yield one unless  $n = m = 1$ .<sup>16</sup>

Fortunately, the use of subjet radii leads to results which are not only collinear-safe, but also more stable to non-perturbative effects. On physical grounds, this is because subjets are collections of particles that approximate high-energy partons produced during a hard process, while hadronization is a low-energy phenomenon. More technically, a subjet radius is a type of infrared cutoff: in a scattering process of characteristic energy scale  $Q$ , subjets with  $r_{\text{sub}} \ll 1$  contained within jets of radius  $R_{\text{jet}}$  probe physics at scales between  $Q R_{\text{jet}}$  and  $Q r_{\text{sub}}$ . Therefore, if  $Q r_{\text{sub}} \gg \Lambda$ , where  $\Lambda$  is an energy scale of non-perturbative physics, we are in a regime where perturbation theory is valid and non-perturbative effects are suppressed by powers of  $\Lambda/(Q r_{\text{sub}})$ .

For example, hadronization effects in the mass EWOC are controlled by powers of  $\Lambda_{\text{QCD}}/(Q r_{\text{sub}})$  in the regime of a small subjet radius. In the regime  $Q \sim Q R_{\text{jet}} \gg m \gg Q r_{\text{sub}} \gg \Lambda_{\text{QCD}}$ , where  $m$  is the argument of the mass EWOC, there are also hadronization effects which scale with  $\Lambda_{\text{QCD}}/Q$  and  $\Lambda_{\text{QCD}}/m$  and are suppressed relative to the leading hadronization corrections, which scale with  $\Lambda_{\text{QCD}}/(Q r_{\text{sub}})$ . Similar conclusions hold for a generic EWOC when  $m$  is replaced by the relevant scale.

### 3 Mass extraction with EWOCs

In this section, we examine EWOCs in greater depth by focusing on the mass EWOC. In section 3.2, we apply EWOCs to one of the most important contexts for jet substructure: the characterization of heavy, boosted objects that emerge in proton-proton collisions. In particular, we demonstrate how the EWOC framework can be used to extract the mass of hadronically-decaying boosted  $W$  bosons created in proton-proton collisions, with results that are robust to non-perturbative corrections. In section 3.3 we explore the fixed order and leading-logarithmic structure of mass EWOCs in the theoretically simpler context of light-quark jets produced in electron-positron collisions, and compare them to results obtained with `Pythia 8.309`.

#### 3.1 Observables and samples

The data presented in this work are derived from sample events generated using `Pythia 8.309` [113] with the default Monash tune [136], with jets and subjets clustered using `FastJet` [137]. The code used in our analysis is available at `ResolvedEnergyCorrelators` [112, 138].

In our studies of proton-proton collisions at  $\sqrt{s} = 14$  TeV in section 3.2, we cluster jets using the anti- $k_t$  algorithm [139] with  $R_{\text{jet}} = 0.8$  and cluster subjets using the  $k_t$

<sup>16</sup>It is also possible to define EWOCs with higher energy weights which nonetheless integrate to one, by ensuring that the sum of each energy weighting factor over subjets gives one; for example, one could replace the energy-weighting factor  $x_i^n$  in eq. (2.3) with  $x_i^n / \sum_k x_k^n$ , and similarly replace  $x_j^m$  by  $x_j^m / \sum_k x_k^m$ . However, these EWOCs are theoretically more complicated because the denominator  $\sum_k x_k^n$  depends sensitively on *all* subjets. This definition also suffers from greater hadronization and underlying event corrections, and thus does not offer improvements in the main application discussed here — the estimation of  $m_W$  using the mass EWOC. We thank Jesse Thaler for discussions on this point.

algorithm [140, 141], and use the winner-take-all (WTA)  $p_T$  recombination scheme [142, 143] for both jets and subjets. We restrict our analysis to jets with a maximum pseudorapidity of  $|\eta| < 4$  and a minimum transverse momentum of  $p_T > 500$  GeV unless otherwise stated, ensuring that the associated  $W$  bosons are sufficiently boosted for their decay products to lie within a single jet. The anti- $k_t$  jet algorithm ensures that our jet boundaries are insensitive to the presence of contaminating radiation such as the underlying event. The  $k_t$  subjet algorithm ensures that our subjets are more likely to be correlated with the hard sub-prongs (quarks) of our  $W$  jets.<sup>17</sup> Using a WTA recombination scheme ensures that the mass EWOC only captures correlations between distinct subjets, and is insensitive to the masses of individual subjets;<sup>18</sup> our results for groomed mass distributions instead uses the standard  $E$ -scheme for jet recombination [149].

In our studies involving  $e^+e^-$  collisions at  $\sqrt{s} = 1$  TeV in section 3.3, we cluster jets and subjets with the angular-ordered, electron-positron version of the Cambridge-Aachen (C/A) algorithm [144, 145]. The use of the C/A algorithm is more appropriate for the comparison of `Pythia 8.309` with our analytic results, since the branching structure of the angular-ordered tree of emissions produced by the C/A algorithm mimics the angular-ordered structure of perturbative QCD [150]. We study jets with  $R_{\text{jet}} = 1$  and use the WTA  $|p|$  recombination scheme [143] for both jets and subjets, again ensuring that all subjets are massless. We also restrict our analysis to jets with a minimum energy of  $E > 100$  GeV in order to avoid contributions from extremely soft jets and thereby facilitate comparisons to analytic results.<sup>19</sup>

### 3.2 Mass at the LHC: $pp \rightarrow W^+W^-$

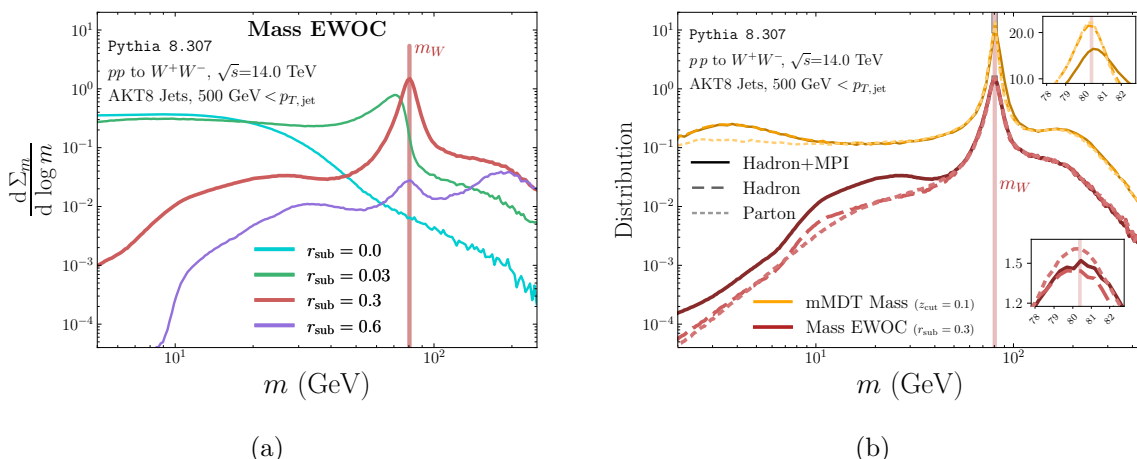
We now use mass EWOCs to extract the mass of the  $W$  boson from  $pp \rightarrow W^+W^-$  events generated in `Pythia 8.309` at parton-level, hadron-level, and in the presence of the underlying event (UE) produced by multiple parton interactions (MPI). The two-pronged hadronic decay of the  $W$  boson,  $W \rightarrow q\bar{q}$ , is particularly well suited for study with the pairwise (or two-point) mass EWOC: subjets are proxies for the quarks emerging from the  $W$  decay, and the pairwise subjet mass becomes a proxy for the mass of the  $W$  boson itself. Furthermore, while we focus on the  $W$  boson in this section, similar methods may be used for the analysis of generic two-pronged hadronic decays, such as hadronic two-prong decays of a hypothetical beyond-the-standard-model  $Z'$ .

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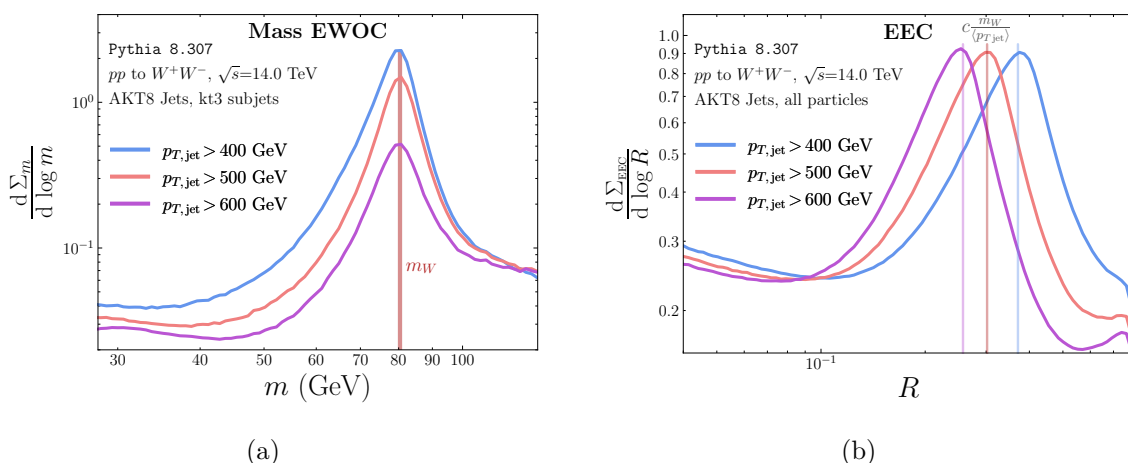
<sup>17</sup>The Cambridge-Aachen (C/A) [144, 145] algorithm, for example, is more likely to cluster the hard, narrow core and the soft, wide-angle radiation produced by a single quark into separate subjets [146–148]. However, contributions of these spurious, low-energy C/A subjets are suppressed by the energy-weighting of the EWOC, and we find very similar results for EWOCs using both  $k_T$  and C/A subjets.

<sup>18</sup>Recombination schemes that produce massive subjets may also offer interesting applications. For example, in the  $E$ -scheme for subjet recombination [149], the mass EWOC interpolates between collinear-unsafe mass correlations — the particle-level mass EWOC with  $r_{\text{sub}} = 0$  — and the jet mass itself — when  $r_{\text{sub}} = R_{\text{jet}}$ .

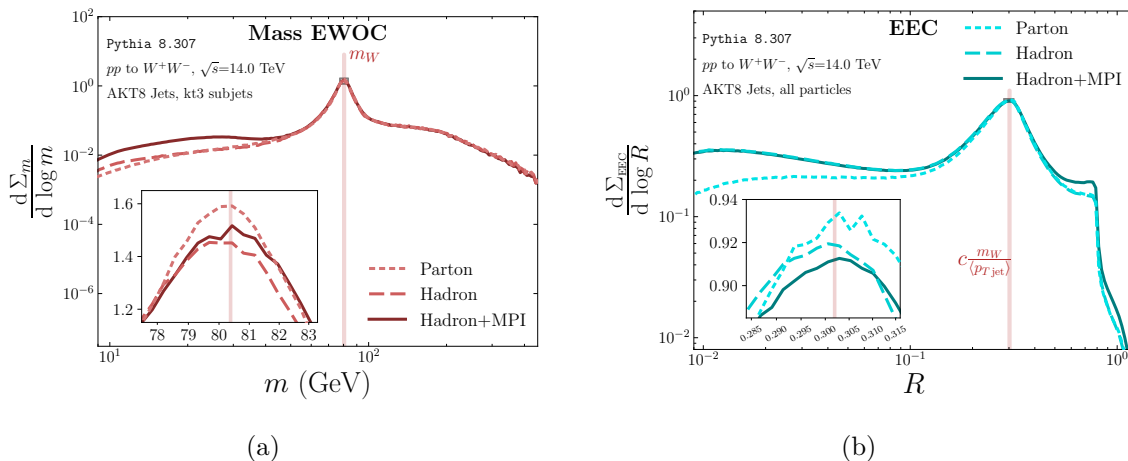
<sup>19</sup>For EWOC computations, we define energy fractions as the ratio of the subjet energy to the energy of the full jet. Therefore, while the energy weighting suppresses contributions from parametrically soft subjets, it does not suppress contributions from soft *jets*. The energy cut ensures that our EWOCs are not entirely swamped by soft jets. Since  $e^+e^-$  collisions usually contain two hard jets of energy  $E \sim \sqrt{s}/2 \gg 100$  GeV, our results are fairly insensitive to changes in this choice of energy cut.



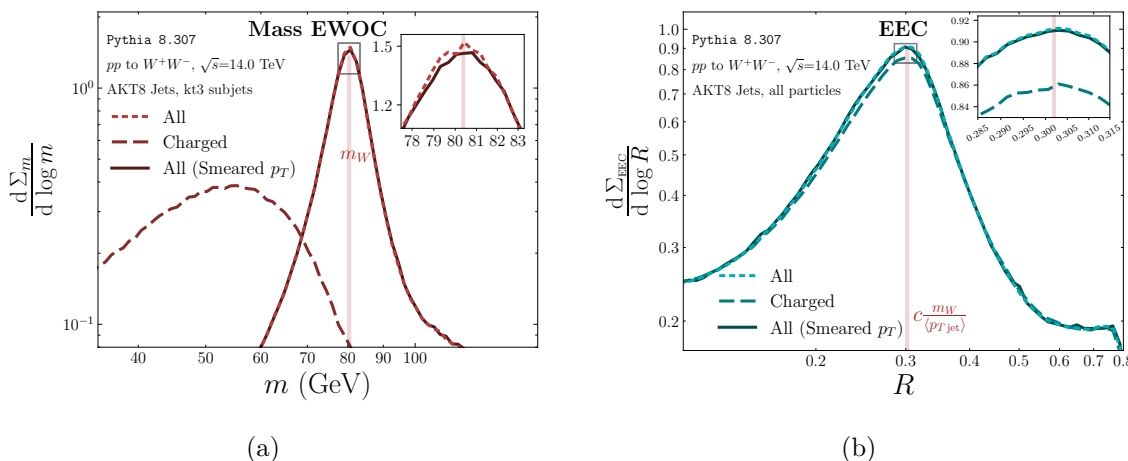
**Figure 3.** Mass EWOCs for  $W$ -boson pair production at the LHC at  $\sqrt{s} = 14$  TeV for anti- $k_T$  jets with  $k_T$  subjets. (a) Mass EWOCs for several subjet radii  $r_{\text{sub}}$ ; the peak at the  $W$  mass is most pronounced if  $r_{\text{sub}}$  is near the mean angular separation between the decay products of the  $W$  boson,  $\Delta\theta \sim 0.3$ . (b) The mass EWOC for  $r_{\text{sub}} = 0.3$  compared to the distribution of the Soft-Drop-groomed  $W$ -jet mass. The mass EWOC near  $m_W$  is more robust to the presence of the underlying event (multiple parton interactions) than the groomed jet mass, though it experiences large corrections due to UE in the small-mass region.



**Figure 4.** Variation in (a) the mass EWOC with  $r_{\text{sub}} = 0.3$  and (b) the EEC, for simulated LHC  $W$ -boson pair production events as the cut on the minimum jet  $p_T$  is varied. The peak position of the mass EWOC is invariant to the choice of minimum  $p_T$  of the  $W$  jets, while the peak position of the EEC changes as the minimum  $p_T$  is varied. Nonetheless, the peak of the EEC is well approximated by  $R_{\text{peak}} \approx c m_W / \langle p_{T,\text{jet}} \rangle$ , where  $\langle p_{T,\text{jet}} \rangle$  is a function of the minimum  $p_T$  cut and  $c \sim 2.5$  is the same for each value of the minimum  $p_T$ .

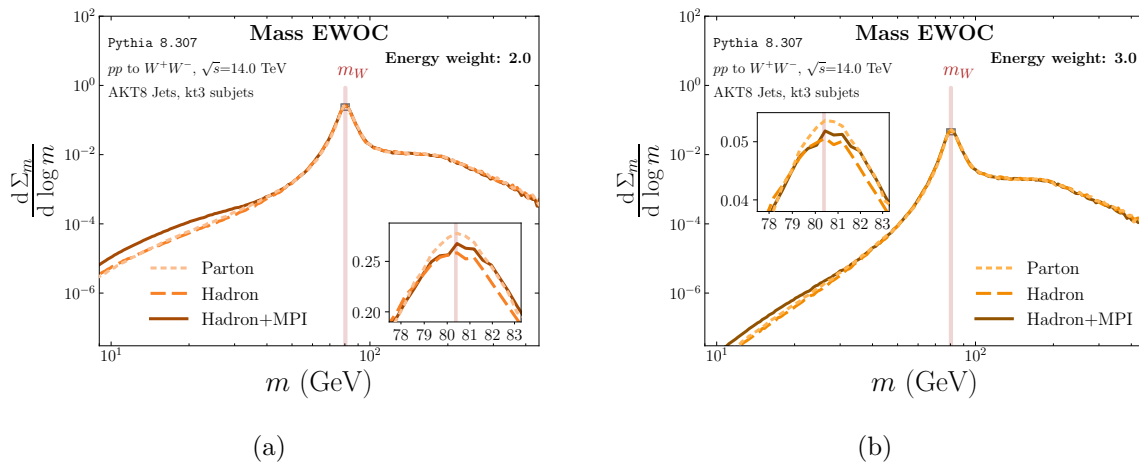


**Figure 5.** Non-perturbative hadronization and UE (MPI) effects on (a) the mass EWOC with  $r_{\text{sub}} = 0.3$ , and (b) the EEC. Both have peaks which are resilient to each source of non-perturbative corrections.



**Figure 6.** Effects of the exclusion of neutral particles, and of a Gaussian model of particle-level momentum smearing emulating the analysis of CMS-SMP-22-015 [57], on (a) the mass EWOC with  $r_{\text{sub}} = 0.3$ , and (b) the EEC. While both the EEC and the mass EWOC are robust to the effects of momentum smearing, only the EEC is qualitatively unchanged by the exclusion of neutral particles. The mass EWOC, on the other hand, is roughly rescaled by a factor of 2/3 when neutral particles are ignored.

Figure 3 shows the mass EWOC at several different values of the  $k_T$  subjet radius as well as a direct comparison of the mass EWOC to the mass distribution of jets groomed with the modified mass drop tagger (mMDT) [114, 115], which is much more robust than the ungroomed jet mass in the estimation of  $m_W$ . The mass EWOC is best at singling out the  $W$ -boson mass when we pick a subjet radius roughly equal to the expected separation between the partonic decay products of the  $W$ :  $r_{\text{sub}} \sim m_W / \langle p_{T,\text{jet}} \rangle \sim 0.3$ . At this tuned subjet radius, the location of the mass EWOC peak, and the corresponding inference of the  $W$  mass, is extremely robust to hadronization and UE. We also note that the mass EWOC at low masses (away from the peak at  $m_W$ ) is more robust to hadronization than the mMDT-groomed



**Figure 7.** Non-perturbative hadronization and underlying event corrections to the mass EWOC with  $r_{\text{sub}} = 0.3$  and non-unity energy weights  $n > 1$  to mitigate low-energy effects (see section 2.3), for (a)  $n = m = 2$  and (b)  $n = m = 3$ . Notably, larger energy weights mitigate the effects of UE at low mass scales. Both distributions, like the mass EWOC with energy weight of unity, are resilient to hadronization effects.

Shift in $m_W$ Determination from the peak of each distribution				<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">More robust</div> <div style="margin-right: 10px;">←</div> <div style="margin-right: 10px;">→</div> <div style="margin-right: 10px;">Less robust</div> </div>
$\Delta$	$n = 1$	$n = 2$	$n = 3$	
Smearing (cf ref. [57])	94 MeV	73 MeV	69 MeV	<div style="display: flex; align-items: center;"> <div style="width: 100px; height: 10px; background-color: #f8d7da; margin-right: 5px;"></div> <div style="width: 80px; height: 10px; background-color: #fff3f3; margin-right: 5px;"></div> <div style="width: 70px; height: 10px; background-color: #fff9c4;"></div> </div>
Parton vs. Hadron	-145 MeV	-127 MeV	-114 MeV	<div style="display: flex; align-items: center;"> <div style="width: 120px; height: 10px; background-color: #f8d7da; margin-right: 5px;"></div> <div style="width: 100px; height: 10px; background-color: #fff3f3; margin-right: 5px;"></div> <div style="width: 80px; height: 10px; background-color: #fff9c4;"></div> </div>
UE (MPI) On/Off	271 MeV	241 MeV	230 MeV	<div style="display: flex; align-items: center;"> <div style="width: 180px; height: 10px; background-color: #f8d7da; margin-right: 5px;"></div> <div style="width: 160px; height: 10px; background-color: #fff3f3; margin-right: 5px;"></div> <div style="width: 140px; height: 10px; background-color: #fff9c4;"></div> </div>
				$0 \quad 5 \cdot 10^{-2} \quad 0.1 \quad 0.15 \quad 0.2 \quad 0.25 \quad 0.3$ $ \Delta  \text{ (GeV)}$

**Table 2.** Shifts in the peaks of mass EWOCs with different energy weightings  $n$  due to detector effects or non-perturbative physics, presented in the style of table 1. We observe that the determination of  $m_W$  is less robust to particle-level momentum smearing when using higher energy weights, but that higher energy weights are more robust to non-perturbative QCD effects.

jet mass distribution. However, since subjets capture all additive contamination due to the UE within the subjet radius, the mass EWOC gains larger UE corrections at low masses than the mMDT-groomed jet mass.

Figure 4 visualizes the effects of changing the selection cuts on the  $W$ -jet samples by varying the minimum  $p_T$  of the  $W$ -jets by 100 GeV about  $p_{T,\min} = 500$  GeV. Though the mass EWOC has a peak that remains at the  $W$ -mass for each value of  $p_{T,\min}$ , the peak of the EEC shifts as one varies the minimum  $p_T$ : changing the allowed  $p_T$  of the jets also changes the associated angular scales between their constituents.

In figures 5 and 7, we examine non-perturbative corrections to EWOCs through the effects of hadronization and UE. Figure 5 compares non-perturbative effects in the mass EWOC and the EEC, focusing on the changes in the peak of each distribution. The peak of both the mass EWOC and the EEC remain nearly unchanged, and have the potential to provide robust determinations of  $m_W$ . Away from the peak, however, both distributions are affected by non-perturbative physics. At small angular scales, the EEC receives relatively large corrections from hadronization but relatively small corrections from UE: hadronization has the potential to change angular scales of hard particles within a jet, while UE provides a background of soft particles which are damped by the energy weighting of the EEC. On the other hand, at small mass scales, the mass EWOC is unchanged by hadronization but receives relatively large corrections from UE: subjets comprise collective degrees of freedom which are relatively unchanged by hadronization by construction, but which gain contributions from UE to their momenta — and therefore their pairwise masses — proportional to the subjet area.

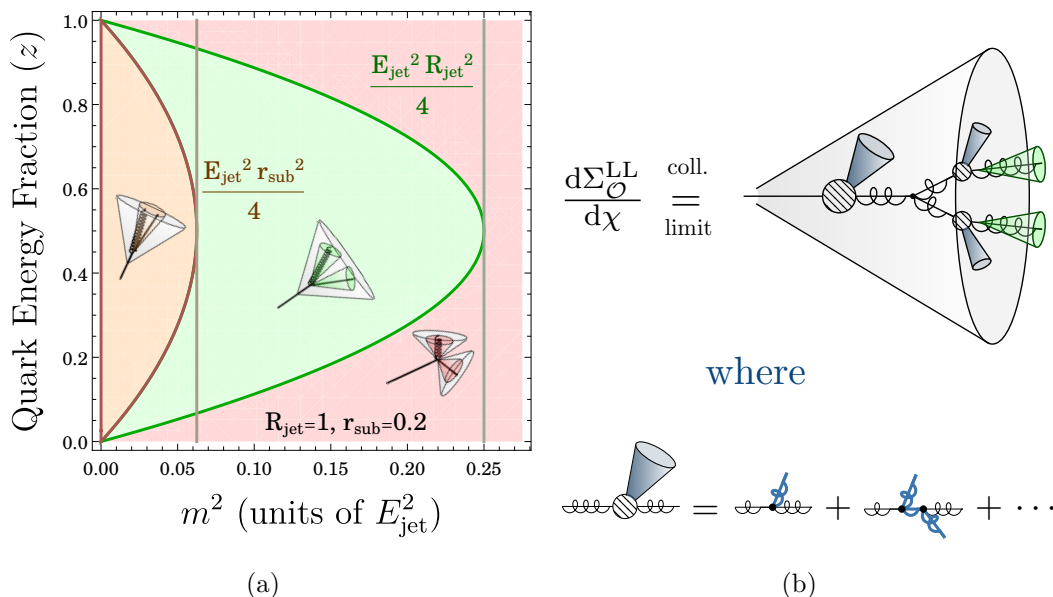
In figure 6, we show changes in the mass EWOC and the EEC due to the exclusion of neutral particles as well as a rough model of experimental detector effects.<sup>20</sup> To model detector effects, we implement a particle-level Gaussian momentum smearing of 5% for neutral particles, 3% for photons, and 1% for charged particles, using the values of the analysis in CMS-SMP-22-015 [57]. We do not implement any angular smearing, however, citing the excellent angular resolution of the CMS detector [57, 111]. We find that the peak of the mass EWOC and of the EEC receive negligible contributions from the smearing of particle momenta. Furthermore, the EEC remains roughly unchanged by the exclusion of neutral particles. However, the exclusion of neutral particles changes the masses of subjets and hence the shape of the mass EWOC, rescaling each by roughly a factor of 2/3.<sup>21</sup>

Finally, in figure 7, we visualize the utility of mass EWOCs with non-unity energy weights to mitigate the effects of UE, as discussed in section 2.3. Figures 7(a) and 7(b) show plots of the mass EWOC with  $r_{\text{sub}} = 0.3$  with energy weights  $n = m = 2$  and  $n = m = 3$ , respectively. Table 2 tabulates the shifts in the peaks of the mass EWOC for these energy weights due to both momentum smearing and non-perturbative effects, and compares these shifts to the analogous shifts in the mass EWOC with unity energy weights. We see that the use of higher

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<sup>20</sup>The EEC can naturally be extended to measurements on charged particles using the track function formalism [66, 77, 78, 151–156], and similar developments for EWOCs may benefit applications relying on the superior resolution of track-based measurements.

<sup>21</sup>This rescaling of energy scales when excluding neutral particles can be argued physically in the “isospin limit”: in the case where all outgoing particles are pions, and the  $\pi^+$ ,  $\pi^-$ , and  $\pi^0$  are all present with similar properties in the final state of a collision, ignoring the neutral pion leads to a loss of  $\sim 1/3$  of the energy of each event.



**Figure 8.** Ingredients in the calculation of EWOCs (a) at leading order (LO) and (b) at all orders. (a) The two-parton phase space can be divided into three regions, in which the two partons are contained either within the same subjet (orange), different subjets (green), or different jets (red). (b) A diagrammatic visualization of a leading-logarithmic computation of EWOCs in the collinear limit, by convolving QCD splitting functions with several instances of parton-to-parton fragmentation, discussed in greater detail in the text. Combined solid/coiled lines indicate a line of arbitrary partonic flavor, blue cones indicate the inclusive emission of a parton, and the three-point vertex denotes a leading-order QCD splitting function.

energy weights generates results that are slightly more resistant to both to our rough model of detector effects and the non-perturbative effects of hadronization and the UE.

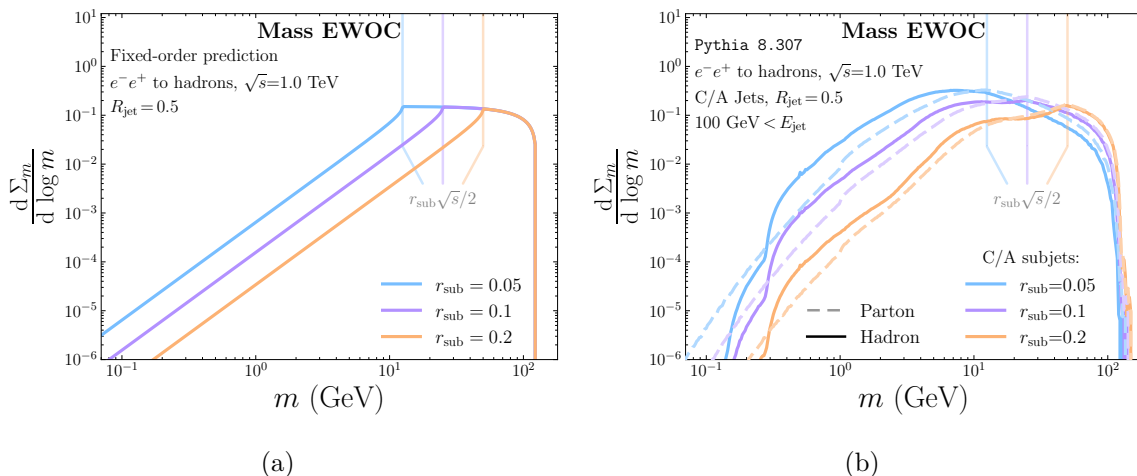
We conclude that the mass EWOC is a robust phenomenological tool for the extraction of mass scales in particle collisions. The use of subjets leads to results which are resilient to non-perturbative and experimental effects, and are quickly and intuitively interpretable as mass scales. Now, we turn towards mass EWOCs in the cleaner laboratory of electron-positron collisions in order to gain a deeper understanding of their analytic structure.

### 3.3 Perturbative results

Now that we have presented the story of EWOCs in a realistic phenomenological example, we compare the features of mass EWOCs in light-quark-initiated jets obtained both perturbatively at leading order (LO) in  $\alpha_s$  and with `Pythia 8.309` in the standard laboratory of  $e^+e^- \rightarrow$  hadrons. We expect that more detailed, leading-logarithmic results will depend on numerical analyses involving parton-to-parton fragmentation (e.g. the semi-inclusive jet function of ref. [157] or the microjet fragmentation functions of refs. [158, 159]), which we briefly outline at the end of this section.

The analysis we explore in this section involves jets consisting of two partons. The corresponding phase space may be separated into three regions as depicted in figure 8(a):

- the region with  $\theta < r_{\text{sub}}$  (the leftmost, orange region of figure 8(a)), where the two partons are grouped into the same subjet;



**Figure 9.** Mass EWOCs for  $e^+e^- \rightarrow$  hadrons (a) at LO and (b) in Pythia 8.309.

- the region with  $r_{\text{sub}} < \theta < R_{\text{jet}}$  (the middle, green region), where the two partons are within the same jet but different subjects;
- the region with  $R_{\text{jet}} < \theta$  (the rightmost, red region), where the two partons are grouped into different jets.

We will use a subjet recombination scheme that assigns zero mass to subjects — in our study of  $e^+e^-$  collisions, the winner-take-all (WTA)  $|\vec{p}|$  recombination scheme is a natural choice — to avoid complications due to subjet mass spectra. The non-trivial contribution to the EWOC then comes from the region of phase space when the two partons are separated by an angle  $r_{\text{sub}} < \theta < R_{\text{jet}}$  and are constructed as distinct subjects within the same jet. The other two regions, in which the partons are reconstructed as distinct jets or as within the same subject, contribute via “contact terms”: delta-functions, due to virtual corrections as well as 2-particle subjet self-correlations, whose combined coefficient is fixed by the fact that the EWOC integrates to one. For example, in the case of mass EWOC,  $d\Sigma_m/dm$ , the contact-term regions produce zero-mass contributions of the form  $(1 + \mathcal{O}(\alpha_s)) \delta(m)$ . We note, however, that if we used a subjet recombination scheme which produced massive subjects, the contact terms would no longer be delta functions at  $m = 0$  and the analysis of the resulting mass EWOC would be more difficult.

For (massless) quark-initiated jets, we find that the LO mass EWOC for  $m > 0$  is given by

$$\frac{d\Sigma_m^{\text{LO}}}{dm}(m > 0) = \frac{3\alpha_s C_F}{8\pi m} \begin{cases} V(m, s R_{\text{jet}}^2), & \sqrt{s} r_{\text{sub}}/4 < m < \sqrt{s} R_{\text{jet}}/4, \\ V(m, s R_{\text{jet}}^2) - V(m, s r_{\text{sub}}^2), & 0 < m < \sqrt{s} r_{\text{sub}}/4, \end{cases} \quad (3.1)$$

where  $V(m, Q^2) \equiv (1 - 8m^2/(3Q^2)) \sqrt{1 - 16m^2/Q^2}$  and  $\sqrt{s}$  is the center-of-mass energy of the collision.  $d\Sigma_m^{\text{LO}}/dm$  is also shown in figure 9(a).<sup>22</sup>

<sup>22</sup>Since the mass EWOC is guaranteed to integrate to one, the correctly normalized mass EWOC (including contributions at  $m = 0$ ) can always be written as  $d\Sigma_m/dm = \delta(m) + [d\Sigma_m/dm(m > 0)]_+$ , where  $[f(m)]_+$  indicates a plus-regularization of the function  $f$  which integrates to zero.

The mass EWOC for quark-initiated jets obtained via `Pythia 8.309` simulations of  $e^+e^- \rightarrow \text{hadrons}$  is shown in figure 9(b). The fixed-order mass EWOC has a kink at the mass scale  $\sqrt{s} r_{\text{sub}}/2$ , the largest mass scale at which the two partons of the fixed-order calculation can be grouped into a single subjet of radius  $r_{\text{sub}}$ , which separates the domain of the mass EWOC into two distinct regions. We expect that this kink is smoothed out by the effects of multiple emissions,<sup>23</sup> and indeed the mass EWOCs obtained with `Pythia 8.309` appear smooth at  $m = \sqrt{s} r_{\text{sub}}/2$ . Nonetheless, the shapes of the fixed-order and `Pythia 8.309` mass EWOCs are roughly the same, and the power laws that roughly govern the fixed-order mass EWOC are also present in the mass EWOC obtained with `Pythia 8.309`.

Having discussed the LO EWOC, we now briefly discuss the ingredients involved in a leading-logarithmic (LL) computation, which requires a more detailed numerical analysis of partonic fragmentation including the possibility of arbitrarily many partonic emissions. As visualized diagrammatically in figure 8(b), at LL accuracy and in the regime  $\sqrt{s} \sim \sqrt{s} R_{\text{jet}} \gg m \gg \sqrt{s} r_{\text{sub}} \gg \Lambda_{\text{QCD}}$ , this entails the computation of intricate convolution integrals of the form

$$\begin{aligned} \frac{d\Sigma_m^{\text{LL}}}{dm} &= \sum_j \int_0^1 dz_1 z_1^2 F_{j \leftarrow q}(z_1; \theta \leftarrow R_{\text{jet}}) \sum_{k,\ell} \frac{\alpha_s}{\pi} \int_0^1 dz' z'(1-z') \int_{r_{\text{sub}}}^{R_{\text{jet}}} \frac{d\theta}{\theta} P_{k\ell \leftarrow j}(z') \\ &\quad \times \sum_c \int_0^1 dz_2 z_2 F_{c \leftarrow k}(z_2; r_{\text{sub}} \leftarrow \theta) \sum_d \int_0^1 dz_3 z_3 F_{d \leftarrow \ell}(z_3; r_{\text{sub}} \leftarrow \theta) \\ &\quad \times \delta\left(m - \frac{\sqrt{s}}{2} z_1 \sqrt{z'(1-z')} \theta \sqrt{z_2 z_3}\right), \end{aligned} \tag{3.2}$$

which adds up the contributions to the mass EWOC from all possible branching histories of a quark-initiated jet (in the collinear limit) and all pairs of final state subjets. In eq. (3.2),  $P_{k\ell \leftarrow j}(z_k)$  is the leading order Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) splitting function [161–163] which encodes the pseudo-probability for a parton of type  $i$  to split into a pair of partons of types  $j$  and  $k$  with the energy fraction  $z_k = 1 - z_\ell = E_k/E_j$ . In QCD,  $\ell$  is fixed by  $j$  and  $k$ , and we can write  $P_{k\ell \leftarrow j}(z) = p_{k \leftarrow j}(z)$ .<sup>24</sup> The  $F_{j \leftarrow i}(z; \theta_j \leftarrow \theta_i)$  are parton-to-parton fragmentation functions which obey the DGLAP evolution equations and encode the probability density that a parton of type  $i$ , probed at an angular resolution  $\theta_i$ , contains a parton of type  $j$  with energy fraction  $z$  when probed at an angular resolution of  $\theta_j < \theta_i$ . The use of parton-to-parton fragmentation functions resums both observable-related logarithms (such as  $\ln m^2/s$ ) and (sub)jet-radius logarithms ( $\ln R_{\text{jet}}$  and  $\ln r_{\text{sub}}$ ). The resummed form of the parton-to-parton fragmentation functions required for the computation of eq. (3.2) is, to our knowledge, obtained only through numeric solutions to the DGLAP equation in the existing literature.<sup>25</sup>

In the case of the EEC, the argument of the delta function in eq. (3.2) depends only on the angle  $\theta$ , and not on any of the energy fractions. In this case, the momentum-fraction

<sup>23</sup>This expectation is supported by our rough initial studies of the analytic behavior of EWOCs at all orders, but we delegate a more rigorous justification to future work; similar results involving all-orders smoothing of kinks found in fixed-order distributions of jet substructure observables can be found in e.g. ref. [160].

<sup>24</sup>For example,  $P_{qg \leftarrow q}$  is non-zero, but  $P_{gg \leftarrow q}$  is zero.

<sup>25</sup>Beyond LL accuracy, the resummation of parton-to-parton fragmentation functions is not described by standard DGLAP, see e.g. refs. [164–167].

integrals decouple and become *Mellin moments*  $\hat{F}_{j\leftarrow i}(\kappa) \equiv \int_0^1 dz z^{\kappa-1} F_{j\leftarrow i}(z)$  of the parton-to-parton fragmentation functions, whose evolution is determined by the Mellin-space form of the DGLAP evolution equations,

$$\frac{d}{d \log r} \hat{F}_{j\leftarrow i}(\kappa; r \leftarrow R) = \sum_k \frac{\alpha_s}{\pi} \hat{F}_{j\leftarrow k}(\kappa; r \leftarrow R) \hat{p}_{k\leftarrow i}(\kappa). \quad (3.3)$$

This leads quickly to the leading-logarithmic form

$$\hat{F}_{j\leftarrow i}(\kappa; r \leftarrow R) = \exp[\alpha_s \hat{p}(\kappa) \log(r/R)/\pi]_{j\leftarrow i}$$

for the parton-to-parton fragmentation functions,<sup>26</sup> and reproduces the leading-logarithmic EEC obtained via jet calculus [168–170]. However, for more generic EWOCs, the momentum-fraction integrals do not decouple, and numerical integration of eq. (3.2) will be required to obtain more precise, leading-logarithmic results.

## 4 Discussion and conclusions

In this work, we introduced Energy-Weighted Observable Correlations (EWOCs) as a flexible new tool for studying the patterns of radiation within hadronic jets produced in particle collisions, generalizing the more familiar Energy-Energy Correlator (EEC). We examined the formal features of EWOCs, their utility in the extraction of mass scales in particle collisions, and their analytic structure.

We began by motivating and defining EWOCs as generalizations of the EEC which can be chosen to probe a variety of different multi-particle correlations, such as the dominant mass scales or formation times involved in jet production in a particular jet sample. We highlighted that collinear safety requires generic EWOCs to be equipped with an additional collinear cutoff, such as a subjet radius. Consequently, the subjet EWOCs on which we focus in this work are more robust to non-perturbative effects such as hadronization. We also found that EWOCs, unlike the particle-level EEC, remain collinear-safe even when using non-unity energy weights, allowing for additional suppression of obfuscating soft physics such as hadronization and the underlying event (UE).

We then restricted ourselves to the study of the mass EWOC. We showed that the mass EWOC provided an estimate of the  $W$  boson mass which was even more robust to the effects of hadronization and UE than the analogous estimate using the EEC (and comparable to the estimate of  $m_W$  using groomed jet masses). We also showed that EWOCs with higher energy weights were more robust to hadronization, UE, and a rough model of smearing due to experimental detectors. We then presented the mass EWOC at leading order in  $e^+e^- \rightarrow$  hadrons — the simplest testbed of perturbative QCD. We conclude that EWOCs provide new perspectives into physical correlations within jets which are robust to soft radiation, and are flexible probes of physics in a variety of contexts due to the user-defined observable and subjet radius.

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<sup>26</sup>See, for example, table III of ref. [168] for expressions for the  $\hat{P}_{j\ell\leftarrow i}(\kappa)$ .

There are several straightforward avenues for future work: Extending the analysis of section 3 to real collision data and including the effects of QCD backgrounds would provide a more realistic test of the utility of the mass EWOC in extracting the  $W$  mass and beyond. The most precise existing measurements of the  $W$  boson mass involve leptonic decays of the  $W$  boson and require a detailed understanding of recoil and missing energy [171]. The use of the mass EWOC allows us instead to study the mass of the  $W$  boson through its hadronic decays, and without sensitivity to recoil.<sup>27</sup> The study of more complicated observables with EWOCs in more variegated contexts is also a promising approach towards extending the utility of EWOCs; for example, the study of formation-time EWOCs in heavy-ion collisions may elucidate time scales associated with the interactions of high-energy jets and the quark-gluon plasma. More broadly, the use of energy flow polynomials [172] as EWOC observables could provide a powerful new tool for studying a variety of correlations within jets.

A particularly simple and phenomenologically interesting extension of the analysis presented in this work is the study of EWOCs to three-point observables; the three-point mass EWOC, for example, could be used to extract the top quark mass from LHC data. We also note that EWOCs may have applications in searches for physics beyond the Standard Model (BSM). In particular, for models which predict the production of SM jets from the decay of BSM particles, the substructure of the SM jets may be used to probe the properties of the BSM particles. It is possible that our proof of concept for the mass EWOC as a tool for extracting the  $W$ -boson mass from LHC collisions in this work may be extended to the extraction of mass scales involved in BSM decays.

This paper serves as an invitation to further explore the landscape of EWOCs and their applications. We expect that EWOCs and their generalizations can contribute to the bridge between experimental and theoretical particle physics, providing new insights into nature from particle collisions.

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<sup>27</sup>We thank Christoph Paus for emphasizing this feature to us.

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