Estimation and Inference with the Efficient Method of Moments: With Applications to Stochastic Volatility Models and Option Pricing

van der Sluis, P.J.

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Chapter 1

Introduction

1.1 Motivation

The Efficient Method of Moments (EMM) is a recent simulation-based estimation technique for models for which standard direct maximum likelihood techniques are infeasible or analytically intractable, but from which one can simulate data on a computer. Examples are general-equilibrium models, auction models and Stochastic Volatility (SV) models. As is apparent from its name EMM is a moment-based estimation technique. The adjective efficient stems from the fact that for a specific choice of the moments the EMM estimator is first-order asymptotic efficient, so EMM is a GMM-type estimation technique that may do as well as maximum likelihood. The common practice in the GMM literature is to select a few low-order moments on an ad hoc basis. Recognizing the need for higher statistical efficiency, Gallant and Tauchen (1996b) propose EMM in an article entitled—"Which Moments to Match?". The answer to this question is given in the paper: the moments should be chosen as the score vector of an auxiliary probability model that fits the data well. In case this auxiliary model is chosen well Gallant and Long (1997) show that maximum-likelihood efficiency can be obtained or at least approached. These asymptotic results can be corroborated in specific cases by using Monte Carlo experiments. The semblance of EMM to GMM could also be exploited for generalizing specification tests that were originally developed for GMM to the EMM case.

One of the most successful applications of the EMM framework is in the field of SV models. These models form a class of models for which standard maximum likelihood techniques are infeasible. These models are used to model changing variance and covariance in (high frequency) financial time series, such as stock returns, exchange-rate movements and interest-rate movements. SV models relate

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directly to the diffusions used in theoretical finance from which they partly origi­
inate. Though the literature on SV models started in the early 1970s with Clark
(1973) and their theoretical implications were well understood by the end of the
1980s through Hull and White (1987), empirical analysis with SV models was only
recently developed. For the larger part this is due to the fact that only by the early
1990s computing speed was sufficient for the development of computationally in-
tensive simulation-based estimation techniques, such as EMM. Throughout this
thesis SV models serve as an illustration of the EMM framework.

The aim of this thesis is fourfold: First, to provide a reliable estimation tech­
nique for a broad class of SV models. In order to achieve this aim different im-
plementations of EMM are introduced and judged. Second, to provide reliable hy-
pothesis tests and tests for mis-specification for EMM. Particularly tests for struc-
tural stability are considered in this thesis. We introduce these tests for EMM and
illustrate these tests in the context of SV models. Third, to implement and test
option-pricing models under stochastic volatility within the EMM framework, and
fourth, to assess volatility forecasting with SV models using EMM.

1.2 Outline

This thesis falls into three parts: The first part, consisting of the Chapters 2 and 3,
focuses on SV models and estimation of SV models via EMM. Chapter 2 provides
a short introduction to financial time series and option pricing and introduces sev­
eral types of univariate and multivariate SV models. Chapter 3 provides the theory
of estimation and testing with EMM and some applications and Monte Carlo re­
results in the field of SV models. The second part, consisting of the Chapters 4 and
5, focuses on EMM-based tests for structural stability in SV models. In this part
these tests are derived, applied and evaluated. Chapter 4 deals with structural sta­
bility tests with known breakpoint. Chapter 5 considers structural stability tests
with unknown breakpoint. In these chapters the set-up is chosen in such a way
that the theory is given in general terms, but the applications are specifically given
for SV models. Finally, the third part, consisting of Chapters 6 and 7, focuses res­
pectively on option pricing and volatility forecasting with SV models using EMM.
These chapters have a more applied nature. Finally, Chapter 8 contains a summary
of the most important results and an outlook for further research in this area.

Going into more detail concerning the structure of the thesis: Chapter 2 con­
tains a short description of models for financial time series. It subsequently deals
with the characteristics of financial time series and the models that have been pro­
posed in the literature for dealing with these characteristics. The main characteris­
tic that is dealt with is time-varying volatility. Next, several classes of SV models
are introduced. First, we introduce SV models with Gaussian errors. These mod­
els usually fall short to describe the excess kurtosis in financial time series. Therefore we next introduce SV models with Student-$t$ errors. Due to the leverage effect asymmetry is often present in this type of data. Therefore we introduce asymmetric SV models with Gaussian errors. These models are also expanded to the non-Gaussian case. Finally, we develop multivariate generalizations of the symmetric and asymmetric SV models with Gaussian errors. In order to compare EMM with alternative estimation techniques in Chapter 3, we next give a concise overview of some alternative estimation techniques for SV models. We conclude this chapter with a short introduction to the theory of option pricing, where in particular the role of stochastic volatility is addressed.

Chapter 3 contains the theory, a Monte Carlo study and some applications of EMM. An extension of the EMM methodology, called reprojection, is discussed here as well. This extension is necessary for implementing option pricing under stochastic volatility. This chapter is partly based on van der Sluis (1997a, 1998a). The key concept in the EMM methodology is to choose a specific auxiliary model—in a way to be explained below—for the model we seek to estimate. Estimation of this auxiliary model is the first step in the EMM methodology. By minimizing, in a certain metric, a quadratic form of the scores of the auxiliary model at the estimated auxiliary parameter values using simulations from the structural model, we can calibrate the parameters of the structural model such that dynamic properties of the data and structural model, as perceived by the scores of the auxiliary model, match. This is the second step in the EMM methodology. In order to justify the claim of asymptotic efficiency of the EMM estimators, we need to impose several assumptions on the auxiliary model such that it (asymptotically) embeds the structural model. In this thesis we choose the Semi-NonParametric (SNP) family of densities of Gallant and Nychka (1987). This is an expansion of Hermite polynomials about the Gaussian density. To improve the small-sample properties of EMM we need a good leading term in the Hermite expansion. For example, this leading term can be a parametric model for the conditional mean and conditional variance. The non-Gaussianity and time structure that remain in the residuals are left over to the Hermite terms. In case of SV models, we choose as a leading term the EGARCH$^2$ model of Nelson (1991) for the conditional variance. Next we discuss reprojection, because the option pricing models of Chapter 6 and the evaluation of volatility forecasts of Chapter 7 need as an input the latent volatilities from the SV models. A major point of critique on the EMM method was that it does not provide a representation of the observations in terms of their past. For SV models this means: extraction of the latent volatility series. Reprojection meets this critique by characterising the dynamic response of a partially observed non-linear model on its observed past. In the same way one can obtain a representation of the

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$^2$Exponential Generalized AutoRegressive Conditional Heteroskedasticity
latent variables in terms of the observations from the past and the present. Reproduction is the third step in EMM: projection of a long simulated series of the estimated structural model on the auxiliary model. In this chapter, we next consider hypothesis tests and tests for mis-specification in the EMM framework. Testing in an EMM framework bears many similarities with testing in a GMM framework, so we build on the theory of GMM-based testing for developing EMM-based testing procedures. Concerning estimation, Monte Carlo experiments are conducted for different auxiliary models, i.e. with different leading terms and different orders of the Hermite polynomials, and for different structural models. Several univariate SV models are fitted to daily returns of the Standard & Poor's 500 Stock Index (S&P500) for the period 1963–1993 using a variety of score generators.

Chapter 4 develops tests for structural stability with known breakpoint for EMM. It is based on van der Sluis (1997b, 1998a). The tests fall into three equivalence classes: the Wald/LR/LM tests, the Hansen tests, and the Prediction tests. Each equivalence class contains tests that are asymptotically locally most powerful for a specific alternative. These alternatives are: (i) variation in the parameters for the Wald/LR/LM test; (ii) violation of the moment conditions both before and after the breakpoint for the Hansen test; (iii) violation of the moment conditions after the breakpoint for the Prediction tests. Only the class of Prediction tests is computationally attractive in the sense that only one EMM estimator is needed, viz. the EMM estimator for the sample. Therefore, we propose computationally attractive modifications of the Wald/LR/LM tests and Hansen tests that retain the property of being asymptotically locally most powerful. Its computational attractiveness is comparable to that of the Prediction test. A Monte Carlo study investigates the small-sample properties of the computationally attractive tests in the context of univariate SV models. Next, the tests are applied to SV models for daily exchange-rate movements of the British Pound versus the Canadian Dollar 1988–1996 and daily returns of the S&P500 index 1981-1993. We set the breakpoint for the exchange-rate movements to Black Wednesday, when Britain left the ERM. For the S&P500 index, we set the breakpoint at Black Monday 1987.

Chapter 5 develops tests for structural stability with unknown breakpoint for EMM. It is based on van der Sluis (1998b). As in Chapter 4, the tests fall apart into three equivalence classes. Each equivalence class contains tests that each consider a specific aspect of mis-specification of the structural model. The difference with Chapter 4 is that the breakpoint is unknown. Therefore, the location of the breakpoint is a nuisance parameter. The asymptotic distribution of the tests cannot be derived by standard theory, since under the null of no structural stability the nuisance parameter is not defined. As suggested in the literature, exponentially weighted test statistics, arithmetically weighted test statistics and the supremum of the test statistics are considered. The exponentially weighted and arithmetically weighted tests have certain optimality properties. The supremum tests have only weak optimal-
ity properties, but have the advantage of giving an intuitively plausible estimator of the breakpoint. The generalization of the Hansen test for structural stability with known breakpoint to the case of unknown breakpoint is a novelty in the literature of moment-based inference. Therefore the asymptotic distribution of this test is derived. The tests are applied to an asymmetric SV model with Gaussian errors for the S&P500 index data of Chapter 3.

Chapter 6 is based on Jiang and van der Sluis (1998c, 1999). Here the EMM methodology is applied to the pricing of options. While the stochastic volatility (SV) generalization has been shown to improve the explanatory power over the Black-Scholes model, empirical implications of SV models on option pricing have not yet been adequately tested. The purpose of this chapter is to first estimate a multivariate SV model using EMM from observations of underlying state variables and then investigate the respective effects of stochastic interest rates, systematic volatility and idiosyncratic volatility on option prices. We use a series of daily US 3-month Treasury bill rates and daily 3Com Corporation stock prices. The data covers the period from March 12, 1986 to August 18, 1997. Option prices are computed using both reprojected underlying historical volatilities and implied stochastic volatility risk to gauge each model’s performance through direct comparison with observed market option prices. The option pricing formulae are tested using option prices on the same underlying stock for the period June 19, 1997 through August 18, 1997.

Chapter 7 is based on Jiang and van der Sluis (1998a). This chapter evaluates the performance of volatility forecasting based on univariate and multivariate stochastic volatility models. The data consists of the daily returns of four technology stocks: 3Com, Applied Material, Cisco, and Oracle which are all traded at Nasdaq, over the period from February 16, 1990 to January 5, 1997. Part of the data was treated as ex-post in volatility forecasting. It is shown that the choice of the squared asset return or squared return residual with mis-specified trend as proxy of ex-post volatility directly leads to the extremely low explanatory power of the common regression analysis. It is argued that since the measure of volatility is always model-dependent, the volatility-forecasting performance should be evaluated in a consistent model framework.

Chapter 8 contains a summary of the thesis and gives some suggestions for further research.

Finally, Appendix A contains a description of the computer programs that have been used and gives explicit formulae for the auxiliary models employed. It is partly based on van der Sluis (1997a).

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3Electronic stock market in the US. It is the fastest growing stock market in the US and is the leading American market for foreign listings.
CHAPTER 1

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