Heterogeneity of Hazard Rates in Insurance.
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Chapter 7
Conclusions

In this thesis, two themes were combined in insurance mathematics and insurance economics. In insurance models, the themes, hazard rates and heterogeneity, have not often been studied together before. The research has lead to five self-consistent chapters in which the problems of heterogeneity in models with hazard rates were discussed. We came to some interesting solutions.

Chapters 2 and 3, both dealing with life insurance, are to a large extent founded on the quantitative measures of solidarity developed by De Wit & Van Eeghen (1984). They base the subsidizing solidarity on the difference between the actual loss suffered and the loss that would have occurred in case of equivalence on the level of an individual contract. This difference, called the parameter effect, is fixed for non-life contracts but random for life contracts, except when the contract is paid by single premium. In Chapter 2, the subsidizing solidarity defined for a life insurance agreement has been based on the expected values of the parameter effects. This has not only been done for the entire contract term but also for parts of it. For this situation, we have defined parameter effects for parts of the contract term by specifying appropriate amounts at risk and risk premiums.

In our analysis in Chapter 2, we have based ourselves on the general life insurance treaty developed by Norberg (1990, 1991, 1992). We have also considered a special case thereof, namely a hierarchical Markov chain, which has been discussed in Wolthuis (1994). Then the solidarity measures can not only be allocated to parts of the contract period, but also to states such as "Alive" and "Disabled for the third time". The formulas reduce even more in complexity if only the classical two states "Alive" and "Dead" apply, and there are only premium and benefit payments at integer times, i.e. at the beginning or the end of a policy year. The measures for the subsidizing solidarity are then relatively easy to compute.

The quantity in the case just mentioned is applied in Chapter 3, introducing a system based on sharing the aggregate mortality result allocated to a certain period (which can very well be a policy year). It appears that the system designed here can contribute to a lower level of subsidizing solidarity, namely in case of division among the deaths’ heirs if the actual number of deaths is at least above average. In order to share only positive mortality profits, the amount at risk should then be of negative sign. This involves insurance agreements with negative amounts at risk, such as annuities. In Chapter 3, it has also been demonstrated that there are cases where the subsidizing solidarity reduces
to zero, but then there is no insurance at all either. In any case, the system at least has one benefit from the insurer's point of view, namely that it has a decreasing impact on the variance of the aggregate loss. This variance is minimal if the entire mortality result is shared.

In Chapter 3, the insurer has complete information on all individual mortality rates. In Chapter 4, the information is incomplete. We studied two different situations, assuming that the insurer knows the minimum possible and the maximum possible mortality rate. In the first case the average mortality rate is known. If additionally only positive mortality profits are divided, we showed that it is in many cases safe to assume that this average mortality rate pertains to all individuals. In the second case, the mortality rates are random and specified as in the model discussed in Tong (1989), Bäuerle (1997) and Bäuerle & Müller (1998). Then deriving the safest risk premium boils down to making the safest assumption about the group structure of the portfolio. By group structure we mean the number of groups to which the individuals can be allocated and the sizes of these groups. In that case, it is in general hard to draw conclusions. For some specific examples it has turned out that, in case of a portfolio small in size and low mortality rates, it is safe to assume that all individuals stem from different groups, provided that only positive mortality profits are shared.

Problems to obtain conclusions in general have also been experienced in Chapter 5, discussing imperfect information in combination with adverse selection and the screening device of a probationary period. Whereas the results obtained by Rothschild & Stiglitz (1976), Wilson (1977), Miyazaki (1977), Spence (1978) and Stiglitz (1977), all regarding the monetary deductible as a screening instrument, are quite straightforward, this does not apply for the probationary period. This is to a large extent due to the fact that so many different specifications of the distribution functions of time-at-accident (or, equivalently, so many different specifications of the time-dependent hazard rate functions) of both risk classes are possible. However, we found out that the equilibria resulting in case of a probationary period do have much in common with the equilibria derived by the authors just mentioned, if a so-called partial stochastic order between the distribution functions of a high risk and a low risk class individual holds (this means that the probability of causing an accident before any relevant point of time for the high risks is at least as high as the corresponding probability for the low risks). It has been proved that in that case the high risks always purchase full coverage, i.e. insurance without a probationary period, irrespective of the market type (whether it is monopolistic or fully competitive). This property also holds for the monetary deductible as a screening instrument.

If there is one monopolistic insurer, the low risks will always pay the maximal premium they are willing to pay, whatever probationary period applies to them. If the utility of the individual is of exponential form, it has been shown that the insurer's strategy yielding maximal profit is under certain conditions similar to the one derived by Stiglitz (1977) for the monetary deductible. This means that the low risks always pay the maximal premium they are willing to pay and that they will not buy any insurance (as the premium is too high) if the actual proportion of high risks in the population is sufficiently high. One important difference compared with Stiglitz' conclusions is, however, that it may be optimal to offer the same contract of full coverage to both groups. This is the case if the
proportion of high risks in the population is sufficiently low.

Concerning the situation of a competitive insurance market, we made the crucial assumption that all companies take into account their competitors' rational reactions to their own strategy. This, together with the assumptions made before, implies that an equilibrium in the market always exists. The high risks will pay a premium which is at most actuarially fair. Since each insurer must break even on average, the low risks must therefore pay a premium being at least actuarially fair. This implies that there may (still) be subsidizing solidarity: the low risks subsidize the high risks. Just as in the case of a monopolistic insurance market, the exponential utility function has been taken as the main example. Under this specification, the equilibrium resulting has much in common with the one derived in Miyazaki (1977) and Spence (1978), provided some restrictions hold. This means that, if the actual proportion of high risks in the population exceeds a certain critical level, all individuals will pay actuarially fair premiums. Otherwise, the probationary period for the low risks will be lower than in the case just dealt with, but the low risks will subsidize the high risks. Just as in the case of a monopolistic insurer, an equilibrium may involve a pooling contract of full coverage - where the low risks subsidize the high risks - if the proportion of high risks in the population falls below another certain critical level. Such an equilibrium does not exist in the monetary deductible case.

In Chapter 6, a statistical model based on hazard rates and observed heterogeneity is applied to predict future claim numbers. The hazard rate is in this context equal to the probability of reporting a claim in some year, conditionally given that it has not been reported before. The chain ladder method and the separation method (where the heterogeneity involves the year of origin and the calendar year of report, respectively) have been treated as special cases of the semiparametric Cox model we used, and it has turned out that the parameters are relatively easy to estimate by means of maximum likelihood. In addition, the model allows for including other exogenous factors, such as age and region.

To conclude, it can be stated that the aim of this study has been fulfilled. The topics of hazard rates and heterogeneity can very well be combined in actuarial models. To actuarial researchers it is advised to develop generalized models comprising both a time dimension and a heterogeneity component. This study may help them to start up.

Some models in this thesis are more convenient for application in actuarial practice in the short term than others. We recommend non-life actuaries to apply the semiparametric Cox model in Chapter 6 for predicting future claim numbers. Actuaries employed at individual life companies are encouraged to develop systems of mortality profit sharing (dealt with in Chapters 3 and 4) based on the assumption that an insurer has only incomplete information available about the risk profile of individuals.