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Matrix Theory S Matrix

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The technology required for eikonal scattering amplitude calculations in matrix theory is developed. Using the entire supersymmetric completion of the v^4/r^7 matrix theory potential we compute the graviton-graviton scattering amplitude and find agreement with 11 dimensional supergravity at tree level. [S0031-9007(98)07213-5]

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M theory, the 11 dimensional quantum theory underlying perturbative strings, has in recent years headlined dramatic changes in our understanding of string theory. At large distances M theory reduces (by definition) to 11 dimensional supergravity. According to the matrix theory conjecture of [1] the microscopic degrees of freedom of M theory are described by the large N limit of a *quantum mechanical* supersymmetric $U(N)$ Yang-Mills model. The model itself arises, on the one hand, as the regulating theory of the 11 dimensional supermembrane [2] and on the other as the short distance description of D0-branes [3,4]. An essential feature of the model is the existence of asymptotic particle states carrying the quantum numbers of the 11 dimensional graviton supermultiplet [1,5].

A principal test of the matrix conjecture is the comparison of scattering amplitudes in the Yang-Mills quantum mechanics with those of 11 dimensional supergravity. To date, typical matrix theory scattering experiments involve the comparison of classical gravity source-probe actions with the background field effective action of super Yang-Mills theory in $(1+0)$ dimensions evaluated on straight line configurations (see [6] for an exhaustive list of references). However, a matrix theory computation yielding true S -matrix elements, depending on momenta and polarizations of the external particles, has remained elusive. In this Letter we carry out precisely such a computation.

To this end we construct a matrix theory analog of the Lehmann, Symanzik, and Zimmermann (LSZ) reduction formula which relates the S matrix to the background field expansion of the matrix theory path integral. In essence, we have found that the S -matrix elements formed from the asymptotic supergraviton states of [5] induce exactly the boundary conditions in the matrix path integral satisfied by straight line diagonal background field configurations.

In fact, in order to obtain the polarization dependence of scattering amplitudes in matrix theory, it is necessary to expand the effective potential in both bosonic and fermionic background fields. There exist, scattered in the literature, some partial results for the fermionic part of the one loop matrix theory effective potential [7,8]. Here

we present the full result which is based on the work of [9,10]. Rather than a matrix theory Feynman diagram *tour de force*, all leading D-brane spin-dependent interactions are obtained by a string theory computation employing the Green-Schwarz boundary state formalism [11].

Combining the effective potential and our matrix theory LSZ reduction formula it is then possible to compute eikonal S -matrix elements. As an example we consider a graviton-graviton scattering process and find that the matrix theory scattering amplitude agrees with that of 11 dimensional supergravity.

Before presenting our results and formalism, a few remarks are in order. Throughout this paper we work in the $N = 2$ sector of the matrix model. Since our computations are, for the time being, restricted to the one loop leading terms of matrix theory which are protected by supersymmetry, there is no need to take the large- N limit. The demonstrated impressive agreement of supergravity and matrix theory amplitudes at finite N indeed confirms this claim. Despite the fact that this agreement is expected by supersymmetry, our results clearly show that matrix theory is aware of the tensorial structure of *Lorentz invariant* 11 dimensional supergravity. Moreover, the formalism developed in the present Letter permits the computation of more general scattering amplitudes that will be crucial to understand the range of validity of matrix theory. (A more detailed analysis of the results presented here will appear in a forthcoming paper.)

LSZ for matrix theory.—The $N = 2$ matrix theory Hamiltonian

$$H = \frac{1}{2}P_\mu^0 P_\mu^0 + \left[\frac{1}{2}\vec{P}_\mu \cdot \vec{P}_\mu + \frac{1}{4}(\vec{X}_\mu \times \vec{X}_\nu)^2 + \frac{i}{2}\vec{X}_\mu \cdot \vec{\theta} \gamma_\mu \times \vec{\theta} \right] \quad (1)$$

is a sum of an interacting $SU(2)$ part describing relative motions and a free $U(1)$ piece pertaining to the center of mass. We use a vector notation for the adjoint representation of $SU(2)$, $\vec{X}_\mu = (Y_\mu^I, x_\mu)$ and $\vec{\theta} = (\theta^I, \theta^3)$ (with $I = 1, 2$ and $\mu = 1, \dots, 9$) and may choose a gauge in which $Y_9^I = 0$. The model has a potential with flat directions along a valley floor in the Cartan sector x_μ and θ^3 . The remaining degrees of freedom transverse to the valley are supersymmetric harmonic oscillators in

the variables Y_μ^I ($\mu \neq 9$) and θ^I . Upon introducing a large gauge invariant distance $x = (\vec{X}_9 \cdot \vec{X}_9)^{1/2} = x_9$ as the separation of a pair of particles, the Hamiltonian (1) was shown [5] to possess asymptotic two particle states of the form

$$|p_\mu^1, \mathcal{H}^1; p_\mu^2, \mathcal{H}^2\rangle = |0_B, 0_F\rangle \frac{1}{x_9} e^{i(p^1-p^2)x} e^{i(p_1+p_2)x^0} \\ \times |\mathcal{H}^1\rangle_{\theta^0+\theta^3} |\mathcal{H}^2\rangle_{\theta^0-\theta^3}. \quad (2)$$

Here $p_\mu^{1,2}$ and $\mathcal{H}^{1,2}$ are, respectively, the momenta and polarizations of the two particles. The state $|0_B, 0_F\rangle$ is the ground state of the superharmonic oscillators and the polarization states are the $\underline{44} \oplus \underline{84} \oplus \underline{128}$ representation of the $\theta^0 \pm \theta^3$ variables, corresponding to the graviton, three-form tensor, and gravitino, respectively.

For the computation of scattering amplitudes one may now form the S matrix in the usual fashion $S_{fi} = \langle \text{out} | \exp\{-iHT\} | \text{in} \rangle$ with the desired ingoing and outgoing quantum numbers according to (2). (The asymptotic states above are constructed with respect to a large separation in the same direction for both ingoing and outgoing particles, i.e., eikonal kinematics. More general kinematical situations are handled by introducing a rotation operator into the S matrix [12].) The object of interest is then the vacuum to vacuum transition amplitude

$$e^{i\Gamma(x'_\mu, x_\mu, \theta^3)} = {}_{x'_\mu} \langle 0_B, 0_F | \exp\{-iHT\} | 0_B, 0_F \rangle_{x_\mu}. \quad (3)$$

Note that the ground states actually depend on the Cartan variables x_μ and x'_μ through the oscillator mass. Also, both the left and right hand sides depend on the operator θ^3 .

Our key observation is rather simple. In field theory one is accustomed to expand around a vanishing vacuum expectation value when computing the vacuum to vacuum transition amplitude for some field composed of oscillator modes. In quantum mechanics the idea is of course exactly the same, and therefore if one is to represent (3) by a path integral one should expand the super oscillators transverse to the valley about a vanishing vacuum expectation value. One may then write the matrix theory S matrix in terms of a path integral with the stated boundary conditions

$$e^{i\Gamma(v_\mu, b_\mu, \theta^3)} = \int_{\vec{X}_\mu=(0,0,x_\mu), \vec{\theta}=(0,0,\theta^3)}^{\vec{X}_\mu=(0,0,x'_\mu), \vec{\theta}=(0,0,\theta^3)} \mathcal{D}(\vec{X}_\mu, \vec{A}, \vec{b}, \vec{c}, \vec{\theta}) \\ \times \exp\left(i \int_{-T/2}^{T/2} L_{\text{SYM}}\right). \quad (4)$$

The Lagrangian L_{SYM} is that of the supersymmetric Yang-Mills quantum mechanics with appropriate gauge fixing to which end we have introduced ghosts \vec{b}, \vec{c} and the Lagrange multiplier gauge field \vec{A} . The effective action $\Gamma(v_\mu, b_\mu, \theta^3)$ is most easily computed via an expansion about classical trajectories $X_\mu^c(t) \equiv x_\mu^c(t) = b_\mu + v_\mu t$ and constant $\theta^3(t) = \theta^3$ which yields the

quoted boundary conditions through the identification $b_\mu = (x'_\mu + x_\mu)/2$ and $v_\mu = (x'_\mu - x_\mu)/T$.

Up to an overall normalization \mathcal{N} , our LSZ reduction formula for matrix theory is simply

$$S_{fi} = \delta^9(k'_\mu - k_\mu) e^{-ik_\mu k_\mu T/2} \\ \int d^9 x' d^9 x \mathcal{N} \exp(-iw_\mu x'_\mu + iu_\mu x_\mu) \\ \times \langle \mathcal{H}^3 | \langle \mathcal{H}^4 | e^{i\Gamma(v_\mu, b_\mu, \theta^3)} | \mathcal{H}^1 \rangle | \mathcal{H}^2 \rangle. \quad (5)$$

The leading factor expresses momentum conservation for the center of mass where we have denoted $k_\mu = p_\mu^1 + p_\mu^2$ and $k'_\mu = p_\mu^3 + p_\mu^4$ for the ingoing and outgoing particles, respectively, and similarly for the relative momenta $u_\mu = (p_\mu^1 - p_\mu^2)/2$ and $w_\mu = (p_\mu^4 - p_\mu^3)/2$.

In a loopwise expansion of the matrix theory path integral one finds $\Gamma(v_\mu, b_\mu, \theta^3) = v_\mu v_\mu T/2 + \Gamma_{(1)} + \Gamma_{(2)} + \dots$ of which we consider only the first two terms in order to compare our results with tree-level supergravity. Inserting this expansion into (5) and changing variables $d^9 x' d^9 x \rightarrow d^9(Tv)d^9 b$, the integral over Tv_μ may be performed via stationary phase. Dropping the normalization and the overall center of mass piece the S matrix then reads

$$S_{fi} = e^{-i[(u+w)/2]^2 T/2} \int d^9 b e^{-iq_\mu b_\mu} \langle \mathcal{H}^3 | \langle \mathcal{H}^4 | \\ \times e^{i\Gamma_{(\text{loops})}[v_\mu=(u_\mu+w_\mu)/2, b_\mu, \theta^3]} | \mathcal{H}^1 \rangle | \mathcal{H}^2 \rangle, \quad (6)$$

where $q_\mu = w_\mu - u_\mu$. It is important to note that in (6) the variables θ^3 are operators $\{\theta_\alpha^3, \theta_\beta^3\} = \delta_{\alpha\beta}$ whose expectation between polarization states $|\mathcal{H}\rangle$ yields the spin dependence of the scattering amplitude.

The loopwise expansion of the effective action should be valid for the eikonal regime, i.e., large impact parameter b_μ or small momentum transfer q_μ . As we shall see below, this limit is dominated by t -channel physics on the supergravity side.

D0-brane computation of the matrix theory effective potential.—We must now determine the one-loop effective matrix potential $\Gamma(v, b, \theta^3)$, namely, the v^4/r^7 term and its supersymmetric completion. Fortunately the bulk of this computation has already been performed in string theory by [9,10] who applied the Green-Schwarz boundary state formalism of [11] to a one-loop annulus computation for a pair of moving D0-branes. They found that the leading spin interactions are dictated by a simple zero mode analysis and their form is, in particular, scale independent. This observation allows one to extrapolate the results of [9,10] to short distances and suggests a matrix theory description for tree-level supergravity interactions.

Following [9,10], supersymmetric D0-brane interactions are computed from the correlator

$$\mathcal{V} = \frac{1}{16} \int_0^\infty dt \langle B, \vec{x} = 0 | e^{-2\pi t \alpha' p^+(P^- i \partial / \partial x^+)} \\ \times e^{(\eta Q^- + \bar{\eta} \bar{Q}^-)} e^{V_B} | B, \vec{y} = \vec{b} \rangle \quad (7)$$

with Q^-, \tilde{Q}^- being the SO(8) supercharges broken by the presence of the D-brane, $|B\rangle$ the boundary state associated with D0-branes, and $V_B \oint_{\tau=0} d\sigma (X^{[1} \partial_\sigma X^{i]} + \frac{1}{2} S \gamma^{li} S)$ is the boost operator where the direction 1 has to be identified with the time (see [9,10] for details). Expanding (7) and using the results in section four of [10], one finds the following compact form for the leading one-loop matrix theory potential (normalizing to one the v^4 term and setting $\alpha' = 1$)

$$\mathcal{V}_{1\text{-loop}} = \left[v^4 + 2i v^2 v_m (\theta \gamma^{mn} \theta) \partial_n - 2v_p v_q (\theta \gamma^{pm} \theta) (\theta \gamma^{qn} \theta) \partial_m \partial_n - \frac{4i}{9} v_q (\theta \gamma^{qm} \theta) (\theta \gamma^{nk} \theta) (\theta \gamma^{pk} \theta) \partial_m \partial_n \partial_p \right. \\ \left. + \frac{2}{63} (\theta \gamma^{ml} \theta) (\theta \gamma^{nl} \theta) (\theta \gamma^{pk} \theta) (\theta \gamma^{qk} \theta) \partial_m \partial_n \partial_p \partial_q \right] \frac{1}{r^7}, \tag{8}$$

where $\theta = (\eta^a, \tilde{\eta}^{\dot{a}})$ should be identified with $\theta^3/2$ of the last section and all indices are nine dimensional. The general structure of this potential was noted in [13] and its first, second, and last terms were calculated in [14], [7], and [8], respectively. (It would be interesting to establish the supersymmetry transformations of this potential. However, acceleration terms, set to zero in our setup, should then be included [15].)

Results.—Our matrix computation is completed by taking the quantum mechanical expectation of the effective potential (8) between the polarization states of (6). Clearly one can now study any amplitude involving gravitons, three-form tensors, and gravitini. We choose to compute a $h_1 + h_2 \rightarrow h_4 + h_3$ graviton-graviton process, and thus prepare states

$$|\text{in}\rangle = \frac{1}{256} h_{mn}^1 (\lambda_1^\dagger \gamma_m \lambda_1^\dagger) (\lambda_1^\dagger \gamma_n \lambda_1^\dagger) h_{pq}^2 (\lambda_2^\dagger \gamma_p \lambda_2^\dagger) (\lambda_2^\dagger \gamma_q \lambda_2^\dagger) |-\rangle, \tag{9}$$

$$\langle \text{out}| = \frac{1}{256} \langle -| h_{mn}^4 (\lambda_1 \gamma_m \lambda_1) (\lambda_1 \gamma_n \lambda_1) h_{pq}^3 (\lambda_2 \gamma_p \lambda_2) (\lambda_2 \gamma_q \lambda_2).$$

Note that (following [5]) we have complexified the Majorana center of mass and Cartan spinors θ^0 and θ^3 in terms of SO(7) spinors $\lambda^{1,2} = (\theta_+^0 \pm \theta_+^3 + i\theta_-^0 \pm i\theta_-^3)/2$, where \pm denotes projection with respect to γ_9 . Actually the polarizations in (9) are seven dimensional but may be generalized to the nine dimensional case at the end of the calculation. We stress that these maneuvers are

purely technical and our final results are SO(9) covariant. The creation and destruction operators $\lambda_{1,2}^\dagger$ and $\lambda_{1,2}$ annihilate the states $\langle -|$ and $|-\rangle$, respectively.

The resulting one-loop eikonal matrix theory graviton-graviton scattering amplitude is composed of 66 terms and [denoting, e.g., $(qh_1 h_4 v) = q_\mu h_{\mu\nu}^1 h_{\nu\rho}^4 v_\rho$ and $(h_1 h_4) = h_{\mu\nu}^1 h_{\nu\mu}^4$] is given by

$$\mathcal{A} = \frac{1}{q^2} \left\{ \frac{1}{2} (h_1 h_4) (h_2 h_3) v^4 + 2[(qh_3 h_2 v) (h_1 h_4) - (qh_2 h_3 v) (h_1 h_4)] v^2 \right. \\ + (v h_2 v) (q h_3 q) (h_1 h_4) + (v h_3 v) (q h_2 q) (h_1 h_4) - 2(q h_2 v) (q h_3 v) (h_1 h_4) \\ - 2(q h_1 h_4 v) (q h_3 h_2 v) + (q h_1 h_4 v) (q h_2 h_3 v) + (q h_4 h_1 v) (q h_3 h_2 v) \\ + \frac{1}{2} [(q h_1 h_4 h_3 h_2 q) - 2(q h_1 h_4 h_2 h_3 q) + (q h_4 h_1 h_2 h_3 q) - 2(q h_2 h_3 q) (h_1 h_4)] v^2 \\ - (q h_2 v) (q h_3 q) (h_1 h_4) + (q h_2 q) (q h_3 v) (h_1 h_4) - (q h_1 q) (q h_2 h_3 h_4 v) + (q h_1 q) (q h_3 h_2 h_4 v) \\ - (q h_4 q) (q h_2 h_3 h_1 v) + (q h_4 q) (q h_3 h_2 h_1 v) - (q h_1 v) (q h_4 h_2 h_3 q) + (q h_1 v) (q h_4 h_3 h_2 q) \\ - (q h_4 v) (q h_1 h_2 h_3 q) + (q h_4 v) (q h_1 h_3 h_2 q) + (q h_1 h_4 q) (q h_2 h_3 v) - (q h_1 h_4 q) (q h_3 h_2 v) \\ + \frac{1}{8} [(q h_1 q) (q h_2 q) (h_3 h_4) + 2(q h_1 q) (q h_4 q) (h_2 h_3) + 2(q h_1 q) (q h_3 q) (h_2 h_4) \\ + (q h_3 q) (q h_4 q) (h_1 h_2)] + \frac{1}{2} [(q h_1 q) (q h_4 h_2 h_3 q) - (q h_1 q) (q h_2 h_4 h_3 q) \\ - (q h_1 q) (q h_4 h_3 h_2 q) - (q h_4 q) (q h_1 h_2 h_3 q) + (q h_4 q) (q h_1 h_3 h_2 q) - (q h_4 q) (q h_2 h_1 h_3 q)] \\ + \frac{1}{4} [(q h_1 h_3 q) (q h_4 h_2 q) + (q h_1 h_2 q) (q h_4 h_3 q) + (q h_1 h_4 q) (q h_2 h_3 q)] \\ \left. + [h_1 \longleftrightarrow h_2, h_3 \longleftrightarrow h_4]. \right\} \tag{10}$$

We have neglected all terms within the curly brackets proportional to $q^2 \equiv q_\mu q_\mu$, i.e., those that cancel the $1/q^2$ pole. These correspond to contact interactions in the D0-brane computation, whereas this calculation is valid only for noncoincident branes.

D = 11 supergravity.—The above leading order result for eikonal scattering in matrix theory is easily shown to agree with the corresponding 11 dimensional field theoretical amplitude. Tree-level graviton-graviton scattering is dimension independent and has been computed in [16]. We have double-checked that work by a type IIA string theory computation

and will not display the explicit result here which depends on 11 momenta p_M^i (with $i = 1, \dots, 4$) and polarizations h_{MN}^i subject to the de Donder gauge condition $p_N^i h_M^{iN} - (1/2)p_M^i h_N^{iN} = 0$ (no sum on i). Matrix theory, on the other hand, is formulated in terms of on shell degrees of freedom only, namely, transverse physical polarizations and euclidean nine momenta.

Going to light-cone variables for the eleven momenta p_M^i we take the case of vanishing p^- momentum exchange, i.e., the scenario of our matrix computation,

$$\begin{aligned} p_M^1 &= (-\frac{1}{2}(v_\mu - q_\mu/2)^2, 1, v_\mu - q_\mu/2), \\ p_M^2 &= (-\frac{1}{2}(v_\mu - q_\mu/2)^2, 1, -v_\mu + q_\mu/2), \\ p_M^4 &= (-\frac{1}{2}(v_\mu + q_\mu/2)^2, 1, v_\mu + q_\mu/2), \\ p_M^3 &= (-\frac{1}{2}(v_\mu + q_\mu/2)^2, 1, -v_\mu - q_\mu/2). \end{aligned} \quad (11)$$

[We denote $p_\pm = p^\mp = (p^{10} \pm p^0)/\sqrt{2}$ and our metric convention is $\eta_{MN} = \text{diag}(-, +, \dots, +)$.] By transverse Galilean invariance we have set to zero the nine dimensional center of mass momentum. We measure momenta in units of p^- which we set to one. For this kinematical situation conservation of p^+ momentum clearly implies $v_\mu q_\mu = 0$. Note that the vectors u_μ and w_μ of (5) are simply $u_\mu = v_\mu - q_\mu/2$ and $w_\mu = v_\mu + q_\mu/2$.

We reduce to physical polarizations by using the residual gauge freedom to set $h_{+M}^i = 0$ and solve the de Donder gauge condition in terms of the transverse traceless polarizations $h_{\mu\nu}^i$ for which one finds $h_{-M}^i = -p_\nu^i h_{\nu M}^i$.

Agreement with the matrix result (10) is then achieved by taking the eikonal limit $v_\mu \gg q_\mu$ of the gravity amplitude in which the t -pole contributions dominate. (In the above parametrization, the Mandelstam variables are $t = q_\mu^2 = -2p_M^1 p_4^M$, $s = 4v_\mu^2 + q_\mu^2 = -2p_M^1 p_2^M$, and $u = 4v_\mu^2 = -2p_M^1 p_3^M = s - t$.) One then reproduces

exactly (10) as long as any pieces canceling the t pole (i.e., the aforementioned q^2 terms) are neglected.

Although we have presented here only a matrix scattering amplitude restricted to the eikonal regime, we nevertheless believe the agreement found is rather impressive.

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