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Cochlear models and minimum phase^{a)}

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In this paper the extent to which the response of a linear cochlear model has the “minimum-phase” property is discussed, along with the topic of what it should mean when experiments confirm or deny the validity of minimum-phase in the response of the actual cochlea. This paper shows that short-wave, long-wave and three-dimensional cochlear models of the “classical” type, in which the operation of the cochlear partition is described by a *local* function (namely, a driving-point impedance) that produces a cochlear map (from frequency to place), have a response that is minimum-phase, or very close to minimum-phase. Conversely, when the response is found to be non-minimum-phase, the best-fitting model cannot be a classical one. © 1997 Acoustical Society of America. [S0001-4966(97)03012-9]

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INTRODUCTION

Understanding the operation of the cochlea can be approached from (at least) two directions. In one strategy the emphasis is on experimenting, and a very large variety of stimulus signals are employed in order to separate and unravel parts of the mechanism involved. In the other approach a mathematical model of the cochlea is formulated, and the computed response of the model is compared with results of appropriately chosen experiments. An intermediate way is possible, in which general properties of models are studied and predictions are made as to what to look for in experiments. The present paper attempts to follow this strategy. It is discussed to which extent the response of a cochlear model has the “minimum-phase” property, and what it should mean when experiments demonstrate the validity or the absence of minimum-phase in the response of the actual cochlea.

Consider a model of the cochlea in which the fluid in the channels interacts mechanically with the cochlear partition, and assume that the model is *linear* and *stable*. Many models treated in the literature can be defined as “classical” models; they have the property that the mechanical reaction of the cochlear partition at the longitudinal location x depends only on the movements of the basilar membrane (BM) at the *same* location x (de Boer, 1991, 1996, 1997a). For a linear model this means that the mechanics of the partition is completely described by the *driving-point impedance* of the partition. Because the mechanical properties of the BM dominate over those of other elements, it is usual to speak of the *BM impedance*, and we will call it $Z_{BM}(x, \omega)$, where x is the longitudinal coordinate and ω the radian frequency. We assume that $Z_{BM}(x, \omega)$ shows resonance for the radian frequency ω at the location x_ω which depends on ω so that the model is capable of transforming (mapping) “frequency” into “place.”

In a classical model the BM impedance must be a real-

izable driving-point impedance, that is, the *BM impedance* $Z_{BM}(x, \omega)$ and its inverse, the *BM admittance* $Y_{BM}(x, \omega)$, must both be physically realizable (the associated impulse responses must not start before zero time). This implies that neither $Z_{BM}(x, \omega)$ nor $Y_{BM}(x, \omega)$ can have *poles* in the right half of the complex s plane.¹ This implies, too, that $Z_{BM}(x, \omega)$ does not have zeros in the right-half s plane.

Recently, “non-classical” models have appeared in the literature (Hubbard, 1993; Steele *et al.*, 1993; Geisler and Sang, 1995). In these models the mechanical reaction of the cochlear partition (or of the BM) at one location is additionally controlled by BM movements elsewhere, at other locations x . In such non-classical models the concept of BM impedance $Z_{BM}(x, \omega)$ can formally be retained, by defining it in terms of the quotient of the pressure (just above the BM) $p(x, \omega)$ and the BM velocity $v_{BM}(x, \omega)$. We will call this impedance the *effective BM impedance* and denote it by $Z_{BM}^{(eff)}(x, \omega)$. The effective impedance can, of course, only be found *after* the model solution has been computed. Because this impedance is no longer determined by *local* mechanical properties, it is no longer restrained to be a driving-point impedance. And the effective BM impedance may not even be a realizable transfer impedance, that is, this function or its inverse, or both, may have poles in the right-half of the complex s plane. Apart from this aspect of realizability, classical and non-classical models are equivalent as has recently been shown by the author (de Boer, 1997a).

A concept that, in linear-filter theory, is related to realizability is that of “minimum-phase.” By definition, a minimum-phase filter function has no *zeros* in the right-half of the s plane. When a filter response is minimum-phase, *the phase delay produced by the filter is smaller than that of any other filter that has the same amplitude-versus-frequency response*. A condensed proof of this property is given in the Appendix. In a minimum-phase filter there is a unique relation between *amplitude response* and *phase response* (both considered as functions of frequency), each of these is fully determined by the other one, namely, via the Hilbert transform. From the response of a filter that is *not* minimum-phase, a part can be isolated that has the characteristics of a pure delay. That delay may depend on frequency, but is *not*

^{a)}Note: The proof of Section I formed part of a poster presentation at the 1996 ARO Midwinter meeting (de Boer and Nuttall, 1996).

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associated with variations of the amplitude with frequency.

In the case of the cochlea we have the situation where a traveling wave is present which also shows a pronounced frequency selectivity, and it is attractive to try to separate the two aspects, wave propagation and frequency selectivity. Is one fully determined by the other, and vice versa, or is there some freedom? Such questions may be answered from a deeper study of minimum-phase in cochlear models. In the present paper the question is addressed: *Does a model of the cochlea have a minimum-phase response?* It is found that the property of minimum-phase is intricately connected with realizability of the impedance $Z_{BM}(x, \omega)$ of the model as a driving-point impedance. We will first consider a *classical* model. When, as is the case in a classical model, $Z_{BM}(x, \omega)$ is a realizable driving-point impedance, the response of a *short-wave* classical model of the cochlea is minimum-phase (Section I). In good approximation, the response of a *long-wave* classical model is also minimum-phase, and the same is true for a *three-dimensional* classical model (Section II). In the case of a *non-classical* model the effective BM impedance $Z_{BM}^{(eff)}(x, \omega)$ is not necessarily a driving-point impedance. For this class of models the following is proven. When the effective BM impedance does not have zeros in the right-half of the s plane, the response is minimum-phase, or very close to it, otherwise, it is not minimum-phase.

The theory described in this paper is based on *linear* models. Application of the results to experimental data is only justified in those cases where the response of the cochlea can be considered as linear or where nonlinear effects can be neglected. This is the case for stimulation of the normal cochlea with extremely weak signals (say, below 30 dB SPL), or for the cochlea in a dead animal. For higher levels of stimulation nonlinear effects do appear. In those cases one can resort to the use of noise bands as stimulus signals and computing input–output *cross-correlation functions* (ccfs). The present author has shown that the ccf for wide-band noise, determined for a *nonlinear* model, is equal to that of a *linear* model of the same structure with different parameters (de Boer, 1997b). The latter model is called the ‘‘comparison model’’ and the cited paper describes how it has to be constructed. From experimentally determined ccfs one can then derive the properties of the comparison model (de Boer and Nuttall, 1997). The results derived in the present paper apply equally well to the comparison model because it is linear. We repeat that one of the requirements that the model has to meet is that it maps frequency to place.

I. THEORY: SHORT-WAVE CLASSICAL MODEL

Consider a classical linear model of the cochlea as described in the Introduction. Two elongated fluid-filled channels are separated by a movable partition that is completely described by its local mechanical impedance $Z_{BM}(x, \omega)$. That impedance has to be realizable as a driving-point impedance (see the Introduction). Choose a long rectangular box for the shape of the model. Assume that the height of the channels (in the z direction) is ‘‘infinite,’’ i.e., so large that fluid movements near the movable partition do not reach the upper (or lower) wall of the channels; this is the *short-wave* case

(cf. de Boer, 1991, Section 2.3; de Boer, 1996, Section 5.2). Assume also that there are only waves going in one direction, from stapes toward helicotrema, i.e., in the direction of increasing x . For this case the fluid pressure $p(x, \omega)$ just above the basilar membrane for a wave with radian frequency ω obeys the following differential equation (de Boer, 1991, Eq. 2.3.n; de Boer, 1996, Eq. 5.2.3):

$$\frac{dp(x, \omega)}{dx} + \frac{2\omega\rho}{Z_{BM}(x, \omega)}p(x, \omega) = 0. \quad (1)$$

Here, ρ is the density of the fluid. The solution to this equation is of the form

$$p(x, \omega) = p_0 \exp\left(-i \int_0^x k(x', \omega) dx'\right), \quad (2)$$

where the *local wave number* $k(x, \omega)$ is given by

$$k(x, \omega) = \frac{-2i\omega\rho}{Z_{BM}(x, \omega)}, \quad (3)$$

and p_0 is a constant. To investigate the properties of $p(x, \omega)$ as a function of ω at a fixed location x is not simple because $p(x, \omega)$ is a function of both x and ω and Eq. (2) involves an integration over x . Therefore, we have to assume some form of relation between these two variables. Take the case where $Z_{BM}(x, \omega)$ depends on x and ω in such a way that a variation of x can be offset by a variation of ω . In particular, assume that *place is logarithmically related to frequency*. This means that, when x and ω are varied in such a way that

$$u = \frac{\exp(-\alpha x/2)}{\omega} \quad (4)$$

does *not* vary (where α is a constant parameter), the impedance changes only by a factor, namely, the factor ω , and the wave equation [Eq. (1)] remains exactly the same. The response pattern, $p(x, \omega)$ as a function of x , then retains the same form, it is only shifted over x (apart from a small influence of the boundary condition at $x=0$ upon the level). We assume that this property holds true for all values of ω , even for complex ones, but for the moment we only consider real values of ω . Applying the property to the value of u where $Z_{BM}(x, \omega)$ shows resonance, we find that $u = \text{constant}$ reduces to the following expression:

$$\omega_{\text{res}}(x) = \omega_0 \exp(-\alpha x/2), \quad (5)$$

where $\omega_{\text{res}}(x)$ is the radian resonance frequency corresponding to location x and ω_0 the radian resonance frequency corresponding to $x=0$. Under conditions of constant u , variations dx and $d\omega$ of x and ω are linked by

$$(\alpha/2)dx = -d\omega/\omega. \quad (6)$$

Combine this with Eq. (3) and substitute in Eq. (2):

$$\begin{aligned} p(x, \omega) &= p_0 \exp\left(-\int_0^x \frac{2\omega\rho}{Z_{BM}(x', \omega)} dx'\right) \\ &= p_0 \exp\left(\frac{4}{\alpha} \int_{\omega_0}^{\omega} \frac{\omega\rho}{\omega' Z_{BM}(x, \omega')} d\omega'\right). \end{aligned} \quad (7)$$

This relation can be interpreted in more than one way. If we think in terms of resonance frequencies, the integration is to be carried out from ω_0 to ω , i.e., over the range of radian

resonance frequencies that corresponds to going from 0 to x . Note, however, that the result is now solely a function of ω at location x , therefore, complex values of ω (and ω_0) can be considered too, and the full mechanism of complex-function theory can be applied.

A filter transfer function $F(\omega)$ is defined as minimum-phase when it has no zeros in the right-half of the complex s plane. Equivalently, this is the case when its logarithm has no poles in that half-plane. A realizable driving-point impedance or admittance, like the BM impedance function $Z_{\text{BM}}(x, \omega)$ or its inverse, has zeros nor poles in the right-half of the s plane. The same is true for the end values of the integration in Eq. (7). Hence the logarithm of the function $[p(x, \omega)/p_0]$ for the short-wave model has no singularities in the right-half of the s plane, and the function $[p(x, \omega)/p_0]$ itself is a minimum-phase function. The same applies to the BM velocity $v_{\text{BM}}(x, \omega)$. Note that this holds true only when $Z_{\text{BM}}(x, \omega)$ is a realizable driving-point impedance. In more general terms, *the response of a classical short-wave model is minimum-phase when $Z_{\text{BM}}(x, \omega)$ has no zeros in the right-half s plane.* The only proviso is that there should be only uni-directional waves in the model.

II. THEORY: LONG-WAVE CLASSICAL MODEL

Next consider the case in which it is assumed that the height of the model is so small that the sound pressure at the upper wall is only slightly smaller in magnitude than the pressure just above the BM. This assumption leads to what is called the *long-wave* approximation (de Boer, 1991, Section 2.2; de Boer, 1996, Section 4.2). We will show that the response of the classical long-wave model is also very close to being minimum-phase. The equation for the long-wave model reads (de Boer, 1991, Eq. 2.1.b; de Boer, 1996, Eq. 4.2.4):

$$\frac{d^2 p(x, \omega)}{dx^2} - \frac{2i\omega\rho}{h_{\text{eff}}Z_{\text{BM}}(x, \omega)}p(x, \omega) = 0. \quad (8)$$

The parameter h_{eff} is the effective height of the model's cochlear channels. Take the Liouville–Green (or WKB) expression for an approximate solution (de Boer, 1996, Section 4.3):

$$p(x, \omega) = p_0(x, \omega) \exp\left(-i \int_0^x k(x', \omega) dx'\right), \quad (9)$$

where the local wave number $k(x, \omega)$ is given by

$$k^2(x, \omega) = \frac{-2i\omega\rho}{h_{\text{eff}}Z_{\text{BM}}(x, \omega)}, \quad (10)$$

and $p_0(x, \omega)$ is a slowly varying function of x and ω . Equation (10) implies that singularities of $k(x, \omega)$ can only occur in the quadrant where $[1/Z_{\text{BM}}(x, \omega)]$ has singularities. In a classical model $[1/Z_{\text{BM}}(x, \omega)]$ has no singularities in the right-half of the s plane because $Z_{\text{BM}}(x, \omega)$ is a driving-point impedance. The factor $p_0(x, \omega)$ is proportional to $[k^{-1/2}(x, \omega)]$ (de Boer, 1996, Eq. 4.3.4), and will have no zeros in the right-half of the s plane either. Therefore, in the Liouville–Green (or WKB) approximation, the pressure

$p(x, \omega)$ is minimum-phase, and the BM velocity $v_{\text{BM}}(x, \omega)$ will be minimum-phase too.

However, the *actual* response of a long-wave model may deviate somewhat from the Liouville–Green (LG) solution, and may show a small extra delay component. It is unlikely that such a component can be detected in computed model responses because the LG approximation is such a “good” one (de Boer and Viergever, 1982).

For the more general case of a *three-dimensional* model we can again start from the LG approximation (cf. Steele and Taber, 1979; de Boer and Viergever, 1982). Equation (10) is replaced by a similar one (Eq. 6 in de Boer and Viergever, 1982), in which $k^2(x, \omega)$ is replaced by a function that goes to infinity as $k(x, \omega)$ [compare the short-wave case of Eq. (3)]. The same argumentation can then be used to show that, in the LG approximation, the response of a three-dimensional classical model is minimum-phase. Again, we have to keep in mind that there should only be uni-directional waves in the model.

III. IMPLICATIONS

The first theorem derived (Sec. I) implies that in the short-wave classical model of the cochlea, when the cochlear partition exhibits any particular form of resonance or filtering involving a cochlear mapping from frequency to place, *the traveling time of the cochlear wave is entirely determined by the type of resonance or filtering* and has no component that is specifically attributable to wave propagation without resonance or filtering.² Conversely, the type of resonance exhibited by the model is so intimately interwoven with wave propagation that it cannot be separated from it. The same is true, within the Liouville–Green approximation, for both the classical long-wave model and the classical three-dimensional model (Sec. II). In general terms this result can be expressed by saying that it is not possible to *separate* the parts played by wave propagation and frequency-specific filtering from an analysis of the model response; one part cannot be modified without affecting the other, and all details of the response are due to the two processes involved. It is repeated that all this holds true only for classical models.

For a *non-classical* model the effective BM impedance $Z_{\text{BM}}^{(\text{eff})}(x, \omega)$ is not a “given” parameter which controls the response. The effective impedance can only be computed *after* the model solution has been obtained. This impedance may turn out to have zeros (or poles) in the right-half of the s plane. If that is the case, the response of the model may or may not be not minimum-phase.³ In the converse sense, if we would find the response of a cochlear model to be non-minimum-phase, the BM impedance $Z_{\text{BM}}^{(\text{eff})}(x, \omega)$ must have at least one zero pair in the right-half of the s plane; that impedance then cannot be realized as a driving-point impedance, and the model must have been a non-classical one.

In the derivation it has been assumed that the cochlear map is logarithmic [Eq. (5)] over the whole range of x . For a model that is closer to reality, the mapping should be different in the basal and apical parts. It should be clear, however, that the major point of the derivation remains valid when a

slowly varying function of x is included in the transformation Eq. (5).

IV. DISCUSSION OF EXPERIMENTAL ASPECTS

From the foregoing discussion, one can understand that the question of whether the response of the actual cochlea is minimum-phase or not, relates to the problem whether the operation of the cochlea is to be described by a classical or a non-classical model. In experimental terms there is conflicting evidence on this issue. De Boer and Nuttall (1996) reported that input–output cross-correlation functions of the cochlea (measured from the basal turn of the guinea-pig cochlea) can be matched very well by artificial response functions from *minimum-phase* filters. This would mean that, for this match to be obtained, it was not necessary to include an extra (traveling-wave) delay. In different terms: the response showed, within the accuracy of this analysis, minimum-phase character. On the other hand, Recio *et al.* (1997) reported that their “first-order Wiener kernels,” which they measured in the basal turn of the chinchilla cochlea, and which are equivalent to input–output cross-correlation functions, are non-minimum-phase. It may be that the matching technique employed by de Boer and Nuttall (1996) has been too crude, or that the analysis by Recio *et al.* (1997) is in error. At any rate, in digital signal processing minimum-phase analysis is a difficult and tricky procedure.⁴ To confound the issue more, de Boer and Nuttall (1997) reported that the effective BM impedance, which they derived by “inverse analysis” from their experimental data, shows clear signs of containing a non-realizable component. This property would entail that the cochlear response is non-minimum-phase. And in its turn this property would imply that the cochlea functions according to a non-classical model. Further study in this field is clearly indicated.

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APPENDIX: THE MEANING OF “MINIMUM-PHASE”

Consider a given response function $F(\omega)$, and consider all realizable response functions $G_i(\omega)$ ($i=0,1,2,3,\dots$) that have the same magnitude as $F(\omega)$. Take one such function $G_i(\omega)$. It is characterized by a specific pole-zero pattern in the complex s plane in which all poles and zeros occur in conjugate complex pairs, and the pole pairs are in the left-

half of the s plane. Assume that one pair of conjugate complex zeros is added in the *right*-half of the s plane. Then, this pair must be offset by a pole pair in the *left*-half of the s plane, *symmetrically* located with respect to the imaginary axis, in order to ensure that $|G_i(\omega)|$ remains the same. The added zero- and pole pairs together represent an all-pass filter that implies a (possibly frequency-dependent) delay. Hence by the addition of such a combination of a pole and a zero pair the phase lag of $G_i(\omega)$ will always *increase*.

Therefore, the function $G_0(\omega)$ that has *no* zeros in the right-half of the s plane is the one that has the *smallest* phase lag among all functions $G_i(\omega)$ with $|G_i(\omega)|=|F(\omega)|$. The function $G_0(\omega)$ is, by definition, a minimum-phase function. The logarithm of a minimum-phase function has no singularities in the right-half of the s plane, and that is the property used in the proof in the main text.

¹The complex s plane is defined so that for real ω , s equals $i\omega$.

²In the case where the cochlear partition does not show resonance, our derivation does not apply, and we have the trivial case of “pure” wave propagation.

³This depends on the sign of the integral in Eq. (7).

⁴Minimum-phase analysis is only useful when *all* signals involved are narrow-band signals. For a minimum-phase response the phase function corresponding to a given amplitude function is the Hilbert transform of the logarithm of the amplitude. First, this logarithm is not guaranteed to be a narrow-band (or even a low-pass) function, and second, the kernel of the Hilbert transform does not have a finite energy. Both factors can easily lead to divergence problems in digital computations, and one should always be aware of these pitfalls.

de Boer, E. (1991). “Auditory physics. Physical principles in hearing theory. III,” *Phys. Rep.* **203**, 127–229.

de Boer, E. (1996). “Mechanics of the cochlea: modeling efforts,” in *The Cochlea*, edited by P. Dallos, A. N. Popper, and R. R. Fay (Springer-Verlag, New York), pp. 258–317.

de Boer, E. (1997a). “Classical and non-classical models of the cochlea,” *J. Acoust. Soc. Am.* **101**, 2148–2150.

de Boer, E. (1997b). “Connecting frequency selectivity and nonlinearity for models of the cochlea,” *Aud. Neurosci.* **3**, 377–388.

de Boer, E., and Viergever, M. A. (1982). “Validity of the Liouville-Green (or WKB) method for cochlear mechanics,” *Hearing Res.* **8**, 131–155.

de Boer, E., and Nuttall, A. L. (1996). “Cochlear travel time and minimum phase,” *ARO Mid-Winter Meeting Abstracts* **19**, 57 (no. 228).

de Boer, E., and Nuttall, A. L. (1997). “On cochlear cross-correlation functions: connecting nonlinearity and ‘activity,’” in *Diversity in Auditory Mechanisms*, edited by E. R. Lewis, G. R. Long, R. F. Lyon, P. M. Narins, C. R. Steele, and E. Hecht-Poinar (World Scientific, Singapore), pp. 291–297.

Geisler, C. D., and Sang, C. (1995). “A cochlear model using feed-forward outer-hair-cell forces,” *Hearing Res.* **86**, 132–146.

Hubbard, A. E. (1993). “A traveling wave-amplifier model of the cochlea,” *Science* **259**, 68–71.

Recio, A., Narayan, S. S., and Ruggero, M. A. (1997). “Wiener-kernel analysis of basilar-membrane responses to white noise,” in *Diversity in Auditory Mechanisms*, edited by E. R. Lewis, G. R. Long, R. F. Lyon, P. M. Narins, C. R. Steele, and E. Hecht-Poinar (World Scientific, Singapore), pp. 325–331.

Steele, C. R., and Taber, L. A. (1979). “Comparison of WKB and finite difference calculations for a two-dimensional cochlear model,” *J. Acoust. Soc. Am.* **65**, 1001–1006.

Steele, C. R., Baker, G., Tolomeo, J., and Zetes, D. (1993). “Electromechanical models of the outer hair cell,” in *Biophysics of Hair-Cell Sensory Systems*, edited by H. Duifhuis, J. W. Horst, P. van Dijk, and S. M. van Netten (World Scientific, Singapore), pp. 207–214.