Estimation and Inference with the Efficient Method of Moments: With Applications to Stochastic Volatility Models and Option Pricing
van der Sluis, P.J.

Citation for published version (APA):
van der Sluis, P. J. (1999). Estimation and Inference with the Efficient Method of Moments: With Applications to Stochastic Volatility Models and Option Pricing Amsterdam: Thela Thesis. TI Research Series nr. 204
Chapter 6

Option Pricing

The purpose of this chapter is to first estimate a multivariate SV model using the efficient method of moments (EMM) technique from observations of underlying state variables and then investigate the respective effect of stochastic interest rates, systematic volatility and idiosyncratic volatility on option prices. We compute option prices using reprojected underlying historical volatilities and implied stochastic volatility risk to gauge each model's performance through direct comparison with observed market option prices. This chapter draws on Jiang and van der Sluis (1998c, 1999).

The structure of this chapter is as follows. Section 6.1 provides an introduction. Section 6.2 outlines the general multivariate SV model. Section 6.3 reports the estimation results of the general model and various submodels. Section 6.4 compares among different models the performance in pricing options and analyses the effect of each individual factor. Section 6.5 concludes.

6.1 Introduction

Acknowledging the fact that volatility is changing over time in time series of asset returns as well as in the empirical variances implied from option prices through the Black and Scholes (1973) model, there have been numerous recent studies on option pricing with time-varying volatility. Many authors have proposed to model asset-return dynamics and to price options using SV models. Examples of these models in continuous time include Hull and White (1987), Johnson and Shanno (1987), Wiggins (1987), Scott (1987, 1991, 1997), Baily and Stulz (1989), Chesney and Scott (1989), Melino and Turnbull (1990), Stein and Stein (1991), Heston (1993), Bates (1996b, 1996a), and Bakshi, Cao and Chen (1997), and examples in
While the stochastic volatility generalization has been shown to improve over the Black-Scholes model in terms of the explanatory power for asset-return dynamics, its empirical implications on option pricing have not yet been adequately tested due to the difficulty involved in the estimation. Can such generalization help resolve well-known systematic empirical biases associated with the Black-Scholes model, such as the volatility smiles (e.g. Rubinstein (1985)) and asymmetry of such smiles (e.g. Stein (1989), Clewlow and Xu (1993) and Taylor and Xu(1994a, 1994b))? How substantial is the gain, if any, from such generalization compared to simpler models? The purpose of this chapter is to answer the above questions by studying the empirical performance of SV models in pricing stock options, and investigating the respective effect of stochastic interest rates, systematic volatility and idiosyncratic volatility on option prices in a multivariate SV model framework. We specify and implement a dynamic-equilibrium model for asset returns extended in the line of Rubinstein (1976), Brennan (1979), and Amin and Ng (1993). Our model incorporates both the effects of idiosyncratic volatility and systematic volatility of the underlying stock returns into option valuation and at the same time allows interest rates to be stochastic. In addition, we model the short-term interest-rate dynamics and stock-return dynamics simultaneously and allow for asymmetry of conditional volatility in both stock-return and interest-rate dynamics.

The first objective that is addressed in this chapter is to estimate the parameters of a multivariate SV model. Instead of implying parameter values from market option prices through option pricing formulas, we directly estimate the model specified under the objective measure from the observations of underlying state variables. By doing so, the underlying model specification can be tested in the first hand for how well it represents the true data generating process (DGP), and various risk factors, such as systematic volatility risk and interest-rate risk, are identified from historical movements of underlying state variables. We employ the EMM estimation technique to estimate some candidate multivariate SV models for daily stock returns and daily short-term interest rates.

The second objective of this chapter is to examine the effects of different elements considered in the model on stock option prices through direct comparison with observed market option prices. Inclusion of both a systematic component and an idiosyncratic component in the model provides information for whether extra predictability or uncertainty is more helpful for pricing options. In gauging the empirical performance of alternative option pricing models, we use both the relative difference and the implied Black-Scholes volatility to measure option pricing.

1 Another branch on the literature has focused on the pricing of options using GARCH-type models, see e.g. Duan (1995) and Engle, Kane and Noh (1997).
errors as the latter is less sensitive to the maturity and moneyness of options. Our model setup contains many option pricing models in the literature as special cases, for instance: (i) the SV model of stock returns (without systematic volatility risk) with stochastic interest rates; (ii) the SV model of stock returns with non-stochastic risk-free interest rates; (iii) the stochastic interest-rate model with constant conditional stock-return volatility; and (iv) the Black-Scholes model with both constant interest rate and constant conditional stock-return volatility. We focus our comparison of the general model setup with the above four submodels.

The framework in this chapter is different in spirit from the implied methodology often used in the finance literature. First, only the risk-neutral specification of the underlying model is implied in the option prices, thus only a subset of the parameters can be estimated (or backed-out) from the option prices. Second, as Bates (1996b) points out, the major problem of the implied estimation method is the lack of associated statistical theory, thus the implied methodology based on solely the information contained in option prices is purely objective-driven, it is rather a test of stability of a certain relationship (the option pricing formula) between different input factors (the implied parameter values) and the output (the option prices). Third, as a result, the implied methodology can at best offer a test of the joint hypothesis, it fails going any further to test the model specification or the efficiency of the market.

Our methodology is also different from other research based on observations of underlying state variables. First, different from the method of moments or GMM used in Wiggins (1987), Scott (1987), Chesney and Scott (1989), Jorion (1995), Melino and Turnbull (1990), the efficient method of moments (EMM) has been shown in Chapter 3 to yield efficient estimates of SV models in finite samples, and the parameter estimates are not sensitive to the choice of particular moments. Second, our model allows for a richer structure for the state variable dynamics, for instance the simultaneous modelling of stock-returns and interest-rate dynamics, the systematic effect considered in this chapter, and asymmetry of conditional volatility for both stock-return and interest-rate dynamics.

In judging the empirical performance of alternative models in pricing options, we perform two tests. First, we assume, as in Hull and White (1987) among others, that stochastic volatility is diversifiable and therefore has zero risk premium. Based on the historical volatility obtained through reprojec­tion, we calculate option prices with given maturities and moneyness. The model-predicted option prices are compared to the observed market option prices in terms of relative percentage differences and implied Black-Scholes volatility. Second, we assume, following Melino and Turnbull (1990), a non-zero risk premium for stochastic volatility, which is estimated from observed option prices in the previous day. The estimates are used in the following day's volatility process to calculate option prices, which again are compared to the observed market option prices. Throughout the
comparison, all our models only rely on information available at given time, thus the study can be viewed as out-of-sample comparison. In particular, in the first comparison, all models rely only on information contained in the underlying state variables (i.e. the primitive information), while in the second comparison, the models use information contained in both the underlying state variables and the observed (previous day’s) market option prices (i.e. the derivative information).

6.2 Model

The basic model is as follows. Let $S_t$ denote the price of the stock at time $t$ and $r_t$ the interest rate at time $t$, we model the dynamics of daily stock returns and daily interest-rate changes simultaneously as a multivariate SV process. Suppose $r_t$ is also explanatory to the trend or conditional mean of stock returns, then the de-trended or the unexplained stock return $y_{s,t}$ is defined as

$$ y_{s,t} = 100 \times \Delta \ln S_t - \mu_S - \phi_S r_{t-1} $$

(6.1)

and the de-trended or the unexplained interest-rate change $y_{r,t}$ is defined as

$$ y_{r,t} = 100 \times \Delta \ln r_t - \mu_r - 100 \times \phi_r \ln r_{t-1} $$

(6.2)

and, $y_{s,t}$ and $y_{r,t}$ are modelled as SV processes

$$ y_{s,t} = \sigma_{s,t} z_{s,t} $$

(6.3)

$$ y_{r,t} = \sigma_{r,t} z_{r,t} $$

(6.4)

where

$$ \ln \sigma_{s,t+1}^2 = \alpha \ln r_t + \omega_s + \gamma_s \ln \sigma_{s,t}^2 + \sigma_s \eta_{s,t}, \quad |\gamma_s| < 1 $$

(6.5)

$$ \ln \sigma_{r,t+1}^2 = \omega_r + \gamma_r \ln \sigma_{r,t}^2 + \sigma_r \eta_{r,t}, \quad |\gamma_r| < 1 $$

(6.6)

and

$$ \begin{bmatrix} \hat{z}_{s,t} \\ \hat{z}_{r,t} \end{bmatrix} \sim iIn(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \lambda_1 \\ \lambda_1 & 1 \end{bmatrix}) $$

(6.7)

so that $\text{Cor}(z_{s,t}, z_{r,t}) = \lambda_1$. Note that this is a bi-variate AMSV(1) of (2.26) to (2.28) with the extension of the inclusion of $\alpha \ln r_t$ in (6.5). The asymmetry, i.e. correlation between $\eta_{s,t}$ and $z_{s,t}$ and between $\eta_{r,t}$ and $z_{r,t}$, is modelled as follows through $\lambda_2$ and $\lambda_3$

$$ \eta_{s,t} = \lambda_2 \hat{z}_{s,t} + \sqrt{1 - \lambda_2^2} u_t $$

(6.8)

$$ \eta_{r,t} = \lambda_3 \hat{z}_{r,t} + \sqrt{1 - \lambda_3^2} v_t $$

(6.9)
where $u_t$ and $v_t$ are assumed to be $IN(0, 1)$. For the same reason as in (2.30) this leads to the restriction

$$\text{Cor}(\eta_s, \eta_r) = \lambda_4 = \lambda_1 \lambda_2 \lambda_3 \quad (6.10)$$

The SV model specified above offers a flexible distributional structure in which the correlation between volatility and stock returns serves to control the level of asymmetry and the volatility variation coefficients serve to control the level of kurtosis. Specific features of the above model include the following. First of all, the above model setup is specified in discrete time and includes continuous-time models as special cases in the limit. Second, the above model is specified to catch the possible systematic effects through parameters $\phi_S$ in the trend and $\alpha$ in the conditional volatility. It is only the systematic state variable that affects the individual stock-returns’ volatility, not the other way around. Third, the model deals with logarithmic interest rates so that the nominal interest rates are restricted to be positive, as negative nominal interest rates are ruled out by a simple arbitrage argument. The interest-rate model admits mean-reversion in the drift and allows for stochastic conditional volatility. Fourth, the above model specification allows the movements of de-trended return processes to be correlated through random noises $z_s,t$ and $z_r,t$ via their correlation $\lambda_1$. Finally, the parameters $\lambda_2$ and $\lambda_3$ are to measure the asymmetry of conditional volatility for stock returns and interest rates. When $z_s,t$ and $\eta_s,t$ are allowed to be correlated with each other, the model can pick up the kind of asymmetric behaviour which is often observed in stock price changes. In particular, a negative correlation between $\eta_s,t$ and $z_s,t$ ($\lambda_2 < 0$) induces the leverage effect (see Section 2.2.2). It is noted that the above model specification will be tested against alternative specifications.

### 6.3 Estimation

#### 6.3.1 Estimation Technique

Estimation in this chapter is done using EmmPack of van der Sluis (1997a) and similar procedures written by the author on which details are given in Appendix A. The leading term in the SNP expansion is a multivariate generalization of the EGARCH model as motivated in Section 3.2.1.

In principle one should simultaneously estimate all structural parameters, i.e. the mean parameters $\mu_S, \mu_r, \phi_S$ and $\phi_r$ in (6.1) to (6.2) and the volatility parameters of $y_s,t$ and $y_r,t$ as given in (6.5) to (6.8). Though this is optimal, it is too cumbersome and not necessary given the low order of autocorrelation in stock returns. Therefore estimation is carried out in the following (sub-optimal) way:
(i) Estimate $\mu_S$ and $\phi_S$, retrieve $y_{s,t}$; Estimate $\mu_r$ and $\phi_r$, retrieve $y_{r,t}$, both using standard regression techniques;

(ii) Simultaneously estimate parameters of the SV model via EMM.

As we have mentioned, EMM estimation of stochastic volatility models is rather time-consuming. Moreover many of the above stochastic volatility models have never actually been efficiently estimated. Therefore we use the auxiliary model, i.e. the MEGARCH model, as a guidance for which of the above SV models would be considered for our data set, that is, we view the auxiliary model as a pendant to the structural bivariate AMSV model as given in (6.5) to (6.8). We explicitly give here the bivariate version of the MEGARCH model in (3.30) to (3.32).

\[
\begin{bmatrix}
  y_{s,t} \\
  y_{r,t}
\end{bmatrix}
= 
\begin{bmatrix}
  \sigma_{s,t} & 0 \\
  0 & \sigma_{r,t}
\end{bmatrix}
\begin{bmatrix}
  \zeta_{s,t} \\
  \zeta_{r,t}
\end{bmatrix}
\]

\[
\begin{bmatrix}
  \ln h^2_{s,t} \\
  \ln h^2_{r,t}
\end{bmatrix}
= 
\begin{bmatrix}
  \phi \\
  0
\end{bmatrix}
\begin{bmatrix}
  T_{t-1} \\
  0
\end{bmatrix}
+ 
\begin{bmatrix}
  \rho_{ss} & \rho_{sr} \\
  \rho_{rs} & \rho_{rr}
\end{bmatrix}
\begin{bmatrix}
  \zeta_{s,t} \\
  \zeta_{r,t}
\end{bmatrix}
+ 
(1 + \begin{bmatrix}
  \alpha_{ss} & \alpha_{sr} \\
  \alpha_{rs} & \alpha_{rr}
\end{bmatrix})
\begin{bmatrix}
  K_{1,ss} & K_{1,sr} \\
  K_{1,rs} & K_{1,rr}
\end{bmatrix}
\begin{bmatrix}
  \zeta_{s,t-1} \\
  \zeta_{r,t-1}
\end{bmatrix}
+ 
\begin{bmatrix}
  K_{2,ss} & K_{2,sr} \\
  K_{2,rs} & K_{2,rr}
\end{bmatrix}
\begin{bmatrix}
  (|\zeta_{s,t-1}| - \sqrt{2/\pi}) \\
  (|\zeta_{r,t-1}| - \sqrt{2/\pi})
\end{bmatrix}
\]

\[
\begin{bmatrix}
  \zeta_{s,t} \\
  \zeta_{r,t}
\end{bmatrix}
\sim 
N(0, \begin{bmatrix}
  1 & \delta \\
  \delta & 1
\end{bmatrix})
\]

where some parameters will be restricted, namely $\alpha_{rs}$, $\alpha_{sr}$, $K_{sr,i}$ and $K_{r,s,i}$ for $i = 1, 2$ will be a priori set as zero in the application.

The parameter $\delta$ in the MEGARCH model corresponds to $\lambda_1$ in the SV model. The $\kappa$'s, possibly in combination with some of the parameters of the polynomial, correspond to $\lambda_2$ and $\lambda_3$. Recall that this latter correspondence was investigated in a Monte Carlo study in Chapter 3 with very encouraging results. Furthermore, note that in (6.11) we include the interest-rate level $r_t$ in the volatility process of the stock returns parallel to (6.5) in the SV model. The parameter $\psi$ in the auxiliary EGARCH model therefore corresponds to $\alpha$ in the SV model.

As in (3.18) the MEGARCH model is expanded with a semiparametric density which allows for nonnormality. In Section 6.3.2 it is argued how to pick a suitable order for the Hermite polynomial for a Gaussian SV model. The efficient moments for the SV model will come initially from the auxiliary model: bi-variate SNP density with bi-variate EGARCH leading terms. For an extensive evaluation of this bi-variate EGARCH model and even of higher dimensional EGARCH models, see Chapter 3. This model will also serve to test the specification of the structural SV
6.3. **ESTIMATION**

(a) Static Properties of Original Series

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Max</th>
<th>Min</th>
<th>Cor((t, r_{t-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta \ln(r_t) )</td>
<td>2835</td>
<td>-9.759 \times 10^{-3}</td>
<td>1.042</td>
<td>-0.549</td>
<td>11.848</td>
<td>6.936</td>
<td>-9.137</td>
<td>-0.043</td>
</tr>
<tr>
<td>( \Delta \ln(S_t) )</td>
<td>2835</td>
<td>1.051 \times 10^{-1}</td>
<td>3.719</td>
<td>-1.109</td>
<td>14.655</td>
<td>21.48</td>
<td>-36.67</td>
<td></td>
</tr>
</tbody>
</table>

(b) Static Properties of Filtered Interest Rates and Stock Returns

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Max</th>
<th>Min</th>
<th>Cor((t, r_{t-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y_{rt} )</td>
<td>2834</td>
<td>-6.10 \times 10^{-10}</td>
<td>1.042</td>
<td>-0.556</td>
<td>11.80</td>
<td>6.948</td>
<td>-9.154</td>
<td></td>
</tr>
<tr>
<td>( \ln(Y_{1S}^2) )</td>
<td>2834</td>
<td>-2.199</td>
<td>2.995</td>
<td>-1.241</td>
<td>1.800</td>
<td>4.429</td>
<td>-20.16</td>
<td></td>
</tr>
<tr>
<td>( \ln(Y_{1S}^2) )</td>
<td>2834</td>
<td>3.86 \times 10^{-4}</td>
<td>3.716</td>
<td>-1.107</td>
<td>14.56</td>
<td>21.34</td>
<td>-36.56</td>
<td></td>
</tr>
<tr>
<td>( \ln(Y_{1S}^2) )</td>
<td>2834</td>
<td>0.766</td>
<td>2.525</td>
<td>-1.201</td>
<td>2.032</td>
<td>7.196</td>
<td>-14.19</td>
<td>-0.063</td>
</tr>
<tr>
<td>( \ln(Y_{2S}^2) )</td>
<td>2834</td>
<td>4.38 \times 10^{-4}</td>
<td>3.719</td>
<td>-1.110</td>
<td>14.65</td>
<td>21.37</td>
<td>-36.77</td>
<td></td>
</tr>
<tr>
<td>( \ln(Y_{2S}^2) )</td>
<td>2834</td>
<td>0.794</td>
<td>2.409</td>
<td>-0.850</td>
<td>0.279</td>
<td>7.209</td>
<td>-6.673</td>
<td>-0.046</td>
</tr>
</tbody>
</table>

(c) Dynamic Properties of Filtered Interest Rates and Stock Returns (autocorrelations \(\times 10^{-1}\))

<table>
<thead>
<tr>
<th></th>
<th>(\rho(1))</th>
<th>(\rho(2))</th>
<th>(\rho(3))</th>
<th>(\rho(4))</th>
<th>(\rho(5))</th>
<th>(\rho(10))</th>
<th>(\rho(15))</th>
<th>(\rho(20))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y_{rt} )</td>
<td>1.270</td>
<td>-0.118</td>
<td>-0.441</td>
<td>0.292</td>
<td>-0.271</td>
<td>0.250</td>
<td>0.837</td>
<td>0.197</td>
</tr>
<tr>
<td>( \ln(Y_{1S}^2) )</td>
<td>1.120</td>
<td>0.981</td>
<td>0.662</td>
<td>0.795</td>
<td>1.180</td>
<td>0.216</td>
<td>0.199</td>
<td>0.419</td>
</tr>
<tr>
<td>(Y_{1S} )</td>
<td>0.653</td>
<td>-0.469</td>
<td>-0.554</td>
<td>0.050</td>
<td>0.011</td>
<td>-0.316</td>
<td>0.414</td>
<td>-0.233</td>
</tr>
<tr>
<td>( \ln(Y_{1S}^2) )</td>
<td>0.644</td>
<td>0.479</td>
<td>-0.064</td>
<td>0.460</td>
<td>0.455</td>
<td>0.219</td>
<td>0.305</td>
<td>0.782</td>
</tr>
<tr>
<td>(Y_{2S} )</td>
<td>0.670</td>
<td>-0.451</td>
<td>-0.537</td>
<td>0.066</td>
<td>0.028</td>
<td>-0.346</td>
<td>0.430</td>
<td>-0.218</td>
</tr>
<tr>
<td>( \ln(Y_{2S}^2) )</td>
<td>0.594</td>
<td>0.495</td>
<td>-0.090</td>
<td>0.363</td>
<td>0.285</td>
<td>0.193</td>
<td>0.247</td>
<td>0.653</td>
</tr>
</tbody>
</table>

Note: \(Y1\) represents the filtered series with systematic effects on stock returns, while \(Y2\) without systematic effects.

Table 6.1: Summary statistics of interest rates and stock returns.

6.3.2 **Description of the Data**

Summary statistics of both interest rates and stock returns are reported in Table 6.1. A time-series plot and salient features of both data sets can be found in Figures 6.1 and 6.2. The interest rates used in this chapter as a proxy of the riskless rates are daily US 3-month Treasury bill rates and the underlying stock considered in this chapter is 3Com Corporation which is listed in NASDAQ. Both the stock and its options are actively traded. The stock claims no dividend and thus theoretically all options on the stock can be valued as European type options. The data covers the period from March 12, 1986 to August 18, 1997 providing 2,860 observations. From Table 6.1, we can see that both the first difference of logarithmic interest rates and that of logarithmic stock prices (i.e. the daily stock returns) are skewed to the left and have positive excess kurtosis \((>> 3)\) suggesting skewed and fat-tailed distributions. Similarly, the filtered interest rates as well as the filtered stock returns, both with systematic effect and without systematic effect, are
Figure 6.1: Salient features of daily US 3-month Treasury bill rates March 1986–August 1997. Top left displays a time plot of the return series; Top right displays a correlogram of the returns; Bottom left displays the empirical density of the returns, a Normal approximation and an estimated density; Bottom right displays a QQ-plot of the returns versus the Normal.
Figure 6.2: Salient features of daily 3Com returns March 1986–August 1997. Top left displays a time plot of the return series; Top right displays a correlogram of the returns; Bottom left displays the empirical density of the returns, a Normal approximation and an estimated density; Bottom right displays a QQ-plot of the returns versus the Normal.
Table 6.2: Estimates of “mean” parameters. The numbers in brackets are t-ratios of the above estimates. The blank cell indicates the parameter is pre-set at zero in the corresponding model.

<table>
<thead>
<tr>
<th>Model</th>
<th>Stock Return Parameter</th>
<th>Interest Rate Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_S$</td>
<td>$\phi_S$</td>
</tr>
<tr>
<td>With Systematic Effect</td>
<td>0.667</td>
<td>-10.29</td>
</tr>
<tr>
<td></td>
<td>(2.60)</td>
<td>(-2.28)</td>
</tr>
<tr>
<td>Without Systematic Effect</td>
<td>0.105</td>
<td>-0.215</td>
</tr>
<tr>
<td></td>
<td>(1.505)</td>
<td>(-1.119)</td>
</tr>
</tbody>
</table>

also skewed to the left and have positive excess kurtosis. However, the logarithmic squared filtered series, as proxy of the logarithmic conditional volatility, all have negative excess kurtosis and appear to justify the Gaussian noise specified in the volatility process. As far as dynamic properties are concerned, the filtered interest rates and stock returns as well as logarithmic squared filtered series are all temporally correlated. For the logarithmic squared filtered series, the first-order autocorrelations are in general low, but higher-order autocorrelations are of similar magnitudes as the first-order autocorrelations. This would suggest that all series are roughly ARMA(1, 1) or equivalently AR(1) with measurement error, which is consistent with the first order autoregressive SV model specification. Estimates of trend parameters in the general model are reported in Table 6.2. For stock returns, interest rate has significant explanatory power, suggesting the presence of systematic effect or certain predictability of stock returns. For logarithmic interest rates, there is an insignificant linear mean-reversion, which is consistent with many findings in the literature.

Since the score-generator should give a good description of the data, we further look at the data through specification of the score generator or auxiliary model. We use the score-generator as a guide for the structural model, as there is a clear relationship between the parameters of the auxiliary model and the structural model. If some auxiliary parameters in the score-generator are not significantly different from zero, we set the corresponding structural parameters in the SV model a priori equal to zero. Various model selection criteria and t-statistics of individual parameters of a wide variety of different auxiliary models that were proposed in Section 6.3 indicate the following. Multivariate MEGARCH(1,1) models are all clearly rejected against two univariate EGARCH models on basis of the model selection criteria and the t-values of the parameter $\delta$. We therefore set the corresponding SV parameter $\lambda_1$ a priori equal to zero. Through (6.10) this implies $\lambda_4 = 0$. The parameter $\psi$ was marginally significant at a 5% level. On basis of the BIC, however, inclusion of this parameter is not justified. This rejects that the short-term interest rate is correlated with conditional volatility of the stock returns. A direct explanation of this finding is that either the volatility of the stock returns truly does not
have a systematic component or the short-term interest rate serves as a poor proxy of the systematic factor. We believe the latter conjecture to be true as we re-ran the model with other stock returns and invariably found $\psi$ insignificantly different from zero. We therefore set its corresponding parameter $\alpha$ a priori equal to zero. The cross terms $\gamma_{12,1}$ and $\gamma_{21,1}$ were significantly different from zero albeit small, again on basis of the BIC inclusion of these parameters was not justified. Therefore we included no cross terms between $\ln \sigma_{s,t}^2$ and $\ln \sigma_{r,t}^2$ in (6.5) and (6.6). As far as the choice of a suitable order for the Hermite polynomial in the SNP expansion, we observe that for all models $K_x$ should be equal to zero, and, more importantly, according to the most conservative criterion, i.e. the BIC, $K_z > 10$. This is undesirable. For the choice of the size of $K_z$, our argument is as follows. The results in Chapter 3 which studied the cases with sample sizes 1,000 and 1,500 indicate that, for these sample sizes, $K_z$ of 4 or 5 was found to be BIC optimal. For our sample which consists of about 3,000 observations, the BIC is in favour of Hermite polynomials of order $K_z$ larger than 10. However, Monte Carlo results in Chapter 3 suggest that for sample sizes of 3,000, convergence problems occur in a substantial number of cases for such high-order polynomials and that under the null of a Gaussian SV model, setting $K_z = 0$ will yield virtually efficient EMM estimates, which are not necessarily dominated by setting $K_z > 0$.

Still we can learn something from the fitted SNP densities with $K_z > 0$. Consider the conditional density implied by the ML estimates for $K_z = 6$ and 10 for both data sets in Figures 6.3 and 6.4. Clearly, there is evidence in the data that a Gaussian EGARCH model is not adequate as was also indicated by model selection criteria and a Likelihood Ratio test. It also appears that for $K_z > 6$ the SNP density starts to put probability mass at outliers. For descriptive purposes such high orders in the auxiliary model can be desirable, however, since under the null of Gaussian SV we cannot get such outliers, there is no need to consider these. Therefore we decided for these sample sizes to set the Hermite polynomial equal to zero. To check the validity of this argument we performed EMM estimation using the EGARCH-H(6,0) as well to see whether the results would differ from the ones with EGARCH-H(0,0), and it turns out that the parameter estimates differ only slightly. However the values of the individual components of the $J$-test corresponding to the parameters of the Hermite polynomial cause rejection of the SV model by the $J$-test. This can also be seen by comparing the sample properties of the data with the sample properties of the SV model in the optimum. Further research should therefore include this fact by using a structural model with fatter-tailed noise or a jump component. However, such a non-Gaussian SV model will make option pricing much more complicated.
CHAPTER 6. OPTION PRICING

6.3.3 Structural Models and Estimation Results

The general model, specified in Section 6.2, assumes stochastic volatility for both the stock-returns and interest-rate dynamics as well as a systematic effect on stock returns. This model nests the Amin and Ng (1993) model as a special case when $\lambda_2 = 0$ and $\lambda_3 = 0$. Below we discuss four alternative model specifications:

- **Submodel 1**: No systematic effect, i.e. $\phi_s = 0$ and $\alpha = 0$: a bi-variate stochastic volatility model;
- **Submodel 2**: No stochastic interest rates, i.e. interest rate is constant, $r_t = r$, which is the Hull-White model and the Baily and Stulz (1989) model;
- **Submodel 3**: Constant stock-return volatility but stochastic interest rate, $\sigma_s = \sigma$, which is the Merton (1973), Turnbull and Milne (1991) and Amin and Jarrow (1991) model;
- **Submodel 4**: Constant stock-return volatility and constant interest rate, $\sigma_s = \sigma, r_t = r$, which is the Black-Scholes model.

---

Figure 6.3: Estimated conditional densities for EGARCH(1,1)-H(6,0) and EGARCH(1,1)-H(10,0) models for interest-rate returns.
The results reported here are all for $K_x = 0$ and $K_z = 0$. As mentioned in Section 6.3.2 the models have also been estimated setting $K_z = 6$ but no substantial differences were found in the estimation results.

- General model: The estimates for the mean terms are given in Table 6.2. We obtained the following estimates for the symmetric SV model using the EGARCH(1,1)-$H(0,0)$ score generator with $\kappa_2 = 0$,

\begin{align}
  y_t &= \sigma_t z_t \\
  \ln \sigma_{t+1}^2 &= .005 + .955 \ln \sigma_t^2 + .218 \eta_t \\
  \ln \sigma_{t+1}^2 &= .161 + .940 \ln \sigma_t^2 + .161 \eta_t
\end{align}

for the interest rates and

\begin{align}
  y_t &= \sigma_t z_t \\
  \ln \sigma_{t+1}^2 &= .005 + .955 \ln \sigma_t^2 + .218 \eta_t \\
  \ln \sigma_{t+1}^2 &= .161 + .940 \ln \sigma_t^2 + .161 \eta_t
\end{align}

for the stock prices. In order to obtain the filtered series, we used $L_1 = 29$ for the interest rate $L_1 = 29$ lags for the stock prices in (3.39). For the asymmetric model we use the EGARCH(1,1)-$H(0,0)$ as a score generator to obtain

![Figure 6.4: Estimated conditional densities for EGARCH(1,1)-$H(6,0)$ and EGARCH(1,1)-$H(10,0)$ models for stock returns.](image)
the following estimates

\[ y_t = \sigma_t z_t \]  

(6.18)

\[ \ln \sigma_{t+1}^2 = .004 + .959 \ln \sigma_t^2 + .222 \eta_t \]  

(6.19)

\[ \text{Cor}(z_t, \eta_t) = - .270 \]  

(6.20)

for the interest rates and

\[ y_t = \sigma_t z_t \]  

(6.21)

\[ \ln \sigma_{t+1}^2 = .175 + .935 \ln \sigma_t^2 + .161 \eta_t \]  

(6.22)

\[ \text{Cor}(z_t, \eta_t) = - .424 \]  

(6.23)

for the stock prices.

It is noted that similar to other financial time series, the persistence parameter is close to unity. The asymmetry is moderate for both series and significantly different from zero. The leverage effect is somewhat higher for the stock returns than for the interest-rate changes. For the purpose of reprojection, we incorporate asymmetry and the AIC advocates to use \( L_1 = 31 \) and \( L_2 = 20 \) in (3.40) for the interest movements and \( L_1 = 28 \) and \( L_2 = 28 \) for the stock returns. The filtered series for the stock returns using the symmetric and asymmetric models are displayed in Figure 6.5. Filtered series for the interest rates are displayed in Figure 6.6.

- **Submodel 1:** The mean terms are given in 6.2. We obtained the following estimates for the symmetric SV model using the EGARCH(1,1)-H(0,0) score generator with \( \kappa_2 = 0 \),

\[ y_t = \sigma_t z_t \]  

(6.24)

\[ \ln \sigma_t^2 = .004 + .959 \ln \sigma_{t-1}^2 + .217 \eta_t \]  

(6.25)

for the interest rates and

\[ y_t = \sigma_t z_t \]  

(6.26)

\[ \ln \sigma_t^2 = .149 + .944 \ln \sigma_{t-1}^2 + .148 \eta_t \]  

(6.27)
Figure 6.5: Filtered stock returns volatility for the SARMAV(1,0) and ASARMAV(1,0) models using reprojection.

for the stock prices. In order to obtain the filtered series, we used the same choices as above for the reprojection. For the asymmetric model we used the EGARCH(1,1)-H(0,0) as a score generator to obtain the following estimates

\[ y_t = \sigma_t z_t \]  
(6.28)

\[ \ln \sigma_{t+1}^2 = 0.004 + 0.959 \ln \sigma_t^2 + 0.223 \eta_t \]  
(6.29)

\[ \text{Cor}(z_t, \eta_t) = -0.275 \]  
(-158)

for the interest rates and

\[ y_t = \sigma_t z_t \]  
(6.31)

\[ \ln \sigma_{t+1}^2 = 0.154 + 0.944 \ln \sigma_t^2 + 0.147 \eta_t \]  
(6.32)

\[ \text{Cor}(z_t, \eta_t) = -0.557 \]  
(-247)
for the stock prices. The estimates do not differ much from the ones obtained for the general model. For the reprojection used the same choices as above. To save space, the filtered series for the submodel have not been displayed. The series resemble the series for the general model very much as displayed in Figures 6.5 and 6.6.

- Other Submodels: Estimation of other submodels is fairly straightforward. Submodel 2 takes the SV part of the stock returns. Submodel 3 takes the SV part of the interest rates.

Table 6.3 reports the results of Hansen J-test using EMM. As we see all the models have been accepted at a 5% level. Although a P-value is a monotone function of the actual evidence against $H_0$, it is very dangerous to choose the best model of these specifications on basis of the P-values; see Berger and Delampady (1987). An LR-test of the asymmetric SV model versus the symmetric SV model cannot be deduced from the difference in criterion values, since the criterion values are based on different moment conditions, i.e. an EGARCH(1,1)-$H(0,0)$ and an EGARCH(1,1)-$H(0,0)$ with $\kappa_2 = 0$. However from the $t$-values corresponding to the asymmetry parameter we can deduce that the null hypothesis of symmetry will
6.4. PRICING OF STOCK OPTIONS

<table>
<thead>
<tr>
<th>(a) Interest rates</th>
<th>Asymmetric General Model</th>
<th>Symmetric General Model</th>
<th>Asymmetric Submodel 1</th>
<th>Symmetric Submodel 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>J-test</td>
<td>.471</td>
<td>2.96</td>
<td>.471</td>
<td>2.96</td>
</tr>
<tr>
<td>df</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>P-value</td>
<td>.493</td>
<td>.085</td>
<td>.493</td>
<td>.085</td>
</tr>
</tbody>
</table>

Table 6.3: Test statistics for the SV models. The entries corresponding to the EGARCH parameters denote the individual $t$-values of the $J$-test.

certainly be rejected in favour of the alternative asymmetric model. For Submodel 1 we obtain similar results.

For the $J$-test with one degree of freedom it is not useful to consider the individual components of the test statistic as in (3.21). In this case the individual $t$-values are all about the same. This is a consequence of the fact that the individual $t$-values are asymptotically equal with probability one in case of only one degree of freedom in the test. As noted in Section 6.3.2 a $J$-test from the EGARCH(1,1)-H(6,0) model leads to rejection of all Gaussian SV models. By inspection of the individual components of the $J$-test we find that in this case the rejection can completely be attributed to the Hermite polynomial. This essentially means that the Gaussian SV model cannot account for the error structure beyond the EGARCH structure that is imposed by the Hermite polynomials. As noted before, a possible solution is to consider non-Gaussian SV models or SV models with jumps, but this will not be pursued here.

6.4 Pricing of Stock Options

The effects of SV on option prices have been examined by simulation studies in e.g. Hull and White (1987), Johnson and Shanno (1987), Baily and Stulz (1989), Stein and Stein (1991), Heston (1993) as well as empirical studies in e.g. Scott (1987), Wiggins (1987), Chesney and Scott (1989), Melino and Turnbull (1990), and Bakshi et al. (1997). In this section we investigate the implications of model specification on option prices through direct comparison with observed market option prices, with the Black-Scholes model as a benchmark. It is documented in the literature that the Black-Scholes model generates systematic biases in pricing op-
tions, with respect to the call option’s exercise prices, its time to expiration, and the underlying common stock’s volatility. Since there is a one-to-one relationship between volatility and option price through the Black-Scholes formula, the volatility is often used to quote the value of an option. An equivalent measure for the mispricing of Black-Scholes model is thus the implied or implicit volatility, i.e. the volatility which generates the corresponding option price. The Black-Scholes model imposes a flat term structure of volatility, i.e. the volatility is constant across both maturity and strike prices of options. Thus the use of implied volatility as the measure of pricing errors is less sensitive to the maturity and moneyness of options.

6.4.1 Description of the Option Data

The sample of market option quotes covers the period of June 19, 1997 through August 18, 1997, which overlaps with the last part of the sample of stock returns. Since we do not rely solely on option prices to obtain the parameter estimates through fitting the option pricing formula, such a sample size is adequate for our comparison purpose. The intradaily bid-ask quotes for the stock options are extracted from the CBOE\(^2\) database. To ease computational burden, for each business day in the sample only one reported bid-ask quote during the last half hour of the trading session (i.e. between 3:30 - 4:00 PM Eastern standard time) of each option contract is used in the empirical test. The main considerations for the choice of the particular bid-ask quote include: i) The movements of stock price is relatively stable around the point of time so that the option quotes are well adjusted; ii) Option quotes which do not satisfy arbitrage restrictions are excluded. The stock prices are calculated as average of bid-ask quotes which are simultaneously observed as the option’s bid-ask quotes. Therefore they are not transaction data and the data set used in this study avoids the issue of non-synchronous prices.

The sampling properties of the option data set are reported in Table 6.4. The data only include options with at least 5 days to expiration to reduce biases induced by liquidity-related issues. We divide the option data into several categories according to either moneyness or time to expiration. In this thesis, we use a slightly different definition of moneyness for options from the conventional one\(^3\). Following Ghysels et al. (1996) we define

\[
x_t = \ln(S_t/K) - \int_t^T r_s \, ds
\]  

(6.34)

\(^2\)Chicago Board Options Exchange.

\(^3\)In practice, it is more common to call an option as at-the-money/in-the-money/out-of-the-money when \(S_t = K\), \(S_t > K\) and \(S_t < K\) respectively. For American type options with possibility of early exercise, it is more convenient to compare \(S_t\) with \(K\), while for European type options and from an economic point of view, it is more appealing to compare \(S_t\) with the present value of the strike price \(K\).
6.4. PRICING OF STOCK OPTIONS

Table 6.4: Sample properties of stock call option prices. In each cell from top to bottom are: the average bid-ask midpoint call option prices with standard error in parentheses; the average effective bid-ask spread (ask price minus the bid-ask midpoint) with standard error in parentheses; and the number of option price observations (in curly brackets) for each moneyness-maturity category. The option price sample covers the period of June 19, 1997 through August 18, 1997 with 2120 observations in total. In calculating the moneyness, we use US 3-month T-bill rates for options with maturity less than 4 months and 6-month T-bill rates for options with maturity longer than 4 months.

Technically if \( x_t = 0 \), the current stock price \( S_t \) coincides with the present value of the strike price \( K \), the option is called at-the-money; if \( x_t > 0 \) (respectively \( x_t < 0 \)), the option is called in-the-money (respectively out-of-the-money). In our partition, a call option is said to be at-the-money (ATM) if \(-0.03 < x < 0.05\); out-of-the-money (OTM) if \( x < -0.03 \); and in-the-money (ITM) if \( x > 0.05 \). A finer partition resulted in six moneyness categories as in Table 6.4. According to the time to expiration, an option contract can be classified as: i) short-term \((T - t \leq 30 \text{ days})\); ii) medium-term \((30 < T - t < 180 \text{ days})\); and iii) long-term \((T - t \geq 180 \text{ days})\). The partition according to moneyness and maturity results in 18 categories as in Table 6.4. For each category, the average bid-ask midpoint price and its standard error, the average effective bid-ask spread (i.e. the ask price
minus the bid-ask midpoint) and its standard deviation, as well as the number of observations in the category are reported. Note that among 2120 total observations, about 26.56% are OTM options, 12.69% are ATM options, 60.75% are ITM options; 26.23% are short-term options, 49.01% are medium-term options, and 24.76% are long-term options. The average price ranges from $0.223 for short-term deep out-of-the-money options to $25.93 for long-term deep in-the-money options, and the average effective bid-ask spread ranges from $0.066 for short-term deep out-of-the-money options to $0.375 for long-term deep in-the-money options. Figure 6.7 plots the implied Black-Scholes volatility against moneyness for options with different terms of maturity. The implied Black-Scholes volatilities are backed out from each option quote using the corresponding stock price, time to expiration, and the current yield of US treasury instruments with maturity closest to the maturity of the option. Namely, we use 3-month T-bill rates for options with maturity less than 4 months, and 6-month T-bill rates for options with maturity longer than 4 months. The yields are hand-collected from the Wall Street Journal over the sample period and the discount rates are converted to annualized compound rates. It is noted that the Black-Scholes implied volatility exhibits obvious U-shaped patterns (smiles) as the call option goes from deep OTM to ATM and then to deep ITM, with the deepest ITM call option implied volatilities taking the highest values. The volatility smiles are more pronounced and more sensitive to the term to expiration for short-term options than for the medium-term and long-term options. Furthermore, the volatility smiles are obviously skewed to the left, indicating a downside risk anticipated by option traders. The asymmetry indicates a possible skewness due to the leverage effect or due to the fact that option traders expect a negative random jump in the dynamics of stock returns. These findings of clear moneyness- and maturity-related biases associated with the Black-Scholes model are consistent with the findings for many other securities in the literature (see e.g. Rubinstein (1985), Clewlow and Xu (1993), Taylor and Xu (1994a)).

6.4.2 Testing Option Pricing Models

Under the “risk-neutral” distribution of the general framework, a European call option on a non-dividend paying stock that pays off \( \max(S_T - X, 0) \) at maturity \( T \) for exercise price \( X \) is priced as

\[
C_0(S_0, r_0, \sigma_{r_0}, \sigma_{S_0}; T, X) = E_0^*[\max(S_T - X, 0) \exp(- \int_0^T r_t dt) S_0, r_0, \sigma_{r_0}, \sigma_{S_0}] \quad (6.35)
\]

where \( E_0^* \) is the expectation with respect to the “risk-neutral” specification for the state variables conditional on all information at \( t = 0 \). In particular, when \( \lambda_2 = 0 \)
### 6.4. PRICING OF STOCK OPTIONS

#### (1) Options with Maturities Less Than or Equal to 30 Days

- Implied BS Volatility

#### (2) Options with Maturities > 30 Days but < 180 Days

- Implied BS Volatility

#### (3) Options with Maturities More Than or Equal to 180 Days

- Implied BS Volatility

Figure 6.7: Implied Black-Scholes volatility from observed option prices on 3Com, June 19, 1997–August 18, 1997. On the x-axis is the degree of moneyness and on the y-axis is the implied BS volatility.
in the general model setup, i.e. Assumption 2 of Amin and Ng (1993) is satisfied, the option pricing formula can be derived as in (2.56). The call option price is the expected Black-Scholes price with the expectation taken with respect to the stochastic variance over the life of the option, i.e. the European call option prices depend on the average expected volatility over the length of the option contract. Furthermore, if stock volatility is also constant, we obtain the Black-Scholes formula. Since the underlying stock we consider in this chapter claims no dividend, all options on the stock can be valued as European-type options. Option prices given in the formula can be computed based on direct simulations. Our analysis for the implications of model specification on option prices is outlined as follows.

Two different tests are conducted for alternative models. First we assume, as in Hull and White (1987) among others, that stochastic volatility risk is diversifiable and therefore has zero risk premium. The underlying volatilities are directly estimated for Submodels 3 and 4 as constants, and are obtained through reprojection methods for the general model and Submodels 1 and 2. Based on the historical volatility, we calculate option prices with given maturities and moneyness. The model-generated option prices are compared to the observed market option prices in terms of relative percentage differences and implied Black-Scholes volatility. Second, we assume a non-zero risk premium for stochastic volatility, which is estimated from observed option prices in the previous day. The estimates are used in the following day’s volatility process to calculate option prices, which again are compared to the observed market option prices. Throughout the comparison, all the models only rely on information available at given time, thus the study can be viewed as out-of-sample comparison. In particular, in the first comparison, all models rely only on information contained in the underlying state variables, while in the second comparison, the models use both information contained in the underlying state variables (i.e. the primitive information) and in the observed (previous day’s) market option prices (i.e. the derivative information). Our study is clearly different from those which use option prices to imply all parameter values of the “risk-neutral” model, e.g. Bakshi et al. (1997). In their analysis, all the parameters and underlying volatility are estimated through fitting the option pricing model into observed option prices. Then these implied parameters and underlying volatility are used to predict the same set of option prices. Obviously models with more factors (or more parameters) are given extra advantage. In our comparison, the risk factors are identified from underlying asset-return process and the preference parameters for option traders are inferred from observed market option prices.

For options with early exercise potential, i.e. the American options, one way to approximate its price is to compute the Barone-Adesi and Whaley (1987) early-exercise premium, treating it as if the stock volatility and the yield-curve were time-invariant. Adding this early-exercise adjustment component to the European option price should result in a reasonable approximations of the corresponding American option price, see e.g. Bates (1996a).
6.4.3 Comparison based on Diversifiable Stochastic Volatility Risk

In this section, we assume that the risk premium in both interest-rate and stock-return processes as well as the conditional volatility processes are all zero. The SV option prices are calculated based on Monte Carlo simulation using (6.35) for asymmetric models and both (2.56) and (6.35) for symmetric models. The approximation error from the Monte Carlo simulations can be reduced to any desirable level by increasing the number of simulations. The estimation error involved in our study is also minimal as we rely on large number of observations over long sampling period to estimate model parameters. In our simulation, 100,000 sampling paths are simulated to reduce the Monte Carlo error and to reflect accurately the fat-tail behaviour of the asset-return distributions, and the antithetic variable technique is used to reduce the variation of option prices; see Boyle, Broadie and Glasserman (1997). The results show that option prices generated using different methods are almost the same, with the largest differences less than a penny for even long-term deep ITM options. The accuracy is further reflected in the small standard derivations of the simulated option prices. Since the option prices are conditional on the initial volatilities that are unobserved, another approximation error stems from the estimation error of the volatilities. As argued in Section 3.2.3 this error is not systematic.

Option pricing biases are compared to the observed market prices based on the mean relative percentage option pricing error (MRE) and the mean absolute relative option pricing error (MARE), given by

\[ MRE = \frac{1}{n} \sum_{i=1}^{n} \frac{C_i^M - C_i}{C_i} \]  
\[ MARE = \frac{1}{n} \sum_{i=1}^{n} \frac{|C_i^M - C_i|}{C_i} \]

where \( n \) is the number of options used in the comparison, \( C_i \) and \( C_i^M \) represents respectively the observed market option price and the theoretical model option price. The MRE statistic measures the average relative biases of the model option prices, while the MARE statistic measures the dispersion of relative biases of the model prices. The difference between MARE and MRE suggests the direction of the bias of the model prices, namely when MARE and MRE are of the same absolute values, it suggests that the model systematically misprices the options to the same direction as the sign of MRE, while when MARE is much larger than MRE in absolute magnitude, it suggests that the model is inaccurate in pricing options but the mispricing is less systematic. Since the percentage errors are very sensitive to the magnitude of option prices which are determined by both moneyness and length
of maturity, we also calculate MRE and MARE for each of the 18 moneyness-maturity categories in Table 6.4.

Table 6.5 reports the relative pricing errors (%) based on underlying volatility for alternative models. In each cell, from top to bottom are the MRE (mean relative error) and MARE (mean absolute relative error) statistics for: 1. The asymmetric general SV model with $\lambda_2 \neq 0, \lambda_3 \neq 0$; 2. The symmetric general SV model with $\lambda_2 = \lambda_3 = 0$; 3. The asymmetric submodel I with $\lambda_2 \neq 0, \lambda_3 \neq 0$; 4. The symmetric submodel I with $\lambda_2 = \lambda_3 = 0$; 5. The asymmetric submodel II with $\lambda_3 \neq 0$; 6. The symmetric submodel II $\lambda_3 = 0$; 7. Submodel III; and 8. Submodel IV. The conclusions we draw from the above comparison are summarized as follows. First, all models appear to perform very poorly in pricing options, especially the long-term options. The models in general over-price medium- and long-term options, and under-price short-term deep ITM and deep OTM options. For short-term options, our results are consistent with simulation results in e.g. Hull and White (1987) and others, i.e. the symmetric SV models tend to predict lower prices than the Black-Scholes model for ATM options and higher prices than the Black-Scholes model for deep ITM options. Since the simulation results in the next section suggest the existence of a non-zero risk premium for the stochastic volatility, the overall overpricing of all SV models may be due to our assumption of zero risk premium for conditional volatility; Second, the effect of stochastic interest rates on option prices is minimal in both cases of stochastic stock-return volatility and constant stock-return volatility, i.e. the differences between Submodels I and II and those between Submodels III and IV; Third, the systematic effect on the “mean” of stock returns, namely the additional predictability of stock returns, has a noticeable effect on option prices as evidenced in the simulation results between the general model and Submodel I. This is due to the fact that the reprojected underlying volatilities are different in magnitude under alternative specifications of the “mean” functions. As discussed in Lo and Wang (1995), predictability of asset returns can have significant impact on option prices, even though the exact effect is far from being clear; Fourth, SV models overall underperform the Black-Scholes model, even though all the models share similar patterns of mispricing as the Black-Scholes model, i.e. underpricing of short-term deep ITM and OTM options and overpricing of long-term and short-term ATM options. While the asymmetric SV models do outperform all other models for pricing short-term options, overall they underperform both the Black-Scholes model and the symmetric SV models, i.e. they tend to have higher relative option pricing errors; Finally, a further look at the implied Black-Scholes volatility of the asymmetric model prices, however, reveals that the implied volatility curve of the asymmetric models against maturity, reported in Figure 6.8, has a curvature closer to the implied volatility from observed market options prices in its shape, suggesting such pricing biases may be easier to correct.
Table 6.5: Relative pricing errors (%) of alternative models with diversifiable stochastic volatility Risk. In each cell, from top to bottom are the MRE (mean relative error) and MARE (mean absolute relative error) statistics for: 1. The asymmetric general SV model with $\lambda_2 \neq 0, \lambda_3 \neq 0$; 2. The symmetric general SV model with $\lambda_2 = \lambda_3 = 0$; 3. The asymmetric submodel I with $\lambda_2 \neq 0, \lambda_3 = 0$; 4. The symmetric submodel I with $\lambda_2 = \lambda_3 = 0$; 5. The asymmetric submodel II with $\lambda_3 \neq 0$; 6. The symmetric submodel II with $\lambda_3 = 0$; 7. Submodel III; and 8. Submodel IV.
<table>
<thead>
<tr>
<th></th>
<th>ITM</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.05 &lt; x ≤ 0.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.46 3.20</td>
<td>11.42 11.44</td>
<td>17.47 17.47</td>
<td>9.83 10.16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.24 2.09</td>
<td>6.94 7.08</td>
<td>13.56 13.56</td>
<td>6.17 6.78</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.32 2.51</td>
<td>10.65 10.71</td>
<td>17.27 17.27</td>
<td>9.12 9.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.22 2.08</td>
<td>6.83 6.98</td>
<td>13.45 13.45</td>
<td>6.09 6.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.32 2.50</td>
<td>10.64 10.70</td>
<td>17.25 17.25</td>
<td>9.11 9.49</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.22 2.08</td>
<td>6.84 6.99</td>
<td>13.46 13.46</td>
<td>6.09 6.71</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.04 1.83</td>
<td>4.28 4.47</td>
<td>8.81 8.81</td>
<td>3.85 4.47</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.04 1.83</td>
<td>4.23 4.43</td>
<td>8.73 8.73</td>
<td>3.81 4.44</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>x &gt; 0.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.45 0.55</td>
<td>1.67 1.86</td>
<td>3.88 3.90</td>
<td>1.86 2.12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.56 0.64</td>
<td>0.51 1.04</td>
<td>2.00 2.34</td>
<td>0.62 1.24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.49 0.58</td>
<td>1.66 1.86</td>
<td>3.58 3.65</td>
<td>1.66 2.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.56 0.65</td>
<td>0.49 1.02</td>
<td>1.95 2.30</td>
<td>0.60 1.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.49 0.58</td>
<td>1.66 1.86</td>
<td>3.58 3.64</td>
<td>1.66 1.99</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.56 0.65</td>
<td>0.49 1.02</td>
<td>1.95 2.30</td>
<td>0.60 1.22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.57 0.65</td>
<td>-0.06 0.74</td>
<td>0.77 1.35</td>
<td>0.02 0.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.56 0.65</td>
<td>-0.06 0.74</td>
<td>0.76 1.34</td>
<td>0.02 0.85</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Overall</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.12 13.00</td>
<td>19.24 19.75</td>
<td>41.03 41.14</td>
<td>19.91 22.93</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.86 10.16</td>
<td>15.78 16.30</td>
<td>40.49 40.60</td>
<td>19.02 20.72</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.70 10.00</td>
<td>17.27 17.76</td>
<td>39.45 39.47</td>
<td>18.88 21.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.65 10.11</td>
<td>15.55 16.07</td>
<td>39.99 40.10</td>
<td>18.73 20.47</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.67 10.00</td>
<td>17.25 17.74</td>
<td>39.38 39.40</td>
<td>18.85 21.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3.65 10.11</td>
<td>15.56 16.08</td>
<td>40.01 40.11</td>
<td>18.74 20.48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.65 9.60</td>
<td>10.46 11.08</td>
<td>25.92 26.10</td>
<td>12.41 14.33</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.57 9.57</td>
<td>10.39 11.01</td>
<td>25.78 25.96</td>
<td>12.32 14.25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.5 continued.
6.4. PRICING OF STOCK OPTIONS

(1) Options with Maturities Less Than or Equal to 30 Days

- Asymmetric SV
- Symmetric SV
- Black-Scholes

(2) Options with Maturities > 30 Days but < 180 Days

- Asymmetric SV
- Symmetric SV
- Black-Scholes

(3) Options with Maturities More Than or Equal to 180 Days

- Asymmetric SV
- Symmetric SV
- Black-Scholes

Figure 6.8: Implied Black-Scholes volatility from model-predicted option prices based on diversifiable stochastic volatility.
6.4.4 Comparison based on Implied Stochastic Volatility Risk

Since the short-term interest rate fails to be a good proxy of the systematic volatility of the market, the empirically insignificant correlation between interest-rate changes and stock returns cannot lead to conclude that there is no systematic component in the stock-return volatility. The assumption that the stock-return volatility is diversifiable, post in the previous section, is very likely to be invalid. In this section, we assume that there is a non-zero risk for stochastic stock-return volatility, and we use market option prices observed in the previous day \((t - 1)\) to imply such risk which is then used in the following day’s \((t)\) volatility processes. Since the estimation only uses option prices at a single point in time, we can assume a general functional form for the price of stochastic volatility risk, namely \(\lambda_t(\sigma_s)\) and \(\lambda_{t+\tau}(\sigma_s) = \lambda_t(\sigma_s), \forall \tau \geq 0\). For simplicity and for the reason that stochastic interest rates only have limited effect on option prices on the asset considered in this chapter, we assume that both the stochastic interest-rate volatility and stochastic interest rate have zero risk premium. Thus, for each pricing model, a parameter \(\theta_t = \lambda_t(\sigma_s), \text{i.e. the implied volatility risk for stochastic volatility models or} \theta_t = \sigma_{s,t}, \text{i.e. the implied volatility for constant volatility models can be obtained by minimizing the sum of squared error (SSE), i.e.}

\[
\hat{\theta}_{t-1} = \arg\min_{\theta_{t-1}} \sum_i \left(C_{t-1}^M(S_{t-1}, r_{t-1}, \theta_{t-1}; T_i, X_i) - C_{t-1}(T_i, X_i)\right)^2
\]  

(6.38)

where \(C_{t-1}(T_i, X_i)\) is the option price observed at \(t - 1\) with maturity date \(T_i\) and strike price \(X_i\). The implied volatility risk or volatility at \(t - 1\) are then used to price the options at \(t\)

\[
C_t^M(S_t, r_t, \sigma_{r,t}, \sigma_{s,t}; T, X) = \mathbb{E}_t^* [\max(S_T - X, 0) \exp\{-\int_0^T r_t dt\} | S_t, r_t, \hat{\theta}_{t-1}]
\]

(6.39)

For the SV models, the implied volatility risk can be interpreted as the option traders’ revealed preference from observed market option prices, while the implied volatility in the constant conditional volatility model is purely \textit{ad hoc} and inconsistent with the underlying model setup even though it is a common practice in the literature. To estimate \(\theta_{t-1}\) through (6.38) is straightforward for constant conditional volatility models with closed-form option pricing formula, but involves two problems for stochastic conditional volatility models. First, when the closed form solution of option prices is not available, the optimization involves enormous amount of simulation; Second, when the theoretical model price is replaced by the average simulated option prices, the estimate of \(\theta_{t-1}\) is biased for finite number of simulations. The bias can be reduced by increasing the number of simulations, which induces extra computational burden. Note that the adjustment of stochastic
6.4. PRICING OF STOCK OPTIONS

volatility risk alters only the drift term of the SV process in the objective measure to the following risk-neutral specification:

\[
\ln \tilde{\sigma}_{s,t+1} = \omega_s + \lambda_t(\sigma_s) + \gamma_s \ln \tilde{\sigma}_{s,t} + \sigma_s \eta_{s,t}, \quad |\gamma_s| < 1
\]  

(6.40)

From our discussion on the statistical properties of SV models in Section 2.2.2, we notice that, given the value of \(\lambda_{t+\tau}(\sigma_s) = \lambda_t(\sigma_s) = \lambda_s, \forall \tau \geq 0\) (i.e. once it is implied from option prices at a particular point of time, the volatility risk is treated as a constant thereafter), \(\text{Var}[\tilde{\sigma}_{s,t}] = \text{exp}\{E[\ln \tilde{\sigma}_{s,t}] + \text{Var}[\ln \tilde{\sigma}_{s,t}]/2\}\) where \(E[\ln \tilde{\sigma}_{s,t}] = (\omega_s + \lambda_s)/(1 - \gamma_s), \text{Var}[\ln \tilde{\sigma}_{s,t}] = \sigma_s^2/(1 - \gamma_s^2)\). It suggests that the parameter \(\lambda_t(\sigma_s)\) can be inferred from the unconditional variance of the SV process. Based on our simulations, the unconditional volatility of the symmetric SV model is approximately the same as the average implied Black-Scholes volatility of long-term options \((T - t \geq 180)\), and that of the asymmetric SV model with negative correlation is slightly higher than the average implied Black-Scholes volatility of long-term options \((T - t \geq 180)\). Thus, we can simply match the unconditional volatility of the SV model to the average implied Black-Scholes volatility from observed long-term options at each day to infer the implied stochastic volatility risk. The downward bias for the asymmetric model is adjusted based on simulations. Our results suggest that, similar to the findings in Melino and Turnbull (1990), there exists a non-zero risk premium for stochastic volatility of stock returns. The price of volatility risk \(\lambda_t(\sigma_s)\) appears to be consistently negative and rather stable over time. This finding is also consistent with the conjecture in Lamoureux and Lastrapes (1993) and explains why the implied volatility is an inefficient forecast of the underlying volatility.

Table 6.6 reports the relative pricing errors (%) based on implied volatility or volatility risk for alternative models. In each cell, from top to bottom are the MRE (mean relative error) and MARE (mean absolute relative error) statistics for various models as listed in Table 6.5. The basic conclusions we draw from the comparison are summarized as follows. First, all models have substantially reduced the pricing errors due to the use of implied volatility or volatility risk. The Black-Scholes model exhibits similar patterns of mispricing, namely underpricing of short-maturity options and over-pricing of long-maturity options, and overpricing of deep OTM options and underpricing deep ITM options. The pricing errors of long-term deep ITM options are dramatically decreased due to the larger weights put on these options in the minimization of the sum of squared option pricing errors; Second, the interest rate still only has minimal impact on option prices for both the cases of stochastic stock-return volatility and constant stock-return volatility; Third, all SV models outperform non-SV models due to the introduction of non-zero risk premium for conditional volatility. Compared to the Black-Scholes model, the symmetric SV models have overall lower pricing errors; Fourth, the asymmetric SV models further outperform the symmetric
### Moneyness

\[ x = \ln\left(\frac{S}{K}B(t, T)\right) \]

### Days-to-Expiration

\[ T-t \in [5, 215] \]

<table>
<thead>
<tr>
<th>Moneyness</th>
<th>Days-to-Expiration</th>
<th>( T-t \in [5, 215] )</th>
<th>( T-t \geq 180 )</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x \leq -0.20 )</td>
<td>( T-t \leq 30 )</td>
<td>20.18 41.40</td>
<td>6.41 15.76</td>
<td>2.69 13.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-27.99 46.16</td>
<td>25.80 29.20</td>
<td>23.35 23.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-25.34 45.83</td>
<td>3.90 13.18</td>
<td>-3.15 13.74</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-28.64 46.44</td>
<td>25.16 28.64</td>
<td>22.67 23.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-25.33 45.82</td>
<td>3.90 13.18</td>
<td>-3.26 13.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-28.64 46.44</td>
<td>25.19 28.68</td>
<td>22.69 23.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-42.12 46.45</td>
<td>17.76 21.48</td>
<td>50.34 50.72</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-42.12 46.46</td>
<td>17.79 21.51</td>
<td>50.35 50.72</td>
</tr>
<tr>
<td>( -0.20 &lt; x \leq -0.03 )</td>
<td>( T-t \leq 30 )</td>
<td>-14.21 21.12</td>
<td>3.17 8.64</td>
<td>7.38 7.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-12.42 21.65</td>
<td>10.73 12.68</td>
<td>11.53 11.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-16.74 22.61</td>
<td>1.48 8.18</td>
<td>6.14 7.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-12.16 21.61</td>
<td>10.49 12.50</td>
<td>11.26 11.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-16.70 22.61</td>
<td>1.47 8.18</td>
<td>6.08 6.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-12.16 21.61</td>
<td>10.50 12.51</td>
<td>11.27 11.39</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-10.07 20.42</td>
<td>11.60 12.78</td>
<td>23.64 23.64</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-10.07 20.42</td>
<td>11.60 12.78</td>
<td>23.65 23.65</td>
</tr>
<tr>
<td>( -0.03 &lt; x \leq 0.00 )</td>
<td>( T-t \leq 30 )</td>
<td>1.08 10.45</td>
<td>1.12 5.91</td>
<td>2.93 2.93</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.61 10.16</td>
<td>4.19 7.05</td>
<td>4.04 4.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.06 10.47</td>
<td>0.89 5.73</td>
<td>2.63 2.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.57 10.14</td>
<td>4.07 7.01</td>
<td>3.86 3.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.09 10.51</td>
<td>0.87 5.72</td>
<td>2.60 2.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.57 10.14</td>
<td>4.08 7.01</td>
<td>3.87 3.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.58 13.49</td>
<td>10.03 11.58</td>
<td>9.86 9.86</td>
</tr>
<tr>
<td></td>
<td></td>
<td>7.58 13.49</td>
<td>10.03 11.59</td>
<td>9.86 9.86</td>
</tr>
<tr>
<td>( 0.00 &lt; x \leq 0.05 )</td>
<td>( T-t \leq 30 )</td>
<td>4.18 9.10</td>
<td>2.48 4.66</td>
<td>4.33 4.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.41 9.50</td>
<td>3.37 5.20</td>
<td>5.21 5.79</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.36 9.20</td>
<td>2.38 4.57</td>
<td>4.11 4.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.37 9.49</td>
<td>3.21 5.10</td>
<td>5.08 5.67</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.38 9.22</td>
<td>2.38 4.85</td>
<td>4.07 4.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.37 9.49</td>
<td>3.21 5.10</td>
<td>5.09 5.68</td>
</tr>
</tbody>
</table>

Table 6.6: Relative pricing errors (%) of alternative models with implied volatility or volatility risk. Refer to Table 6.5 for a description.
6.4. PRICING OF STOCK OPTIONS

<table>
<thead>
<tr>
<th></th>
<th>ITM</th>
<th>$0.05 &lt; x \leq 0.30$</th>
<th>$x &gt; 0.30$</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.42 1.76</td>
<td>1.52 2.59</td>
<td>3.01 3.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.72 1.87</td>
<td>1.00 2.47</td>
<td>2.71 3.07</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.31 1.79</td>
<td>1.73 2.66</td>
<td>3.11 3.22</td>
</tr>
<tr>
<td></td>
<td>ITM</td>
<td>-0.74 1.87</td>
<td>0.94 2.45</td>
<td>2.64 3.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.31 1.79</td>
<td>1.73 2.66</td>
<td>3.08 3.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.73 1.87</td>
<td>0.95 2.45</td>
<td>2.65 3.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.80 1.91</td>
<td>2.09 3.12</td>
<td>7.63 7.70</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.81 1.92</td>
<td>2.10 3.13</td>
<td>7.40 7.46</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.46 0.59</td>
<td>-0.06 0.68</td>
<td>0.10 0.77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.58 0.67</td>
<td>-0.46 0.75</td>
<td>-0.48 0.90</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.46 0.59</td>
<td>0.04 0.70</td>
<td>0.23 0.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.59 0.67</td>
<td>-0.47 0.75</td>
<td>-0.50 0.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.46 0.59</td>
<td>0.04 0.70</td>
<td>0.23 0.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.59 0.67</td>
<td>-0.47 0.75</td>
<td>-0.49 0.91</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.58 0.67</td>
<td>-0.40 0.73</td>
<td>0.52 1.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.58 0.67</td>
<td>-0.40 0.73</td>
<td>0.52 1.47</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.36 8.18</td>
<td>1.69 4.27</td>
<td>2.59 6.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.57 8.83</td>
<td>4.53 6.21</td>
<td>9.19 9.89</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.51 8.37</td>
<td>0.99 3.98</td>
<td>0.70 6.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.67 8.85</td>
<td>4.40 6.12</td>
<td>8.91 9.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.50 8.37</td>
<td>0.99 3.98</td>
<td>0.65 6.17</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.67 8.85</td>
<td>4.41 6.12</td>
<td>8.92 9.66</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2.69 8.75</td>
<td>4.50 5.86</td>
<td>20.61 21.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-2.69 8.75</td>
<td>4.51 5.87</td>
<td>20.62 21.05</td>
</tr>
</tbody>
</table>

Table 6.6 continued.
SV models significantly, especially for deep OTM and deep ITM and long-term options. Again, measured by the implied volatility the asymmetric SV models exhibit implied volatility very close to those implied from observed option prices, its plot has the same curvature as that in Figure 6.8 but with different level; Finally, the asymmetric models, however, still exhibit systematic pricing errors, namely underpricing of short-term deep OTM options, overpricing of long-term deep OTM options, and underpricing of deep ITM options. This is consistent with our diagnostics of the SV model specification, i.e. the SV models fails to capture the short-term kurtosis of asset returns. And recall the salient features of the stock returns reported in Figure 6.1, the big downside risk anticipated by option traders may be related to the historical large negative returns. These large negative returns induce a very long but thin left tail, which the SV models fail to capture. More importantly, such consistent findings in the diagnostics of the underlying model specification and the performance of option pricing model only suggest that the option pricing errors of the SV models do not provide sufficient evidence to reject the hypothesis of market efficiency. It should be noted that while statistically these pricing errors appear to be large, as high as 20% for short-term OTM options, its economic implications may not be so significant. For instance, for short-term deep OTM options, a 20% relative pricing error correspond to an absolute error of $0.05 on the average, which is smaller than the average effective bid-ask spread. Furthermore, the MARE statistics, a measure of the dispersion of the relative pricing errors, are not reduced as much as the MRE statistics.

6.5 Conclusion

In this chapter we specify an SV process in a multivariate framework to simultaneously model the dynamics of stock returns and interest rates. The model assumes a systematic component in the stock-return volatility and “leverage effect” for both stock-return and interest-rate processes. The proposed model is first estimated using the EMM technique based on observations of underlying state variables. The estimated model is then utilized to investigate the respective effect of systematic volatility, idiosyncratic volatility, and stochastic interest rates on option prices. The empirical results are summarized as follows.

While theory predicts that the short-term interest rates are strongly related to the systematic volatility of the consumption process, our empirical results suggest that the short-term interest rate fails to be a good proxy of the systematic factor. However, the short-term interest rate is significantly correlated with the “mean” of the stock returns, suggesting stock return is predictable to certain extent. Such predictability is shown to have a noticeable impact on option prices as the repro-
6.5. CONCLUSION

jected underlying volatilities are different in magnitude in alternative model specifications. While allowing for stochastic volatility can reduce the pricing errors and allowing for asymmetric volatility or “leverage effect” does help to explain the skewness of the volatility “smile”, allowing for stochastic interest rates has minimal impact on option prices in our case. Similar to Melino and Turnbull (1990), our empirical findings strongly suggest the existence of a non-zero risk premium for stochastic volatility of stock returns. Based on implied volatility risk, the SV models can largely reduce the option pricing errors, suggesting the importance of incorporating the information in the options market in pricing options. Both the model diagnostics and option pricing errors in our study suggest that the Gaussian SV model is not sufficient in modelling short-term kurtosis of asset returns, an SV model with fatter-tailed noise or jump component may have better explanatory power. An important implication of the consistent findings in the diagnostics of the underlying model specification and the performance of option pricing model is that the option pricing errors of the SV models do not provide sufficient evidence to reject the hypothesis of market efficiency. Finally, the failure of short-term interest rate as a valid proxy of systematic volatility component suggests that in the future study, an alternative state variable, say a market index, should be used to study the impact of systematic volatility on option prices. Our empirical results also suggest that normality of the stochastic volatility model may not be adequate for this data set and other data sets as well. These results have been verified in Jiang and van der Sluis (1998b) with the same methodology using S&P500 data (not reported here). We leave it in our future research to explore a richer structural model, for example the jump-diffusion and/or the SV model with Student-t disturbances, to describe the dynamics of asset returns.
6.5 Conclusion

In this chapter, we specify a SV process in a multifactor framework to simultaneously model the dynamics of stock returns and interest rates. The model assumes a syndromic composition in the stock-return volatility, along with the effect of both stock-return and interest-rate processes. The proposed model is estimated using the LMM technique based on observations of underlying variables. The estimated model is then utilized to investigate the respective effect of systematic volatility, idiosyncratic volatility, and mean of the interest rates on option prices. The estimated results are summarized as follows.

While theory predicts that the short-term interest rate is strongly related to the systematic volatility of the consumption process, our empirical results suggest that the short-term interest rate fails to be a good proxy of the systematic factor. However, the short-term interest rate is significantly correlated with the "sustainable" growth process, suggesting stock returns are dependent on consumption. With possibilities to choose to have a non-stochastic jump on option prices due to