Heterogeneity of Hazard Rates in Insurance.
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Chapter 1

Introduction

1.1 Motivation

As the title indicates, in this thesis three themes are combined. Generally stated, a hazard rate is the probability of a certain event at some point of time, conditionally given what has happened before that time. As the description shows, there is always a time dimension when dealing with hazard rates. An elementary example of a hazard rate is found in elementary life contingencies: the probability of dying in a certain year conditionally given survival until the beginning of that year. A hazard rate can apply to a human life or an item such as a light bulb.

We are also considering the theme of heterogeneity. This means that we take into account the fact that the hazard rates for similar items are not the same. Not all individuals aged 60 have the same probability to survive the next 20 years, and not all unemployed with the same employment history have the same chance to find a job within some years' time.

The word insurance, finally, indicates the field of application of the first two concepts. In this respect, the items to which hazard rates pertain can for instance be contracts in an insurance portfolio and potentially insured individuals.

Up to now, only few contributions have appeared in the literature in which all three elements mentioned are considered. Heterogeneity models based on hazard rates exist and have found many applications in several fields such as reliability theory, medical statistics, labor economics and demography, but not so many in insurance mathematics and insurance economics.

Life insurance mathematics is largely based on the hazard rate approach, since life contracts usually comprise several years and there are always probabilities applying (e.g. the probability to die or to become disabled). On the other hand in life contingencies, the heterogeneity aspect plays only a minor role. Instead, the traditional assumption is that of homogeneity, i.e. that all individual persons, when they are identical with respect to the rating factors applied (usually these are restricted to age, gender, state of health and sometimes smoking habits), and to their life history, represent identical risks. In other words, differences in risk profiles between contracts allocated to the same risk class are ignored. An exception is Norberg (1989), discussing experience rating in group
life business, thereby explicitly taking into account the differences in mortality patterns between collective contracts. Furthermore, Groot (1996) discusses solidarity aspects for some elementary cases in individual life contingencies.

In non-life insurance mathematics, the concept of heterogeneity is commonplace. Many non-life contracts are subject to a more refined risk classification scheme than life policies, thus accounting for more heterogeneity in a portfolio. Moreover, some non-life actuarial models explicitly take unobserved heterogeneity between contracts into account. In these models, it is assumed that each individual contract has its own random risk parameter. These models are used for experience rating techniques. Examples are bonus-malus systems and credibility models.

Up to now, non-life actuaries have made little use of models based on hazard rates. This can partly be understood by the fact that many non-life actuarial models study risks over a single period, omitting the time element. But even in non-life models with a time component, the approach followed is usually not by means of hazard rates. An exception is Keiding et al. (1998).

By combining the concepts of "heterogeneity" and "hazard rates" in one approach, one can develop more generalized insurance models and, as a consequence, tackle many problems in insurance mathematics and insurance economics. The main aim of this thesis is to consider insurance issues where both the themes "hazard rates" and "heterogeneity" play an important role. Both life and non-life topics will be considered. Life applications can be found in Chapters 2 to 4. Chapter 6 considers a typical non-life issue, while the model in Chapter 5 can apply to both life and non-life business.

1.2 Outline of the thesis

As mentioned in the previous section, in non-life insurance mathematics models exist which take account of unobserved heterogeneity between contracts. In such cases we are then in fact dealing with a so-called urn-of-urns model, where the outcome of a risk corresponding to a contract, which is often the aggregate claim amount in the contract period, is the result of a two stage process. Imagine a large urn containing some smaller urns. Each smaller urn corresponds to a certain risk profile. The first stage involves the draw of one of these smaller urns. This draw determines the risk profile of the contract. Then, in the second stage, the actual outcome is drawn from the inner urn with the given risk profile. The result of the first stage draw is not observable to the insurer and therefore, at the time of inception of the policy, all contracts involved in such a process pay the same average premium. As a consequence, some risks (the "good" ones) subsidize other ("bad") ones. However, as time passes and claims data are generated, premiums for each contract can be adapted to the claims experience and so the cross-subsidization, or subsidizing solidarity, between contracts gradually diminishes, which in return decreases the chance of adverse selection. Besides, as shown in De Wit & Van Eeghen (1984), in any contract period the subsidizing solidarity can be quantified, giving the insurer an indication of the danger of adverse selection.
The urn-of-urns model defined in the previous paragraph will take a central place in Chapter 2. It will be applied to a general individual life insurance contract introduced in Norberg (1990, 1991, 1992). In this context, the hazard rates involve the probabilities to make a transition from one state to another at some point of time during the contract term, conditionally given the event history of the individual insured. An example is the probability for an active person to become disabled at the beginning of the second policy year, conditionally given that he has never got disabled before. The heterogeneity is assumed to be unobserved. For instance, age and gender may be observable risk factors, but not the life-style of the individual. The aim of Chapter 2 is to consider experience rating as described above and to derive measures of subsidizing solidarity for individual life contracts, in the same way as done in De Wit & Van Eeghen (1984). These measures will be defined not only on the level of the entire contract term but also on the level of parts of it, because life insurers are interested in quantities accounting to policy years. Not only Norberg’s model in general will be discussed but also special cases thereof, where the quantities derived may reduce in complexity. Regarding the elementary life model, where only the two states "Alive" and "Dead" apply, part of all this work has been done in Spreeuw (1996) and Spreeuw & Wolthuis (1997). Both papers mentioned have highlighted the complications that arise in life insurance, as a consequence of the fact that premium payment is often not restricted to time-at-issue (in other words, not all contracts are paid by single premium).

In individual life insurance, experience rating actually makes no sense as there is no difference between updating the premiums to experience and not updating at all. This will also be confirmed in Chapter 2. Hence, if an insurance company wants to decrease cross-subsidization between contracts, it has to find out other ways. In Chapters 3 and 4, such a method is proposed. We introduce an individual life insurance system in a discrete time framework, based on sharing the aggregate mortality result realized in a certain period among either the survivors or the deaths’ heirs in a certain portfolio. For the sake of simplicity, all individual contracts have the same amount at risk in the period mentioned. The mortality result for the given period times a fixed predetermined proportion will, under certain conditions, at the end of the period be equally divided among either the survivors or the deaths’ heirs. Each individual is charged at the beginning of the period an average risk premium. The portfolio is heterogeneous, which means that the mortality rates related to the contracts differ. The system can be used for any part of the contract term. If this part is a policy year, for instance, the hazard rates are to be interpreted as the probabilities of dying in that year, conditionally given survival up to the beginning of that year.

In Chapter 3, it will be analyzed whether the sharing system has a decreasing impact on the subsidizing solidarity, defined and quantified in Chapter 2. It is assumed that the insurer knows the mortality rate of each individual. This somewhat artificial assumption is made in order to examine whether applying the system may lead to a lower subsidizing solidarity under any circumstances. One special case is based on Spreeuw (1998c), who considers distribution to the survivors, if any, and an insured portfolio consisting of only two risk classes. In Chapter 3, it will also be studied how the system affects the variability in the insurer’s portfolio results, expressed by the variance of the aggregate loss.
In Chapter 4, it will be demonstrated how safe bounds for the average risk premium can be obtained if the information available to the insurer is less complete than in Chapter 3. For two situations, it will be shown that, provided some specific information is known, safe bounds of the average risk premium can be obtained by means of the majorization order. In both cases, it is assumed that the insurer knows the minimum and maximum possible individual mortality rate in the portfolio considered. In the first case, the average mortality rate is assumed to be known. In the second situation, it is supposed that the individuals can be allocated to several groups. In addition, the mortality rates are taken to be stochastic themselves, being a function of an individual risk factor, a group specific risk factor and an overall risk factor, just as in Tong (1989), Bäuerle (1997) and Bäuerle & Müller (1998). One example treated is derived from Spreeuw (1998b), where the case of "division among the survivors when there are survivors" is considered.

Adverse selection, based on the expected utility hypothesis, will be a major theme in Chapter 5. A model is considered based on a population consisting of two risk groups, namely the high risks, being the individuals with a high probability of making an accident, and the low risks, those with a low probability of making an accident. So there is heterogeneity between the two risk types. It is supposed that each individual in the population knows which risk type applies to him and that, on the other hand, the insurance company knows which proportion of individuals belongs to which of the two risk classes, but cannot monitor the risk profile per individual. By assumption, having an accident results in the loss of a certain monetary amount. Literature has shown that insurance firms may be able to cope with the problem of adverse selection by offering two different insurance contracts, one giving full coverage, should the accident occur, and one involving a certain monetary deductible. These contracts are designed in such a way that a certain self-selection mechanism among the individuals is induced. The high risks will prefer the contract with full coverage while the low risks will choose the one with partial coverage. Equilibria in an insurance market may result. In this context, an equilibrium is defined as the set of contracts which will be offered by each insurer in the market as no firm is induced to offer other policies. The screening device of a monetary deductible has been analyzed for both a competitive (Rothschild & Stiglitz, 1976, Wilson, 1977, Miyazaki, 1977, and Spence, 1978) and a monopolistic (Stiglitz, 1977) insurance market. However, such an instrument may be ineffective in case of life insurance (where "making an accident" means "dying") as the crucial assumption, that insurers perfectly share their knowledge about the purchase of insurance by any individual, may not hold. This is the reason why, in Chapter 5, the probationary period, during which time there is no coverage in case of an accident, is discussed as an alternative to the monetary deductible. Compared with the papers just mentioned, a time dimension is added. Besides it is assumed that each individual can make an accident only once. Hence, the time-dependent hazard rates are in this respect equal to the probabilities of making an accident, conditionally given that no accident has been made before from time-at-issue on. Both the monopolistic and the competitive insurance market will be considered and it will be analyzed how the equilibria resulting from the probationary period as a screening device differ from those obtained from the monetary deductible case. Part of this research, regarding a monopolistic insurer, was carried out in Spreeuw (1998a).
Chapter 6 indicates that an approach by means of hazard rates can be used to tackle a typical non-life problem, namely that of predicting future claim numbers. Up to now, almost exclusively parametric models appeared in the literature, which carry the risk that they may be misspecified. Assuming that each contract can report at most one claim (so the probabilities of reporting a claim in some development year, conditionally given that no claim has been reported before, serve as hazard rates) statistical models to predict claim numbers, based on hazard rates, are at the disposal of the actuary. A major one of these is the Cox model which has the property of being semiparametric, so, contrary to the parametric models, not all quantities to be estimated have to be specified. Hence the chance of a misspecification is less when applying the Cox model compared to applying a parametric model. If, except for "development time", being the time dimension, either "year of origin" or "calendar year of reporting" is considered to be a significant factor, hence reflecting the heterogeneity in the portfolio, the discrete time version of the Cox model reduces to either the chain ladder method or the separation method, respectively. The chapter is an extended and improved version of Spreeuw & Goovaerts (1998).

Chapter 7 gives an overview of the main conclusions on the five chapters considered.

1.3 The common themes

The chapters that follow are quite diverse, and it may be tempting for the reader to conclude that this thesis is nothing but a set of extended versions of papers. Nevertheless, as stated before, the chapters do have some topics in common:

1. In all chapters, a hazard rate approach applies. In each case we are dealing with an event history which is unique throughout time and cannot be repeated again.

In Chapter 2, the hazard rate approach involves the several states visited and left by the insured. In Chapters 3 and 4, an individual may die, after which the corresponding contract is removed from the portfolio. In Chapter 5, the assumption is made that an insured can have at most one accident during the contract period, while, as mentioned in the previous section, it is presumed in Chapter 6 that each contract can report at most one claim.

2. In all chapters there is heterogeneity with respect to risk profile.

In Chapters 2, 4 and 5, this heterogeneity is assumed to be unobservable for the insurer. On the other hand, in Chapter 3 the insurer knows the mortality rate of each individual in the heterogeneous portfolio. In Chapter 6 the insurer has perfect information on year of origin and calendar year of reporting and hence the heterogeneity is observed, at least regarding the chain ladder and separation method.

3. All chapters involve both life and non-life topics.

Chapters 2 to 4, dealing with life insurance, are based for an important part on the non-life approach in De Wit & Van Eeghen (1984). The contracts considered in Chapter 5 on monetary deductibles were primarily developed for non-life applications. The contracts with a probationary period may concern both non-life (e.g.
medical and dental insurance) and life applications. Chapter 6, finally, involves a non-life application, but the assumption is made that a contract can report a claim only once. Hence, the number of contracts which might report a claim in the future, gradually decreases in size. This phenomenon is typical for portfolios of individual life contracts where only the states "Alive" and "Dead" apply, because when an individual dies, the contract related will (sooner or later) be removed from the portfolio.

In this respect, the approach in this thesis has something in common with the one in Kling (1993), considering life insurance from a non-life point of view.