Dynamics of Price Formation in Financial Markets

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Market Development under Costly Information

5.1 Introduction

Many exchanges have expanded considerably over the past years[1]. This expansion is not only seen in the number and types of products that are traded, but also in the number of traders who participate in the trading process. The latter in particular applies to electronic exchanges. An example is the index futures market on the EUREX, which has almost tripled its number of traders over the past two years. A compelling question is how such an increase in agents that facilitate trade affects market properties. This issue is the main focus of this chapter, where we specifically address the impact of market growth on market depth given the presence of information asymmetry. Within our model this information friction arises due to the availability of private information which agents can acquire at a fixed cost. It is shown that, while in homogeneously informed economies market depth increases with the number of traders, with information asymmetry, market depth may decrease under such development.

The information friction in our model arises due to the possibility of agents to become informed at a certain cost. Consequently, information friction occurs if it is not beneficial for all traders to become informed, but only for a fraction to do so. The benefits to becoming informed relative to staying uninformed typically depend on the variance of the per trader excess supply caused by the presence of liquidity traders. The latter ultimately pay the price for the provision of immediacy by the rational traders, who can demand a surplus from the liquidity traders with their perfectly inelastic demand schedules. Therefore, with an increase in the number of traders the returns to informed agents decrease. Eventually then, a smaller fraction of agents decides to become informed, leading to a decrease in the effective competitiveness of the market, and a decrease in market depth.

We utilize a multi-period noisy rational expectations model to derive these results. We assume a market in which a risky and a riskless asset can be traded. The risky asset has a fundamental value associated that stochastically evolves through time. Three types of traders actively exchange the risky asset: liquidity traders who need to trade for exogenous reasons;

[1] This sometimes to the detriment of other exchanges (e.g. EUREX vs LIFFE).

uninformed traders who in each period observe a public signal regarding the fundamental value; and informed traders. The latter can additionally observe the next periods public signal, a privilege that is costly. The asset traded is not liquidated, and does not generate any distributions. Hence, traders can only gain through re-trade, realizing capital gains. From a mathematical perspective, the model is a special case of the generic model of chapter 3. As such, we can use the results therein to characterize the equilibrium. The properties of the equilibrium can however not be stated in explicit closed formula, and we need to consider the comparative statics of the market numerically.

In the process, we also consider the impact of longer time horizons. We show that, conform earlier results (Vives[1995]), multi-horizon agents trade more aggressively than their myopic colleagues. The reason is the dynamic diversification agents can exploit when having longer time horizons and the ability of intermediate trade. However, we also show that when agents get closer to their consumption horizon their trading aggressiveness decreases. A result that differs from models (Vives[1995], Brennan and Cao[1996]) that assume liquidation of the asset coinciding with the consumption horizon of investors. This underscores the absence of resolution of uncertainty within the re-trade setup.

Our model has many similarities with Slezak[1994]. He also considers a multi-period noisy rational expectations model with informed traders that perceive the public signal one period before the uninformed. Also, liquidity supply follows a similar process in his model. The main difference is his assumption of a liquidation date at which all investors consume. In our model, the asset is not liquidated while investors live for a finite number of periods. Additionally, the trader community we have is a heterogeneously composed of traders with different time horizons. The most profound difference, however, is the endogenous determination of the fraction of informed given a fixed cost to the acquisition of information.

This chapter is setup as follows. Section 2 introduces the model. In Section 3 the equilibrium is derived. The dependency of the equilibrium on the length of the time horizons and on the fraction of informed is discussed in section 4. Section 5 considers the interior equilibrium that arises given a certain cost to becoming informed. The impact of market growth in the costly information economy is the focus of section 6. Section 7 concludes.

5.2 The Model

We adopt the model of the previous chapter, and study its single-asset, two-type investor special case. We repeat the basic setup, as well as our notation for completeness. The additional structure we impose is outlined as well.

A. Assets

We assume that two assets are traded, a risky asset whose value varies stochastically, and a riskless asset that yields a fixed zero return. The risky asset has a fundamental value associated with it, which we denote by $\tilde{F}_t$. This fundamental value evolves stochastically through time. Specifically, we assume that $\tilde{F}_t$ follows a random walk, i.e. $\tilde{F}_{t+1} = \tilde{F}_t + \tilde{\delta}_{t+1}$, with $\delta_{t+1} \sim N[0, V]$. In each period, investors can trade the risky asset at the market clearing price denoted by $\tilde{P}_t$. We assume that the risky asset does not generate any distributions, and is not liquidated within a reasonable amount of time. Hence, agents can only benefit from trading by making capital gains through their investment decisions.

B. Investors

We assume that in each period a new generation of traders is born, which replaces an old generation that liquidates its positions and consumes. Each individual is assumed to live for T periods. Consequently, at each point in time there are T generations actively trading, and the distribution of these generations is perfectly uniform[2]. For simplicity we assume that each trader is born with a zero endowment. The total number of rational traders is finite and given by N. Investors submit a demand schedule in each period that maximizes their CARA utility function at their time horizon, rationally anticipating the possibility to trade at intermediate stages. For tractability reasons, we assume that each trader has the same risk tolerance level which equals unity[3]. As is usual within the rational expectations approach, all traders behave as price takers.

Additionally, there is a group of liquidity traders present, that causes the total per capita supply to vary over time. The absolute variance of this quantity is given by Δ^2. For exposition purposes, we mainly use the *per capita*[4] excess supply, denoted by $\tilde{Z}_t$, with variance U $(= \Delta^2/N^2)$.

C. Information Structure, Costly Information Acquisition

In each period, a public signal is revealed regarding the true value which is observed by all

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[2]Indeed, it implies that at each point in time each generation makes up a fraction T^{-1} of the total trader community.

[3]This is without loss of generality. Would we want to explicitly incorporate this quantity, a simple transformation of variances of the exogenous shocks would suffice.

[4]The term *per capita excess supply* should be interpreted as the *per trader excess supply.*

traders. Additionally, there is a group of informed traders that have a subscription to a signal in each period that reveals the public signal of the next period. Hence, an informed trader can front-run the public signal during his entire life. Initially, we assume that informed traders are born informed. Next, we assume information can be acquired at a certain cost c and that traders can choose to become informed. The number of informed and uninformed traders is given by N_I, N_U respectively. We denote the fraction of informed traders by w $(= N_I N^{-1})$. This fraction arises endogenously when we assume that traders have the option to become informed.

D. Equilibrium

The rational expectations equilibrium is defined through the existence of a linear pricing function of the form

$$\tilde{P}_t = a\tilde{F}_{t-1} + b\tilde{F}_t - c\tilde{Z}_t$$

at which the market clears, and in which all non-liquidity traders act optimally conditional on their information set. Ultimately, we also impose the restriction that given a cost to becoming informed, each trader makes the optimal decision to become informed or stay uninformed at the beginning of his life as a trader.

5.3 Equilibrium

Given the generic theorems derived in chapter 3, we can immediately derive the equilibrium conditions by implementing the specifics of this economy.

A. Traders Demands and Aggressiveness

Let us first introduce additional notation. Of primary importance in this environment is what we call the trading aggressiveness of each investor. As stated in the previous chapter (see lemma 3.1)), the demand of an investor with time horizon T_i, and belonging to the group $i \in$ {Informed,Uninformed} is given by

$$d_{i,t} = \beta_{i,t}\left(\mathrm{E}[\tilde{P}_{t+1}|\mathcal{I}_{i,t}] - \tilde{P}_t\right)$$

The parameter $\beta_{i,t}$ is recursively determined by

$$\beta_{i,t} = G_i^{11} - (G_i^{12})^2(G_i^{22} + \beta_{i,t+1})^{-1} \tag{5.1}$$

with the boundary condition $\beta_{i,T_{i-1}}^{-1} = \mathrm{var}^i_{T_{i-1}}(\tilde{P}_{T_i} - \tilde{P}_{T_i-1})$, and G_i^{kl} the kl-th partition of size

$n \times n$ of the matrix G_i, defined by

$$G_{i,t} = \begin{pmatrix} \mathrm{var}_t^i(P_{t+1} - P_t) & \mathrm{cov}_t^i(P_{t+1} - P_t, \hat{\Pi}_{t+2}^{i,t+1}) \\ \mathrm{cov}_t^i(P_{t+1} - P_t, \hat{\Pi}_{t+2}^{i,t+1}) & \mathrm{var}_t^i(\hat{\Pi}_{t+2}^{i,t+1}) \end{pmatrix}^{-1}$$

with $\hat{\Pi}_{t+2}^{i,t+1} = \mathrm{E}_{t+1}^i[P_{t+2} - P_{t+1}]$. The matrix G_i is independent of time in our steady state economy.

The parameter $\beta_{i,t}$ plays a profound role. It measures the trading aggressiveness of trader i, and is named accordingly. This parameter is the inverse of an uncertainty value which prior to consumption equals the conditional variance of the future price change.

Note that within a generation, the trading aggressiveness is equal across equally informed investors. Of special interest is the aggregation of the trading aggressiveness across generations for the two-types, informed and uninformed. For ease of exposition, we therefore define the following two quantities:

$$\beta_I \equiv w^{-1} \int_{i \in \inf} \beta_{i,\tau} di, \text{ and } \beta_U \equiv (1-w)^{-1} \int_{i \in un\inf} \beta_{i,\tau} di$$

that measure the average trading aggressiveness of the informed and uninformed respectively. Note that these quantities are time-independent. We have that

$$\beta_I = \frac{1}{T} \sum_{\tau=1}^{T} \beta_{I,\tau}, \text{ and } \beta_U = \frac{1}{T} \sum_{\tau=1}^{T} \beta_{U,\tau}$$

Indeed, the time dependency cancels due to the integration over generations. Additionally, we define $\beta \equiv w\beta_I + (1-w)\beta_U$ as the average trading aggressiveness of the market.

B. Equilibrium

Given these definitions we can state the following theorem.

Theorem 5.1 *If a linear equilibrium exists for a given fraction w of informed, it is given by*

$$\tilde{P}_t = \tilde{F}_{t-1} + \pi_\delta \tilde{\delta}_t - \pi_z \tilde{Z}_t$$

where

$$\pi_\delta = w\beta_I \frac{U + w\beta\beta_I V}{\beta\left(U + w^2\beta_I^2 V\right)}, \text{ and } \pi_z = \frac{U + w\beta\beta_I V}{\beta\left(U + w^2\beta_I^2 V\right)} \tag{5.2}$$

Proof. Using theorem (3.1) in chapter 3, we know that

$$\pi_\delta = \left(Q^2R^2U^{-1} + Q + V^{-1}\right)^{-1}\left(Q^2R^2U^{-1} + Q\right)$$

and $\pi_z = \pi_\delta R^{-1} Q^{-1}$. Q and R can readily be calculated given the relations (3.10) and (3.11), resulting in the following expressions

$$\begin{aligned} R &= (1-w)\,(V^{-1}+w^2\beta_I^2 U^{-1})^{-1}\beta_U \\ Q &= w(1-w)^{-1}(V^{-1}+w^2\beta_I^2 U^{-1})\,\beta_U^{-1}\beta_I \end{aligned}$$

where we used the definitions for β_I and β_U. The expressions in the theorem then follow upon substitution and using the definition for the average trading aggressiveness β. ■

The parameter π_δ measures the response of the price to the private information of informed traders. As such, it can be depicted as the news response rate. Note from the expression (5.2) for π_δ that it is bounded between 0 and 1, corresponding to $w=0$ and $w=1$ respectively. The parameter π_z measures the impact of the per capita excess supply on the market clearing price, and can be characterized as a liquidity cost. This parameter is bounded between 0 and infinity. Note that if the trading aggressiveness increases, π_z decreases, as intuition would suggest.

Unfortunately, the equilibrium cannot be solved explicitly due to the complicated nature of the trading aggressiveness parameters β_i that are recursively determined through (5.1). The properties of the equilibrium are however easily derived using numerical procedures. To determine an equilibrium explicitly, we apply a simple scheme that utilizes the fixed point theorem. For each investor, the trading aggressiveness can be calculated using the recursion relation (5.1) and the matrix G_i, which in turn can be determined given the information set of each investor and his conjecture for the coefficients of the price function. In the appendix it is shown that

$$\begin{aligned} G_I &= \begin{pmatrix} \pi_\delta^2 V + \pi_z^2 U & \pi_\delta(1-\pi_\delta)V - \pi_z^2 U \\ \pi_\delta(1-\pi_\delta)V - \pi_z^2 U & (1-\pi_\delta)^2 V + \pi_z^2 U \end{pmatrix}^{-1}, \text{ and} \\ G_U &= \begin{pmatrix} \pi_\delta^2 V + \pi_z^2 U + \frac{\pi_z^2 UV}{\pi_z^2 U + \pi_\delta^2 V} & \pi_\delta(1-\pi_\delta)V - \pi_z^2 U \\ \pi_\delta(1-\pi_\delta)V - \pi_z^2 U & \frac{\left(\pi_\delta(1-\pi_\delta)V - \pi_z^2 U\right)^2}{\pi_z^2 U + V\pi_\delta^2} \end{pmatrix}^{-1} \end{aligned}$$

for the informed and uninformed respectively. The numerical procedure given these matrices is straightforward. We initialize the recursive scheme by setting $\pi_\delta = 1$ and $\pi_z = 0$. We then calculate the $G's$, from which the quantities β_I and β_U are derived. These values are implemented in the specification for π_δ and π_z according to (5.2). The procedure is repeated until a convergence is established. This convergence is guaranteed given the fixed point theorem if an equilibrium exists.

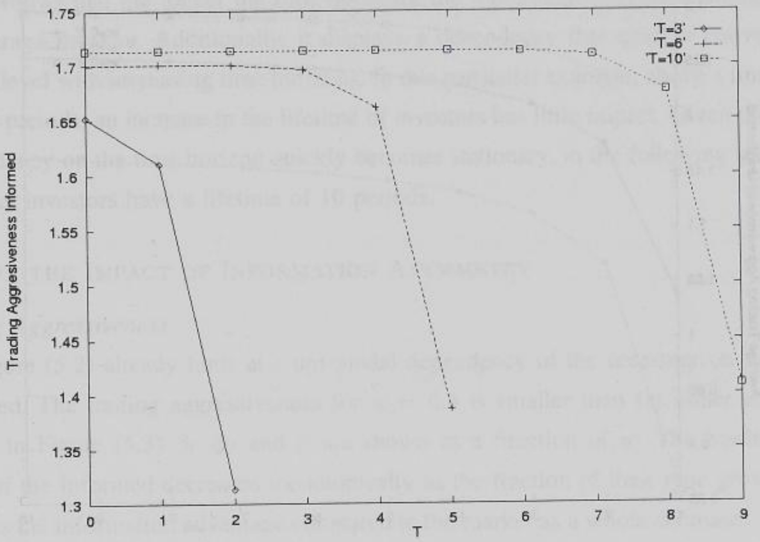

FIGURE 5.1. The trading aggressiveness of informed investors as a function of their age for a market with time horizons 3, 6, and 10 respectively. The other parameters are chosen as follows : $w = 0.4$, $U = 0.5$, and $V = 0.5$. The trading aggressiveness decreases when the agent has less periods till consumption.

5.4 Properties of the Equilibrium

5.4.1 On the Impact of Longer Time Horizons

A Investors Trading Aggressiveness

We allow agents' time horizons to extend over multiple periods, and, consequently, at each point in time, the market is composed of traders with different times until consumption. This heterogeneity impacts the market clearing price through the average trading aggressiveness parameters β_i. We consider the latter as a function of the time horizon in more detail.

In Figure (5.1) the trading aggressiveness of an informed[5] investor as a function of his age is plotted for different time horizons of the economy. Evidently, as time increases, the trading activity of the investor becomes less aggressive. The reason is that with longer time till consumption, the possibility to re-trade in each period, gives the investor a diversification ability and consequently allows him to take larger positions. This effect is also noted by Vives[1995] who shows, within a different setup, that myopic investors always trade less aggressively than

[5]The trading aggressiveness of the uninformed has the same type of dependency, though they always trade less aggressively than informed investors due to their greater uncertainty regarding future price realizations.

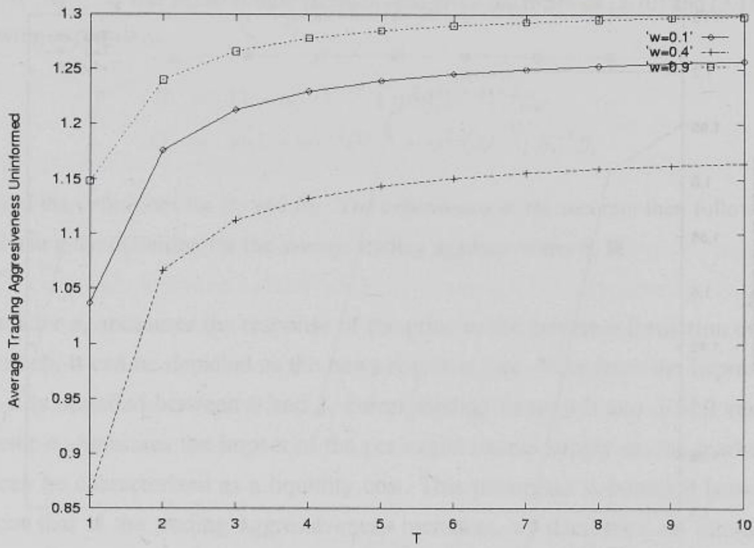

FIGURE 5.2. Average trading aggressiveness of the uninformed, β_U, as a function of the time horizon of the market for $w = 0.1$, $w = 0.4$, and $w = 0.9$. We chose $V = U = 0.5$. The average trading aggressiveness increases with the life-time of investors. The figure implies a uni-modal dependency of this quantity on the fraction of informed in the market.

investors with longer time horizons. A feature which is different in his work is that nvestors increase their trading activity when nearing their consumption horizon. This can be ascribed to the resolution of uncertainty in his model (and in the Brennan and Cao[1996] model). Here, however, this resolution of uncertainty is not present, and therefore investors reduce the magnitude of their trading activity getting closer to the point of consumption. In fact, the posterior uncertainty regarding the fundamental value of the asset is constant. Also observe that the trading aggressiveness is almost stationary if the age of the investor is relatively small compared to his total lifetime. Only when the consumption horizon is near, the trading aggressiveness changes significantly.

B. Average Trading Aggressiveness

The quantity that ultimately enters the market clearing condition is the average trading aggressiveness of each group, β_I and β_U. In the Figure (5.2) the dependency[6] of β_U on the time horizon is shown for three values of w. A few observations are worth noting. First of all,

[6]For the informed, similar dependencies can be found as a function of the time horizon.

the graph shows that the longer the time horizons, the more aggressive the trading behavior of the average investor. Additionally, it displays a dependency that quickly converges to a stationary level with increasing time horizons. In this particular example, above a time horizon of about 5 periods, an increase in the lifetime of investors has little impact. Given the fact that the dependency on the time horizon quickly becomes stationary, in the following sections, we assume that investors have a lifetime of 10 periods.

5.4.2 On the Impact of Information Asymmetry

A. Trading Aggressiveness

The Figure (5.2) already hints at a uni-modal dependency of the economy on the fraction of informed. The trading aggressiveness for $w = 0.4$ is smaller than for either $w = 0.1$ or $w = 0.9$. In Figure (5.3) β_I, β_U and β are shown as a function of w. The trading aggressiveness of the informed decreases monotonically as the fraction of their type grows. This is intuitive as the information advantage compared to the market as a whole decreases. The uninformed investors trading behavior is a more complex function of w. Initially, if all agents are uninformed the trading aggressiveness is high. However, with increasing information friction, the aggressiveness of the uninformed decreases until at some point they can benefit from the increasing competition between informed investors. Eventually, the latter effect dominates, upon which their trading aggressiveness increases accordingly.

B. Market Depth

A quantity that plays a central role in our discussion is the so-called relative market depth λ which we define as the inverse of the liquidity cost parameter π_z, i.e. $\lambda \equiv \pi_z^{-1}$. We explicitly call it the relative market depth, as it measures the impact of the per capita excess supply on the price realization. The higher λ, the less market prices are affected by liquidity shocks. Information friction has a significant impact on this quantity which can be observed from Figure (5.4) where the market depth is shown as a function of w. Increasing the fraction of informed, starting with a completely uninformed market, initially decreases market depth due to the increase in information friction. At a certain point, the information friction is maximal after which it declines back to zero. The result is that for larger values of w, increasing the fraction of informed merely increases the competition between informed agents leading to an increase in market depth.

Slezak[1994] also finds a uni-modal dependency for market depth (though he considers its inverse, the liquidity cost). As such, the impact of a heterogeneous distribution of generations,

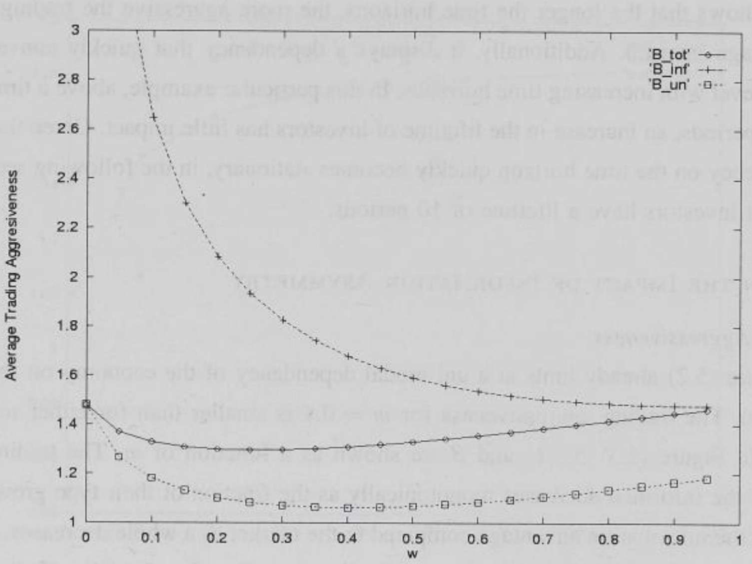

FIGURE 5.3. Average trading aggressiveness as a function of the fraction of informed in the economy for the informed (β_I), the uninformed (β_U) and the market as a whole (β) for $V = 0.5$, and $U = 0.5$.

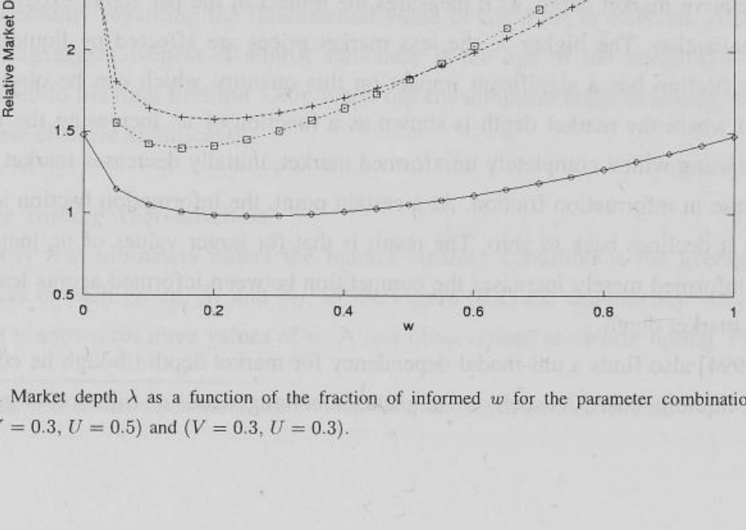

FIGURE 5.4. Market depth λ as a function of the fraction of informed w for the parameter combinations ($V = 0.5$, $U = 0.5$), ($V = 0.3$, $U = 0.5$) and ($V = 0.3$, $U = 0.3$).

and the finiteness of agents' lifetimes, does not have a profound impact on the qualitative features of the equilibrium. The variance of price changes as a function of the fraction of informed he additionally considers, has the same features in our model. It again is a uni-modal function of the fraction of informed.

C. Investors Utilities

Of interest is the welfare of investors as a function of the information friction in the market. The measure for this quantity for investor i is the ex ante expected utility, which we denote by $\mathcal{V}_i$, prior to his first trading period, i.e.

$$\mathcal{V}_i = \max_{\{d_t\}} \mathrm{E}_i \left[- \exp\left(-\tilde{W}_{T_i} \right) \right]$$

where $\tilde{W}_T$ is the wealth at the time horizon of investor i, and the expectation is taken conditional on the information prior to trading. The ex ante expected utility is defined conditional on the optimal policy of the investor.

In the appendix it is shown that the ex ante expected utility of being uninformed, is given by

$$\mathcal{V}_U = - \prod_{n=1}^{T-1} \left(1 + \beta_{U,n} \frac{(\pi_\delta(1-\pi_\delta)V - \pi_z^2 U)^2}{\pi_z^2 U + \pi_\delta^2 V} \right)^{-\frac{1}{2}}$$

while for the informed, we have:

$$\mathcal{V}_I = - \prod_{n=1}^{T-1} \left(1 + \beta_{I,n} \left((1-\pi_\delta)^2 V + \pi_\delta^2 U \right) \right)^{-\frac{1}{2}}$$

In Figure (5.5), $\mathcal{V}_I$, $\mathcal{V}_U$, and $\mathcal{V}$ ($\equiv w\mathcal{V}_I + (1-w)\mathcal{V}_U$) are shown as a function of the fraction of informed. Indeed, informed always have a higher expected utility as logic would demand. The equivalence between completely uninformed and informed markets is underscored by the fact that the ex ante expected utility of the informed in the perfectly informed economy equals that of the uninformed in the completely uninformed market. The average utility of agents varies slightly as the fraction of informed increases, however, it almost retains the level of its homogeneous boundaries.

The dependency of the ex ante utilities of agents on the per capita excess supply plays a profound role. All agents, informed and uninformed, ultimately derive their fortune from trading through the sacrifice of the liquidity traders. It is therefore natural to assume that, on average, their ex ante expected utility will decrease if the activity of liquidity traders decreases. Additionally, since informed agents receive the greater chunk of the generous contribution of this group, they will be affected most significantly given a decrease in the variance of the per capita excess supply. Figure (5.6) acknowledges this observation.

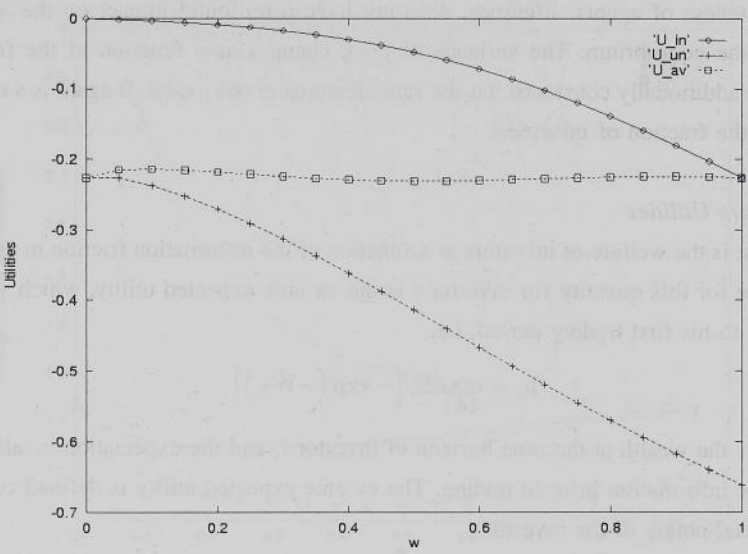

FIGURE 5.5. Ex ante expected utilities for the informed, the uninformed and the average of these expectations as a function of w for $U = 0.5$ and $V = 0.5$. Both types benefit most, when the fraction of informed is as small as possible.

5.5 Endogenous Information Acquisition

The preceding analysis imposes the fraction of informed agents in the economy exogenously. Agents may however decide to become informed, given an information acquisition cost c. A decision that will depend on the relative advantage of being informed compared to being uninformed. More formally, an agent will become informed if

$$e^{-c}\mathcal{V}_I(w') > \mathcal{V}_U(w)$$

where w' is given by $w(1 + N^{-1})$. In equilibrium, the fraction of informed w^{eq} is therefore determined endogenously by the interior solution to

$$e^{-c}\mathcal{V}_I(w^{eq}) = \mathcal{V}_U(w^{eq}),$$

unless c is either too large or very small leading to the boundary solutions $w = 0$, or $w = 1$ respectively.

As pointed out by Grossman and Stiglitz[1980], given a fixed cost for the acquisition of information, an interior equilibrium may arise due to the diminishing relative returns to informedness, when more agents decide to become informed. In our model this intuitive relation

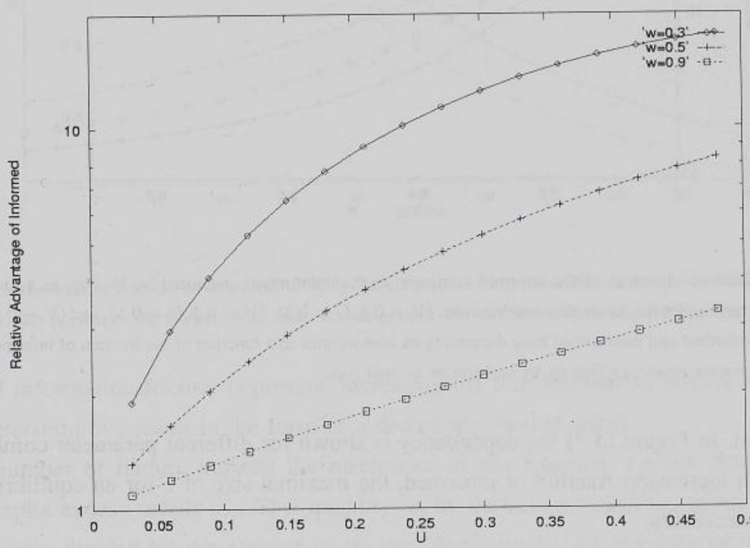

FIGURE 5.6. Relative advantage of the informed over the uninformed measured by $\mathcal{V}_U/\mathcal{V}_I$, as a function of the variance of the per capita excess supply U for $w = 0.3$, $w = 0.5$, $w = 0.9$ with $U = 0.5$, and $V = 0.5$. The higher the per capita excess supply, the more informed agents benefit relative to the uninformed. The ex ante utilities of both types are increasing in the per capita excess supply.

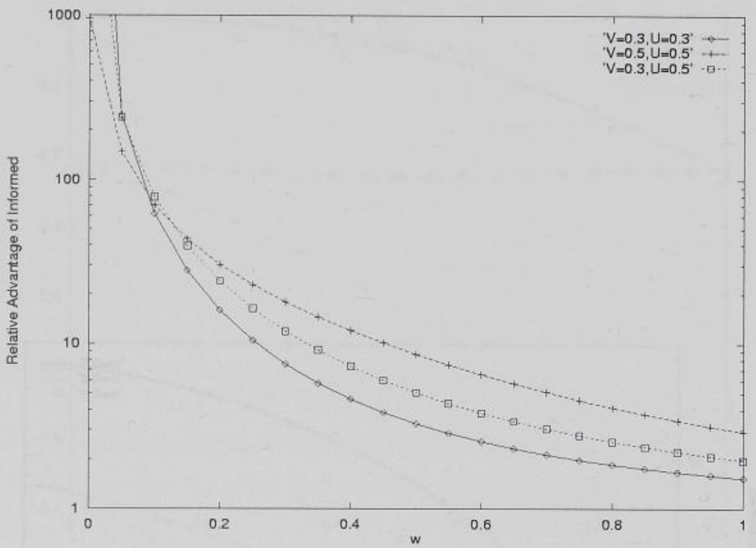

FIGURE 5.7. Relative advantage of the informed compared to the uninformed, measured by $\mathcal{V}_U/\mathcal{V}_I$, as a function of the fraction of informed w, for the parameters combinations $(V = 0.3, U = 0.3)$, $(V = 0.3, U = 0.5)$, and $(V = 0.5, U = 0.5)$. Although both informed and uninformed have decreasing ex ante utilities as a function of the fraction of informed, informed investors are impacted more significantly by an increase in their type.

holds as well. In Figure (5.7) the dependency is shown for different parameter combinations. Clearly, with increasing fraction of informed, the maximal size of c for an equilibrium to be sustainable decreases.

5.6 Market Development under Costly Information

In this section we consider the interior equilibrium that arises given a fixed cost to the acquisition of information, and its dependency on the development of a market measured by the total number of traders[7]. It is intuitive to assume that with increasing number of traders, the competitiveness of a market tends to increase. As we show in this section, though ceteris paribus this indeed holds, with endogenous information acquisition this may not be the case.

[7]Note that this association with market development is in line with the definition of market development in the previous chapter. In chapter 4, it was argued that $\zeta = A(UV)^{-1/2}$ was a relative risk parameter, and effectively measures the development of a market. Would we translate this parameter to this model, we would have $\zeta = (V\Delta^2/N^2)^{-1/2} = N/(\Delta^2\sqrt{V})$. Hence, ζ scales with the number of traders.

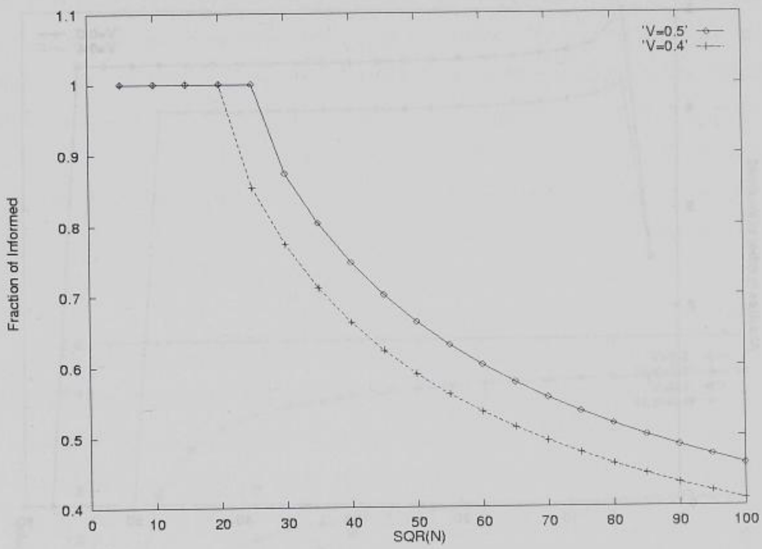

FIGURE 5.8. The interior fraction of informed, w^{eq}, that arises given a fixed information cost c, as a function of the total number of traders present in the market. With many traders present it is less attractive to become informed.

In fact, if information friction is present, increasing the total number of traders tends to lead to a counter-intuitive result in the form of a decreasing market depth.

The number of traders impacts the mechanics of our financial market directly through the per capita excess supply $\tilde{Z}_t$. This quantity is, by definition of course, given by the total excess supply divided by the number of traders. Accordingly, the variance of this quantity, U, depends on the number of traders as well, through the relation $U = \Delta^2/N^2$, where Δ^2 is the variance of the total excess supply in the market. Hence, increasing the number of traders leads to a decrease in the variance of the per capita excess supply.

A Number of Traders vs Fraction of Informed

Increasing the number of traders impacts the comparative advantage of being informed over uninformed. As the previous section showed, with decreasing variance of liquidity shocks U, the relative advantage of being informed decreases as well. If the returns for the informed increase, necessarily the interior fraction of informed should be less or equal given the new level of U. In Figure (5.8[8]) the interior fraction of informed w^{eq} is plotted as a function of

[8] In the graphs in this section we have assumed a cost $c = 0.5$.

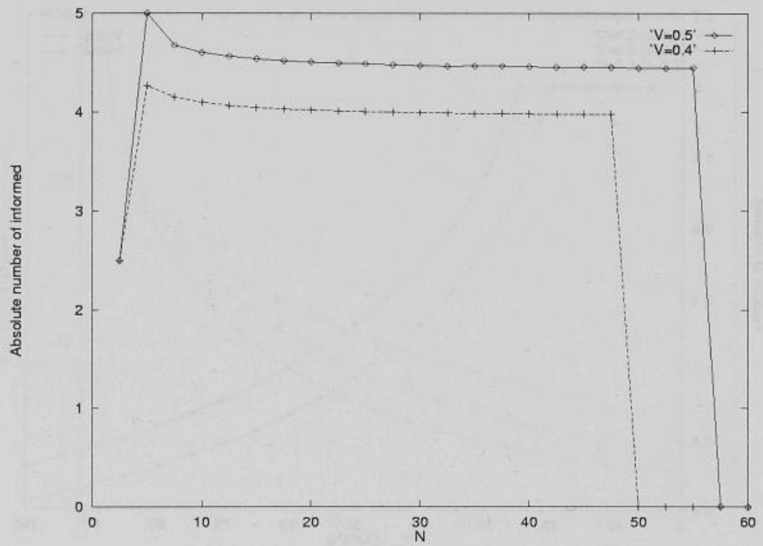

FIGURE 5.9. Absolute number of informed traders as a function of the total number of traders under costly information acquisition.

the total number of traders. For $N^2 < 10$, competition is weak enough for all traders to become informed. However, an increase in the number of traders makes the informed status less attractive, until at some point ($N^2 = 20 - 30$ in the figure) some traders decide to remain uninformed. Increasing the number of traders from that point on leads to a decrease in the fraction of informed.

The absolute number of informed traders is plotted in Figure (5.9). Initially, the number of informed equals the number of traders and grows accordingly. At the break point, the absolute number starts to decrease until the second break point where no trader decides to become informed.

B. Market Depth and Market Development

Although the decreasing fraction of informed, as a function of the number of traders, may be fairly intuitive, market depth has a more complicated dependency on the development of a market. As stated earlier, we measure market depth through the inverse of the liquidity cost, i.e. market depth $\lambda \equiv \pi_z^{-1}$. Figure (5.10) displays this quantity as a function of the total number of traders, N. Additionally, its homogeneous counterpart is included in the graph. Initially, the fraction of informed remains constant. The only effect of increasing the number of traders is

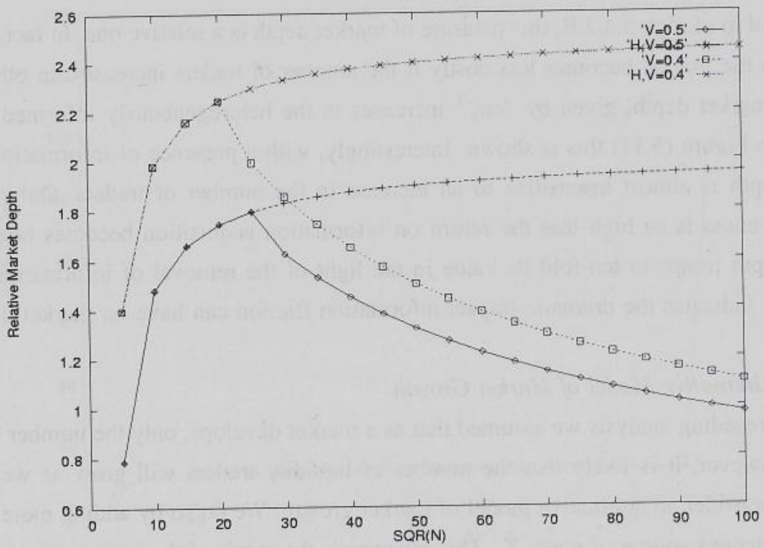

FIGURE 5.10. Market depth as a function of the total number of traders in the market under costly information acquisition, and in a homogeneously informed economy for $V = 0.3$ and $V = 0.5$ respectively. The costly information and homogeneous economy initially display the same dependency, however at a certain breakpoint, market depth in the costly information economy decreases.

the increased competition among homogeneous types, leading to an increase in market depth. However, as the benefit of being informed decreases, ultimately an information friction is introduced, and the dependency reverses. Instead of increasing the effective competitiveness of agents, a growing number of traders leads to less well informed agents on average, and more information friction in the market. The result is that relative market depth diminishes with an increase in the number of traders. Note that the homogeneous counterpart, which is also shown in the graph, initially has the very same dependency, as expected, but at the break point the two diverge. In a homogeneously informed economy, increasing the number of traders leads to an increase in market depth.

As noted in section 5.4.2.B, this measure of market depth is a relative one. In fact, to liquidity traders the market becomes less costly if the number of traders increases. In other words, the total market depth, given by $N\pi_z^{-1}$ increases in the heterogeneously informed economy as well. In Figure (5.11) this is shown. Interestingly, with a presence of information friction, market depth is almost insensitive to an increase in the number of traders. Only when the competitiveness is so high that the return on information acquisition becomes negative, the market depth jumps to ten-fold its value in the light of the removal of information friction. The graph indicates the dramatic impact information friction can have on market depth.

C. An Alternative Model of Market Growth

In the preceding analysis we assumed that as a market develops, only the number of traders grows. However, it is likely that the number of liquidity traders will grow as well. Let us therefore consider an alternative model of market growth. We do so by adding more structure to the exogenous source of noise $\tilde{Z}_t$. This quantity is the result of the aggregation over many individual noisy demands. As a first approximation, it is not unreasonable to assume that we can decompose individual liquidity demands as

$$\tilde{z}_{i,t} = \tilde{\varepsilon}_t + \tilde{\eta}_{i,t}$$

where $\tilde{\varepsilon}_t$ is a common noise term which reflects the correlation of liquidity shocks across investors, and $\tilde{\eta}_{i,t}$ is the liquidity component specific to liquidity investor i which is uncorrelated with other liquidity shocks. Assume that both noise terms are normally distributed with $\tilde{\varepsilon}_t \sim N[0, \sigma_\varepsilon^2]$, and $\tilde{\eta}_{i,t} \sim N[0, \sigma_\eta^2]$. Note that this implies time-independence of the second moments, and identical variance of liquidity shocks across liquidity investors. With N traders and M liquidity traders, the per capita (per trader) excess supply is then given by

$$\tilde{Z}_t = N^{-1} \sum_{i=1}^{M} (\tilde{\varepsilon}_t + \tilde{\eta}_{i,t}) = N^{-1}(\alpha M \tilde{\varepsilon}_t + \sum_{i=1}^{M} \tilde{\eta}_{i,t}) \ .$$

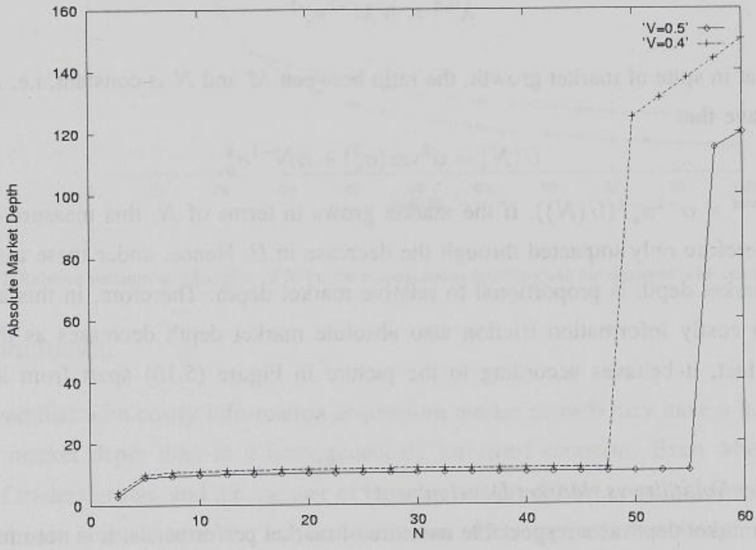

FIGURE 5.11. Absolute market depth as a function of the number of traders in the market. Initially, market depth increases as in the homogeneous economy until the breakpoint where information costs have impact. From that point on, absolute market depth is almost insensitive to an increase in the size of the trader community. Ultimately, competition is so high that no trader can afford to become informed, after which the absolute market depth jumps to about ten-times its original value.

Accordingly, the variance of per supply noise can be calculated to be

$$\begin{aligned} U &= \mathrm{var}(\tilde{Z}_t) = N^{-2}\mathrm{var}(M\tilde{\varepsilon}_t + \sum_{i=1}^{M}\tilde{\eta}_{i,t}) = N^{-2}(M^2\sigma_\varepsilon^2 + \mathrm{var}(\sum_{i=1}^{M}\tilde{\eta}_{i,t})) \\ &= N^{-2}(M^2\mathrm{var}(\sigma_\varepsilon^2) + \alpha M\sigma_\eta^2) = N^{-2}(M^2\mathrm{var}(\sigma_\varepsilon^2) + M\sigma_\eta^2) \end{aligned}$$

To the individual liquidity trader, the relevant measure of market depth is given by his individual impact on the price, which on average is given by $NM^{-1}\tilde{Z}_t$. Hence, market depth for the individual liquidity trader is captured through λ^{ind}, with

$$\lambda^{ind} = NM^{-1}\pi_z^{-1}$$

Assume that in spite of market growth, the ratio between M and N is constant, i.e. $M = \alpha N$. We then have that

$$U(N) = \alpha^2\mathrm{var}(\sigma_\varepsilon^2) + \alpha N^{-1}\sigma_\eta^2$$

and that $\lambda^{ind} = \alpha^{-1}\pi_z^{-1}(U(N))$. If the market grows in terms of N, this measure of market depth is therefore only impacted through the decrease in U. Hence, under these assumptions absolute market depth is proportional to relative market depth. Therefore, in this alternative setup, with costly information friction also absolute market depth decreases as the market grows. In fact, it behaves according to the picture in Figure (5.10) apart from a constant factor α^{-1}.

D. Market Volatility vs. Market Development

Though market depth is a respectable measure of market performance, it is not immediately observable. We therefore study the dependency of an easily measurable quantity, in the form of the unconditional variance of returns. This quantity is easily calculated and is given by

$$\mathrm{var}(\tilde{P}_{t+1} - \tilde{P}_t) = (1-\pi_\delta)^2V + \pi_\delta^2V + 2\pi_z^2U$$

As a function of N, for a homogeneously informed economy and an economy with costly information the variance relative to V is plotted in Figure (5.12). Though after the break point, the variance of the informed economy decreases less slowly than for the homogeneous case, the difference is not very significant. This indicates that even though the relative market depth may be ten-fold higher in the economy with costly information, an observable quantity such as the variance of price changes cannot reveal this inefficiency.

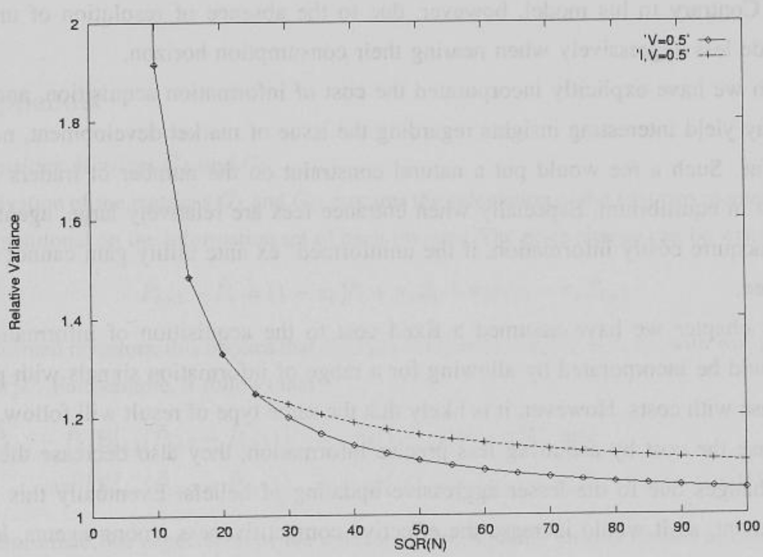

FIGURE 5.12. Relative variance as a function of N for the homogeneous economy and the economy with costly information.

5.7 Conclusion

We showed that with costly information acquisition market growth may have a less intuitive effect on market depth than in a homogeneously informed economy. Even when only the number of traders grows, and the number of liquidity traders is kept constant, absolute market depth is almost insensitive to the increase in traders. This applies, even though supply risk decreases steadily. The reason is that if the number of traders increases, informed traders compete less effectively given that less agents decide to become informed. Moreover, the information friction in the market increases. The aggregate result is a decreasing relative market depth as a function of the number of traders. When the number of liquidity traders increases as a market develops, also absolute market depth in the costly information economy is seen to decrease. Hence, information dispersion may dominate when market size develops. These results contrast the usual relation between market development and market depth. It implies that one should not underestimate the important role of information acquisition costs. As we have shown, endogenizing information acquisition efforts may result in equilibria that exhibit quite different comparative statics.

Additionally, we considered the impact of the length of time horizons on the trading behavior of investors. Conform Vives[1995], agents trade more aggressively with longer time horizons. Contrary to his model, however, due to the absence of resolution of uncertainty, agents trade less aggressively when nearing their consumption horizon.

Though we have explicitly incorporated the cost of information acquisition, another type of cost may yield interesting insights regarding the issue of market development, namely the entrance fee. Such a fee would put a natural constraint on the number of traders that enter the market in equilibrium. Especially when entrance fees are relatively large, agents may be forced to acquire costly information, if the uninformed' ex ante utility gain cannot offset the entrance fee.

In this chapter we have assumed a fixed cost to the acquisition of information. More realism could be incorporated by allowing for a range of information signals with precisions that increase with costs. However, it is likely that the same type of result will follow. If agents can decrease the cost by acquiring less precise information, they also decrease the variance of price changes due to the lesser aggressive updating of beliefs. Eventually this comes to their detriment, as it would increase the effective competitiveness among agents, leading to a decrease in the returns to being informed. Ultimately this leads to a decline in the fraction of informed investors, while the information friction in the economy increases.

5.A Appendix

A. The Transition Matrices G_I and G_U

The derivation of the matrices G_I and G_U requires the calculation of the first two moments of price changes conditional on the information set of each investor. The price change can be written as

$$\tilde{P}_{t+1} - \tilde{P}_t = (1-\pi_\delta)\tilde{\delta}_t + \pi_z \tilde{Z}_t + \pi_\delta \tilde{\delta}_{t+1} - \pi_z \tilde{Z}_{t+1}$$

For the informed investors, this implies that $\mathrm{E}_t^I[\tilde{P}_{t+1} - \tilde{P}_t] = (1-\pi_\delta)\tilde{\delta}_t + \pi_z \tilde{Z}_t$, with $\mathrm{var}_t^I(\tilde{P}_{t+1} - \tilde{P}_t) = \pi_\delta^2 V + \pi_z^2 U$. Furthermore, it follows that

$$\begin{aligned}
\mathrm{cov}_t^I\left(\tilde{P}_{t+1} - \tilde{P}_t, \mathrm{E}_{t+1}^I(\tilde{P}_{t+2} - \tilde{P}_{t+1})\right) &= \pi_\delta(1-\pi_\delta)V - \pi_z^2 U, \text{ and} \\
\mathrm{var}_t^I\left(\mathrm{E}_{t+1}^I(\tilde{P}_{t+2} - \tilde{P}_{t+1})\right) &= \mathrm{var}_t^I((1-\pi_\delta)\delta_{t+1} + \pi_z Z_{t+1}) = (1-\pi_\delta)^2 V + \pi_z^2 U
\end{aligned}$$

For the uninformed, the expectation of the innovation and liquidity term is needed to calculate these values. For the innovation $\tilde{\delta}_t$, the estimates of the uninformed are given by

$$\mathrm{E}_t^U[\tilde{\delta}_t] = \frac{V}{(\pi_\delta^{-1}\pi_z)^2 U + V}(\tilde{\delta}_t - \pi_z/\pi_\delta Z_t), \text{ with } \mathrm{var}_t^U[\tilde{\delta}_t] = \frac{\pi_z^2 UV}{\pi_z^2 U + \pi_\delta^2 V}$$

and for the per capita excess supply shock $\tilde{Z}_t$,

$$\mathrm{E}_t^U[\tilde{Z}_t] = \frac{-\pi_z/\pi_\delta U}{(\pi_\delta^{-1}\pi_z)^2 U + V}(\tilde{\delta}_t - \pi_z/\pi_\delta \tilde{Z}_t)$$

Hence, we have that

$$\mathrm{E}_t^U(\tilde{P}_{t+1} - \tilde{P}_t) = (1-\pi_\delta)\mathrm{E}_t^U \tilde{\delta}_t + \pi_z \mathrm{E}_t^U \tilde{Z}_t = \left(\tilde{\delta}_t \pi_\delta - \pi_z \tilde{Z}_t\right) \frac{\pi_\delta(1-\pi_\delta)V - \pi_z^2 U}{\pi_z^2 U + V\pi_\delta^2}$$

Furthermore,

$$\begin{aligned}
\mathrm{var}_t^U\left(\tilde{P}_{t+1} - \tilde{P}_t\right) &= \pi_\delta^2 V + \pi_z^2 U + \frac{\pi_z^2 UV}{\pi_z^2 U + V\pi_\delta^2}, \\
\mathrm{cov}_t^U\left(\tilde{P}_{t+1} - \tilde{P}_t, \mathrm{E}_{t+1}^i(\tilde{P}_{t+2} - \tilde{P}_{t+1})\right) &= \pi_\delta(1-\pi_\delta)V - \pi_z^2 U, \text{ and,} \\
\mathrm{var}_t^U\left(\mathrm{E}_{t+1}^i(\tilde{P}_{t+2} - \tilde{P}_{t+1})\right) &= \frac{(\pi_\delta(1-\pi_\delta)V - \pi_z^2 U)^2}{\pi_z^2 U + V\pi_\delta^2}.
\end{aligned}$$

Armed with these expressions, the relations for G_I and G_U follow immediately.

B. Ex Ante Expected Utilities

According to chapter 3, we can write the ex ante expected utility of agent i as

$$\mathcal{V}_i^T = -\sqrt{\frac{1}{|\mathrm{var}_t^{i,un}(\hat{\Pi}_{t+1}^{i,t})||\mathrm{var}_t^{i,un}(\hat{\Pi}_{t+1}^{i,t})^{-1} + \beta_{i,t}|}} \prod_{n=t}^{T-1} \sqrt{\frac{|G_{i,n}|}{|G_{i,n} + c_n|}} \exp[-r_i^{-1} W_{i,t}]$$

where c_i is defined as

$$c_{i,t} = \begin{pmatrix} 0 & 0 \\ 0 & \beta_{i,t} \end{pmatrix}$$

with $\mathrm{var}_t^{i,un}(\hat{\Pi}_{t+1}^{i,t})$ the unconditional variance of the first period's payoff. This quantity is equivalent to $\mathrm{var}_t^i(\hat{\Pi}_{t+2}^{i,t+1})$ in our model. For all investors, the quantity $\frac{|G_i|}{|G_i+c_n|}$ can easily be calculated to be

$$\frac{|G_i|}{|G_i + c_n|} = \frac{1}{1 + \beta_{i,t}^{-1} \mathrm{var}_t^i(\mathrm{E}_{t+1}^i(\tilde{P}_{t+2} - \tilde{P}_{t+1}))}$$

Hence, we obtain for investor i :

$$\mathcal{V}_{i,0} = -\prod_{n=t}^{T-1} \sqrt{\frac{1}{1 + \beta_{i,t}^{-1} \mathrm{var}_t^i(\mathrm{E}_{t+1}^i(\tilde{P}_{t+2} - \tilde{P}_{t+1}))}}.$$

Substituting the functional forms found for $\mathrm{E}_{t+1}^i(\tilde{P}_{t+2} - \tilde{P}_{t+1})$, the expressions reported in the text follow. ■