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Dynamics of Price Formation in Financial Markets

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On the Impact of Information Friction in Re-Trade Economies

4.1 Introduction

Information asymmetry is commonly modeled by assuming the existence of two types of agents: informed and uninformed investors. To the uninformed investors, insiders play a dual role. Their presence increases the informativeness of prices with respect to fundamentals, which usually allows the uninformed to predict their future wealth with higher precision. However, additionally insiders are the cause of an adverse selection problem: uninformed agents face the risk of trading with insiders. This dual character prohibits any definite conclusion regarding the cumulative effect of information asymmetry. The key problem is that the individuals' information quality has relevance in two ways. First, information quality is important on an absolute scale: agents can better predict future realizations given a higher information precision regarding fundamentals. Second, the individuals' information quality is valued relative to the informedness of other agents. This heterogeneity in information quality gives rise to the adverse selection effect. It follows that two measures for the informedness of an economy are relevant: the absolute informedness regarding fundamentals and the dispersion of information precision across investors. In re-trade economies¹, however, information dispersion dominates. The reason is that in these markets, agents care solely about the ability to predict future prices, and not fundamentals. This creates a less significant role for absolute informedness of agents. A presence of insiders may increase the information content of prices regarding current level of fundamentals, at the same time, however, the presence of insiders in subsequent periods makes knowledge about the current level fundamentals a less precise predictor for future prices. These two effects more or less offset each other. Hence, in the type of models we study, increasing or decreasing the fraction of insiders mainly has relevance in its impact on the dispersion of information across investors. This feature of re-trade economies allows us to disentangle the dual character of insider trading, and focus exclu-

¹That is, re-trade economies as opposed to markets in which the asset is liquidated, or where information about fundamentals is realized, for instance in the form of dividends.

sively on its adverse selection component. That relative informedness matters, as opposed to absolute informedness, is underscored by the fact that the latter is not a measurable quantity. A necessary condition is clearly that the true value of the asset should be revealed. Indeed, within the context of stock markets, such an event is not likely to occur.

That information dispersion plays a unique role in our model, is emphasized by a feature which was already noted in the previous chapter: markets that are homogeneously informed, be it perfectly informed or uninformed, are equivalent in its pricing of risk, and have the same market statistics. In these markets, the competition between agents is maximal. On the other hand, in heterogeneously informed markets, market properties are impacted by information dispersion as well. It causes agents to compete less effectively, leading to higher risk premia and lower market depth. In this chapter, it is shown that if markets are relatively *uninformed*, an increase in insider trading decreases the overall competition among agents. If markets are relatively *informed*, the reverse is true. The implication follows that there is a critical fraction of informed for which this switch in the dependency of market statistics on the fraction of informed takes place. This occurs when informed and uninformed equally impact the price function, or, in other words, if the aggregate trading aggressiveness of informed and uninformed are equally high.

We relate the parameters of our model to the development of markets. Specifically, we associate market development with a high *relative* risk tolerance level. Using this measure, we show that a change in information friction has a less profound impact on variance and volume if markets are better developed. However, when considering market depth the opposite conclusion can be made. In more developed markets, an extreme sensitivity of liquidity cost on the fraction of informed is observed. This implies that if markets are highly developed, insider regulations have relatively more added value to liquidity traders.

The framework we utilize is based upon the noisy rational expectations paradigm as developed by Grossman[1976], Grossman and Stiglitz[1980], using the large market extension of Hellwig[1980]. In our multi-period environment, we assume that in each period the fundamental value of the asset experiences a shock whose variance is constant through time. In each period a public signal is revealed to the traders, which we assume to exist in three types: (1) Informed traders, who already know the future public signal, (2) Uninformed traders who rationally extract information from prices, and, (3) Liquidity traders who cause the per capita excess supply to vary stochastically through time. Within our model, insider trading, front running and dual trading are equivalent; the information advantage of informed investors owes to their receipt of information one period earlier than the uninformed. The model is a special case of the generic framework presented in the previous chapter. We use the theorems

derived therein to extract the equilibrium conditions.

Many authors have considered the impact of information friction on market statistics. Our approach differs in that we model an economy in which the asset is re-traded. Accordingly, some of our results differ from other work that considers this subject. For instance, Leland [1992] shows in a two-period model how insider trading affects price formation and its implications for investment decisions. In his model, the presence of insiders always reduces future uncertainty, whereas in our model future (price) uncertainty has an uni-modal dependency on the fraction of informed. In Wang[1994] it is shown that an increase in information friction reduces volume. In our model the opposite happens. In a re-trade economy, information friction induces additional trade between investors with different information precisions.

This chapter is organized as follows. Section 2 introduces the assumptions underlying the model. Section 3 discusses the equilibrium. In section 4 the impact of insider trading on various market statistics is considered. We conclude with a discussion of our results in section 5.

4.2 The Model

The model we utilize is a special case of the model developed in the previous chapter. For tractability, we consider the single asset case, and assume that agents act myopically². For completeness, in the following, we report the setup of the model.

A The Investors

We assume that there is a continuum of investors, indexed by $i \in [0, 1]$. The investors are myopic, and maximize their expected CARA utility function one-period hence. We assume that the investors are of two types, informed and uninformed. All investors receive a public signal (at each date) regarding the true value of the asset traded. The informed, however, already perfectly know the public signal that is to be revealed in the next period. Consequently, they can front-run future information signals. A fraction w of the investors is assumed informed. Furthermore, investors have identical risk tolerance level, which we denote by A .

B. The Liquidity Traders

Liquidity traders are present in the market causing the information signal revealed by the market clearing price to be only partially revealing. We assume that their cumulative impact

²In the next chapter, the model is considered when the economy consists of agents with multi-period horizons.

causes the per capita excess supply, \tilde{Z}_t , to be i.i.d. normally distributed in each period. This distribution is assumed to have zero mean and variance U , hence $\tilde{Z}_t \sim N[0, U]$.

C. The Fundamental Value

The risky asset is not liquidated in any nearby future period. It has, however, a fundamental value associated, which follows a pure random walk, i.e.

$$\tilde{F}_{t+1} = \tilde{F}_t + \tilde{\delta}_{t+1}$$

where $\tilde{\delta}_t \sim N[0, V]$.

The public signal which is revealed in each period concerns the previous true value of the asset, that is, at t all agents learn \tilde{F}_{t-1} . The informed agents however learn \tilde{F}_t as well³.

D. The Market Clearing Price

The market clearing price is assumed to be linear in the information of the informed and uninformed and the liquidity shocks to the market, i.e. we conjecture a pricing functional

$$\tilde{P}_t = \tilde{F}_{t-1} + \pi_\delta \tilde{\delta}_t - \pi_z \tilde{Z}_t \quad (4.1)$$

Following Slezak[1995] we refer to π_δ and π_z as the (news) response rate and the liquidity cost, respectively. The parameter π_δ effectively measures the sensitivity of price to the private information of the informed investors. The parameter π_z represents its sensitivity with respect to liquidity shocks in the market, and compensates rational investors for providing immediacy to the liquidity traders.

In order to tie down the equilibrium uniquely, we need to impose a boundary condition. We do so by assuming that at period T , with T infinite, the asset is liquidated at its true value \tilde{F}_T .

E. Additional Notation

Although the model has an abstract setup, it is tempting to ultimately translate this structure to a classification of markets. A key role in our analysis is played by the quantity ζ , which is defined as

$$\zeta = A(UV)^{-1/2}, \quad (4.2)$$

³Indeed, usually informed are modeled as if they can perfectly predict a future price (as in Kyle[1985]). However, it is more realistic to assume that even when one combines all knowledge in an economy, it can only imperfectly forecast a future value.

and can be interpreted as a relative risk tolerance level. The motivation for this interpretation is as follows. First note that the standard deviation of supply noise is dimensionless, while the standard deviation of the shocks to the fundamental value is measured in consumption units. The product of the two, therefore, proxies the variance of the supply turnover measured in consumption units. As a result, the fraction ζ , measures the average risk tolerance level, A , relative to the standard deviation of liquidity supply in consumption units. As such, it characterizes the *relative* risk tolerance of the market.

We relate a high value of ζ to a high development of a financial market. The reason is that the more developed a financial market is, the more short-term traders will be present relative to the total supply in the market, to engage in the (generally) profitable business of trading. A few arguments back this contention. One argument is that when a market develops, first market making activity needs to be shown to be profitable, upon which more agents will enter the market having observed the performance of other agents. A second argument is that an increase in the efficiency of a trading system will decrease trading costs, lowering the barriers for other traders. Finally, if competition increases between exchanges that list similar, or the same, securities, entrance fees to the exchange tend to be lowered, again increasing the number of rational traders. These scenarios all lead to the same result: a decrease in the variance of the per capita (trader) supply noise. However, given that supply is dimensionless, we need to relate it to the risk tolerance level and express it in consumption units. Hence, the interpretation of ζ as both a proxy for relative risk tolerance and the development of a market.

Additionally, we define

$$\alpha \equiv \sqrt{U/V} \pi_z. \quad (4.3)$$

As will be shown, α can be interpreted as a measure of the information friction between informed and uninformed agents. Observe that this parameter is dimensionless.

4.3 Solution to the Pricing Problem

The model we have adopted is a special case of the model proposed in chapter 3. The equilibrium is therefore readily found by applying the theorems derived therein. In the appendix, the necessary additional calculus is performed that leads to the equilibrium specified in the following theorem.

Theorem 4.1 *In the steady state economy, the pricing function is given by (4.1) with $\pi_z = \sqrt{V/U}\alpha$, and π_δ and α functions of w and ζ only, and given by the solution to*

$$\pi_\delta = w \left((1-w) \left(1 + \frac{\pi_\delta}{w\zeta\alpha} + \frac{\pi_\delta^2}{w\alpha^2} \right)^{-1} + w \right)^{-1}$$

and

$$\alpha = \frac{\pi_\delta(\pi_\delta^2 + \alpha^2)}{w\zeta}$$

Equilibrium exists for all values of w if $\zeta > 2\frac{1}{4}$.

Proof. See appendix 4.A. ■

Observe an elegant feature of this equilibrium : both π_δ and α can be expressed in w , the fraction of informed, and ζ , the relative risk tolerance of the market, only. Additionally, note that the existence of equilibrium is a sufficient condition. It implies that if markets are risk tolerant enough compared to the variance of supply turnover, equilibrium will exist. The set of equations for this quantity can be shown to be of third order. Hence, it is explicitly solvable. The solution is however not very illuminating. We therefore consider some of the dependencies of π_δ and α on the exogenous parameters in a more formal manner.

Lemma 4.1 *In equilibrium, the news response rate $\pi_\delta(w, \zeta)$ is bounded between 0 and 1 and strictly increasing in w and ζ .*

Proof. See appendix 4.A. ■

Indeed, π_δ measures the relative impact of informed investors on price. Increasing their number naturally increases their impact. That the response rate increases with the average risk aversion parameter, ζ , is to be expected as well. Informed agents will trade more aggressively given a higher relative risk tolerance, leading to more informational efficiency. The coefficients' dependency on ζ is displayed in figure (4.1).

Note that for any given π_δ and ζ , we can uniquely pin down the fraction of informed w . This bijective property implies that we can write α equivalently in terms of π_δ and ζ . As is shown in the appendix, the dependency of α in terms of the response rate can actually be written as $\alpha = \alpha(y, \zeta)$ with $y = \pi_\delta - \pi_\delta^2$, and is seen to be monotonically increasing in y . This observation leads to the property stated in the lemma.

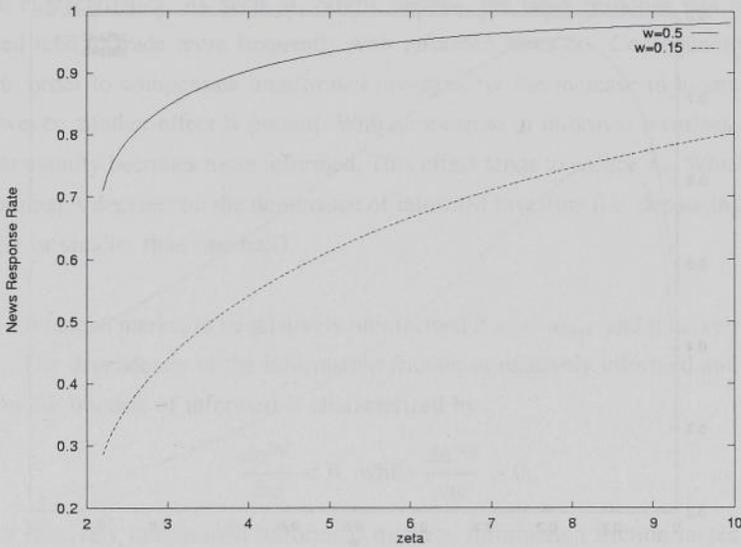


FIGURE 4.1. The response rate π_δ as a function of ζ for $w = 0.15$ and $w = 0.5$. This news sensitivity parameter increases with the relative risk tolerance of agents.

Lemma 4.2 *The quantity $\alpha(w, \zeta)$ is uni-modal in the fraction of informed, w , and maximal for w_{crit} which solves*

$$\pi_\delta(w_{crit}, \zeta) = \frac{1}{2}. \quad (4.4)$$

Proof. See appendix 4.A. ■

Interestingly, α is, ceteris paribus, only dependent on w through $y = \pi_\delta - \pi_\delta^2$. Therefore, not the fraction of informed is important to the parameter α , but rather the dispersion of information across the economy. We therefore associate α with the degree of information friction in the market. The lemma shows that if the price is impacted equally by informed and uninformed, information friction is maximal. Figure (4.2) illustrates the uni-modal dependency of information friction on the fraction of the informed. Observe the extreme sensitivity to an increase in the fraction of informed agents for relatively uninformed markets. Another implication of this pattern concerns the liquidity cost parameter π_z . The latter is directly related to the information friction α through

$$\pi_z = \sqrt{VU^{-1}}\alpha(w, \zeta). \quad (4.5)$$

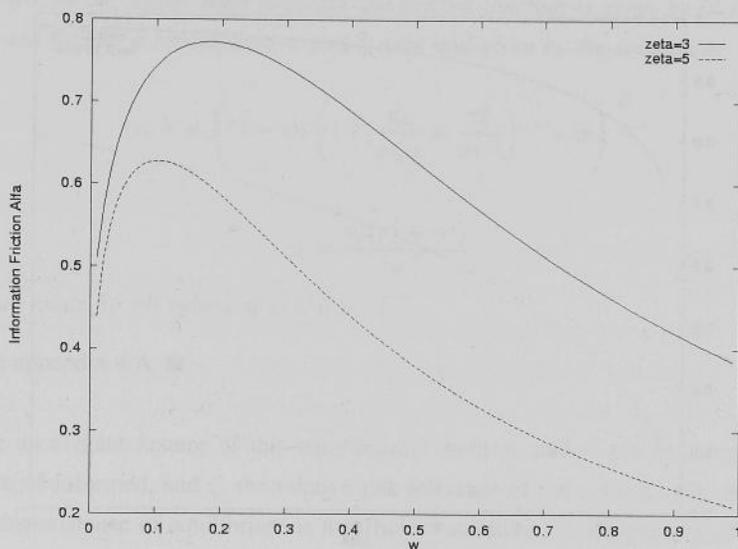


FIGURE 4.2. The information friction parameter, α as a function of w for $\zeta = 3$ and $\zeta = 5$. The information friction is maximized if informed and uninformed impact prices equally.

Combining this expression with the lemmas 4.1 and 4.2, we immediately obtain the following corollary.

Corollary 4.1 *The liquidity cost in the market is uni-modal in the fraction of informed w , and reaches its maximum for $w_{crit}(\zeta)$.*

Proof. Follows directly from (4.5) and theorem (4.2). ■

The corollary supports our association of w_{crit} with maximal information friction. At this point liquidity costs are highest. Although we have derived the uni-modal dependency from the properties of the information friction parameter α , it can also be understood intuitively by considering the characteristics of the trading strategies of informed and uninformed investors. Their demands can be written as

$$\begin{aligned} d_{i,t}^{inf} &\propto (1 - \pi_\delta)\bar{\delta}_t + \pi_z\tilde{Z}_t, \\ d_{i,t}^{unf} &\propto -\pi_\delta\bar{\delta}_t + \pi_z\tilde{Z}_t \end{aligned}$$

respectively. Observe how investors respond to the news innovation process $\bar{\delta}_t$. Informed tend to buy (sell) on good (bad) news, while the uninformed investors' demand function exhibits

contrarian characteristics. As such, if, ceteris paribus, the news response rate π_δ increases, uninformed tend to trade more frequently with informed investors. Consequently, π_z should increase in order to compensate uninformed investors for the increase in losses. Simultaneously, however, another effect is present. With an increase in informed investors, the average trading community becomes more informed. This effect tends to reduce π_z . Which of the two effects dominates depends on the dominance of informed investors (i.e. depending on whether π_δ is larger or smaller than one-half).

Define a financial market to be relatively uninformed if $w < w_{crit}$, and relatively informed if $w > w_{crit}$. The dependency of the information friction in relatively informed and uninformed markets on the fraction of informed is characterized by

$$\frac{d\alpha^{inf}}{dw} < 0, \text{ while } \frac{d\alpha^{unf}}{dw} > 0.$$

Hence, for relatively uninformed (informed) markets, information friction increases with the fraction informed (uninformed).

It is easily shown that the critical fraction, w_{crit} , depends on the relative risk aversion parameter ζ only. In the appendix, it is shown that the following theorem applies to this dependency.

Theorem 4.2 *The critical value of informed investors, w_{crit} , where information friction is maximal, is decreasing in the relative risk tolerance of the market ζ .*

Proof. Follows directly from (4.4) and the fact that π_δ is increasing in w and increasing with ζ (lemma 4.1). ■

The negative monotonic dependency of w_{crit} as a function of ζ is verified in Figure (4.3). Observe that, given our interpretation of ζ , the theorem implies that the critical value of w , decreases with the relative risk tolerance of a financial market. As laid out in the discussion in section 2.E, we can relate higher values of ζ with more developed markets. Hence, theorem 4.2 tells us that in more developed markets, maximal information friction occurs for a smaller fraction of informed.

4.4 Information Friction and Market Statistics

In this section we consider the dependency of markets statistics on the fraction of insiders present.

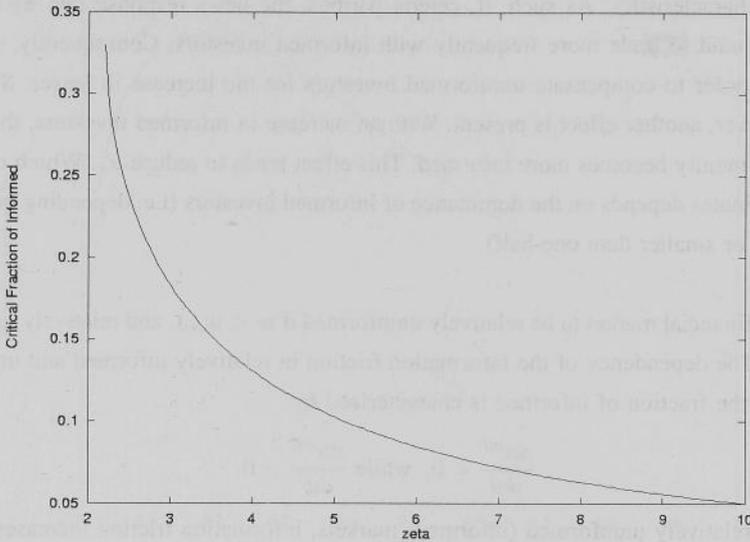


FIGURE 4.3. The critical fraction of informed w_{crit} as a function of ζ . More risk tolerant markets have a lower critical value.

A. Volatility

If information friction increases, a natural consequence is that risk premia increase. One would expect therefore that price volatility increases with information friction. Consider therefore the relative variance of price changes. This quantity is given by

$$\text{var}^{\text{rel}}[\Delta \tilde{P}_t] \equiv V^{-1} \text{var}[\Delta \tilde{P}_t] = (1 - \pi_\delta)^2 + \pi_\delta^2 + 2\alpha^2$$

It is indeed uni-modal in the fraction of informed and maximal for $w = w_{crit}$. Note that this happens even though the contribution of news variance, represented by the terms $(1 - \pi_\delta)^2 + \pi_\delta^2$, is minimized at this point. As a function of w , we obtain the graph in Figure (4.4). Observe the significant increase at the critical point compared to the homogeneously informed economies for $\zeta = 3$. This result somewhat over-emphasizes the impact of information friction. The reason is that this variance concerns short-term price movements where liquidity noise has a profound impact. The relative contribution of this component diminishes with longer time horizons. This effect is readily noted by considering the variance of long-term price changes. Measuring the return over K periods, we have

$$P_{t+K} - P_t = \sum_{i=1}^K \delta_{t-1+i} + \pi_\delta \bar{\delta}_{t+K} - \pi_\delta \bar{\delta}_t - \pi_z (\tilde{Z}_{t+K} - \tilde{Z}_t)$$

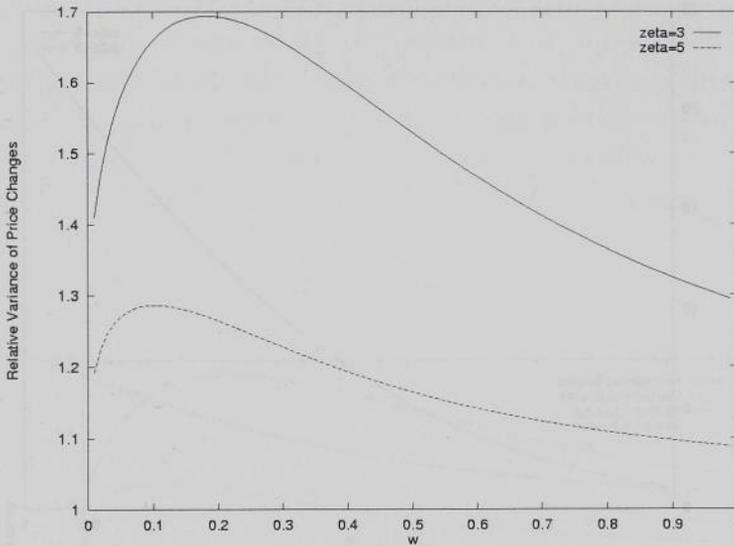


FIGURE 4.4. Relative variance of short term price changes as a function of fraction of informed for $\zeta = 3$, $\zeta = 5$. The quantity is maximal under maximal information friction.

In this case, we obtain for the relative variance that ($K > 1$) $\text{var}^{\text{rel}}[P_{t+K} - P_t] = K + 2\pi_\delta^2 + 2\alpha^2$, which implies that the contribution of the information friction parameter can be written as $2\alpha^2(K + 2\pi_\delta^2)^{-1}$. Hence, the relative impact of information friction decreases with the number of periods K over which the returns are measured.

B. Informational Efficiency

An important measure of market performance is informational efficiency. Usually, this efficiency is associated in relation to the transparency of price to fundamentals. Consider how the uninformed investor updates his information precision regarding fundamentals. The ex post information precision compared to the ex ante information precision, denoted by Γ_{rel} can be written as

$$\Gamma^{\text{rel}} \equiv \frac{\text{var}[\bar{\delta}_t | \mathcal{I}_t^{\text{unf}}, \bar{P}_t]}{\text{var}[\delta_t | \mathcal{I}_t^{\text{unf}}]} = (1 + \pi_\delta^2 / \alpha^2)$$

Because of the uni-modal dependency of α on the fraction of informed and π_δ strictly increasing in w , the formula cannot readily reveal its dependency on the fraction of informed. However, as shown in graph (4.5), the intuitive relation is present that with increasing fraction of informed, the informativeness of prices increases. Additionally, it implies that the informa-

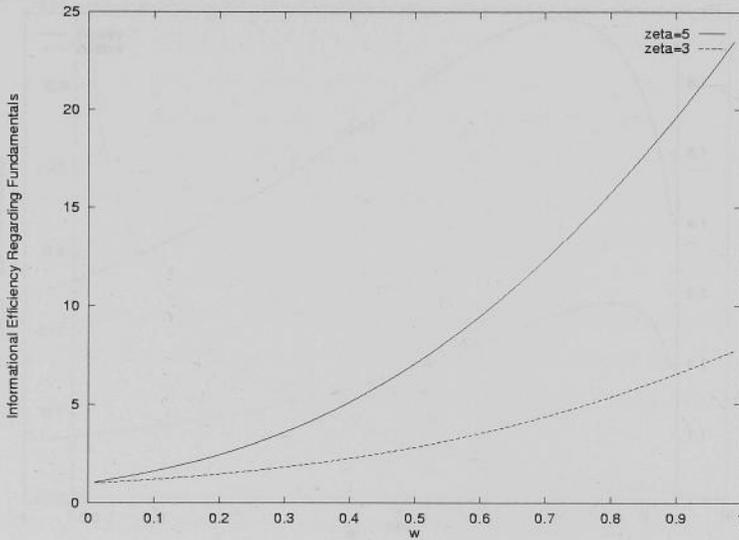


FIGURE 4.5. Price efficiency with respect to fundamentals as a function of the fraction of informed for $\zeta = 3$ and $\zeta = 5$. Insiders always increase the informative content regarding fundamentals.

tional efficiency increases with ζ . Again, this is intuitive. The more risk tolerant traders are, the more aggressively they compete, which leads to higher information efficiency. It should be noted however, that the uninformed investors are actually not interested in the revelation of fundamentals per se, but rather the predictive content of price concerning future prices. Let us therefore consider the uncertainty regarding future price realizations. For informed investors, this uncertainty is given by

$$\text{var}^{\text{inf}}[\tilde{P}_{t+1}] = V(\pi_{\delta}^2 + \alpha^2)$$

while for uninformed investors, we have

$$\text{var}^{\text{unf}}[\tilde{P}_{t+1}] = V \frac{\alpha^2 + (\alpha^2 + \pi_{\delta}^2)^2}{\alpha^2 + \pi_{\delta}^2}$$

In Figure (4.6) these two quantities are plotted relative to V as a function of the fraction of informed. The graph indicates that to insiders the future price uncertainty always increases with their presence. The reason is of course that today's fundamentals have less predictive power if more insiders are present in the next period. To uninformed, the liquidity component that affects the price in the next period is a determinant factor. Given that the latter increases with information friction, uni-modal dependency of the uninformed uncertainty on w results.

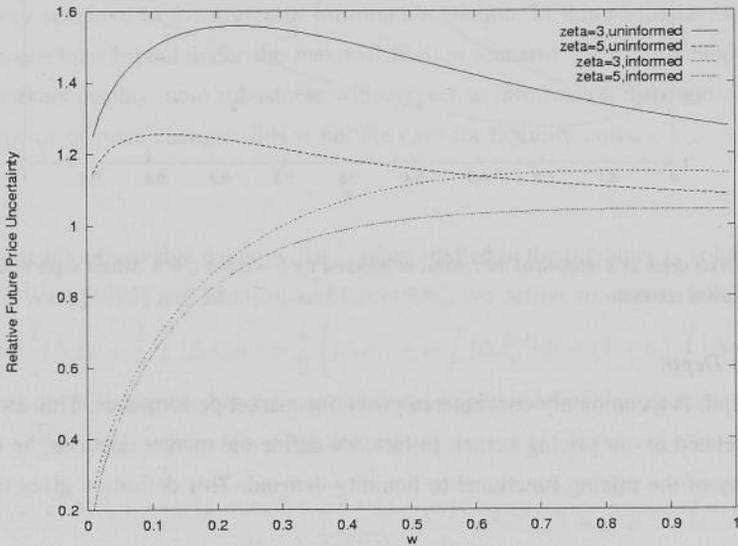


FIGURE 4.6. Future price certainty of informed and uninformed as a function of the fraction of informed for $\zeta = 3$ and $\zeta = 5$. Although informed increase the informative content of prices with respect to fundamentals, in relatively uninformed markets insiders only increase future uncertainty.

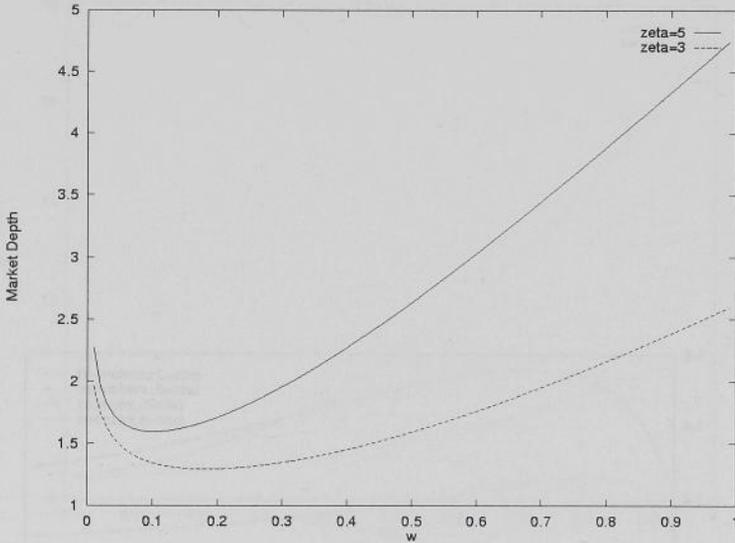


FIGURE 4.7. Market depth as a function of the fraction of informed for $\zeta = 3$ and $\zeta = 5$. Market depth is smallest when information friction is maximal.

C. Market Depth

Market depth is a commonly encountered proxy for market performance. This measure can be directly related to our pricing kernel. In fact, we define the market depth as the inverse of the sensitivity of the pricing functional to liquidity demand. This definition gives us that

$$\lambda = \pi_z^{-1} = \sqrt{UV^{-1}}\alpha(w, \zeta).$$

We have plotted this definition of market depth as a function of w in Figure (4.7). Two observations are worth noting. First of all, as can be seen for the extreme cases, i.e. perfectly informed and completely uninformed, the dependency of the market depth on the risk tolerance level is positive: increasing the variance of shocks to liquidity increases the depth of the market. This relation is intuitive and commonly found. The dependency on the variance of liquidity shocks U , is not revealed by the graph, where we assumed this variance to be constant. Observe, however, that for constant relative risk tolerance ζ , an increase in U leads to an increase in market depth. Brennan and Cao[1996] also model a competitive multi-period market, and report the same dependency of market depth on risk tolerance and supply variance. In Subrahmanyam[1991], within a Kyle-type of environment this result is also seen to hold.

Another dependency that can be derived from the graph, concerns the fraction of informed. Clearly, an uninformed economy is extremely sensitive to an increase in the fraction of informed trading. Market depth almost jumps to a fifth of its original size. Hence, we may conjecture that uninformed economies are much less resilient against changes in the fraction of informed, compared to a informed economy. That market depth is uni-modal in the fraction of informed is also noted by Subrahmanyam[1991]. Indeed, market depth is determined by the average competitiveness of the market. If the market is relatively well-informed, an increase in informed trading leads to more competition, leading to an increase in market depth.

Interestingly, for higher developed markets (corresponding to $\zeta = 5$ in the figure) market depth is very sensitive to the degree of information friction. In this particular case, liquidity costs are more than tripled under the maximal friction scenario. Hence, although higher developed markets display more robustness with respect to information dispersion if measured by the variance of price changes, this is not the case for liquidity costs.

D. Volume

An empirically observable quantity that is often studied in the literature is volume. Following He and Wang[1995] and Brennan and Cao[1996], we define volume as follows⁴

$$\text{vol}_t = \frac{1}{2} |\Delta \tilde{Z}_t| + \frac{1}{2} \int |\Delta d_{it}| di = \frac{1}{2} \left\{ |\Delta \tilde{Z}_t| + w \int |\Delta \tilde{d}_{it}^{\text{inf}}| di + (1-w) \int |\Delta \tilde{d}_{it}^{\text{un}}| di \right\}$$

In the appendix it is shown that the unconditional expectation of this quantity is given by $E^u[\text{vol}_t] = \sqrt{\frac{U}{\pi}} f(w, \zeta)$, where $f(w, \zeta)$ is defined as

$$f(w, \zeta) \equiv 1 + \frac{\pi_\delta}{\alpha} \sqrt{(1 - \pi_\delta)^2 + \alpha^2} + (1-w) \zeta \frac{\alpha^2 + \pi_\delta^2 - \pi_\delta}{\alpha^2 + (\pi_\delta^2 + \alpha^2)^2} \sqrt{(\pi_\delta^2 + \alpha^2)}$$

Figure (4.8) displays the functional dependency of this factor on w . Clearly, volume increases with the information friction in the market. The increase is most profound when ζ is largest. Hence, for relatively less developed markets, a change in information friction may cause a significant increase in volume. Wang[1993] also considers volume in a heterogeneously informed economy. He finds that volume decreases with increasing information asymmetry. This result is due to the dual character of the informed investors in his model. They additionally represent liquidity traders. Hence, uninformed can only trade with informed agents. Increasing the uncertainty of uninformed decreases their trading aggressiveness, and in turn leads to a decrease

⁴Note that volume equals infinity in a infinite agent model. Therefore, we define volume as volume per capita. Our definition of volume is slightly different from the definition Brennan and Cao[1996] use. While Brennan and Cao[1996] assume that traders are randomly endowed with the risky asset, we assume a presence of liquidity traders. Our definition is identical to the one used in He and Wang[1995].

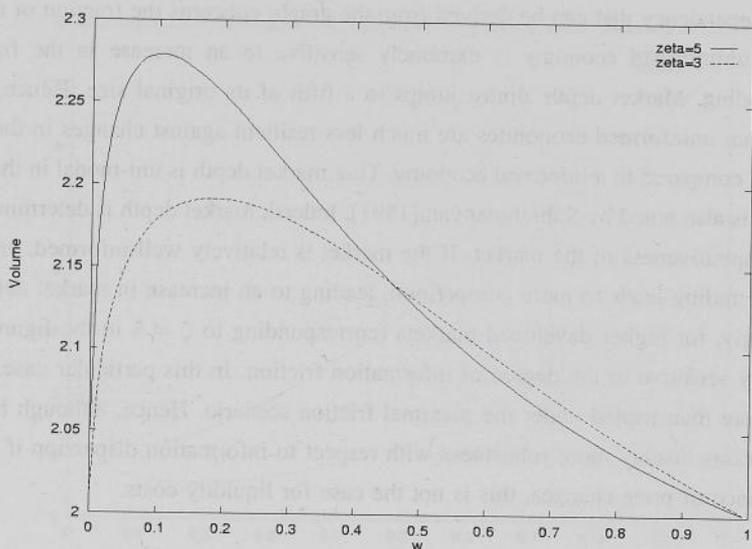


FIGURE 4.8. The volume factor as a function of the fraction of informed for $\zeta = 3$ and $\zeta = 5$. Volume has a complex dependency on the fraction of informed, although it generally increases with the level of information friction.

in volume. In our model, if information asymmetry is absent, rational investors only trade with liquidity traders. With information asymmetry additional trade occurs between informed and uninformed investors. In fact, the increase in volume in heterogeneously informed economies can be ascribed entirely to trades between informed and uninformed investors. This observation is in line with our earlier explanation for the uni-model pattern found for the liquidity cost parameter. When the frequency of trades between informed and uninformed is largest, uninformed need to be compensated for these extra losses. Hence, a natural consequence is that when volume is largest also liquidity costs are maximal.

In He and Wang[1995] intertemporal patterns of volume are considered. We have assumed a stationary economy, keeping the variance of exogenous shocks constant. As such, the conditional pre-trade expectation of volume is constant through time. Note however, that the model developed in chapter 3 allows for a solution even if we assume time-dependency of the exogenous parameters.

4.5 Concluding Remarks

In re-trade models investors do not care about fundamentals, but only about price realizations. Consequently, the information content of price is valued according to its ability to predict future prices. If price conveys more information about fundamentals due to a larger presence of insiders, additionally its sensitivity to news increases. Hence, although inference of fundamentals may be more precise, future price risk may increase due to a high news sensitivity of future price realizations. This feature of re-trade models magnifies the role of information dispersion as opposed to the absolute degree of informativeness. It allowed us to disentangle the dual character of insider trading and focus on its adverse selection component. We showed that volume, variance and market depth, have a uni-modal dependency on the fraction of informed in the market. All are found extremal for a certain critical fraction of informed, where information friction is highest.

We have related our exogenous parameters to the development of a market. It was shown that the critical fraction of informed is decreasing in the degree of development of markets. Interestingly, although higher developed markets are less sensitive to information friction when measured by the variance of price changes, this does not apply to the costs of liquidity. These are actually more sensitive to information dispersion than in less developed markets. The implication is, that insider regulations may have relatively more benefits in terms of cost reduction for liquidity traders, than in less developed markets.

In order to derive more normative implications, welfare analysis is needed. Though we have not explicitly derived ex ante utilities of investors, it is easily shown that all traders benefit when the information friction in the market is as large as possible. The reduced competition between agents effectively allows them to extract a larger rents from liquidity traders. Hence, the most attractive market for our short-term traders is the least attractive market for the liquidity traders. This trade-off between the welfare of rational traders and traders makes the extraction of definitive normative statements non-trivial. We may however resolve this problem by incorporating supply noise by endowing traders randomly with a position in the asset. This would allow for more insights in how insider trading impacts social welfare.

4.A Appendix

A Proof of Theorem 4.1, Lemma 4.2 and Lemma 4.2

From theorem 3.3 it follows how the pricing coefficients are related between subsequent periods for the multi-asset version of the model. In this special case, it is immediately implied that the pricing functional is given by

$$\tilde{P}_t = \tilde{F}_{t-1} + \pi_\delta \tilde{\delta}_t - \pi_z \tilde{Z}_t$$

with its coefficients determined by the relations

$$\begin{aligned}\pi_\delta &= w \left((1-w) \left(1 + V\Sigma^{-1} + wA^2V\Sigma^{-2}U^{-1} \right)^{-1} + w \right)^{-1}, \\ \pi_z &= (wA)^{-1}\pi_\delta\Sigma,\end{aligned}$$

where $\Sigma = \pi_\delta^2 V + \pi_z^2 U$. Using the definitions (4.2) and (4.3), the common uncertainty of the future price, Σ , is given by $\Sigma = V(\pi_\delta^2 + \pi_z^2 UV^{-1}) = V(\pi_\delta^2 + \alpha^2)$. Substitution leads to

$$\begin{aligned}\pi_\delta &= w \left((1-w) \left(1 + (\pi_\delta^2 + \alpha^2)^{-1} + w\zeta^2(\pi_\delta^2 + \alpha^2)^{-2} \right)^{-1} + w \right)^{-1}, \\ \alpha &= (w\zeta)^{-1}\pi_\delta(\pi_\delta^2 + \alpha^2).\end{aligned}\tag{4.6}$$

Using relation (4.6), we obtain

$$\pi_\delta = w \left((1-w) \left(1 + \frac{\pi_\delta}{w\zeta\alpha} + \frac{\pi_\delta^2}{w\alpha^2} \right)^{-1} + w \right)^{-1},\tag{4.7}$$

$$\alpha = (w\zeta)^{-1}\pi_\delta(\pi_\delta^2 + \alpha^2).\tag{4.8}$$

Note that these equations imply that the quantities π_δ and α are functions of w and ζ only. Equation (4.7) can be written as

$$\pi_\delta = 1 - (1-w) \frac{1}{1 + (\zeta x)^{-1} + x^{-2}} \equiv f(x; \zeta, w)\tag{4.9}$$

where we defined $x = x(\pi_\delta; w, \zeta) \equiv \pi_\delta^{-1}\alpha$. Note that it follows immediately that $\pi_\delta \in [0, 1]$. In terms of x , relation (4.8) becomes $x = (w\zeta)^{-1}\pi_\delta^2(1 + x^2)$, which is solved by

$$x(\pi_\delta; w, \zeta) = \frac{1}{2} \frac{w\zeta}{\pi_\delta^2} \left(1 - \sqrt{(1 - 4\pi_\delta^4/w^2\zeta^2)} \right)$$

Straightforward calculus shows that $x(\pi_\delta; w, \zeta)$ is strictly increasing in π_δ , and strictly decreasing in w and ζ . Moreover, $f(x; \zeta, w)$ is strictly increasing in w and decreasing in x and in ζ . We can now easily show that π_δ is increasing in w . Assume $w < w'$, and that the opposite is true, i.e. $\pi_\delta > \pi'_\delta$. We then have that $x \equiv x(\pi_\delta; w, c) > x(\pi'_\delta; w, c) > x(\pi'_\delta; w', c) \equiv x'$, which implies that

$$\pi_\delta = f(x; \zeta, w) < f(x'; \zeta, w) < f(x'; \zeta, w') = \pi'_\delta$$

and violates our assumption that $\pi_\delta > \pi'_\delta$. Hence, π_δ is strictly increasing in w and bounded between 0 and 1.

Next, consider the dependency on ζ . By taking direct derivatives, the product $\zeta x(x; \zeta, w)$ is seen to be decreasing in ζ . Therefore, $f(x; \zeta, w)$, specified by (4.9), is strictly increasing in ζ . It immediately follows, using the same technique (*reductio ad absurdum*) that the solution π_δ is strictly increasing in ζ .

Next, we consider the existence of an equilibrium, and simultaneously the dependencies of $\alpha(w, \zeta)$. Combining relations (4.7) and (4.8), after some algebra it can be shown that α is the solution to

$$\alpha = g(\alpha; y, \zeta) \equiv -\alpha^3 + \zeta\alpha^2 + \alpha y - \zeta y \quad (4.10)$$

with $y \equiv \pi_\delta - \pi_\delta^2$. Note that y is uni-modal in π_δ reaching its maximum value for $\pi_\delta = \frac{1}{2}$, hence $y \in [0, \frac{1}{4}]$. It is readily derived that for $\alpha \in [0, \frac{2}{3}\zeta)$, $g(\alpha; y, \zeta)$ is strictly increasing in α . We can now derive a sufficient condition for an equilibrium to exist. For any y , an equilibrium exists if $g(\frac{2}{3}\zeta; y, \zeta) \geq \frac{2}{3}\zeta$. Using the definition for $g(\alpha; y, \zeta)$, it follows that this condition is always satisfied if $\zeta \geq 2\frac{1}{4}$. Furthermore, inspection of (4.10) reveals that g is strictly decreasing in y . Hence, it immediately follows that the solution α is increasing in y . ■

B. Information Efficiency

In chapter 3 (relation (3.14)), it was shown that the ex post precision, Γ_i , of investor i , who receives a signal with precision S_i^{-1} prior to trading, is given by

$$\Gamma_i = V^{-1} + S_i^{-1} + w^2 A^2 \Sigma^{-2} U^{-1} = S_i^{-1} + \frac{1}{V} \left(1 + \frac{\pi_\delta^2}{\alpha^2}\right)$$

For informed investors, this expression indeed evaluates to infinity. For uninformed investors, whose information precision $(S^{\text{unf}})^{-1} = 0$, we obtain $\Gamma^{\text{unf}} = \frac{1}{V} \left(1 + \frac{\pi_\delta^2}{\alpha^2}\right)$. The relative information precision measure is then readily found by scaling this quantity with a factor V .

C. Volume

The demand of an investor with information precision S_i can be written as

$$d_{i,t} = A \left(\Sigma + \Gamma_i^{-1}\right)^{-1} \left(\frac{1}{\Gamma_i S_i} (\tilde{Y}_{it} - \tilde{F}_{t-1}) + \left(1 - \frac{\pi_\delta}{\Gamma_i \pi_\delta^2 U}\right) (\tilde{F}_{t-1} - \tilde{P}_t)\right) \quad (4.11)$$

For informed and uninformed, this implies

$$\begin{aligned} d_{i,t}^{\text{inf}} &= A \Sigma^{-1} (\tilde{F}_t - \tilde{P}_t) \equiv L^{\text{inf}} (\tilde{F}_t - \tilde{P}_t) \\ d_{i,t}^{\text{unf}} &= A \left(\Sigma + (\Gamma^{\text{unf}})^{-1}\right)^{-1} \left(1 - \frac{\pi_\delta}{\Gamma^{\text{unf}} \pi_\delta^2 U}\right) (\tilde{F}_{t-1} - \tilde{P}_t) \equiv L^{\text{unf}} (\tilde{F}_{t-1} - \tilde{P}_t) \end{aligned}$$

Volume is given by

$$\text{vol}_t = \frac{1}{2} |\Delta \tilde{Z}_t| + \frac{1}{2} \int L_i^{\text{inf}} |\Delta \tilde{F}_t - \Delta \tilde{P}_t| di + \frac{1}{2} \int L_i^{\text{unf}} |\Delta \tilde{F}_{t-1} - \Delta \tilde{P}_t| di$$

$$= \frac{1}{2}|\tilde{Z}_t - \tilde{Z}_{t-1}| + \frac{1}{2}wL^{\text{inf}}|(1 - \pi_\delta)(\tilde{\delta}_t - \tilde{\delta}_{t-1}) + \pi_z(\tilde{Z}_t - \tilde{Z}_{t-1})| \\ + \frac{1}{2}(1 - w)L^{\text{unf}}| - \pi_\delta(\tilde{\delta}_t - \tilde{\delta}_{t-1}) + \pi_z(\tilde{Z}_t - \tilde{Z}_{t-1})|$$

Using standard results from distribution theory⁵, unconditional expected volume is given by

$$E^u[\text{vol}_t] = \sqrt{\frac{1}{\pi}} \left(\sqrt{U} + wL^{\text{inf}}\sqrt{(1 - \pi_\delta)^2V + \pi_z^2U} + (1 - w)L^{\text{unf}}\sqrt{\pi_\delta^2V + \pi_z^2U} \right)$$

The quantities L^{inf} and L^{unf} are given by,

$$L^{\text{inf}} = \frac{A}{V} \frac{\pi_\delta}{w\zeta\alpha}, \text{ and } L^{\text{unf}} = \frac{A}{V} \frac{(\alpha^2 + \pi_\delta^2 - \pi_\delta)}{\alpha^2 + (\pi_\delta^2 + \alpha^2)^2}$$

It follows then that

$$E^u[\text{vol}_t] = \frac{1}{\sqrt{\pi}}\sqrt{U} \left[1 + \left(\frac{\pi_\delta}{\alpha} \sqrt{(1 - \pi_\delta)^2 + \alpha^2} + (1 - w)\zeta \frac{(\alpha^2 + \pi_\delta^2 - \pi_\delta)}{\alpha^2 + (\pi_\delta^2 + \alpha^2)^2} \sqrt{\pi_\delta^2 + \alpha^2} \right) \right] \\ \equiv \frac{1}{\sqrt{\pi}}\sqrt{U} (1 + f(w, \zeta))$$

where we implicitly defined $f(w, \zeta)$. ■

⁵Namely that if $x \sim N[0, \sigma^2]$, $E[|x|] = \sqrt{\frac{2}{\pi}}\sigma$.