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Chen, J.; Schouten, J.A.

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Comment on “A new approach to optimum design in thermoelectric cooling systems” [J. Appl. Phys. 80, 5494 (1996)]

JinCan Chen\textsuperscript{a)}
Department of Physics, Xiamen University, Xiamen 361005, The People’s Republic of China and Van der Waals-Zeeman Laboratory, University of Amsterdam, Valckenierstraat 65, 1018 XE, Amsterdam, The Netherlands

Jan A. Schouten
Van der Waals-Zeeman Laboratory, University of Amsterdam, Valckenierstraat 65, 1018 XE, Amsterdam, The Netherlands

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It is pointed out that there are some errors existing in a recent investigation in this journal [M. Yamanishi, J. Appl. Phys. 80, 5494 (1996)]. The correct results are given so that one can better understand the performance of real thermoelectric cooler systems. © 1997 American Institute of Physics. [S0021-8979(97)05323-1]

The theoretical analysis of the optimal design of thermoelectric cooler (TEC) systems has been one of the most important subjects in the investigation of thermoelectric devices. Several authors have considered the effect of thermal resistances between the TEC and the external heat reservoirs on the performance of the TEC system and obtained a lot of significant results.\textsuperscript{1–4} In a recent article\textsuperscript{5} in this journal, Yamanishi put forward a new cycle model, which is useful for the performance analysis of TEC systems. However, the main Eqs. (43) and (52) in Ref. 5 and the relevant results are incorrect. In addition, there are some other problems in Ref. 5, which need to be discussed further.

(i) Using Eqs. (14)--(24) in Ref. 5 and the same notation, we obtain the coefficient of performance (COP)

\[
\eta = \frac{Q_C}{Q_H - Q_C}
\]

\[
= \frac{q_C}{\theta_H q_H - q_C}
\]

\[
= \frac{j - \frac{1}{2} j^2 \frac{1}{ZT_C} \left( 1 + \frac{2 \beta}{1 - \beta} \right) - \frac{1}{1 - \beta j} (\theta_H - 1)}{j \left( \theta_H \frac{1 + \lambda j}{1 - \beta j} - 1 \right) + \frac{1}{2} j^2 \frac{1}{ZT_C} \frac{2(1 + \lambda + \beta + (\lambda - \beta) j)}{1 - \beta j}}
\]

(1)

and the dimensionless cooling rate

\[
q_C = j(1 - \beta j) - \frac{1}{2} j^2 \frac{1}{ZT_C} \frac{(1 - j + 2 \beta) - (\theta_H - 1)}{(1 + \lambda + \lambda j)(1 - \beta j) + (1 + \lambda j) \beta}
\]

(2)

Comparing Eq. (1) with Eq. (43) in Ref. 5, one can find without difficulty that some calculative errors have been included in Eq. (43) in Ref. 5, so that the relevant results derived in Ref. 5 may not be correct.

(ii) In the optimal design of the TEC, one of the important problems is to determine the structure parameters of the device. It may be easily proven that for given semiconductor materials, when

\[
\frac{l_n/S_n}{l_p/S_p} = \sqrt{\frac{\kappa_p \rho_n}{\kappa_n \rho_p}}
\]

the figure of merit of the device, Z, attains its maximum and the total electrical resistance R and thermal conductance K of a TEC should be, respectively, determined by

\[
R = (\rho_p + \sqrt{\rho_p \rho_n \kappa_n / \kappa_p}) \frac{l_p}{S_p} n,
\]

(4)

\[
K = (\kappa_p + \sqrt{\kappa_p \kappa_n \rho_p / \rho_n}) \frac{S_p}{l_p} n.
\]

(5)

In this case, the geometric configuration of the device is optimal.\textsuperscript{5} It is seen from Eq. (3) that under the assumption \(l_n/S_n = l_p/S_p\) of Ref. 5 the maximum value of Z can be obtained only in the very special case of \(\rho_n/\kappa_n = \rho_p/\kappa_p\). In general, the semiconductor materials have different electrical and thermal conductivities, so that \(\rho_n/\kappa_n\) is not equal to \(\rho_p/\kappa_p\). In the optimum design of the TEC systems, one should take Eq. (3) into account and use the substitution of our Eqs. (4) and (5) for Eqs. (1) and (3) in Ref. 5. Moreover, one should determine further the optimum value of \((S_p/l_p)n\). For given semiconductor materials and specified operating conditions, the parameters \(ZT_C\), \(L_C\), \(L_H\), and \(\theta_H\) are known quantities. It is seen from Eqs. (2) and (1) that the dimensionless current \(j\) is a function of the thermal conductance \(K\) and \(Q_C/T_C\), so the COP may be written as

\[
\eta = \eta[j(Q_C/T_C, K, K)].
\]

(6)

For the general case, the concrete expression of Eq. (6) is complicated. The maximum COP and other relevant parameters may be calculated numerically. For example, when \(\beta/\lambda = L_C/L_H = 5\), \(ZT_C = 0.8\), \(L_C = 1.0\) W/K, \(\theta_H = 1.1\), and \(Q_C/T_C = 0.0015\) W/K, the maximum COP occurs at \(K = 0.0144\) W/K. Then, the optimum value of \((S_p/l_p)n\) is

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\textsuperscript{a}Electronic mail: jcc@xmu.edu.cn
According to Eq. (7) and the technological requirements, one may determine the values of \( S_p/l_p \) and \( n \). For a special case \( \beta = \lambda = 0 \), Eq. (6) may be written as

\[
\eta = \frac{b_1}{(1-b_1 b_2)K-\sqrt{(1-b_2)K^2-2bK-2bb_1}},
\]

where

\[
b = \frac{Q_C}{ZT_C}, \quad b_1 = \frac{1}{\theta_H+1}, \quad b_2 = \frac{2(\theta_H-1)}{ZT_C}.
\]

From Eq. (8), we can find that for a given \( Q_C/T_C \), when the COP is maximum, the optimum value of \( (S_p/l_p)n \) is

\[
\left( \frac{S_p}{l_p} \right)_{opt} = \frac{b/(1-b_2)}{\kappa_p + \sqrt{\kappa_p \kappa_n R_n / \rho_p}} \times \left[ 1 + \sqrt{1 + \frac{1-b_2}{(1-b_1 b_2)^2-(1-b_2)^2}} \right]
\]

\[
= \left( \frac{S_p}{l_p} \right)_{max}.
\]

Obviously, Eq. (10) may provide theoretically an instruction for the optimal design of the TEC. In real TEC systems, the thermal conductances between the TEC and the external heat reservoirs are always finite. The optimum value of \( (S_p/l_p)n \) should be chosen to be smaller than \( [(S_p/l_p)n]_{max} \).

(iii) According to the exergy efficiency of the TEC system defined in Ref. 5, we obtain

\[
\phi = \frac{E_i}{P} = (\theta_H-1) \frac{q_C}{\theta_H q_H - q_C} = (\theta_H-1) \eta.
\]

It is seen from Eq. (11) that the difference between the performance parameters \( \phi \) and \( \eta \) is only a factor \( (\theta_H-1) \), which is independent of the property of the TEC. For a given \( \theta_H \), \( \phi_{max} = (\theta_H-1) \eta_{max} \) and \( j(\phi_{max}) = j(\eta_{max}) \). Thus, it is unnecessary to take \( \phi \) as a new objective function in the optimal analysis of the TEC system, so that in Ref. 5 the discussion concerned is superfluous. Like Eq. (43) in Ref. 5, similar errors are also included in Eq. (52) in Ref. 5.

(iv) Analyzing Eqs. (27), (29), and (32) in Ref. 5, one will find easily that for the general case that \( L_C \) and \( L_H \) are finite, Eq. (32) doesn't represent the maximum dimensionless cooling rate of the TEC system operating in the limit \( \theta_H \rightarrow 1 \) and its physical meaning is not clear, so that the discussion concerned really is not significant. It can be proven from Eq. (2) that for finite \( L_C \) and \( L_H \) , when the dimensionless cooling rate of the TEC system operating in the limit \( \theta_H \rightarrow 1 \) attains its maximum, the corresponding dimensionless current \( j \) is not equal to the \( j_{max} \) in Eq. (29).

It is worthwhile noting that real TEC systems are always operated between two heat reservoirs with a nonzero temperature difference, so that it is not very significant to calculate the maximum dimensionless cooling rate of the TEC system operating between two heat reservoirs with a zero temperature difference. In the optimal analysis of the TEC system, it is more important to calculate the maximum dimensionless cooling rate of the TEC system for a given temperature difference.

It is significant to investigate the dimensionless power input of the TEC system. However, the relevant calculation in Ref. 5 also had a problem which is similar to the calculation of the maximum dimensionless cooling rate discussed above. In fact, one should calculate the dimensionless powers \( p(\eta_{max}) \) at the maximum COP and \( p(q_{c, max}) \) at the maximum dimensionless cooling rate, so that the optimal range of the power input can be determined.