Charged Current Interactions at HERA
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Chapter 2

The Charged Current Process

2.1 Introduction

In deep inelastic scattering (DIS) we probe the constituents of the proton with a virtual boson: \(\gamma, Z^0\) or \(W^\pm\). In the case of neutral boson exchange (\(\gamma, Z^0\)) we speak of Neutral Current (NC) scattering, the exchange of a charged vector boson \(W^+\) or \(W^-\) is called Charged Current (CC) scattering. Figure 2.1 shows the Feynman diagram for this process. We denote with \(l (l')\) the incoming (outgoing) lepton with four momenta \(k (k')\), \(q \equiv (k - k')\), \(P\) the initial state proton with four momentum \(p\) and \(H\) the hadronic final state.

It is conventional to describe the kinematics of the scattering with Lorentz scalars. The four momentum transfer

\[
Q^2 \equiv -(k - k')^2 = -q^2
\]  

(2.1)
gives the length scale at which we probe

\[
\lambda = 1/\sqrt{Q^2}
\]  

(2.2)
The maximum \(Q^2\) is given by the center-of-mass energy squared of the lepton-proton system

\[
s \equiv (p + k)^2
\]  

(2.3)

Further the inelasticity,

\[
y \equiv \frac{q \cdot p}{k \cdot p}
\]  

(2.4)
is, in the proton rest frame, the fraction of the energy transferred from the lepton to the struck quark.

Another convenient variable

\[
x \equiv \frac{Q^2}{2q \cdot p}
\]  

(2.5)
gives the fraction of the proton four momentum carried by the struck quark.

Only two of the variables \(x, Q^2\) and \(y\) are needed to fully describe the kinematics. For \(s \gg m_P^2\) (\(m_P\) is the proton mass) the following relation between the above quantities holds

\[
Q^2 = xys
\]  

(2.6)
The invariant mass of the hadronic final state is given by

\[
W^2 = Q^2 \frac{1-x}{x} + m_p^2
\]  

(2.7)
In terms of these variables the cross section for neutral current scattering is given by

\[
\frac{d^2\sigma_{NC}}{dx dQ^2}(e^\pm p) = \frac{4\pi\alpha^2}{xQ^4} (y^2 x F_1(x, Q^2) + (1 - y) F_2(x, Q^2)) \equiv (y - \frac{y^2}{2}) x F_3(x, Q^2))
\]  

The structure functions $xF_1, F_2$ and $xF_3$ stem from general parameterization of the hadronic tensor [1]. For the naive quark parton model with massless quarks $2xF_1 = F_2$ and the structure functions $F_2$ and $xF_3$ can be written as

\[
F_2(x, Q^2) = \sum_q A_q(Q^2) (x q(x) + x \bar{q}(x))
\]  

\[
xF_3(x, Q^2) = \sum_q B_q(Q^2) (x q(x) - x \bar{q}(x))
\]

with $q(x), \bar{q}(x)$ being the quark and anti-quark densities in the proton respectively and $A_q(Q^2), B_q(Q^2)$ describing the coupling of quark of flavor $q$ to the exchanged vector boson. $A_q(Q^2)$ can be written as

\[
A_q(Q^2) = e_q^2 e_q \frac{1}{4 \sin^2 \theta_W \cos^2 \theta_W} (\frac{Q^2}{Q^2 + M_Z^2})
\]
with $e_l, v_l, a_l$ and $e_q, v_q, a_q$ the charge, vector and axial-vector coupling of the initial lepton and scattered quark respectively.

$B_q(Q^2)$ can be expressed as:

$$
B_q(Q^2) = 2|e_l||e_q|a_la_q\left(\frac{1}{4\sin^2\theta_W \cos^2\theta_W}\right)\left(\frac{Q^2}{Q^2 + M_W^2}\right)
$$

$$
+ 4|v_l|v_qa_q\left(\frac{1}{4\sin^2\theta_W \cos^2\theta_W}\right)\left(\frac{Q^4}{(Q^2 + M_Z^2)^2}\right)
$$

One can identify taking the first term of equation (2.8) and equations (2.11) and (2.12) the propagator terms for the exchange of a $\gamma$ ($1/Q^4$), the $Z^0$ ($1/(Q^2 + M_Z^2)^2$) and the interference of $Z^0$ and $\gamma$ exchange ($1/(Q^2 (Q^2 + M_Z^2))$).

For charged current scattering the cross section is:

$$
\frac{d^2\sigma_{CC}(e^\pm p)}{dx dQ^2} = \frac{\pi\alpha^2}{8\sin^4\theta_W (Q^2 + M_W^2)^2 \cos^2\theta_W} \left((1 + (1 - y)^2)W_2^+ + (1 - (1 - y)^2)W_3^+\right)
$$

$W_2$ and $W_3$ are the sum and difference respectively of the quark and anti-quark densities. Since charge is conserved at the $W^\pm q$ vertex only those quarks which actually contribute to the cross section are taken:

$$
W_2^+ = \sum_i (d_i(x) + \bar{u}_i(x))
$$

$$
W_2^- = \sum_i (u_i(x) + \bar{d}_i(x))
$$

$$
W_3^+ = \sum_i (d_i(x) - \bar{u}_i(x))
$$

$$
W_3^- = \sum_i (u_i(x) - \bar{d}_i(x))
$$

Again one can identify in equation (2.13) the propagator term for the $W^\pm$ exchange ($1/(Q^2 + M_W^2)^2$).

Clearly, when comparing the cross section for charged and neutral current scattering one can readily see that for relatively small $Q^2$ CC scattering is significantly less probable than NC scattering. The CC cross section is relatively flat for $Q^2$ up to the mass of the $W^\pm$ squared.

It is important to note that in the naive quark parton model the structure functions are independent of $Q^2$. When gluon radiation of the quarks and splitting of gluons into quark anti-quark pairs are taken into account (QCD improved quark parton model), the parton density functions become indeed dependent on $Q^2$. The dependence is described in leading order QCD by the DGLAP [2] equations:

$$
\frac{dg_f(x, Q^2)}{d\log Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_z^1 (g_f(x, Q^2) P_{qq}(x/z) + G(z, Q^2) P_{qg}(x/z)) \frac{dz}{z}
$$
Here \( G(x, Q^2) \) is the gluon density in the proton and the “splitting functions” \( P_{ij}(y) \) give the probability of obtaining a parton \( i \) from parton \( j \) where parton \( i \) has a fraction \( y \) of the momentum of parton \( j \).

With these equations it is possible to calculate the parton density functions at all \( Q^2 \) if they are given at a certain \( Q_0^2 \). In next to leading order the equations become more cumbersome and the densities become dependent on the renormalization scheme. Within each scheme, however, the parton densities should be universal functions applicable to any interaction. In particular it should be possible to use parton density functions extracted from NC scattering to calculate CC scattering and visa versa.

### 2.2 Parton Density Functions

NC data from ZEUS and H1 have been used to extract the parton densities (\( q, \bar{q} \) and \( G \)) of the proton. An example of data from ZEUS and NMC are shown in figures 2.2 and 2.3 together with the PDF fit which uses the DGLAP evolution [2] done by the ZEUS experiment. The figures have been taken from [3].

Many other groups have performed fits such as GRV [4], CTEQ [5] and MRS [6] which are available as PDFLIB [7]. These groups have used more data than just DIS but give an equally good description of ZEUS data.

The subject of this thesis is charged current positron proton scattering. As seen from equation (2.14) the charged current process selects a subset of quarks inside the proton. This allows investigation of the flavor decomposition of the parton densities.

Previous data on CC scattering stem from \( \nu \) scattering ([8, 9, 10]). The energy of the \( \nu \) beam is restricted to about 300 GeV which yields an \( s \)-value of about 600 GeV\(^2\) compared to the 90200 GeV\(^2\) available at HERA. An example of the structure functions for \( \nu p \) scattering is shown in figure 2.4. The data have been taken from [8]. The curve shows the CTEQ4D parameterization of the measured structure function. The agreement is not more than reasonable. This is most likely due to the transformation of the measured data to one fixed \( Q^2 \) for which the authors use a power law behavior with a fixed slope below \( x = 0.2 \) and a different slope above \( x = 0.2 \). However the overall features and magnitude are still reasonably described by the parameterization.

Fixed target experiments reach up to a \( Q^2 \) of about 25 GeV\(^2\) for measurements on protons and of \( Q^2 \) of approximately 100 GeV\(^2\) and \( x > 0.1 \) for measurements on Fe targets.

The current parton density functions which fit the NC data predict total cross sections for NC and CC scattering at HERA of

\[
\sigma_{\text{NC}}^{\text{tot}}(Q^2 > 1\text{GeV}^2) = 1.15 \text{ pb} \\
\sigma_{\text{CC}}^{\text{tot}}(Q^2 > 1\text{GeV}^2) = 38.7 \text{ pb}
\]
Figure 2.2: Structure function $F_2$ for various values of $Q^2$ ($60 \text{GeV}^2 < Q^2 < 800 \text{GeV}^2$) as a function of $x$ as measured by the ZEUS collaboration. The curves indicate a QCD NLO fit to the data. The $Q^2$ values are indicated in $\text{GeV}^2$. 
Figure 2.3: Structure function $F_2$ for various values of $Q^2$ ($1200 \text{ GeV}^2 < Q^2 < 5000 \text{ GeV}^2$) as a function of $x$ as measured by the ZEUS collaboration. The curves indicate a QCD NLO fit to the data. The $Q^2$ values are indicated in GeV$^2$. 
2.3 Photoproduction

At very low $Q^2$, deep inelastic scattering turns into quasi-real photon proton scattering. The lepton proton cross section is given in terms of a photon proton cross section by the Weizsäcker-Williams formula [11] for the photon flux $F(y, Q^2)$:

$$\frac{d^2\sigma_{ep}(s)}{dydQ^2} = \sigma_{\text{tot}}^{ep}(ys) \cdot (1 + \delta_{RC}) \cdot F(y, Q^2)$$

$$= \sigma_{\text{tot}}^{ep}(ys) \cdot (1 + \delta_{RC}) \cdot \frac{\alpha}{2\pi Q^2} \left( \frac{1+(1-y)^2}{y} - \frac{2(1-y)}{y} \cdot \frac{(m_e y)^2}{Q^2(1-y)} \right)$$ (2.17)

The factor $(1 + \delta_{RC})$ takes into account QED radiative corrections to the $e-p$ Born cross section. $\delta_{RC}$ is small ($< 5\%$) over most of the phase space. The photon proton cross section has been measured over a wide range of center-of-mass energy and is in the range of 100 $\mu$b to 200 $\mu$b as shown in figure 2.5. This translates into a positron proton cross section with $W > 10$ GeV and for $Q^2$ from the kinematical limit to about 1 GeV$^2$ of 40 $\mu$b.
2.4 Event Kinematics Reconstruction

The ZEUS coordinate system is shown in figure 2.6.

The reconstruction of the kinematic variables for neutral current scattering can easily be done if the outgoing positron scattering angle $\theta$ and energy $E$ are measured. Equations (2.1), (2.4) and (2.5) then read

\[
Q^2 = 2AE(1 + \cos \theta) \tag{2.18}
\]
\[
y = 1 - \frac{E}{2A} (1 - \cos \theta) \tag{2.19}
\]
\[
x = \frac{A}{P} \frac{E(1 + \cos \theta)}{2A - E(1 - \cos \theta)} \tag{2.20}
\]

with $P$ the energy of the incoming proton and $A$ the energy of the incoming positron. From equation equations (2.18) and (2.19) one can easily derive:

\[
Q^2 = \frac{p_{Te}^2}{1 - y} \tag{2.21}
\]
where $p_T$ is the transverse momentum of the scattered positron. Note that the problem is over determined, there are four independent measurement variables, the energy and direction of the scattered lepton and the hadronic system, but only two variables are needed to fully describe the kinematics of the interaction. It is therefore possible to determine the kinematics from any two of the measurement variables [13].

For charged current positron proton scattering the neutrino escapes undetected, so the only detectable particles come from the hadronic final state. In general the struck quark is ejected from the proton and will together with the remnant of the proton hadronize into a final state which typically will consist of a jet proximately in the direction of the struck quark and a jet in the proton remnant direction.

In this case we can determine the kinematic variables from the energy and momenta of the final state particles. Denoting the sum of the four momenta of the final state particles by $p'$ then the momentum transfer vector is given by $q = (p - p')$. The variable $y$ then follows from

$$y = \frac{p \cdot (p - p')}{p \cdot k}$$

$$= \frac{P \sum_h (E_{h} - p_{zh})}{2PA}$$

$$= \frac{\sum_h (E_{h} - p_{zh})}{2A}$$

(2.22)

where the sum runs over all final state particles in the hadronic system. Because of conservation of transverse momentum we can use equation (2.21) to determine $Q^2$ by replacing $p_T^2$ by $p_{Th}^2$:

$$p_{Th}^2 = \left(\sum_h p_{Xh}\right)^2 + \left(\sum_h p_{Yh}\right)^2$$

(2.23)

where again the sum runs over all final state particles in the hadronic system. Finally the value of $x$ can be obtained from equation (2.6). In this way we arrive at the Jacquet-Blondel variables [14] for the reconstruction of the kinematic variables:
\[ y_{jb} = \frac{\sum_i (E_i - P_{zi})}{2 \times E_e} \]  
\[ Q_{jb}^2 = \frac{(\sum_i P_{zi})^2 + (\sum_i P_{yi})^2}{1 - y_{jb}} \]  
\[ x_{jb} = \frac{Q_{jb}^2}{4 \times E_e \times E_p \times y_{jb}} \]

For an ideal detector and if all particles of the hadronic system were measured the above formulas would exactly return the event kinematics variables. In reality however the energy of the particles can only be determined with a finite precision. Moreover the particles lose energy while traversing material before hitting the calorimeter. For this reason the Jacquet-Blondel estimators tend to return a smaller value than the true one for \( Q^2 \) and \( x \).

It is interesting to note that the Jacquet-Blondel estimators are not so sensitive to particles escaping through the beam pipe hole, as these particles usually carry small \( P_t \) and \( (E_{tot} - P_z) \).

It is useful to introduce another variable which is the hadronic angle \( \gamma_{had} \): In the quark parton model \( \gamma_{had} \) gives the polar angle of the struck quark:

\[ \cos \gamma_{had} = \frac{P_t^2 - (E_{tot} - P_z)^2}{P_t^2 + (E_{tot} - P_z)^2} \]