Evanescent-wave mirrors for cold atoms
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Citation for published version (APA):
Voigt, D. (2000). Evanescent-wave mirrors for cold atoms Amsterdam: van der Waals Zeeman Instituut

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# Appendix

## A.1 Useful atom-optical numbers for $^{87}\text{Rb}$

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<th>Spectroscopy: [143,144]</th>
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<td><strong>D1 line</strong></td>
<td><strong>$\lambda_0$</strong></td>
<td>$(5s^2S_{1/2} \rightarrow 5p^2P_{1/2})$</td>
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<tr>
<td></td>
<td>natural lifetime $\tau$</td>
<td>$\equiv 1/\Gamma$</td>
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<tr>
<td></td>
<td>natural linewidth $\Gamma/2\pi$</td>
<td>$\equiv \pi\hbar\Gamma/3\lambda_0^2$</td>
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<tr>
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<td>saturation intensity $I_0$</td>
<td>$\equiv \pi\hbar\Gamma/3\lambda_0^2$</td>
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| **D2 line** | **$\lambda_0$** | $(5s^2S_{1/2} \rightarrow 5p^2P_{3/2})$ | 780.2 nm |
|  | $\tau$ | $\equiv 1/\Gamma$ | 26.24(4) ns |
|  | $\Gamma/2\pi$ | $\equiv \pi\hbar\Gamma/3\lambda_0^2$ | 6.07 MHz |
|  | $I_0$ | $\equiv \pi\hbar\Gamma/3\lambda_0^2$ | 1.67 mW/cm² |

### Laser cooling (D2 line):

- Doppler temperature $T_D \equiv h\Gamma/2k_B$  
  - 146 $\mu$K

- Typ. Doppler velocity $v_D \equiv \sqrt{2k_BT_D/M}$  
  - 4.7 m/s

- Doppler capture velocity $\Gamma/k_L$  
  - 16.7 cm/s

- Recoil temperature $T_R \equiv (hk_L)^2/Mk_B$  
  - 361 nK

- Recoil velocity $v_R \equiv hk_L/M$  
  - 5.88 mm/s

- Recoil frequency $\omega_R \equiv E_R/h = h\omega_R^2/2M$  
  - $2\pi \times 3.77$ kHz

- Thermal De Broglie wavelength $\lambda \equiv h/\sqrt{2\pi Mk_BT}$  
  - 15.5 nm ($T_D$)

- 312 nm ($T_R$)

### Gravitation:

- $Mg/k_B$  
  - 1.03 mK/cm

- $Mg/h$  
  - 21.4 MHz/cm

- $Mg/\mu_B$  
  - 15.3 G/cm

### Atomic collisions: [63]

- $a_{2,2}$ (s-wave scattering, length $a_{F,m}$)  
  - 109(10) $a_0$

- 106(6) $a_0$

### General constants:

- $h/k_B$  
  - 48.0 $\mu$K/MHz

- $k_B/h$  
  - 20.8 kHz/$\mu$K

- $\mu_B/h$  
  - 1.40 MHz/G

- $\mu_B/k_B$  
  - 67.2 $\mu$K/G

- $g$  
  - 0.67 K/T

- 0.98 (cm/s)/ms
A.2 Fresnel coefficients for evanescent waves

Fresnel coefficients are usually derived for the (complex) reflection and transmission coefficient of light incident with an angle \( \theta_i < \theta_c \) at a dielectric interface, where the internal reflection is not total [6,7]. However, these formulas can also be used for total internal reflection (TIR), \( \theta_i > \theta_c \), i.e. with a complex “transmission angle”. Snel’s law, \( \sin \theta_i = n \sin \theta_t > 1 \), then is written as

\[
\cos \theta_t = i \sqrt{n^2 \sin^2 \theta_t - 1}.
\]

The wave vectors and polarisations maintain their common form using the complex angle \( \theta_t \). The wave vectors are

\[
\begin{align*}
k_i &= n k_0 (\sin \theta_i, 0, \cos \theta_i), \\
k_t &= k_0 (\sin \theta_t, 0, \cos \theta_t).
\end{align*}
\]

The polarisations are

\[
\begin{align*}
s_i &= s_t = (0, 1, 0), \\
p_i &= (-\cos \theta_i, 0, \sin \theta_i), \\
p_t &= (-\cos \theta_t, 0, \sin \theta_t).
\end{align*}
\]

Note that \( p_t \) is not normalised in the usual way, \( p_t^* \cdot p_t \neq 1 \). Instead, it obeys the normalisation \( p_t^* \cdot p_t = 1 \). The reflection and transmission coefficients also keep the common form,

\[
\begin{align*}
r_s &= \frac{n \cos \theta_i - \cos \theta_t}{n \cos \theta_i + \cos \theta_t} = \frac{n \cos \theta_i - i \sqrt{n^2 \sin^2 \theta_i - 1}}{n \cos \theta_i + i \sqrt{n^2 \sin^2 \theta_i - 1}}, \\
t_s &= \frac{2n \cos \theta_i}{n \cos \theta_i + \cos \theta_t} = \frac{2n \cos \theta_i}{n \cos \theta_i + i \sqrt{n^2 \sin^2 \theta_i - 1}}, \\
r_p &= \frac{\cos \theta_i - n \cos \theta_t}{\cos \theta_i + n \cos \theta_t} = \frac{\cos \theta_i - i n \sqrt{n^2 \sin^2 \theta_i - 1}}{\cos \theta_i + i n \sqrt{n^2 \sin^2 \theta_i - 1}}, \\
t_p &= \frac{2n \cos \theta_i}{\cos \theta_i + n \cos \theta_t} = \frac{2n \cos \theta_i}{\cos \theta_i + i n \sqrt{n^2 \sin^2 \theta_i - 1}}.
\end{align*}
\]

However the transmission occurs into the evanescent wave, \( t_s \) and \( t_p \) are proportionality factors between the incident and the evanescent field amplitude. Indeed we find \(|r_s, p| = 1\).
A.3 Light forces and scattering rate

A.3.1 Two-level atoms

A detailed description of the atom-light interaction can be found, e.g., in Ref. [2]. We assume an atom with a ground state \(|g\rangle\) and an excited state \(|e\rangle\) of lifetime \(\tau = 1/\Gamma\), where \(\Gamma\) is the natural transition linewidth, e.g. with \(\Gamma/2\pi = 6.1\) MHz for the rubidium D2 line. The states are separated by \(\hbar\omega_0\), and the detuning of a laser frequency \(\omega_L\) is defined as \(\delta = \omega_L - \omega_0\). A useful expression in the description of the atom-light coupling is the Rabi frequency for a given laser intensity \(I_L\):

\[
\Omega_R = \Gamma \sqrt{\frac{I_L}{2I_0}}.
\]  
\[(A.11)\]

It describes the resonant (\(\delta = 0\)) cycling frequency between the ground and excited state population. The saturation intensity is defined as \(I_0 = \pi\hbar c\Gamma/3\lambda_0^3\), with the optical wavelength \(\lambda_0\).

Scattering rate and spontaneous force.— The atomic scattering rate is obtained by solving the “Optical Bloch Equations” (OBE). These describe the evolution of the density operator \(\hat{\sigma}\) of an atom coupled to the light field. We obtain the OBE’s for the Bloch vector, \((u, v, w)\), by the elimination of the fast evolution \(\propto \exp(i\omega_L t)\) of the laser oscillation in the “rotating-wave approximation”, and using the transform \(\{\sigma_{ge}, \sigma_{eg}, \sigma_{ee}, \sigma_{ee}\} = \{\sigma_{ge} \exp(-i\omega_L t), \sigma_{eg} \exp(i\omega_L t), \sigma_{eg}, \sigma_{ee}\}\):

\[
u = \frac{1}{2} (\sigma_{ge} + \sigma_{eg}), \quad \dot{u} = \delta v - \frac{\Gamma}{2} u, \quad (A.12)
\]

\[
v = \frac{1}{2i} (\sigma_{ge} - \sigma_{eg}), \quad \dot{v} = -\delta u - \Omega_R w - \frac{\Gamma}{2} v, \quad (A.13)
\]

\[
w = \frac{1}{2} (\sigma_{ee} - \sigma_{ge}), \quad \dot{w} = \Omega_R v - \Gamma w - \frac{\Gamma}{2}. \quad (A.14)
\]

The component \(w\) describes half the population inversion between the atomic states. A useful notation is also the saturation parameter:

\[
s_0 = \frac{1}{2} \frac{\Omega_R^2}{\delta^2 + \left(\frac{\Gamma}{2}\right)^2} = \frac{1}{1 + \left(\frac{\delta^2}{\Gamma^2}\right) + \frac{\hbar I_L}{I_0}}. \quad (A.15)
\]

As a steady-state solution for the scattering rate \(\Gamma'\), i.e. the excited state population \(\sigma_{ee}^{(st)}\), we find:

\[
\Gamma' = \Gamma \sigma_{ee}^{(st)} = \frac{\Gamma}{2} \frac{s_0}{1 + s_0} = \frac{\Gamma}{2} \frac{1}{1 + \left(\frac{\delta^2}{\Gamma^2}\right) + \frac{\hbar I_L}{I_0}}. \quad (A.16)
\]

The recoil, \(\hbar k_L\), from absorbed photons causes radiation pressure or, the “spontaneous light force”, \(F_{sp} = \hbar k_L \Gamma'\). This force saturates for \(s_0 \gg 1\) as \(F_{sp} = \hbar k_L \Gamma/2\).
Far off-resonance dipole potentials.— In the limit of large detuning, $|\delta| \gg \Gamma$, we can approximate the saturation parameter by $s_0 \approx (\Gamma / 2\delta)^2 I_L / I_0$. If also $|\delta| \gg \Omega_R$, the eigenstates of the atom-light interaction approach the uncoupled states, $|g\rangle$ and $|e\rangle$, and the coupling to the field effectively causes a “light shift” of these states or, a “dipole potential”. If in this limit also the saturation parameter is small, $s_0 \ll 1$, the (ground state) light shift and the scattering rate are given as:

$$\frac{1}{\hbar} U_{\text{dip}} \approx \frac{1}{2} s_0 \delta \approx \frac{\Omega_R^2}{4\delta} = \frac{\Gamma^2}{8\delta} \frac{I_L}{I_0},$$

$$\Gamma' \approx \frac{1}{2} s_0 \Gamma' \approx \frac{\Omega_R^2 \Gamma}{4\delta^2} = \frac{\Gamma^3}{8\delta^2} \frac{I_L}{I_0}.$$  \hspace{1cm} (A.17, A.18)

The ratio of light shift and scattering rate is now simply $U_{\text{dip}} / \hbar \Gamma' \approx \delta / \Gamma$. Since the light shift is usually spatially varying, its gradient represents the “dipole force”, $F_{\text{dip}}(r) = -\nabla U_{\text{dip}}(r)$.

A.3.2 Multilevel atoms — rubidium hyperfine structure

In the interaction of a multilevel atom with a laser field, the polarisation state of the light has to be considered together with the coupling strengths of the various optical transitions between atomic sublevels. For example, the D2 line of $^{87}\text{Rb}$ is a $J_g = 1/2 \rightarrow J_e = 3/2$ transition. The coupling to the nuclear spin, $I = 3/2$, results in the hyperfine structure with the ground and excited states $F_g = \{1,2\}$ and $F_e = \{0,1,2,3\}$, respectively, shown in Fig. 3.7.

Spherical polarisation basis.— The expression (A.11) for the Rabi frequency has its origin in the coupling of the atomic dipole moment to the electric field, written as $E = (1/2)\hat{\epsilon} E \exp(-i\omega t) + c.c.$:

$$\Omega_R = 2 \frac{\mathbf{d} \cdot \hat{\epsilon} E}{\hbar}. \hspace{1cm} (A.19)$$

The matrix element of the dipole operator $\mathbf{D}$ is here $\mathbf{d} = \langle g|\mathbf{D}|e\rangle$, the electric field amplitude is $E$, and the field polarisation is given by the unit vector $\hat{\epsilon}$.

It may be useful to work in a spherical basis $\{\hat{\epsilon}_-, \hat{\epsilon}_0, \hat{\epsilon}_+\}$, that is defined in the cartesian basis $\{\hat{x}, \hat{y}, \hat{z}\}$ as:

$$\hat{\epsilon}_- = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -i \end{pmatrix}, \quad \hat{\epsilon}_0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \hat{\epsilon}_+ = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 \\ 0 \\ -i \end{pmatrix}. \hspace{1cm} (A.20)$$

These basis vectors describe $\sigma^-$, $\pi$, and $\sigma^+$-polarised light with respect to the deliberately chosen $y$-direction. The dipole operator can now be expressed in the spherical basis, $D_j = \mathbf{D} \cdot \hat{\epsilon}_j$, where $j = \{0, \pm 1\}$. 

Reduced dipole matrix elements.— Writing ground and exited state of a rubidium atom as \(|F_g, m_g\rangle\) and \(|F_e, m_e\rangle\), the Wigner-Eckart theorem is applied to factorise the dipole matrix element:

\[
\langle F_g, m_g | D | F_e, m_e \rangle = \langle F_g | D | F_e \rangle \langle F_e, m_e, 1, j | F_g, m_g \rangle .
\]  

(A.21)

The first term is the "reduced dipole matrix element", \(D_{F_g,F_e}\). It is independent of the atomic orientation, i.e. polarisation and sublevel structure. The second term is a Clebsch-Gordan coefficient, describing the coupling of the sublevels to the spherical polarisation component \(j\) of the light field.

The reduced matrix elements are calculated starting from the matrix element \(D_{2,3}\) for the closed transition of the rubidium D2 line, which is equivalent to the reduced matrix element of a two-level atom. This can be expressed using the Eqs. (A.11), (A.19), the saturation intensity, and the relation \(I_s = (1/2)\varepsilon_0 c|\mathcal{E}|^2\):

\[
D_{2,3} = \sqrt{\frac{3\varepsilon_0 \lambda_0^3}{8\pi^2}} = 2.53 \times 10^{-29} \text{ C m}.
\]  

(A.22)

The reduced matrix elements of the other hyperfine transitions are calculated by

\[
D_{F_e,F_e} = D_{2,3} (-1)^{F_e+J_e+I+1} \sqrt{(2J_e + 1)(2F_g + 1)} \begin{bmatrix} F_g & F_e & 1 \\ J_e & J_g & I \end{bmatrix}_{ij} ,
\]  

(A.23)

\[
d_{F_e,F_e} = \frac{D_{F_e,F_e}}{D_{2,3}} .
\]  

(A.24)

where \(d_{F_e,F_e}\) is a dimensionless expression relative to the closed transition. The Racah "6j" symbol and the Clebsch-Gordan coefficients can be calculated, using e.g. the Mathematica software package (Wolfran Research).

Light-shift Hamiltonian for rubidium.— In order to calculate the light-shift Hamiltonian for a given polarisation \(\mathcal{E}\), it is useful to define a "reduced light-shift Hamiltonian" \(\Lambda(F_g, F_e, \mathcal{E})\) with matrix elements \(\Lambda_{m_e,m_{e'}}(F_g, F_e, \mathcal{E})\), which result from the angular part of Eq. (A.21), see e.g. Ref. [79].

We therefore define a tensor \(\mathcal{C}(F_g, F_e)\) with the Clebsch-Gordan coefficients as elements, \(\mathcal{C}_{m_e,m_{e'}}(F_g, F_e) = \langle F_g, m_g, 1, j | F_e, m_e \rangle\). The elements of the polarisability tensor \(\mathcal{A}(F_g, F_e)\) are therewith defined as:

\[
\mathcal{A}_{j',m_{e'},e'}(F_g, F_e) = \sum_{m_e} \mathcal{C}_{m_e,m_{e'}}(F_g, F_e) \mathcal{C}_{m_e,m_{e''}}(F_g, F_e) .
\]  

(A.25)

They describe the coupling to the light field in terms of an excitation of an atom from \(|F_g, m_g\rangle\) to \(|F_e, m_e + j\rangle\) by the component \(\varepsilon_j\) of the polarisation \(\mathcal{E}\), followed by a (stimulated) deexcitation to \(|F_g, m_g' = m_g + j - j'\rangle\) by the component \(\varepsilon_{j'}\).
The reduced light-shift Hamiltonian is now defined in the spherical polarisation basis as:

\[ \Lambda(F_g, F_e, \hat{\epsilon}) = \hat{\epsilon}^\dagger \cdot \mathbf{A}(F_g, F_e) \cdot \hat{\epsilon}. \] (A.26)

In low-saturation and for large detuning, the light-shift Hamiltonian for a rubidium atom in the ground state \( F_g \) can be written similarly to the 2-level expression of Eq. (A.17):

\[ \mathbf{U}_{F_g} = \frac{\hbar^2}{8} \frac{I_L}{I_0} \sum_{F_e} \frac{d_{F_g,F_e}^2 \Lambda(F_g, F_e, \hat{\epsilon})}{\delta_{F_g,F_e}}. \] (A.27)

This expression has to be calculated for both the D1 and the D2 line, summing over \( F_e^{(D1)} = \{1, 2\} \) and \( F_e^{(D2)} = \{0, 1, 2, 3\} \), respectively. Also the reduced dipole matrix elements have to be calculated for both lines. The total light shift is obtained as \( \mathbf{U}_{F_g}^{(tot)} = \mathbf{U}_{F_g}^{(D1)} + \mathbf{U}_{F_g}^{(D2)} \). However, either the detunings \( \delta_{F_g,F_e}^{(D1)} \) or \( \delta_{F_g,F_e}^{(D2)} \) are usually small compared to the splitting of 7.2 THz (or 15 nm) between the D-lines. Thus calculating the dominant contribution may be sufficient.

### A.3.3 Transition matrix elements for \(^{87}\text{Rb}\)

**Figure A.1:** Transition matrix elements of the D1 line: \( 60(d_{F_g,F_e}^{(D1)} C_{m_g,m_e}(F_g, F_e))^2 \).
Figure A.2: Transition matrix elements of the D2 line: $60 \left( d_{F_g,F_e}^{(D2)} C_{m_0,m_{g,j}}(F_g,F_e) \right)^2$. 
A.4 Analysis of absorption images

Due to the vertical symmetry axis of our mirror configuration, a factorised spatial density of $N$ atoms is assumed,

$$\rho(r) = N \rho_x(x) \rho_y(y) \rho_z(z), \quad N = \iiint \rho(r) dr^3, \quad (A.28)$$

$$\rho_x(x) = \frac{1}{\sqrt{2\pi} \sigma_x} \exp(-\frac{x^2}{2\sigma_x^2}), \quad (A.29)$$

$$\rho_y(y) \quad \text{similarly with} \quad \sigma_y = \frac{\sigma_x}{\chi}. \quad (A.30)$$

The absorption measurements project the distributions onto the $xz$-plane, while integrating in the $y$-direction (the line-of-sight). The coordinates in the $xz$-plane can be defined as $r' = (x, z)$. An absorption image, $A(r') = I_d(r')/I_l(r')$, is the ratio of the detected probe laser intensity, $I_d(r')$, and the incident intensity, $I_l(r')$.

For a coordinate $r'$ in the detection plane, the absorption law is written as:

$$\frac{dI(r)}{dy} = -\rho(r) \sigma(\delta) I(r), \quad (A.31)$$

$$\sigma(\delta) = \frac{3\lambda_0^2}{2\pi} \frac{1}{1 + 4(\frac{\delta}{\Gamma})^2}. \quad (A.32)$$

Here, $\sigma(\delta)$ is the detuning dependent absorption cross section for unity Clebsch-Gordan coefficients [34]. By integration along the $y$-direction, the relation between the absorption image, $A(r')$, and the atomic density is found in the $xz$-projection:

$$D(r') = -\ln A(r') = N \sigma(\delta) \rho_x(x) \rho_z(z). \quad (A.33)$$

From the image data, a line sum along $x$ can be formed, which leads to the vertical atomic density, $\rho_z(z)$:

$$L(z) = \int_{-\infty}^{+\infty} \ln A(r') dz = N \sigma(\delta) \rho_z(z). \quad (A.34)$$

The projected density, $D(r')$, allows to read out the horizontal Gaussian width, $\sigma_x$. Hence, by the Eqs. (A.29) and (A.30), the 3D density from Eq. (A.28) is known.