Spectroscopic diagnostics of pulsation in rotating stars
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Multi-periodic line-profile variability in the binary $\beta$ Cep star $\nu$ Centauri

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Abstract

We present time series of high-resolution spectra of the $\beta$ Cephei star $\nu$ Cen, showing multi-periodic variations in the SiIII $\lambda$ 4552 and $\lambda$ 4567 line profiles. After correction for the binary motion, we analyse the time series of spectra by Fourier transforms for each position in the line profile. For selected frequencies we perform a direct fit of multiple sinusoids. We present the distributions of amplitude and phase across the line profile and use these diagnostics diagrams to show that most of the variability can be explained by multiple non-radial pulsations. We use the diagnostics to derive $\ell$-values of individual pulsation modes in $\nu$ Cen.

7.1 Introduction

The number of $\beta$ Cephei stars which are found to vary multi-periodically is rapidly increasing. In many cases the variability found in these stars is successfully explained by models of stellar oscillations. The presence of multiple non-radial pulsation modes, which penetrate the stars interior, offers the means to test and improve theories of stellar structure and evolution. The $\beta$ Cephei stars are therefore excellent candidates for asteroseismological applications.

The $\beta$ Cephei character of $\nu$ Cen was discovered by Rajamohan (1977) and later confirmed by Kubiak & Seggwiss (1982), Ashoka (1985) and Ashoka & Padmini (1992). In the O-C variations of their measured radial velocities around the orbital solution they detect $\beta$ Cephei-type variation with a pulsation period of $P \sim 0.17$ days.
Chapter 7. Multi-periodic line-profile variability in \( \nu \) Cen

In this Chapter we present a time series of spectra of \( \nu \) Cen (HD120307, \( m_V=3.41 \), B2IV). \( \nu \) Cen is a single-line spectroscopic binary (Palmer 1906). Orbital elements were first determined by Wilson (1914) who obtained a circular orbit with \( P=2.62516 \) days. Double emission surrounding the absorption core of H\( \alpha \) was found by Hendry & Bahng (1981). This indicates that \( \nu \) Cen may be classified as a Be star. It is probable that the emission is not an intrinsic property of the star but the result of binary interaction (Cuypers et al. 1989).

Brown & Verschueren (1997) have determined a projected rotational velocity \( V \sin i \) of \( 65 \pm 6 \) km s\(^{-1}\) using an artificially broadened sharp-lined template spectrum matching the measured FWHM of weak metal lines.

De Geus et al. (1989) have analysed Walraven photometry of the members of the Sco OB 2 association, and for \( \nu \) Cen they derived \( \log T_{\text{eff}}=4.35 \), \( \log g=4.02 \), and \( \log L/L_\odot=3.7 \). These values place \( \nu \) Cen inside the domain of the Hertzsprung-Russell diagram where \( \beta \) Cephei stars are predicted to be unstable against non-radial pulsations (Dziembowski & Pamyatnykh 1993). Fundamental stellar parameters where also derived by Remie and Lamers (1982), who combined various observational data with model atmosphere and stellar evolution codes; they find \( R/R_\odot=6.7 \), \( M/M_\odot=13 \), \( d=190 \) pc, \( \log T_{\text{eff}}=4.352 \), \( \log g=3.90 \) and \( \log L/L_\odot=4.02 \).

We find multi-periodic variations in the Si III \( \lambda 4552 \) Å and \( \lambda 4567 \) Å line profiles. We show features moving from blue to red across the line profiles on time-scales of several hours. These variations resemble those of non-radially pulsating stars. To the best of our knowledge this is the first time that spectroscopic time series of line profiles of \( \nu \) Cen have been presented in the literature.

In Section 7.2 we describe the acquisition and reduction of the data. We measure the equivalent width (EW) and radial velocity variations in Sections 7.3 and 7.4; from the latter we determine orbital parameters to remove the Doppler-shift caused by the orbit of the star. In Section 7.6 we apply three different methods to analyse the intensity variations at each position in the line profile: Fourier analysis, multiple-sinusoid fit and single-sinusoid fits. With these three techniques we create diagnostic diagrams that allow to estimate the \( \ell \)-value of the responsible non-radial pulsation modes (Section 7.7). We discuss the implications and restrictions posed by the explanation of non-radial pulsations as the main cause for the line-profile variability in \( \nu \) Cen. We formulate conclusions in Section 7.9.

### 7.2 Observations and reduction

During a period of 14 nights in March 1998, we obtained 93 high-resolution spectra (\( R=65000 \)) at the ESO La Silla CAT telescope equipped with the CES spectrograph. The spectral range, projected by ESO LongCam on about 2600 useful columns of ESO CCD #38 covers the Si III triplet (\( \lambda 4552, 4567, 4574 \) Å). The first part of our observing run was dedicated to two other \( \beta \) Cephei stars, and we filled the available time taking occasional observations of suspected \( \beta \) Cephei stars. We discovered the complex variability in \( \nu \) Cen during the 5\(^{th}\) observing night of our run and we continued taking
7.2. Observations and reduction

regular observations of ν Cen. The total time span of the acquired time-series is 9.26 days. Exposure times were typically 12 minutes and were kept shorter than 20 minutes, resulting in S/N ratios of the reduced spectra typically between 600 and 1200.

Fig. 7.1. One night of data of the λ 4552 line profile. Spectra are offset according to acquisition time. The line profiles are corrected for orbital velocity shifts. The moving bumps visible in the first half of this night seem to disappear gradually. At the second half of the night the number of moving bumps and their amplitude have decreased. Such behavior can be explained by multiple non-radial pulsation modes: modes with similar ℓ values cooperate in the beginning of the night, while at the end of the night the modes cancel revealing or mimicking a mode with much lower ℓ value.

The spectra were reduced using standard packages in IRAF. Wavelength calibration was done with ThAr calibration spectra. We used the CCD overscan region to determine the bias level. CCD pixel-to-pixel variations were removed by flat fielding with dome flats; dome flats also provided a first rough correction for the continuum shape which is heavily affected by vignetting of the light beam. Bad columns were removed by linear interpolation of pixel intensities in adjacent columns. One-dimensional spectra were extracted after subtracting a global fit to background and scattered light. All spectra were shifted to, and acquisition times were transformed to the barycentric frame. We normalized the spectra by fitting a cubic spline with typically 10–15 segments to the regions in between the obvious absorption lines in the
extracted spectra. For our analysis we use only the \( \lambda 4552 \) and \( \lambda 4567 \) lines, since these two lines of the triplet have the highest quality in our data.

### 7.3 Analysis of the equivalent width

We calculated the Equivalent Width (EW) as described in Chapter 2 (Schrijvers et al. 1997). The integration domains were chosen separately for each observing night, based on where the average line profile for that night reaches the continuum. The measured EW variations are shown in Fig. 7.2.

The EW measured from the \( \lambda 4552 \) and \( \lambda 4567 \) lines show variations that seem unrelated. The fact that both lines are of the same triplet means that if the measured EW were real, i.e. not caused by statistical errors and/or errors from the reduction process, the variations in both lines should be correlated. A correlation diagram of the variations measured in both lines is displayed in Fig. 7.3, which shows that the measured EW variations of these lines are not related. We suspect that the EW variability is mainly caused by errors in the normalization of the spectra.

![Fig. 7.2. Variation of the equivalent width of the \( \lambda 4552 \) (open squares) and the \( \lambda 4567 \) line profile (plus symbols). The correlation between the measured EW of these lines is shown in Fig. 7.3](image)

### 7.4 Radial velocity variations: the orbit

We calculated the radial (centroid) velocity of the line profiles as described in Chapter 2. The radial velocity variations were computed for the rest wavelengths 4552.6 Å and 4567.8 Å. The measured radial velocity variations are shown in Fig. 7.4.

The orbit of \( \nu \) Cen around its close companion is clearly visible in the radial velocities measured from the line profiles (see Fig. 7.4). From these we computed the single-line binary orbit solution by means of the Sterne (1941) method which uses a perturbation of a circular orbit solution to fit the orbital elements. The orbit is known to be circular with a period \( P=2.625 \) days (Wilson 1914), with a radial velocity semi-amplitude \( K=20.6 \) km s\(^{-1}\) (Lucy and Sweeney 1971). When we keep this orbital period fixed in our fit we obtain for the semi-amplitude \( K=22.4\pm0.4 \) km s\(^{-1}\). A fit with the period as free parameter gives \( P=2.622\pm0.018 \) days and the same semi-amplitude \( K \).
7.5 Variations of the velocity moments

As before. For both fits we find an almost circular orbit (e=0.015±0.025). Given the low eccentricity we also fitted a single sinusoid to the radial velocity data.

We tried all three fits to correct for the orbital Doppler shifts but found no markable differences in the results of our analysis of the pulsation modes. For this Chapter we adopted the solution from the $P=2.622$ fit (leading to the epoch of maximum radial velocity $T_0=2450012.095±5.666$ days). The residual radial velocities after correcting for the orbit are displayed in the top panel of Fig. 7.4.

In the rest of this Chapter we will use the orbit-corrected spectra to analyse the intrinsic variability of the primary star in $\nu$ Cen. The line profiles show a behavior that is typical for a star exhibiting multiple non-radial pulsation modes: a complex pattern of bumps that move from the blue to the red side across the line profiles, on timescales of several hours. One night of line-profiles is shown in Fig. 7.1. In some nights the variations seem to have disappeared while in other nights we find variations that are much larger than in this example.

7.5 Variations of the velocity moments

From the orbit-corrected line profiles we calculate the first few velocity moments, defined in Chapter 2. We compute the Fourier transform of the velocity moments and the EW. The results for the $\lambda 4552$ line are displayed in Fig. 7.5. The rapid radial velocity variations are dominated by the frequency of $2.34 \text{ d}^{-1}$, which is also present in the
Chapter 7. Multi-periodic line-profile variability in \( \nu \) Cen

Fig. 7.4. Bottom: Radial velocity variations for \( \lambda \) 4552. Top: O–C diagram for \( \lambda \) 4552, of the residual radial velocities after removal of the binary-orbit solution higher velocity moments.

Rajamohan (1977) found \( \beta \) Cephei-type variation in his O–C diagrams. He detected a period of \( P \approx 0.17 \) days, which was confirmed by Kubiak & Seggewiss (1982), Ashoka (1985) and Ashoka & Padmini (1992). Our radial velocity measurements do not confirm this period: we checked if our O–C variations (top panel of Fig. 7.4) show variations with a similar period, but we could not confirm a period of \( P \approx 0.17 \) days from our data. Actually, the region in frequency space around \( P \approx 0.17 \) shows hardly any variability (see Fig. 7.5). An explanation for this discrepancy could be the fact that the periods in the above 4 publications were determined from sparse data that consisted of only a few tens of spectra, taken over several years (Ashoka & Padmini 1992). Note that the peak-to-peak changes in our O–C residuals are less than 4 \( \text{km s}^{-1} \), which is comparable to the projected slit width on the CCD.

7.5.1 Fourier analysis of the variability across the line profile

To find the frequencies of the pulsations that presumably cause the line-profile variations we analysed the time series with the method described by Gies & Kullavanijaya (1988) and Chapter 2. The method is based on the Doppler imaging principle (Vogt et al. 1987), in which one assumes a mapping of photospheric features (e.g. local velocity, brightness or EW variations) onto line profiles that are Doppler broadened by the rotation of the star. From a Fourier transform applied to each wavelength bin of a time series of observed spectra one obtains the power of variability as a function of frequency (periodogram), for all positions in the line profile. Additionally, the Fourier analysis provides information about the phase change of the periodic variations across the line profile. Using the power and phase information, a number of pulsation parameters can be derived.
Fig. 7.5. Power spectra from the Fourier analysis of the EW and the first 3 velocity moments, after correction of the spectra for the orbital Doppler shifts of \( \nu \) Cen
The shortest timespan between subsequent exposures corresponds to a Nyquist frequency of \( \sim 50 \, \text{d}^{-1} \). The length of the dataset is such that the HWHM of the main power peak in the window function is 0.047 \( \text{d}^{-1} \). For each wavelength bin in the line profiles we Fourier-analysed the variable signal: we computed the Fourier components for frequencies between 0 and 50 \( \text{d}^{-1} \) with a frequency spacing of 0.01 \( \text{d}^{-1} \). We CLEANed the resulting Fourier spectrum of each wavelength bin in order to remove the temporal window function, which is due to incomplete sampling of the variational signal. We used CLEAN parameters \( N_{\text{iterations}}=400 \) and a gain of 0.2 (Roberts et al. 1987). For the SiIII \( \lambda 4552 \) line profile the resulting two-dimensional periodogram is displayed in Fig. 7.6.

The result of our Fourier analysis is again not in agreement with Rajamohan (1977), Kubiak & Seggwiss (1982), Ashoka (1985) and Ashoka & Padmini (1992), who find a single period of \( \sim 0.17 \) days in their radial velocity data. We do not find a dominant peak for periods between 0.135 and 0.215 days.

7.5.2 Towards the intrinsic frequency spectrum of \( \nu \) Cen

Fig. 7.7 shows the one-dimensional periodograms obtained by summing the variational amplitudes of the two-dimensional periodograms over the wavelength range which covers the line profiles. We find that the frequency spectra obtained from the SiIII \( \lambda 4567 \) and \( \lambda 4552 \) line profiles are very similar. The maxima detected in both lines differ less than 0.02 \( \text{d}^{-1} \). We investigated the origin of the peaks in the periodograms.

One-day aliasing

Some of the power has obviously leaked to one-day aliases, which shows that the CLEAN algorithm was not able to fully correct for the window function. The peaks recognized as weak one-day aliases of other (stronger) frequencies are indicated by the thin vertical lines in Fig. 7.7. There are a few remaining ambiguities in the recognition of false peaks caused by aliasing, such as the pairs of peaks at frequencies 4.16 and 5.13 \( \text{d}^{-1} \), and at 2.35 and 3.32 \( \text{d}^{-1} \). For both these two pairs, one peak is most probably a one-day alias of the other.

Neglecting the weak one-day aliases, the remaining frequencies are 2.35, 3.32, 4.16, 4.66, 5.13, 6.42, and 7.95 \( \text{d}^{-1} \). Peaks at frequencies below 2.0 \( \text{d}^{-1} \) are discarded. Based on the periodograms alone, we can not decide whether all peaks at these frequencies are real or that some are a result of data reduction, noise and/or insufficient data sampling. We elaborate on these candidate frequencies in Section 7.5.2.

Iterative prewhitening

We iteratively prewhitened the time series using a single frequency for each iteration. We tried several series with different combinations of frequencies. We find that four iterations are sufficient to remove the seven most prominent peaks selected from the original summed periodogram. Different combinations of four frequencies lead
7.5. Variations of the velocity moments

Fig. 7.6. Top: CLEANed Fourier analysis. For every wavelength bin (horizontal axis) a Fourier analysis of all the spectra is carried out. The power resulting from the Fourier analysis is plotted as a grey-value as a function of temporal frequency (vertical axis). Grey-scale cuts: 0–2·10⁻⁷. Horizontal bars indicate the frequencies used in the further analysis. Bottom: Mean of all spectra. Left: Corresponding window function, which is due to incomplete temporal sampling of the variational signal.
Chapter 7. Multi-periodic line-profile variability in ν Cen

Fig. 7.7. Summed Fourier spectrum. The power (i.e. the square of half the amplitude) in the two-dimensional periodograms of the λ 4552 (solid line) and λ 4567 (dotted line) lines has been summed over the line profile. The thick vertical lines indicate the frequencies used in the further analysis while the peaks identified as weak one-day aliases are indicated by the thin vertical lines.

to similar results. The frequencies 7.95 and 6.42 d\(^{-1}\) are required, plus either 4.16 or 5.13 d\(^{-1}\), and 2.35 or 3.32 d\(^{-1}\). We remove most power from the Fourier transform if we prewhiten with 5.13, 7.95, 2.35 and 6.42 d\(^{-1}\), respectively (see Fig. 7.8).

We suspect that one pulsation mode is responsible for the two peaks at 2.35 and 3.32 d\(^{-1}\), and another mode causes both peaks at 4.16 and 5.13 d\(^{-1}\). Two other pulsation modes give rise to the peaks at 7.95 and 6.42 d\(^{-1}\). The peak at 4.66 d\(^{-1}\) is automatically removed in the prewhitening process, and is most likely a by-product of the other frequencies. Most power at the weak one-day aliases, identified as such in Section 7.5.2, is removed in the four iterations, as expected.

We conclude that the intrinsic frequency spectrum in ν Cen consists of only 4 dominant frequencies. Although the mentioned combination gives the best results, one of the other three mentioned combinations might be the actual frequency spectrum as well. More data is needed to identify the true intrinsic frequency spectrum of ν Cen.

For the rest of our analysis we have created diagnostic diagrams for each of the seven selected peaks from the summed periodogram. We keep in mind that only four peaks are real, and that the other three are artefacts.

7.6 IPS amplitude and phase diagrams

In Chapter 2 we defined a diagnostic tool for the analysis of non-radial pulsations in rotating stars. The method is based on the IPS (Intensity Period Search) diagnostic which consists of two diagrams, the amplitude and the phase diagram, giving a full representation of the line-profile variations caused by each of the individual pulsation modes.

The two diagrams are very different and are each related to distinct properties of the pulsation mode. The amplitude diagram contains information on mode parameters like \(k\), \(i\), \(V_{\text{max}}\), \(\Omega/\omega^{(0)}\), \(W_{\text{r}}\), \((\delta T/T)_{\text{max}}\), \(\alpha_{\text{W}}\), and \(\chi^{(n)}\) (see Chapter 2 and 5 for the meaning of these symbols). The phase diagram is representative for the pulsation in-
Fig. 7.8. Summed Fourier spectra after iteratively prewhitening with the frequencies of the strongest peaks. The isolated plot at the top shows the summed Fourier transform which is also shown in Fig. 7.7. The four lower plots show (from top to bottom) the summed Fourier transforms after subsequently prewhitening with 5.13, 7.95, 2.35 and 6.42 $d^{-1}$, respectively. These frequencies are indicated by the dotted vertical lines. The thick and thin vertical lines have the same meaning as in Fig. 7.7.
Chapter 7. **Multi-periodic line-profile variability in \( \nu \) Cen**

dex \( \ell \) (and \(|m|\) if a harmonic is measured also). In Chapter 3 (Telting & Schrijvers 1997) we established a relation between the measured phase diagram and the \( \ell \)-value of the responsible pulsation mode.

Here, we use two different techniques to calculate IPS diagnostics (Figs. 7.11 and 7.12) for each of the suspected pulsation frequencies. We will use these in Section 7.7 to measure the \( \ell \)-values of these pulsation modes.

### 7.6.1 IPS diagnostics: description of the diagrams

Each diagnostic (see Figs. 7.11 and 7.12) consists of two diagrams. The top diagram displays the amplitude distribution across the line profile. The bottom diagram shows the phase distribution across the line profile. We present the amplitudes in units of \( d_{\text{mean}} \), which is the maximum absorption depth of the mean line-profile, to allow a better comparison with theoretical models. For the \( \lambda \) 4552 line we find \( d_{\text{mean}} = 0.08373 \) in continuum units.

For the region in which we find significant amplitude the diagrams are drawn with a solid line. Outside this region a dotted line is used. The positions of the phase diagram at which we measure \( \Delta \Psi \), are indicated by a square symbol.

The IPS diagrams resulting from our single-sinusoid fits are drawn as a heavy line. The diagrams from the multi-sinusoid approach are drawn with a thinner line. For the IPS diagnostic from the CLEANed Fourier transform we used the thinnest line.

The wavelengths corresponding to \( -V_c \sin i \) and \( V_c \sin i \) are indicated by the marks on the bottom axis of the phase diagrams, were we used \( V_c \sin i = 65 \pm 6 \text{ km s}^{-1} \) from Brown & Verschueren (1997).

### 7.6.2 IPS diagrams from the Fourier transform

We used the Fourier transforms from Section 7.5.1 to investigate the amplitude and phase behavior across the line profile. For each of the 7 selected frequencies we plot the amplitudes and phases at each position in the line profile, leading to the IPS diagrams represented by the thinnest lines in Figs. 7.11 and 7.12.

Most of the IPS amplitude diagrams obtained from the Fourier transform appear non-continuous and some of the IPS phase diagrams show discrete jumps. Some of these diagrams are useless for mode-identification. We therefore adopted a second method to create the diagnostics.

### 7.6.3 IPS diagrams from the direct fit approach

We created IPS diagnostics by fitting sinusoids with fixed frequencies to the intensity variation at each position in the line profiles. The input frequencies were taken from the results of the Fourier analysis. The main advantage of this approach, when compared to creating IPS diagnostics from the CLEANed Fourier transform, is that it does not allow one-day aliases. At certain wavelengths the Fourier transform assigns all power to the one-day alias instead of the main peak, which leads to zero amplitudes and an
irregular phase diagram at that position. This problem does not arise when creating IPS diagnostics by fitting sinusoids with fixed frequencies. We performed fits of the function

$$I(\lambda, t) = \sum_{i=1}^{N} A_i(\lambda) \sin(f_i t + \Psi_i(\lambda))$$  \hspace{1cm} (7.1)$$

to the intensity variations, where $A_i(\lambda)$ and $\Psi_i(\lambda)$ are the free parameters in the fit. For $f_i$ we used different combinations of the 7 selected frequencies from Section 7.5.1. The IPS diagnostics are created by plotting $A_i(\lambda)$ and $\Psi_i(\lambda)$ across the line profile.

For the frequencies below $2 \text{d}^{-1}$ we were not able to obtain IPS diagrams that are in some way similar to those generated with theoretical models. We cannot decide whether these frequencies are caused by pulsations or by other observational effects or stellar phenomena. From this point on we focus on the frequencies above $2 \text{d}^{-1}$.

For each of the 7 selected frequencies we created IPS diagrams in different ways. First we applied a multiple-sinusoid fit (Eq. 7.1) for the combination of 4 frequencies mentioned in Sect. 7.5.2, leading to an amplitude and phase diagram for each of the 4 frequencies (indicated by the lines with medium thickness in Figs. 7.11 and 7.12). A second set of diagnostics was created by fitting a single-sinusoid to the variability for each of the 7 individual frequencies (thick lines in Figs. 7.11 and 7.12).

### 7.6.4 Reconstruction of the time-series

We used Eq. 7.1 to extract IPS amplitude and phase diagrams from the line profiles. We also used the IPS diagrams from the multiple-sinusoid fit approach to reconstruct the data by means of Equation 7.1. The reconstruction, which is presented in Fig. 7.10, shows that 4 IPS diagrams are a reasonable representation of the most obvious variability. Note that including more frequencies in the multiple-sinusoid fit does not noticeably improve the reconstruction.

### 7.7 Mode identifications

We used three different methods to create the IPS diagnostics presented in Figs. 7.11 and 7.12. We find considerable differences between the diagnostics obtained with each of the three methods. Because of one-day aliasing, the amplitude diagrams created from the Fourier transform drop to zero at positions within the line profile, which is reflected by the corresponding phase diagrams behaving irregularly at these positions. We attribute this to an insufficient sampling by our 93 spectra (see Section 7.8.2). The amplitude diagrams from the sinusoid fitting methods are not affected by one-day aliasing effects, which results in more realistic amplitude diagrams and more monotonous phase diagrams. We find that the phase diagrams from the different methods are consistent in the ranges in the line profile where the corresponding amplitude diagrams are non-zero.
Chapter 7. Multi-periodic line-profile variability in ν Cen

Fig. 7.9. Last three nights of the observed time-series of the λ 4552 Å line profile. 
**Bottom:** gray-scale representation of residual spectra (mean subtracted). Intensities less than average are indicated black; bright regions in the profile are indicated by lighter shades. 
**Middle:** Overplot of all residuals. This panel shows that most of the variability is present at the wings of the line-profile. 
**Top:** Mean of all line-profiles
7.7. Mode identifications

Fig. 7.10. Reconstruction of data from the IPS diagrams resulting from our multiple-sinusoid approach. We used the IPS diagrams at 2.35, 5.13, 6.42 and 7.95 d$^{-1}$ to reconstruct the data covering the last three nights. This figure illustrates that the data is well represented by 4 IPS diagrams. Including more than 4 frequencies in the multi-sinusoid fit does not lead to a noticeable improvement of the reconstruction.
Note that the amplitudes from the Fourier transform are systematically lower than those from the fit methods. Apart from power that has leaked to one-day aliases, this is caused by the CLEAN algorithm which not only removes the window function from the periodogram but also some of the power at the real frequencies.

Apart from the difference mentioned in Section 7.6, we recognize another important difference between amplitude and phase diagrams caused by non-radial pulsation. Amplitude diagrams are much more easily affected by problems like insufficient data sampling and imperfections of the data reduction. In contrast, the phase diagrams are rather robust.

Regions in the line profile at which zero amplitude is found do not always hamper a correct retrieval of $\ell$ values for these modes. If the amplitude distribution is interrupted over only a short wavelength interval, it should be obvious whether the phase is connected correctly over the power-lacking region. A failure would result in a $2\pi$ jump that is clearly visible. The phase diagram should be monotonic over these regions.

Amplitude diagrams of the line-profile variations caused by non-radial pulsation modes are expected to extend over a region that is a fraction larger than $2 \times V_c \sin i$. If amplitude is missing at one of the outer wings of the line-profile, a straightforward identification of the pulsation mode is less certain. For such cases, the phase diagram provides a lower limit for $\Delta \Psi$ and, consequently, a lower limit for the $\ell$ value of the mode. In this Chapter we find such a phase diagram at $6.42 \, \text{d}^{-1}$.

### 7.7.1 Determination of $\ell$ values

The IPS diagnostics derived for the 7 frequencies from Tab. 7.1 are displayed in Figs. 7.11 and 7.12. Non-radial pulsation modes would be a plausible explanation for the variability displayed by these diagrams. We measure the maximum blue-to-red phase difference $\Delta \Psi$ from the phase diagrams, which we use to determine an $\ell$ value for each frequency. We measure $\Delta \Psi$ as the maximum phase difference found in the region of the line profile where we find significant amplitudes. See Figure 3.2 for instructive examples on the measurement of $\Delta \Psi$ from different types of phase diagrams. We determined $\Delta \Psi$ for the IPS phase diagrams obtained by all three methods, i.e. Fourier transform, multiple-sinusoid fit and single-sinusoid fits. These values for $\Delta \Psi$ are listed in Table 7.1, provided that a useful phase diagram was obtained.

To estimate the pulsation index $\ell$ from the maximum blue-to-red phase difference $\Delta \Psi$ we use the relations given by Equation 3.8 and Table 3.2. For the present analysis we chose to use the entry for modes with $\ell - |m| < 6$, for which find an 84% certainty of retrieving the correct value of $\ell \pm 1$ (see Table 3.2). This gives the relation

$$\ell = 0.015 + 1.109 |\Delta \Psi|/\pi . \quad (7.2)$$

In the following sections our determination of the $\ell$ values is addressed for each of the selected frequencies.
7.7. Mode identifications

In some of the phase diagrams amplitude is missing at the line wings, i.e. the amplitude diagrams do not extend over a region larger than $2 \times V_e \sin i$. For these cases we obtain lower limits for $\Delta \Psi$, from which we derive a lower limit for $\ell$. We also estimate the upper limit for $\ell$ by assuming that the phase diagram is more of less symmetric.

Table 7.1 lists the $\ell$-values derived with each of the 3 methods and for all selected frequencies, as well as our final $\ell$-determinations.

1) Line-profile variations with $f=4.16$ and $f=5.13$ d$^{-1}$

At both these frequencies we find amplitude and phase diagrams which extend over the full wavelength domain between $-V_e \sin i$ and $+V_e \sin i$.

As already mentioned in Section 7.5.1, the power found at either 4.16 or 5.13 d$^{-1}$ is a one-day alias of the other. We expect that only one pulsation mode is responsible for the detection of these two frequencies. We will nevertheless derive $\ell$-values for both these frequencies. From $\Delta \Psi^{SF}$ we estimate $\ell=6$ for the mode that is responsible for $f=4.16$ and $f=5.13$.

2) Line-profile variation with $f=4.66$ d$^{-1}$

The diagnostics calculated at 4.66 d$^{-1}$ show a non-monotonic phase diagram which is inconsistent with those predicted theoretically, i.e. we do not find much amplitude in the red half of the line profile. This frequency might be the result of beating between 3.32 and 7.95 d$^{-1}$. It turned out in Sect. 7.5.2 that the variability with this frequency is removed by prewhitening with other frequencies. We regard it unlikely that 4.66 d$^{-1}$ is one of the intrinsic frequencies in $\nu$ Cen.

3) Line-profile variation with $f=7.95$ d$^{-1}$

For the diagnostics from the CLEANed Fourier transform, amplitude seems missing at the red wing of the line profile, and the phase diagram behaves erratically in the red half of the line profile. The diagnostics from the single-sinusoid and multiple-sinusoid do not have this problem. We find $\ell=10$ for this frequency.

4) Line-profile variation with $f=6.42$ d$^{-1}$

At this frequency it appears as if amplitude is missing close to the line center and at the blue wing of the line profile. Around the line center there seems to be no problem with the phase diagram, but around $-V_e \sin i$ part of the phase diagram is missing (see Sect. 7.7). The phase diagrams at 6.42 d$^{-1}$ give rise to a lower limit for $\ell$.

We estimate that our measured value for $\Delta \Psi^{SF}$ underestimates the true blue-to-red phase difference by not more than $\pi$. This yields a pulsation mode with $\ell=7$ or $\ell=8$ for this frequency.

5) Line-profile variation with $f=2.35$ and $f=3.32$ d$^{-1}$

The IPS diagrams obtained for 2.35 and 3.32 d$^{-1}$ are far from ideal. Amplitude is
Fig. 7.11. IPS diagnostics for the variations found in the λ 4552 line-profile at the frequencies 5.13, 4.16 and 4.66 d⁻¹. See Sect. 7.6.1 for a general description of the diagrams. Heavy line: single-sinusoid fit, thinner line: multiple-sinusoid fit, thinnest line: CLEANed Fourier transform.
7.8. Multiple pulsation modes in \( \nu \) Cen

missing in the red half of the line profile and the corresponding region of the phase diagrams show irregular behavior, instead of a smooth and gradual change as expected for non-radial pulsations. The \( \ell \)-value that we derive for this presumed mode is tentative.

We derive \( \ell < 6 \) from the phase diagrams at 2.35 and 3.32 d\(^{-1}\). Better data are needed to decide whether this variation is caused by non-radial pulsation.

Note that 3.32 d\(^{-1}\) is, within the half width of the main peak of the window function (0.047 d\(^{-1}\)), the beat frequency of 4.66 and 7.95 d\(^{-1}\). This might indicate that 3.32 and 7.95 d\(^{-1}\) are true frequencies and that 2.35 d\(^{-1}\) is a one-day alias of 3.32 d\(^{-1}\).

![Fig. 7.12. Similar to Fig. 7.11, but for the frequencies 7.95, 6.42, 2.35 and 3.32 d\(^{-1}\). See Sect. 7.6.1 for a general description of the diagrams](image)

7.8 Multiple pulsation modes in \( \nu \) Cen

In the previous section we analysed the amplitude and phase characteristics of the line-profile variability in \( \nu \) Cen. We assumed that the variability is caused by multiple non-radial pulsation modes. Using the method of Chapter 3 we assigned an \( \ell \)-value to each of the presumed pulsation modes. In this section we address whether the assumption of non-radial pulsations is supported by the analysis. We discuss some of the implications.
Chapter 7. Multi-periodic line-profile variability in \( \nu \) Cen

Table 7.1. The frequencies found in the summed Fourier transforms of the SiIII \( \lambda 4552 \) line profile. We find a very similar frequency spectrum for the SiIII \( \lambda 4567 \) line profile, where the maxima deviate less than \( 0.02 \, \text{d}^{-1} \) from those found for the line at \( \lambda 4552 \, \text{Å} \). Only the frequencies which we used for our final analysis and mode-identification are listed here. The frequencies are ordered according to their power in the summed Fourier spectra (see Fig. 7.7). The first column lists the frequency in \( \text{d}^{-1} \). The second gives the period in hours. The maximum blue-to-red phase differences \( \Delta \Psi \), measured from the IPS diagnostics calculated from the Fourier transform (label FT), the multiple-sinusoid fit (label MF) and the single-sinusoid fits (label SF), are in columns 3, 4 and 5 (in units of \( \pi \) radians). These values are read from the phase diagrams in a straightforward manner, i.e. we applied strict rules without taking into account the effects that might hamper a correct identification. Columns 6, 7 and 8 list the fractional \( \ell \) values calculated from columns 3, 4 and 5 by means of Eq. 7.2. The two rightmost columns list the values for \( \Delta \Psi \) (in \( \pi \) radians) and \( \ell \) from our final interpretation of the diagnostics; these were the results of careful consideration of all amplitude and phase diagrams. For some of the diagrams we used two additional criteria (see text) in order to be able to use these diagrams for identification.

<table>
<thead>
<tr>
<th>( f_{\text{obs}} )</th>
<th>( P_{\text{obs}} )</th>
<th>( \Delta \Psi_{i}^{\text{FT}} )</th>
<th>( \Delta \Psi_{i}^{\text{MF}} )</th>
<th>( \Delta \Psi_{i}^{\text{SF}} )</th>
<th>( \ell_{\text{FT}} )</th>
<th>( \ell_{\text{MF}} )</th>
<th>( \ell_{\text{SF}} )</th>
<th>( \Delta \Psi )</th>
<th>( \ell )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.35</td>
<td>10.21</td>
<td>4.07</td>
<td>5.21</td>
<td>5.20</td>
<td>-</td>
<td>5.8</td>
<td>5.8</td>
<td>&lt;5.2</td>
<td>&lt;6</td>
</tr>
<tr>
<td>3.32</td>
<td>7.23</td>
<td>3.08</td>
<td>-</td>
<td>7.22</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4.16</td>
<td>5.77</td>
<td>5.89</td>
<td>-</td>
<td>5.67</td>
<td>6.6</td>
<td>-</td>
<td>6.3</td>
<td>5.7</td>
<td>6</td>
</tr>
<tr>
<td>4.66</td>
<td>5.15</td>
<td>6.68</td>
<td>-</td>
<td>7.85</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5.13</td>
<td>4.68</td>
<td>5.67</td>
<td>5.46</td>
<td>5.49</td>
<td>6.3</td>
<td>6.1</td>
<td>6.1</td>
<td>5.5</td>
<td>6</td>
</tr>
<tr>
<td>6.42</td>
<td>3.74</td>
<td>5.84</td>
<td>5.52</td>
<td>5.80</td>
<td>6.5</td>
<td>6.1</td>
<td>6.4</td>
<td>&gt;6.0</td>
<td>&gt;7</td>
</tr>
<tr>
<td>7.95</td>
<td>3.02</td>
<td>3.97</td>
<td>9.09</td>
<td>9.15</td>
<td>-</td>
<td>10.1</td>
<td>10.2</td>
<td>9.1</td>
<td>10</td>
</tr>
</tbody>
</table>

To investigate the parameter space of the pulsation modes we consider three scenarios regarding the rotation rate of \( \nu \) Cen, relative to the orbital frequency. For the first scenario, \( \nu \) Cen has been tidally synchronized by the companion, i.e. the rotation period is equal to the orbital period. Based on published values for some of the stellar parameters (\( R=6.7 \, R_{\odot} \), \( V_{\sin i}=65 \, \text{km s}^{-1} \)) we find an inclination \( i=30.22^\circ \), rotational velocity \( V_{\zeta}=129 \, \text{km s}^{-1} \), and a rotation rate \( \Omega=0.38 \, \text{d}^{-1} \) for the case of synchronous rotation. For the second scenario we take a sub-synchronous rotation by assuming \( i=60^\circ \), which gives \( V_{\zeta}=75 \, \text{km s}^{-1} \) and \( \Omega=0.22 \, \text{d}^{-1} \). Super-synchronous rotation is accounted for by the third scenario of \( i=20^\circ \), which implies \( V_{\zeta}=190 \, \text{km s}^{-1} \) and \( \Omega=0.56 \, \text{d}^{-1} \). For these three considered cases we use \( \omega=f_{\text{obs}}-\left[m\right]\Omega \) to calculate lower limits for the intrinsic pulsation frequencies. The lower limit \( \omega_{0} \) is the frequency for prograde sectoral modes \( (m=-\ell) \); tesseral modes imply frequencies higher than \( \omega_{0} \). We use the lower limit of the frequency to calculate an upper limit for \( k \) (using \( k=\frac{GM}{R_{\odot}c^{2}} \) and \( M=13M_{\odot} \) from Debye & Lamers 1982). The values for the lower limits \( \omega_{0} \) and upper limits \( k_{\text{up}} \), calculated for the three scenarios, are listed in Table 7.2.
7.8. Multiple pulsation modes in $\nu$ Cen

7.8.1 Frequency considerations

It is generally accepted that non-radial pulsations are a common feature of $\beta$ Cephei stars. The excitation mechanism for pulsations in these stars is firmly established to be the $\kappa$-mechanism (Moskalik & Dziembowski 1992, Gautschy & Saio 1993, Dziembowski & Pamyatnykh 1993).

We investigate if the observed frequencies agree with the theoretically predicted frequencies in the corotating frame. Figures 5 and 6 of Dziembowski & Pamyatnykh (1993) show that pulsation modes with $\ell<6$ can only be $p$-modes, whereas modes with $\ell \geq 6$ can be both $p$-type and $g$-type oscillation (for their $\beta$ Cephei model with $M=12 M_\odot$). The predicted frequency domains for their $\beta$ Cephei star model are $\omega<2.6 \, d^{-1}$ for the $g$ modes and $\omega>5.6 \, d^{-1}$ for the $p$-modes. For all our modes we derived $\ell \geq 6$ which suggest that both $p$ and $g$ mode instability can be expected in $\nu$ Cen. For modes for which we have two possible frequencies, we consider both possibilities.

Of course, since the frequencies listed in Table 7.2 are lower limits, all modes could be $p$ modes if we allow only tesseral modes. Another possibility is that all frequencies in $\nu$ Cen are $g$-modes. Of our three scenarios in Table 7.2, only the super-synchronous rotation ($i=20^\circ$) leads to frequencies that are all in the theoretically expected range for $g$-modes.

Note the commensurability of the frequencies 5.13 and 6.42 $d^{-1}$ which have a ratio of 4:5. This can be a coincidence but if the $|m|$ values of the two responsible pulsation modes have the same ratio this would mean that the two modes have identical super-periods. It could be advocated that these two frequencies are harmonics of the rotation frequency. We checked for an equivalent period ratio that would match more than two commensurate frequencies. We find that the ratios 26:29:32:40 matches 4 of the frequencies very well (4.168:4.649:5.130:6.413 $d^{-1}$). However, we regard it as unlikely that such a ratio would be a real rotationally induced phenomenon. The implied rotation rate would be $\Omega=0.16 \, d^{-1}$, leading to $R>8R_\odot$.

7.8.2 The $k$-values of the modes

It is less easy to bring the derived diagnostic diagrams in line with the pulsation interpretation. Contrary to what is theoretically expected, the amplitude diagrams appear discontinuous. However, with 93 spectra, this could well be caused by a sampling that is insufficient for the multiperiodic variations. We folded the acquisition times with the inverse of each of the 6 frequencies to investigate the sampling of each of the signals. We find that the phases corresponding to the acquisition times are well dispersed. However, the beat frequencies of neighboring frequencies are in a domain that is not properly sampled by the data set. This could explain the decreased or missing amplitude in some areas of the amplitude diagrams.

The observed amplitudes are obviously larger near the line wings than around the line center. This is clearly visible in the overplot of residual (mean subtracted) profiles in Fig. 7.9. Standard models of non-radial pulsations can only explain such behavior if the modeled ratio of the horizontal to the vertical motions, $k$, is substantial (say $k>0.3$).
Table 7.2. Calculated values of the lower limit for the intrinsic pulsation frequency $\omega$ and the upper limit for $k$, for 3 considered cases. For the 3 scenarios of synchronous, sub-synchronous and super-synchronous rotation we calculated the lower limit for the pulsation frequency $\omega_{lo}$ and the upper limit $k_u$. The case marked with * indicates the exception of a retrograde mode for $|m|=\ell$. The values depend on the observed frequencies and $\ell$-values (see Table 7.1) and on estimates for the mass and radius of \(\nu\) Cen ($M=13M_\odot$, $R=6.7R_\odot$, $V_\sin i=65$ km s$^{-1}$)

| $f_i$ | $|m|$ | super-synchr $i=20^\circ$ | synchronous $i=30.22^\circ$ | sub-synchr. $i=60^\circ$ |
|------|------|-----------------|-----------------|---------------|
| 2.35 | $\leq6$ | 3.3 0.99*       | 842 0.062       | 3.07 1.02     |
| 4.16 | $\leq6$ | 4.8 0.82        | 0.92 1.87       | 0.40 2.83     |
| 5.13 | $\leq6$ | 1.0 1.79        | 0.40 2.84       | 0.22 3.80     |
| 6.42 | $\leq8$ | 0.83 1.96       | 0.28 3.37       | 0.15 4.65     |
| 7.95 | $\leq10$ | 0.57 2.38       | 0.19 4.14       | 0.10 5.74     |

Although the amplitude diagrams deviate strongly from the ideal cases, we have the impression that some of the amplitude diagrams agree more with high $k$-values than others. The amplitude diagrams for the presumed pulsation modes at 7.95, 6.42 and 5.13 d$^{-1}$ appear to have more amplitude around the line center than those at 2.35, 3.32, 4.16 and 4.66 d$^{-1}$. As evidenced by Table 7.2 this would agree perfectly with the concept of pulsations: the higher $k$-values expected for the modes with the lower frequencies seem to be sustained by the amplitude diagrams.

The quality of our amplitude diagrams is not high enough to really estimate the $k$-values of the modes, which would lead to an estimate of the stars rotation rate $\Omega$. However, the low $k$-values expected for the case of sub-synchronous rotation seem less compatible than the other two cases (see Tab. 7.2).

### 7.8.3 Harmonic variability due to high pulsational velocity amplitudes

We did not find a harmonic (i.e. $2f_{\text{obs}}$) for any of the investigated frequencies. Harmonics are expected to show up for large line-profile variations if the contribution of velocity fields to the variations, as opposed to the effects of local brightness and EW variations, is large enough (Schrijvers & Telting 1999). The fact that we do not find harmonics does however not make non-radial pulsations a less probable explanation for the observed variability. In fact, for the observed line-profile amplitudes (less than 6% of the absorption depth of the mean line-profile $d_{\text{mean}}$) it would be very hard to detect such harmonics, even if the line-profile variations are solely caused by surface velocity fields.

A consequence of the harmonics remaining undetected is that we only have an
upper limit $|m| \leq \ell$, rather than retrieving $|m|$ from the phase diagram of the harmonic.

7.8.4 Tidal interactions

Waelkens & Rufener (1983) suggested that tidal interactions have a damping effect on $\beta$ Cephei type pulsations, which would explain the observed lower limit of the orbital periods of the binary $\beta$ Cephei stars (at that time the $\beta$ Cephei star with the shortest orbital period was $\alpha$ Vir, with $P=4.015$. This is about three times the theoretical limit, which corresponds to a contact configuration). In an attempt to find $\beta$ Cephei type variability in stars with orbital periods shorter than that of $\alpha$ Vir, they observed 16 short-period binaries that are inside or close to the $\beta$ Cephei star instability strip, but found no new cases with $P<4$. The present work on $\nu$ Cen shows, however, that $\beta$ Cephei stars with shorter orbital periods do exist. Another such case is presented by Telting et al. (1999) for the star $\Psi^2$ Ori, which has an orbital period $P=2.53$. To the best of our knowledge, these two $\beta$ Cephei stars have the shortest orbital period presently known.

7.8.5 Spectroscopic versus photometric variations

Cuypers et al. (1989) found photometric variations in $\nu$ Cen showing the orbital period known from spectroscopy. They looked for the possibility of other frequencies, but their careful investigations did not reveal any sign of variations at other frequencies, with amplitudes exceeding 2 millimags. The result of the present work makes clear why other frequencies were never found in photometry: it turns out that $\nu$ Cen exhibits only high-order ($\ell \geq 6$) pulsation modes, and light variations related to such high $\ell$ values are simply too small to be detected.

This emphasizes that we are again in an era of finding new $\beta$ Cephei variables spectroscopically, rather than photometrically. Of all nearby $\beta$ Cephei stars, most of the photometric variables are already known whereas many line-profile variables might still be undiscovered.

7.9 Conclusions

The main conclusions of this Chapter can be summarized as follows.

We observed complex line-profile variability in $\nu$ Cen.

We Fourier analysed the intensity variations across the line profile and found variational power at a number of different frequencies. Many of these could be identified as one-day aliases of other frequencies.

We find four non-radial pulsation modes in $\nu$ Cen. We derive $\ell$-values between $\ell=6$ and $\ell=10$.

We find that the pulsations in $\nu$ Cen are most likely caused by $g$-mode instabilities, which implies that the star is rotating super-synchronously. Synchronous rotation may
also be acceptable, which implies both $g$- and $p$-modes, as the corresponding $k$ values are compatible with the observed amplitude distributions. We can not exclude that all modes are low-order $p$-modes, but this would require most modes to be tesseral.

The frequency spectrum of $\nu$ Cen remains partly unresolved. We found that only 4 frequencies are sufficient to reproduce the observed variability, but we were unable to firmly identify the true frequencies. For this, more extensive observations of $\nu$ Cen are needed, preferably from continuous multi-site observations since this prevents the appearance of one-day aliases in the frequency spectrum.

References

Breger, M., 1995, Baltic Astronomy v.4, p.423
Wilson, R.E., 1914, Lick Obs. Bull. 8, 130