Government decisions on income redistribution and public production
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2 Public consumption and redistribution

2.1 Introduction

Computable general equilibrium models are, in particular, convenient for the analysis of allocative and redistributive effects of government policies. In Chapter 1 we reviewed models that are equipped for the study of alternative fiscal policies. In most of these models the description of the public sector is rather poor. Government spending is, for example, not further specified and exogenously given [cf. AUERBACH AND KOTLIKOFF (1987) and GOULDER AND SUMMERS (1989)]. In BOVENBERG (1986, 1989) the public sector demands two privately produced goods, one domestic good and one foreign good. Public consumption levels are exogenously given, but public expenditures depend on the prices of the goods. Public consumption is determined in an ad hoc way by the preferences of a public household in SERRA-PUCHE (1984) and BALLARD ET AL. (1985). The latter study also distinguishes public enterprises that provide private goods using a cost minimizing rule. A similar approach for the determination of the level of a pure public consumption good is used in PEREIRA (1994), where in addition attention is paid to the production of the public good. In KELLER (1980) public consumption also follows from an ad hoc public utility function, but there this function is based on individual utility functions.

Apart from the poor representation of the public sector, tax rates and the values of social security variables are exogenous in the above models. In some studies, though, the tax base is endogenous and the tax rate is derived from a balanced budget condition. In COUGHLIN ET AL. (1990b) the public consumption level as well as the tax rates are determined endogenously in a probabilistic voting model. In that paper group specific taxes are assumed. The model misses the feedback with the economic system. In RUTHERFORD AND WINER (1990) the probabilistic voting model is linked with a general equilibrium model. Both the level of the public consumption good and group specific tax rates are endogenously determined. RUTHERFORD AND WINER (1995) base their economic model on BALLARD ET AL. (1985). In the latter paper of Rutherford and Winer, tax rates on capital and labor still follow from the maximization of political support, but the size of public consumption follows from the balanced budget condition and is not an argument in the political support function.

A different approach with respect to government decisions is used in VAN VELTHOVEN AND VAN WINDEN (1986). As discussed in Chapter 1, the public consumption level and the uniform tax rate follow in that paper from the maximization of the value of
a political interest function. In addition, the number of public sector workers is
determined by the public production process. The economic feedback follows from a
Keynesian model.

In this chapter government decisions are also derived from the maximization of the
political interest function. Instead of a Keynesian model, a computable general
equilibrium model is used. In the model, a pure public consumption good enters the
utility functions of the consumers. The production of the public good requires public
capital and labor. The government decides upon the level of public consumption,
which depends on the preferences of representative individuals of social groups,
parameters that define the technology in the public sector, and the political influence
structure. The political influence structure determines to which extent the interests of
individuals of the different social groups are taken into account by the politicians. (see
Chapter 1). Public production costs are financed with a uniform income tax. The tax
affects the income that individuals can spend on other goods. The government takes
account of the negative effect of taxes on income and private consumption when
deciding upon public consumption.

The government not only decides on the level of public consumption, but also on the
redistribution of income. In political economics a median voter model is typically used
for the analysis of income redistribution [cf., e.g., MELTZER AND RICHARD (1981)
and PERSSON AND TABELLINI (1992)]. An exception is BECKER (1983) who uses a
model with pressure groups. In studies using the median voter model, the official tax
system determines the redistribution of income. As will be argued in Section 2.3,
actual redistribution depends not only on the official tax system but also on special
provisions that may not be observed by all individuals. These special provisions are
affected by political influence in so far as this is not reflected by the tax system. They
are inserted in the model to be presented in this chapter by way of lump sum
transfers.

In the private sector, two consumer groups are distinguished: capital owners and
workers. They derive utility from the consumption of private commodities, leisure and
the public consumption good. The model thus allows for an endogenous leisure-labor
supply decision. Utility is maximized subject to a budget constraint, implying that
consumers cannot spend more on private consumption and leisure than their after-tax
full income. The full income of workers consists of the revenues from their labor
endowment and of special provisions. Capital owners receive, in addition, income
from their (exogenously given) capital endowments. The capital owners run the firms.
They produce a homogeneous commodity. Private capital and labor are used as inputs in the production process. The production level of private commodities follows from the maximization of profits. Prices are endogenously determined and follow from the equilibrium conditions on the markets for private commodities, capital and labor. The combination of this general equilibrium model with the political interest function model renders a positive general equilibrium model.

Apart from this introduction, the organization of this chapter is as follows. Behavior of producers and consumers is modeled in Section 2.2. This section also gives the private sector equilibrium. Section 2.3 specifies decisionmaking by the government. Comparative static results are derived and discussed in Section 2.4. The outcomes of the general equilibrium model with endogenous government decisions are compared with the outcomes of a traditional general equilibrium model in Section 2.5. In Section 2.6 the tax system of the model, consisting of a uniform tax rate for the finance of public production costs and a self-financing lump sum redistribution system, is compared with a lump sum tax system. Section 2.6 also addresses the issue of tax reform. Section 2.7 concludes.

2.2 Private sector behavior

2.2.1 Introduction

Decisions in the private sector, that are relevant for the analysis at hand, are made by consumers and producers under conditions of perfect competition. Consumers maximize their utility subject to an income constraint and producers maximize their profits while facing a given production technology. Consumers and producers take the decisions of the public sector as given, while maximizing utility and profits. An equilibrium in the private sector exists if demand equals supply in all commodity and factor markets. The behavior of producers is discussed in Subsection 2.2.2. Then the behavior of consumers is modeled in Subsection 2.2.3. The conditions for an equilibrium in the private sector are presented in Subsection 2.2.4.

2.2.2 Production

Producers, running identical firms, are assumed to operate in a competitive market for a single commodity. They maximize profits under a Cobb-Douglas production
technology, using capital and labor as inputs.\(^1\) Profits are equal to the firm’s revenues minus its input costs. Using aggregate variables, for convenience, producers are confronted with the following objective

\[
\Pi_c = p_c X_c - p_L L_c - p_K K_c
\]

(2.1)

where \(\Pi_c\) refers to profits, \(X_c, L_c\) and \(K_c\) stand for the production level, labor demand and capital demand, respectively, and \(p_c, p_L\) and \(p_K\) denote the respective prices. Production is given by

\[
X_c = \Omega_c L_c^{\delta_{lc}} K_c^{\delta_{kc}}, \quad \delta_{lc} + \delta_{kc} = 1, \quad \Omega_c > 0
\]

(2.2)

where \(\Omega_c\) is a scaling parameter, \(\delta_{lc}\) indicates the production elasticity of labor, and \(\delta_{kc}\) the production elasticity of capital.

From the profit maximization by firms, the following optimal choices with respect to capital and labor input are obtained

\[
K_c = \frac{\delta_{kc} p_c X_c}{p_K}
\]

(2.3)

\[
L_c = \frac{\delta_{lc} p_c X_c}{p_L}
\]

(2.4)

The input conditions indicate that capital costs and labor costs are a fixed part of the production revenues. The constant returns to scale assumption leads to equality between revenues and costs. This result is independent of the production level. Note, finally, that the capital-labor ratio is determined by the input price ratio.

\[\text{2.2.3 Consumption}\]

Consumers are divided into two social groups, capital owners and workers, indexed by \(i = c, w\), respectively. The former also run the firms, and are therefore also called capitalists-entrepreneurs (or entrepreneurs, for short). The income of workers only consists of labor income, while the income of capital owners consists of both labor

\[\text{\(^1\) Empirical support for the assumption that the elasticity of substitution is equal to one can be found in BERNDT (1976).}\]
and capital income. It is assumed that consumers within a social group have identical preferences; among these groups preferences may differ, however. The utility of a consumer depends on the consumption of a private commodity $c_i$, a public consumption good $G_s$, and leisure $\ell_i$. Consumers maximize utility, subject to a budget and a time constraint. Utility functions $U_i$ are assumed to be of the Cobb-Douglas type

$$U_i = c_i^{\alpha_c} \ell_i^{\alpha_l} G_s^{\alpha_G}, \quad \alpha_c + \alpha_l + \alpha_G = 1, \quad i = c, w$$

(2.5)

where the $\alpha_i$'s are the preference weights.

For simplicity, the labor and time endowments are set equal to unity. The time constraint yields $0 \leq \ell_i \leq 1$. The budget constraint demands that expenditures on the private commodity and leisure must be equal to the consumer’s full income

$$p_c c_i + p_l \ell_i = f_i, \quad i = c, w$$

(2.6)

where $p_i$ is the shadowprice of leisure, which is equal to the after-tax wage rate $(1-\tau_h)p_L$, and $f_i$ stands for disposable full income. Full income of workers consists of the income they receive if they fully supply the labor endowment and a group-specific transfer $\sigma_i$, which may be negative. For capital owners it includes, in addition, the income from their capital endowment $k^0_c$. The transfer system, discussed in detail in Section 2.3, self-financing; that is, $\sigma_c N_c + \sigma_w N_w = 0$, where $N_c$ and $N_w$ refer to the number of entrepreneurs and workers, respectively. The disposable (after-tax) full income of entrepreneurs and workers equals

$$f_c = (1-\tau_h)(p_c + p_k k^0_c + \sigma_c)$$

(2.7)

$$f_w = (1-\tau_h)(p_L + \sigma_w)$$

(2.8)

Assuming that there is an interior solution, the demand for the private commodity $c_i$ and for leisure $\ell_i$ is given by

$$c_i = \frac{\alpha_c f_i}{(\alpha_c + \alpha_l)p_c}, \quad i = c, w$$

(2.9)

$$\ell_i = \frac{\alpha_l f_i}{(\alpha_c + \alpha_l)p_L}, \quad i = c, w$$

(2.10)
Thus, the demand for private commodities and leisure depends on disposable full income, the price of the private commodity and the shadowprice of leisure, respectively, and the respective relative preference weight.

Using these results, the indirect utility function for a member of social group $i$ can be written as

$$V_i = \frac{\alpha_i \alpha_i^{\alpha_i} f_i^{\alpha_i} G_i^{\alpha_i}}{(\alpha_i + \alpha_i^{\alpha_i})^{\alpha_i} \beta_c^\alpha \beta_t^\alpha p_c^\alpha p_t^\alpha}, \quad i = c, w$$

(2.11)

Thus, indirect utility will increase if disposable full income increases, if consumption of the public good increases, or if prices $p_c$ and $p_t$ decrease.

### 2.2.4 Private sector equilibrium

An equilibrium for the private sector is obtained if demand equals supply in all commodity and factor markets. In the model presented in this section, there is one commodity market, a capital market, and a labor market. The equilibrium on these markets determines the equilibrium values for the prices $p_c$, $p_K$, and $p_L$. The markets are in equilibrium if

$$X_c = c_c N_c + c_w N_w$$

(2.12)

$$k_c^0 N_c = K_c$$

(2.13)

$$(1 - \ell_c) N_c + (1 - \ell_w) N_w = L_c + L_s$$

(2.14)

where $L_s$ stands for the amount of labor demanded by the government for the production of the public good, and $N_c$ and $N_w$ refer to the numerical strength of the social groups of capital owners and workers, respectively.

Walras’ law makes one of the equilibrium conditions redundant. Therefore, only relative prices matter and one of the prices can be chosen as numéraire. In this chapter the wage rate is the numéraire, $p_L = 1$. Using eqs. (2.2), (2.3), (2.4), (2.7), (2.8) and

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2 Note that this would be a general equilibrium in case of a traditional general equilibrium model with an exogenous public sector.
(2.9), the following expressions can be derived from the equilibrium conditions (2.12) and (2.13) for the price of capital $p_K$ and the commodity price $p_c$

$$p_K = \frac{\delta_K (1 - \tau_h) [\alpha_c (1 + \sigma_c) N_c + \alpha_w (1 + \sigma_w) N_w]}{[1 - \alpha_c \delta_K (1 - \tau_h)] k^0_c N_c}$$ (2.15)

$$p_c = \frac{1}{\Omega_c \delta^\prime_c} \left[ \frac{(1 - \tau_h)[\alpha_c (1 + \sigma_c) N_c + \alpha_w (1 + \sigma_w) N_w]}{[1 - \alpha_c \delta_K (1 - \tau_h)] k^0_c N_c} \right]^\ast_c$$ (2.16)

where $\delta_c = \alpha_c / (\alpha_c + \alpha_w)$ for $i = c, w$. Labor market equilibrium follows from Walras' law.

Note that these prices are only a function of exogenous variables and the government policy variables $\tau_h$, $\sigma_c$ and $\sigma_w$. Prices do not depend on the public good $G_t$ because of the separability of the utility function [cf. eq. (2.5)]. Since all the remaining endogenous private sector variables, $L_c$, $K_c$, $X_c$, $c_i$, $\ell_i$ and $V_i$, can be written as functions of exogenous variables and the prices $p_c$ and $p_K$, they also depend only on exogenous variables and the aforementioned government policy variables. To save space, the expressions for those variables are not presented here.

Note from eq. (2.15) that the relative price of capital, $p_K$, decreases if the income tax rate $\tau_h$ increases. The reason is that in that case the demand for private goods ($c_c$ and $c_w$) decreases [cf. eq. (2.9)], leading to a lower price of the private good and, as a consequence, to a decrease in private production and private demand for labor ($X_c$ and $L_c$, respectively). Because capital is fixed in the private sector, a lower demand for labor decreases the price of capital, as follows from eqs. (2.3) and (2.4). The negative effect on the tax base that follows from the fall in capital income is mitigated by the increase in labor supply of capital owners [cf. eq. (2.10)]. Whether tax revenues and the public demand for labor will increase is ambiguous.

If the transfer $\sigma_c N_c$ to capital owners decreases the transfer $\sigma_w N_w$ to workers increases with the same amount, due to the assumption that the transfer system is self-financing. In that case the price of capital $p_K$ increases if the relative preference for the private commodity is higher for workers than for capital owners: $\delta_{cc} < \delta_{cw}$ (the reverse holds if $\delta_{cc} > \delta_{cw}$). The decrease in $\sigma_c$ causes demand for private commodities and leisure by entrepreneurs to decrease, while the increase in $\sigma_w$ increases the workers' demand for private commodities and leisure. If $\delta_{cc} < \delta_{cw}$ the increase in workers' demand for private commodities more than compensates the lower demand for private commodities.
by capital owners, which causes total demand for private commodities to increase. Labor supply will also increase. Because $\alpha_{cc} < \alpha_{wc}$ implies that $\alpha_{cc} > \alpha_{wl}$, the decrease in $a_c$ leads to a reduction in total demand for leisure. The increase in private demand leads to an increase in the commodity price and this raises private production and private labor demand. The latter increases the relative price of capital, because of the fixed capital input in the private sector. These results will be further discussed in Section 2.4, where the general equilibrium solution of the political economic model is investigated.

2.3 The public sector

In the public sector decisions are (formally) made by politicians who, like the private sector agents, are assumed to take account of their own interests. The behavior of politicians is, however, constrained by the structure of the economy and the reactions of private sector agents and bureaucrats. The reactions of these agents force politicians to take account of their interests, to some extent. In the present model these interests are summarized by the indirect utility functions of capital owners and workers. Following the interest function approach, government decisions are assumed to be in accordance with the maximization of the political interest function, which is a weighted representation of the interests of the representative individuals of the social groups. The weights reflect the effectiveness with which the groups get their interests promoted by the politicians. The political interest function reads

$$P = V_c V_w \cdot \mu_c + \mu_w = 1$$

(2.17)

where $\mu_c$ and $\mu_w$ denote the political influence weights of capital owners and workers, respectively. Using (2.11), the political interest function can be written as

$$P = \frac{\alpha_{cc} \alpha_{cw} \alpha_{wc} \alpha_{wc} \alpha_{cw} \alpha_{wc} f_c (\sigma_c + \sigma_w) f_w (\sigma_w + \sigma_w) G^c_v}{(\alpha_{cc} + \alpha_{wc}) f_c (\sigma_c + \sigma_w) G^c_v \mu_c + \mu_w}$$

(2.18)

where $a_j = \mu_c \alpha_{jc} + \mu_w \alpha_{jw}$, $j = c, \ell, G$. 

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3 For simplicity it is assumed that the preferences of public sector workers (bureaucrats) are similar to the workers' preferences. Because they receive the same wage as private sector workers, public sectors workers are not distinguished as a separate social group. The political influence weight $\mu_w$ is implicitly built up by the relative influence weights of workers and bureaucrats. The analysis for a separate group of bureaucrats is straightforward.
It follows from the optimization problem of the government that policies depend on the decisions of the private sector agents. Conversely, the decisions of private sector agents depend on the decisions of the government. It will be assumed throughout this monograph that private sector agents take government decisions as given. As a consequence, the private sector outcomes are optimal for every given public policy scheme. For the government a similar assumption is adopted. In particular, it is assumed that the government neglects the impact of her decisions on the prices of private commodities, capital and labor, as well as the shadowprice of leisure. The interaction between the public and the private sector is thus modeled in a Nash-Cournot fashion. This differs from the convention in the public economics (optimal taxation) literature, where the public sector is typically regarded as the dominant player. The underlying Stackelberg assumption entails that one player (the dominant player) moves before the other players and that this dominant player knows how the other players will react to its decisions. The Stackelberg assumption is relevant if one player acts before the other players or if one player announces an action before other players act, and commits her- or himself to this announced action. In case of no commitment, the dominant player may have an incentive to depart from the announced strategy and other players may not believe that the dominant player will actually enforce the announced strategy. The Stackelberg equilibrium is time-inconsistent in that case [cf. PERSSON AND TABELLINI (1990); see also Subsection 1.4.3]. In reality it is indeed typical for the public sector to announce its (fiscal) policy before the private sector acts, and to carry out this policy simultaneously with or after the actions of the private sector agents. However, the public sector can not commit herself to the announced policy. Therefore, the Stackelberg assumption does not give an appropriate description of the way the private and the public sector interact [cf. BLACKBURN (1987)]. Furthermore, if the government is a dominant player it must know the reaction function of the private sector. In the model presented here this implies that the government would be able to determine the impact of its policies on prices. In our view, this informational requirement is too severe. The Nash-Cournot assumption seems to give a more realistic description of the interrelation between private and public sector decisions than the Stackelberg assumption.

The governmental decisionmaking process is split up in decisions with respect to public production and redistribution. Production of the public good \( G_s \) requires (public) capital \( K_s \) and labor \( L_s \) as inputs. Assuming a Cobb-Douglas technology, the production function is specified as

\[
G_s = \Omega_s L_s^{\delta_{L_s}} K_s^{\delta_{K_s}}, \quad \delta_{L_s} + \delta_{K_s} = 1, \quad \Omega_s > 0
\]

(2.19)
Government holds a fixed capital stock \( K^0 \) that can be used for public production without further costs. Thus,

\[
K_i = K^0
\]

(2.20)

Labor costs are financed by an income tax \( \tau_h \). The government is not allowed to run deficits. The balanced budget condition requires that

\[
p_L L = \tau_h \left[ p_k k^0 + p_L (1 - \ell_c) + \sigma_c \right] N_c + \tau_h \left[ p_L (1 - \ell_w) + \sigma_w \right] N_w
\]

(2.21)

Apart from producing a public good, the government is able to redistribute income. This is done by a group-specific transfer \( \sigma_i \), \( i = c, w \). It is assumed that the redistribution system is self-financing, which gives the condition

\[
\sigma_c N_c + \sigma_w N_w = 0
\]

(2.22)

The division of the tax-transfer system in a uniform and a group-specific segment is supported by the following two observations. Firstly, following BRENNAN AND BUCHANAN (1980), the uniform income tax can be interpreted as the (quasi-) constitutionalized part of the system. Since tax systems have some duration, the choice of the system is made behind a ‘veil of ignorance’. The insurance motive, regarding the risk of tax exploitation by the government, leads to a uniform tax as a plausible outcome. Day-to-day political decisionmaking, however, determines the actual level of the tax rate. It is allowed here that the daily political tug-of-war may lead to special provisions (captured by \( \sigma \)) affecting the effective tax rate [cf. BECKER (1983)]. In this context it is important to observe that special provisions, in contrast with the official tax rate, "miss the notice of most people". The ignorance with respect to other people's special provisions makes the decisions of an individual independent of differences in these provisions.

A second, and somewhat related, observation is that the uniform income tax may be seen as the official part (the ‘flag’) of the system urged by the value system (ideology) of a democracy, whereas the transfer part crops up under the force field of the existing political influence structure in society, which also determines the level of the uniform tax rate. According to STIGLITZ (1989, p. 29): "In fact, the appearance of equity is often more important than the reality". Lack of information with respect to

\footnote{TULLOCK (1988, p. 473).}
the precise consequences of government programs seems to play an important role here. The combination of a uniform income tax, which serves the ideological purpose, and group-specific transfers, which serve the tactical purpose, is also in DIXIT AND LONDREGAN (1997).

The government thus maximizes the value of the political interest function given by eq. (2.18), subject to a technology constraint [eq. (2.19)], a capital endowment constraint [eq. (2.20)], a budget constraint [eq. (2.21)] and a self-financing constraint for the redistribution system [eq. (2.22)]. The solution of this maximization problem leads to the following behavioral equations for the public sector

\[ \tau_a = \frac{a_G \delta_k}{1 - a_G \delta_k}, \quad a_G = \mu_c \alpha_{Gc} + \mu_w \alpha_{Gw} \]  

\[ L_a = \frac{a_G \delta_k}{1 - a_G \delta_k} \frac{p_k^c K_c + p_L (1 - l_c) N_c + p_L (1 - l_w) N_w}{p_L} \]  

\[ \sigma_c = \frac{\mu_c (1 - \alpha_{Gc})}{1 - a_G} \frac{p_k^c K_c + p_L N_c + p_L N_w}{N_c} - (p_k^c + p_L) \]  

\[ \sigma_w = \frac{\sigma_w}{N_w} = \frac{\mu_w (1 - \alpha_{Gw})}{1 - a_G} \frac{p_k^c K_c + p_L N_c + p_L N_w}{N_w} - p_L \]  

It appears that the tax rate only depends on the relative political influence of the social groups \((\mu_c \text{ and } \mu_w)\), their preferences with respect to the public consumption good \((\alpha_{Gc} \text{ and } \alpha_{Gw})\), and the labor elasticity of production of the public good \((\delta_k)\). The tax rate

Note that the division of the tax-transfer system is also in line with MUSGRAVE'S (1959) distinction between fiscal functions of the government. Major functions are the allocation and the distribution function. Under the allocation function the government collects taxes to finance the production of public goods. This function is represented by the tax segment of the tax-transfer system. The distribution function refers to the adjustment of the distribution of incomes (wealth), which is represented by the redistribution segment of the system. Musgrave's third fiscal function, the stabilization of the economy, is not relevant in a general equilibrium model.
is not influenced by prices or the income distribution. Note that an increase in the influence of a social group does not necessarily lead to a lower tax rate. This will only be true if individuals of that social group have a relatively lower preference for the public consumption good. Of course, the assumption of a uniform tax rate is crucial here. The tax rate is, furthermore, positively related to the labor elasticity of production. This is due to the fact that an increase in $\delta_L$, makes labor more productive and capital less productive in the public sector, which leads to a higher labor input and, as a consequence, to higher labor costs in the public sector (recall that the wage rate is the numéraire). It then follows immediately from the balanced budget condition [eq. (2.21)] that the tax rate will increase. The demand for labor $L_s$ (and, thus, the public production level $G_s$) depends on the tax rate, the total tax base (gross income), and the wage rate.

With respect to the transfer system it is noted that through this system total full income $(p_Kk_cN_c + p_LN_c + p_LN_w)$, is redistributed between capital owners and workers, where a group’s share in the pie depends on the relative political influence and the preference weights of the group at hand. As is obvious from eqs. (2.25) and (2.26), the whole full income of a group is transferred away if this group has no political influence ($\mu_i = 0$) or if it is only interested in the public good ($\alpha_{G} = 1$). This is due to the non-distortive character of the group-specific transfers and to the fact that the government does not take account of excess burdens when choosing its optimal policy (Nash assumption). Of course, a social group’s share of the pie (total full income) depends only on its political influence weight if all individuals have identical preferences ($\alpha_{jc} = \alpha_{jw}$, $j = c, \ell, G$).

From the partial equilibrium solutions for the private sector and the public sector, that were determined in Subsection 2.2 and Subsection 2.3, respectively, the general equilibrium solution for the political economic model can be determined. This solution is given in Appendix 2.A.

2.4 Equilibrium analysis

2.4.1 Introduction

In this subsection the nature of the general equilibrium solution of the political economic model, as presented in Appendix 2.A, is investigated by means of a comparative static analysis. The results are summarized in Tables 2.1 - 2.5.
Section 2.5 the general equilibrium results will be contrasted with the results of a partial equilibrium (traditional general equilibrium) model.

Before presenting the comparative static results, some preliminary remarks are in order. First, as can be checked from eqs. (2.A.1), (2.A.2) and (2.A.3) in Appendix 2.A, it turns out that the price of capital \( p_K \), the private commodity price \( p_c \), and the private output level \( X_c \), are similarly affected by a change in a parameter or an exogenous variable, except for a change in the private production parameters \( \delta_{l,c} \), \( \delta_{K,c} \), and \( \Omega_c \) or the private capital stock \( k^o_c \). Second, because in equilibrium capital is fully employed, the effect of a change in an exogenous variable or a parameter on the private (public) demand for labor is of similar sign as its effect on private (public) output, except for a change in the private (public) capital stock \( k^o_c \), and the private (public) production parameters \( \delta_{l,c} \), \( \delta_{K,c} \), and \( \Omega_c \). Third, since \( \alpha_{c} + \alpha_{fi} + \alpha_{gi} = 1 \), a change in one preference weight (\( \alpha_{j} \)) implies a simultaneous change in at least one other preference weight (for given \( i = c, w \)). Fourth, because \( \mu_c + \mu_w = 1 \), the effect of a change in \( \mu_c \) on an endogenous variable is of equal size, but of opposite sign, compared to the effect on this variable of a change in \( \mu_w \). Fifth, because \( \sigma_c N_c + \sigma_w N_w = 0 \), changes in \( \sigma_c \) and \( \sigma_w \), as a consequence of a change in a parameter or an exogenous variable, differ a fixed factor \( N_c / N_w \) and have the opposite sign.

### 2.4.2 Effects of changes in the political influence structure

The most important parameters from a political economic point of view are the relative political influence of capital owners (\( \mu_c \)) and workers (\( \mu_w \)). The effects of a change in political influence strongly depend on the preferences of these social groups with respect to the private and public good. Therefore, the effects of a change in the political influence structure will be analyzed under different conditions with respect to preferences of capital owners and workers. The results are presented in Table 2.1. Not surprisingly, these results show that if the relative political influence of workers increases the transfer for workers (\( \sigma_w \)) increases, while the transfer for capital owners (\( \sigma_c \)) decreases. In case of identical preferences (\( \alpha_{jc} = \alpha_{jw}, j = c, \ell, G \)) this is the only result (except for the distribution of private commodities and leisure over the social groups), since the tax rate, prices and the production of private and public goods will not change.

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6 Note that these are real variables, expressed in the numéraire.
Table 2.1 Signs of comparative static effects of changes in workers’ political influence $\mu_w$

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<thead>
<tr>
<th>Conditions</th>
<th>$\alpha_j = \alpha_{jw}$</th>
<th>$\alpha_{jc} \leq \alpha_{cw}$</th>
<th>$\alpha_{cc} \geq \alpha_{cw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital price $p_K$</td>
<td>$\pm$</td>
<td>0</td>
<td>$\pm$</td>
</tr>
<tr>
<td>Commodity price $p_c$</td>
<td>$\pm$</td>
<td>0</td>
<td>$\pm$</td>
</tr>
<tr>
<td>Private production $X_c$</td>
<td>$\pm$</td>
<td>0</td>
<td>$\pm$</td>
</tr>
<tr>
<td>Public production $G_s$</td>
<td>$\pm$</td>
<td>0</td>
<td>$+$</td>
</tr>
<tr>
<td>Leisure capital owners $l_c$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>Leisure workers $l_w$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
<tr>
<td>Tax rate $\tau_h$</td>
<td>$\pm$</td>
<td>0</td>
<td>$+$</td>
</tr>
<tr>
<td>Transfer capital owners $a_c$</td>
<td>$-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>Transfer workers $a_w$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
</tr>
</tbody>
</table>

Note: The effects of a change in $\mu_w$ are of equal size, but of opposite sign.

An increase in the influence weight of workers will lead to an increase in the tax rate if and only if workers have a stronger preference for the public good than capital owners [$\alpha_{Gc} < \alpha_{Gw}$, cf. eq. (2.23)]. With increasing $\mu_w$ and $\alpha_{Gc} < \alpha_{Gw}$, the effect on private and public production depends primarily on the preferences of workers and capital owners for private commodities, represented by $\alpha_{cw}$ and $\alpha_{cc}$, respectively. It turns out that $\alpha_{cc} \leq \alpha_{cw}$ is a sufficient condition for an increase in public production $G_s$. The change in private production $X_c$ (and, thus, in the prices $p_K$ and $p_c$; see above) remains ambiguous, however. In that case capital owners have a relatively stronger preference for leisure ($\alpha_{lc} \geq \alpha_{lw}$) and, consequently, show a stronger labor supply effect of full income than workers [cf. eq. (2.10)]. Thus, if the transfer for capital owners ($a_c$) decreases and the transfer for workers ($a_w$) increases, the total supply of labor will increase. This extra labor supply may increase the production of private as well as public goods. If it is still assumed that $\alpha_{Gc} < \alpha_{Gw}$, but that $\alpha_{cc} > \alpha_{cw}$, then the effect of an increase in the political influence of workers on the production of not only the private but also the public good is ambiguous [in contrast with the partial equilibrium result, taking private sector behavior as given; cf. eqs. (2.19) and (2.24)]. In that case $\alpha_{fc} < \alpha_{fw}$, may hold, which leads to a decrease in labor supply. Although the tax rate increases, tax revenues and public production may decline because of a smaller tax base.

We now turn to the situation where $\alpha_{Gc} > \alpha_{Gw}$. In that case an increase in the political influence of workers still leads to an increase (decrease) in the transfer for workers
(capital owners), but the tax rate will now decrease. Since \( \alpha_{Gc} > \alpha_{Gw} \), an interesting question is whether the production of the public good will decrease if \( \mu_w \) increases (as suggested by the partial equilibrium result). It turns out that a sufficient condition for a lower production of the public good is that \( \alpha_{cc} \geq \alpha_{cw} \). However, if \( \alpha_{cc} < \alpha_{cw} \), it is possible that capital owners have a stronger preference for leisure, implying that \( \alpha_{rc} > \alpha_{rw} \). This may lead to a larger supply of labor and in spite of a decrease in the tax rate \( \tau_h \), to an increase in the tax base and the production of the public good. With respect to private production it is noticed that if \( \mu_w \) increases and \( \alpha_{Gc} > \alpha_{Gw} \), a sufficient condition for a decrease in the private production level (and, consequently, in the prices \( p_K \) and \( p_c \)) is that \( \alpha_{cc} > \alpha_{cw} \). In that case labor supply decreases.

In addition we examined the effect of a change in relative political influence on the net (of transfer) average tax rate and the utility of a representative individual of a social group. Using again aggregate variables, for convenience, the total net tax burden of a (representative) individual of social group \( i \) equals

\[
\tau_i = \tau_a [p_k k^0_i + p_i (1 - \ell_i) + \sigma_i] - \sigma_i, \quad i = c, w \tag{2.27}
\]

whereas the gross income of this individual equals

\[
y_i = p_k k^0_i + p_i (1 - \ell_i), \quad i = c, w \tag{2.28}
\]

with \( k^0_w = 0 \). The average tax rate of the individual equals \( \tau_i / y_i \). Using \( p_i = 1 \), it can be proved that the average tax rate for workers decreases if their political influence increases. This result is independent of the relative preference of workers for public goods.\(^7\) For capital owners this is harder to prove, because of the effects on capital income.

Finally, the effect of a change in political influence on utility is investigated. Surprisingly, it turns out that an increase in the political influence of a social group does not necessarily lead to a higher utility for this group. If all individuals have the same preferences \( (\alpha_{jc} = \alpha_{jr}; j = c, \ell, G) \), then an increase in political influence always leads to an increase in utility for the group considered and to a decrease in utility for the other social group. If \( \alpha_{jc} \neq \alpha_{jr} \), then an increase in political influence

\[7\text{In the proof of this result account is taken of the condition that } \ell_w \leq 1.\]
of a social group may lead to a decrease in utility for the members of that group.  

For example, if capital owners have a stronger preference for the public good than workers \((\alpha_{Gc} > \alpha_{Gw})\) an increase in the political influence of workers leads to a decrease in public production and, consequently, in the tax rate. Furthermore, it leads to an increase in the workers’ transfer at the cost of the transfer for capital owners. These effects on the tax rate and transfers lead to an increase in workers’ full income and the government expects that the consequent positive effect on workers’ utility will compensate the negative effect of the decrease in public production. The government neglects, however, the effect of changes in government decisions on prices. This is due to the assumption of Nash behavior for the public sector. In particular, the government neglects two important effects. First, the increase in the price of leisure \(p_l\), resulting from the decrease in the tax rate, has a negative effect on workers’ utility. This effect is stronger if capital owners and workers have a stronger preference for the public good and if the labor elasticity of public production is high, as can be checked from eq. (2.23). Second, the government neglects the effects on \(p_c\) and \(p_K\), that may follow from a change in the demand for leisure. The increase in workers’ full income leads to an increase in the demand for leisure, whereas the effect of an increase in the political influence of workers on the full income of capital owners and, consequently, on the demand for leisure of capital owners is ambiguous. It can be checked, though, that labor supply will decrease \((l_c + l_w\) will increase) if workers and capital owners have identical preferences for private commodities \((\alpha_{cc} = \alpha_{cv}, \text{ implying that } \alpha_{tc} < \alpha_{tw})\). Moreover, it follows that in the situation where \(\alpha_{Gc} > \alpha_{Gw}\) and \(\alpha_{cc} = \alpha_{cv}\), the decrease in labor supply is so strong that it not only compensates for the smaller labor demand in the public sector, but also requires the labor demand in the private sector to decrease. The lower labor input in the private sector causes the price of private commodities \(p_c\) and the price of capital \(p_K\) to decrease. The decrease in the production price has a positive effect on workers’ utility, while the decrease in the price of capital has a negative effect on workers’ utility, through the lower transfer to workers [see eq. (2.26)] and through an additional decrease in public production, because the tax base decreases. Summarizing, the government neglects the negative effects on workers’ utility that follow from the increase in the price of leisure and the decrease in the price of capital, and the positive effect on worker’s utility that follows from the decrease in the price of private commodities. If these negative effects are

---

8 The result that the utility of workers will decrease if their political influence increases is obtained, for instance, for the following parameter configuration: \(\mu_w = 0.9, \mu_e = 0.1, \alpha_{cv} = 0.2, \alpha_{cw} = 0.1, \alpha_{cv} = 0.7, \alpha_{cc} = 0.2, \alpha_{tc} = 0, \alpha_{Gc} = 0.8, \delta_{tc} = 0.9\) and \(\delta_{tw} = 0.9\). For capital owners this result holds if, for instance, \(\mu_w = 0.1, \mu_e = 0.9, \alpha_{cv} = 0.2, \alpha_{cw} = 0, \alpha_{Gc} = 0.8, \alpha_{cc} = 0.2, \alpha_{tc} = 0.1, \alpha_{Gc} = 0.7, \delta_{tc} = 0.9\) and \(\delta_{tw} = 0.9\).
sufficiently strong, the overall effect of the increase in the political influence of workers on their utility will be negative, as the numerical example in the above footnote illustrates. Mutatis mutandis, the same holds for the capital owners.

2.4.3 Effects of changes in preference weights

The effects of a change in the preference weights concerning private commodity consumption and leisure of capital owners and workers (\(\alpha_{cl}\) and \(\alpha_{lu}\)) are rather straightforward. Therefore, we will concentrate here on the effects of a change in these parameters on the lump sum transfers and public production. The effects of changes in preferences of capital owners can be found in Table 2.2, while the effects of a change in workers' preferences are summarized in Table 2.3. With respect to public production it is noticed that an increase in the preference weight concerning private commodity consumption (\(\alpha_{cl}\)), which has in general an ambiguous effect, has a positive effect on public production if this increase does not lead to a change in the preference weight concerning public good consumption (\(d\alpha_{G} = 0\)). Note that in that case the preference weight for leisure (\(\alpha_{lu}\)) decreases, which leads to an increase in the tax base while the tax rate remains the same. With respect to the effects on the group-specific transfers, the asymmetry between the effects of changes in the preference weights \(\alpha_{cc}\) and \(\alpha_{cw}\) is striking: the effect on the group-specific transfers is ambiguous if \(\alpha_{cc}\) changes, while a change in \(\alpha_{cw}\) gives unambiguous results. The amount that is transferred to a social group depends on the political influence weighted preference for private commodities (including leisure), given by \(\mu_{i}(1-\alpha_{G})/(1-\alpha_{G})\), and the pre-tax full income, which is equal to \((p_{K}k_{c}^{D} + p_{L})N_{c}\) for capital owners and \(p_{L}N_{w}\) for workers [cf. eqs. (2.25) and (2.26)]. More precisely, this amount is determined by the difference between \(\mu_{i}(1-\alpha_{G})/(1-\alpha_{G})\) times the aggregate pre-tax full income \((p_{K}k_{c}^{D} + p_{L})N_{c} + p_{L}N_{w}\) and the pre-tax full income of social group \(i\). The contribution of capital owners and workers to the pie (aggregate pre-tax full income) differs, because the former social groups receives capital income \(p_{K}k_{c}^{D}N_{c}\) in addition to labor income.\(^9\) The asymmetry between the effects of changes in the preference weights \(\alpha_{cc}\) and \(\alpha_{cw}\) results because the presence of capital income leads to a different effect of \(\alpha_{cc}\) and \(\alpha_{cw}\) on the net share of the pie that is transferred to the social groups.

\(^9\) This implies that, if capital owners and workers have identical preferences and if the political influence weights are related to the numerical strength of the social group, capital owners and workers receive (per capita) the same share of the pie (see Subsection 2.4.5 for the intuition behind this result). To realize this situation a part of capital income is transferred from capital owners to workers.
Table 2.2 Signs of comparative static effects of changes in preferences of capital owners

<table>
<thead>
<tr>
<th>Conditions</th>
<th>$\alpha_{cc}$</th>
<th>$\alpha_{tc}$</th>
<th>$\alpha_{Gc}$</th>
<th>$\alpha_{Gt}$</th>
<th>$d\alpha_{tc} = -d\alpha_{Gt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital price $p_K$</td>
<td>+</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Commodity price $p_c$</td>
<td>+</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Private production $X_t$</td>
<td>+</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Public production $G_t$</td>
<td>±</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Leisure capital owners $l_c$</td>
<td>±</td>
<td>+</td>
<td>±</td>
<td>±</td>
<td>-</td>
</tr>
<tr>
<td>Leisure workers $l_w$</td>
<td>±</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Tax rate $\tau_h$</td>
<td>—</td>
<td>—</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Transfer capital owners $a_c$</td>
<td>±</td>
<td>+</td>
<td>±</td>
<td>±</td>
<td>—</td>
</tr>
<tr>
<td>Transfer workers $a_w$</td>
<td>±</td>
<td>—</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
</tbody>
</table>

With respect to the effects of changes in the preference weights for the public consumption good ($\alpha_{Gc}$) rather unambiguous results are only obtained if such a change merely affects the preference for leisure ($d\alpha_{Gc} = -d\alpha_{lt}$). In that case all variables are positively affected by an increase in $\alpha_{Gc}$, except for the transfer to capital owners and capital owners’ demand for leisure for which an opposite effect is obtained. If $\alpha_{Gc}$ increases capital owners are willing to pay more for public goods, which leads to an increase in the tax rate. It also leads to a decrease in the political influence weighted preference for private commodities, $\mu_c(1-\alpha_{Gc})/(1-\alpha_{G})$, and, consequently, to a decrease in the transfer $a_c$ and an increase in the workers’ transfer $a_w$, as follows from the redistribution mechanism described in the previous paragraph. The decrease in the transfer to capital owners and their decreased preference for leisure lead to a decrease in the demand for leisure by this group. Although workers will show an increased demand for leisure, because of the increase in their transfer it can be shown that total labor supply will rise in this case.\(^{10}\) The extra labor supply will, however, not be completely absorbed in the public sector (note that the relative preference for private commodities does not change). The larger labor input in the private sector increases the price of capital, which leads to a higher before-tax full income of capital owners, $(p_kk_c^0 + p_L)N_c$. As a consequence, total full income, $(p_kk_c^0 + p_L)N_c + p_LN_w$, will increase. This reinforces the negative effect on the transfer to capital owners, as can be derived from the aforementioned redistribution mechanism.

\(^{10}\) Note that price effects have not entered the story yet.
Table 2.3 Signs of comparative static effects of changes in preferences of workers

<table>
<thead>
<tr>
<th>Conditions</th>
<th>$\alpha_{cw}$</th>
<th>$\alpha_{lw}$</th>
<th>$\alpha_{Gw}$</th>
<th>$\alpha_{Gw}$</th>
<th>$d\alpha_{lw} = -d\alpha_{Gw}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital price $p_K$</td>
<td>+</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Commodity price $p_c$</td>
<td>+</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Private production $X_c$</td>
<td>+</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Public production $G_s$</td>
<td>±</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Leisure capital owners $\ell_c$</td>
<td>±</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Leisure workers $\ell_w$</td>
<td>±</td>
<td>+</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Tax rate $\tau_h$</td>
<td>-</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Transfer capital owners $\sigma_c$</td>
<td>-</td>
<td>±</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
<tr>
<td>Transfer workers $\omega_w$</td>
<td>+</td>
<td>±</td>
<td>±</td>
<td>±</td>
<td>+</td>
</tr>
</tbody>
</table>

In case of an increase in workers’ preference weight for public good consumption, $\alpha_{Gw}$, and an equal sized decrease in their preference weight for leisure, $\alpha_{ew}$, the effects are similar as described above for capital owners, except for the effect on the transfers which are ambiguous now. This ambiguity follows from the fact that the concomitant decrease in the political-influence-weighted preference for private commodities, $\mu_w(1-\alpha_{Gw})/(1-\alpha_G)$, has a negative effect on the transfer to workers and a positive effect on the transfer to capital owners, while the increase in capital income $p_kk_0N_c$ has a positive effect on the former and a negative effect on the latter.

If the preference weight for private commodity consumption ($\alpha_{cd}$) is also affected, then the results depend on the configuration of parameter values.

2.4.4 Effects of changes in production parameters

As can be observed from Table 2.4, a change in the labor elasticities of production ($\delta_{Lc}$ and $\delta_{Ls}$) has a differential effect on the two group-specific transfers: an increase in $\delta_{Lc}$ or $\delta_{Ls}$ leads to a decrease (increase) in the transfer for workers (capital owners). The intuition of this result is that an increase in one of these elasticities makes labor more scarce in relation to capital [cf. eqs. (2.A.6) and (2.A.7)], which leads to a decrease in the relative input price of capital $p_k$ and in capital owners’ before-tax full income $(p_kk_0^0 + p_N)c$. The result then immediately follows from the redistribution mechanism described in Section 2.4.3.
Table 2.4 Signs of comparative static effects of changes in technical coefficients

<table>
<thead>
<tr>
<th></th>
<th>$\delta_{Lc}$</th>
<th>$\delta_{Is}$</th>
<th>$\Omega_c$</th>
<th>$\Omega_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital price $p_K$</td>
<td>$-$</td>
<td>$-$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Commodity price $p_c$</td>
<td>$\pm$</td>
<td>$-$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Private production $X_c$</td>
<td>$\pm$</td>
<td>$-$</td>
<td>$+$</td>
<td>$0$</td>
</tr>
<tr>
<td>Public production $G_s$</td>
<td>$-$</td>
<td>$\pm$</td>
<td>$0$</td>
<td>$+$</td>
</tr>
<tr>
<td>Leisure capital owners $\ell_c$</td>
<td>$-$</td>
<td>$-$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Leisure workers $\ell_w$</td>
<td>$-$</td>
<td>$-$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Tax rate $\tau_h$</td>
<td>$0$</td>
<td>$+$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Transfer capital owners $a_c$</td>
<td>$+$</td>
<td>$+$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Transfer workers $a_w$</td>
<td>$-$</td>
<td>$-$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

An increase in the labor elasticity of production $\delta_{Lc}$ stimulates the input of labor in the private sector $L_c$ [cf. eq. (2.4.6)]. By raising the relative price of labor ($p_K$ decreases) it further encourages the supply of labor [cf. eq. (2.4.5)]. Nevertheless, the demand for labor in the public sector $L_s$ and, consequently, public output $G_s$ will fall in this case because the decrease in $p_K$ negatively affects the tax base [cf. eq. (2.4.8)]. The effect of an increase in $\delta_{Lc}$ on private output $X_c$ is ambiguous, because it makes capital less productive in the private sector ($\delta_{Kc}$ decreases). An unambiguous (negative) relationship is obtained, however, if the total private capital endowment exceeds the total labor endowment ($k^0 N_c > N_c + N_w$). In this situation the decrease in capital productivity dominates.

The same holds, mutatis mutandis, for the effects of a change in $\delta_{Ls}$, the labor elasticity of production in the public sector. Although the effect of a change in $\delta_{Ls}$ on $G_s$ is ambiguous in general for the same reasons as mentioned in the previous paragraph for the effect of $\delta_{Lc}$ on $X_c$, the tax rate is always positively affected. The reason is that the increase in $\delta_{Ls}$ raises the public demand for labor and, thus, the production costs in the public sector, which calls for higher tax revenues to balance the government budget.

The results for the production scaling parameters $\Omega_c$ and $\Omega_s$ are straightforward, given the separability of the utility and political interest functions. Apart from the negative relationship between $\Omega_c$ and the private commodity price $p_c$, changes in the scaling parameters $\Omega_c$ and $\Omega_s$ only (positively) affect the private and public production levels, respectively.
2.4.5 Effects of changes in group size and capital endowments

Table 2.5 shows, not unexpectedly, that an increase in group size (which increases labor endowments) will lead to an increase in private and public production. The reason that the effect of such a change on transfers as well as on the prices is not symmetric for $N_c$ and $N_w$ is that a change in the number of capital owners influences both labor and (private) capital endowments, whereas a change in the number of workers influences only total labor endowments. The effects of changes in capital endowments are straightforward. Apart from the additional negative relationship between $k_c^0$ and $p_k$, the effects of changes in $k_c^0$ and $K_w^0$ are similar in sign as those referring to the scaling parameters $\Omega_c$ and $\Omega_w$, respectively [note that $p_k k_c^0 N_c$ is not affected by a change in $k_c^0$, see eq. (2.A.1)].

In analyzing the effects of a change in group sizes on endogenous variables it was thusfar assumed that there is no relation between group sizes (numerical strength) and political influence. If it is, in line with the probabilistic voting models of COUGHLIN AND NITZAN (1981) and COUGHLIN ET AL. (1990b), assumed that the numerical strengths of social groups affect the influence weights, the impact of a change in group sizes on endogenous variables will alter. According to Coughlin and Nitzan, the maximization of expected plurality in a two party election is equivalent with the maximization of a generalized Nash product if the political influence of a social group equals

$$\mu_i = \frac{N_i}{N_c + N_w}, \quad i = c, w \quad (2.29)$$

The relations between the endogenous variables and numerical strength can now easily be obtained by inserting eq. (2.29) in the equations of the respective endogenous variables as given in Appendix 2.A. It turns out that the effect of a change in either group size is in general ambiguous for all endogenous variables. If individuals have the same preferences ($\alpha_{j_c} = \alpha_{j_w}$, $j = c, w, G$), a change in the number of capital owners has the same effect on the public sector variables as a change in the number of workers, except for the effect on group-specific transfers. An increase in the number of capital owners has, compared with an increase in the number of workers, an extra positive effect on private production, because the private capital stock ($k_c^0 N_c$) grows with the number of capital owners. Furthermore, extra private capital lowers the commodity price and the capital price. Note that extra private capital has no effect on private revenues $p_c X_c$ and capital costs $p_k k_c^0 N_c$ [see eqs. (2.A.1), (2.A.2) and (2.A.3)]. If individuals have identical preferences both private and public production
Table 2.5 Signs of comparative static effects of changes in group size and capital endowments

<table>
<thead>
<tr>
<th></th>
<th>$N_c$</th>
<th>$N_w$</th>
<th>$k^0_c$</th>
<th>$k^0_w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital price $p_K$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$0$</td>
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<tr>
<td>Commodity price $p_c$</td>
<td>$-$</td>
<td>$+$</td>
<td>$-$</td>
<td>$0$</td>
</tr>
<tr>
<td>Private production $X_c$</td>
<td>$+$</td>
<td>$+$</td>
<td>$+$</td>
<td>$0$</td>
</tr>
<tr>
<td>Public production $G_s$</td>
<td>$+$</td>
<td>$+$</td>
<td>$0$</td>
<td>$+$</td>
</tr>
<tr>
<td>Leisure capital owners $l_c$</td>
<td>$-$</td>
<td>$+$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Leisure workers $l_w$</td>
<td>$+$</td>
<td>$-$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Tax rate $r_h$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Transfer capital owners $\sigma_c$</td>
<td>$\pm$</td>
<td>$\pm$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>Transfer workers $\sigma_w$</td>
<td>$+$</td>
<td>$-$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

are positively affected by an increase in the number of capital owners or workers, while there is no effect on the tax rate, which is in line with the intuition. Moreover, in that case the group specific transfer to capital owners is affected by a change in the number of capital owners or workers, while the transfer to workers is not [cf. eqs. (2.29), (2.A.10) and (2.A.11)]. The intuition behind this result is as follows. If individuals have identical preferences and political influence according to their numerical strength an egalitarian government policy is optimal (compare the optimal policy of a utilitarian government if individuals have identical preferences). Because there is only one wage rate, all individuals earn the same wage income, but capital owners receive in addition income from their capital endowments. To reach the egalitarian solution, the government only needs to redistribute capital income. Consequently, the transfer for capital owners will be negative ($\sigma_c < 0$) and the transfer for workers positive ($\sigma_w > 0$), as can be easily checked from eqs. (2.A.10) and (2.A.11). Each individual receives a full income equal to

$$p_K + \frac{p_K k^0_c N_c}{N_c + N_w} = \frac{1 - a_G \delta_{ks}}{1 - a_c \delta_{ks} - a_G \delta_{ks}}$$

which follows from eq. (2.A.1) and the fact that the wage rate is the numéraire. Note that this (after-transfer) full income per individual does not depend on the size of the population ($N_c + N_w$). This is due to the transfer system and the linear homogeneity of the production and utility functions. As a consequence, an extra individual generates extra income, through an increase in the capital price $p_K$. Per capita full income does not change, however, if the number of capital owners or workers
increases. The extra capital income must, thus, be transferred to the individuals that entered. If the number of workers increases income is transferred from capital owners to workers, which leads to an increase in (the absolute value of) the transfer for capital owners, while the individual transfer for workers does not change. If, on the other hand, the number of capital owners increases, the transfer for an individual capital owner can decrease (in absolute value), because a same amount of income (equal to $\sigma_r N_e$) must be transferred from capital owners to workers, while there are more capital owners that can contribute to this amount.

An increase in the number of workers has a negative effect on both private and public production if preferences of workers for both private commodities and the public good are very low ($\alpha_{w_e}$ and $\alpha_{G,w}$ approach zero) and if, in addition, individuals of the other social group have a stronger preference for the public good ($\alpha_{G,c} > \alpha_{G,w}$). Furthermore, in this case the tax rate decreases, total transfers to capital owners decreases and, consequently, total transfers to workers increases, in this case. Because workers have, in this situation, a relatively weak preference for the public good, their increased political influence causes the government to produce less of the public good, which leads to a decrease in government expenditures and, consequently, to a decrease in the tax rate. Private production decreases because workers use their increased after tax disposable full income only for extra leisure whereas capital owners\' after tax disposable full income decreases, which leads to a decrease in their demand for private commodities.

### 2.5 Comparison with partial equilibrium results

In partial equilibrium (traditional general equilibrium) models the effects of exogenous changes in the public sector variables on the endogenous private sector variables are analyzed. However, from a political economic point of view the government cannot freely choose the shocks in tax variables and public production, as these shocks depend on changes in economic and political variables and parameters. The latter changes also have a direct effect on private sector variables. As a consequence, if no account is taken of the reason why a tax variable changes, there may be a serious misspecification in the analysis of the effects of a change in tax policies on private sector variables if. This claim will be illustrated with some examples.

The partial effect of an increase in the tax rate $\tau_h$ on private production $X_c$ (and prices $p_c$ and $p_K$) is negative (see Subsection 2.2.4). So, one would expect from a partial
equilibrium analysis that an increase in the tax rate $\tau_h$ leads to a decrease in private production. However, if the increase in the tax rate follows from a decrease in capital owners' preference for leisure ($\alpha_{lc}$) then private production and the commodity and capital price do not decrease, but will instead increase (cf. Table 2.2 for this general equilibrium effect). This is due to the additional indirect effects through the group-specific transfers and the direct effect of a change in the preference weight on private production and the commodity and capital price.

As a second example, consider the expected positive effect of an increase in the transfer to capital owners on private production if $\delta_{a} > \delta_{m}$ (this follows from analysis in the last paragraph of Subsection 2.2.4). If the increase in the capital owners' transfer follows from an increase in the labor elasticity of production of the public good ($\delta_{lc}$) the general equilibrium effect on private production is negative, as can be checked from Table 2.4. Thus, again, the partial equilibrium analysis gives false information.

2.6 Lump sum taxation and tax reform

2.6.1 Introduction

In Section 2.3 it was argued that the tax-transfer system incorporated in the model can be interpreted as an abstract representation of existing tax regimes. This view was supported by arguments borrowed from BRENNAN AND BUCHANAN (1980) and STIGLITZ (1989). In the present section we will investigate the efficiency of this system.

First, the efficiency of public production is investigated. It is easily shown that the Samuelson first best condition for an efficient supply of the public good, which demands that the sum of the marginal rates of substitution equals the marginal rate of transformation, is only satisfied for the tax-transfer system if individuals of both social

11 Note that one should also take account of the effect of a change in $\alpha_{lc}$ on private production $X_c$ in the traditional general equilibrium case. The analysis of the effect of a simultaneous change in the tax rate and $\alpha_{lc}$ in a traditional general equilibrium framework differs, however, from the analysis in a model as presented here, because the effect on the tax rate of a change in $\alpha_{lc}$ is missed in the former framework.
groups attach zero weight to the consumption of leisure ($\alpha_{t_c} = \alpha_{t_w} = 0$). In order to study the efficiency of the tax-transfer system in detail it will be compared with a non-distorting endogenous lump sum tax system. It will be explained in Subsection 2.6.2 that the government provides more of the public good under the latter system than under the former, and that the lump sum tax system is Pareto superior to the tax-transfer system. The latter result does, however, not carry over to all other tax systems.\footnote{13} In view of the Pareto superiority of the endogenous lump sum tax system compared with the tax-transfer system, the section is closed with a discussion of the feasibility of tax reform. Two issues are highlighted here: the costs of tax reform, and the (quasi-)constitutional and ideological aspects of tax systems.

2.6.2 Comparison with an endogenous lump sum tax system

When introducing a new tax system the government has to reconsider the tax payments it will demand from each individual. If taxes are levied by lump sum taxation, where the lump sums can be different for different social groups, (after-tax) full income of capital owners and workers changes into, respectively

$$f_c^* = p_c k_c + p_L - s_c$$

$$f_w^* = p_L - s_w$$

(2.31)

(2.32)

The indirect utility functions $V_i$ and the political interest function $P$ are now subject to these full incomes.

\footnote{12} This follows from the equations

$$MRS_i = - \frac{d c_i}{d G_i} = \frac{\partial U_i / \partial G_i}{\partial U_i / \partial c_i} = \frac{\alpha_{c} G_i}{c_i}$$

$$MRT = - \frac{d X_c}{d G_s} = \frac{\delta_{c} X_c / L_c}{\delta_{L_c} G_s / L_c}$$

\footnote{13} Note that the redistribution of income as specified in the tax-transfer system has no direct excess burden. If, however, capital owners and workers have different preferences with respect to the public good, income redistribution may have an indirect burden, because it will change the provision of the public good.
The optimization problem of the government changes into the maximization of the political interest function with respect to the lump sums $s_c$ and $s_w$, capital input $K_s$, and labor input $L_s$, subject to the public production technology [eq. (2.19)], the given public capital endowment [eq. (2.20)] and the budget constraint, which changes into $p_L L_s = s_c N_c + s_w N_w$. The optimal values for the lump sum taxes and the demand for labor are

$$s_c = \frac{a_c \delta_{s_c} + \mu_w (1 - \alpha_{Gw})}{1 - a_K \delta_{K_s}} \left( p_K^{k_0} + p_L \right) - \frac{\mu_c (1 - \alpha_{Gc})}{1 - a_K \delta_{K_s}} \frac{p_L N_c}{N_c}$$

(2.33)

$$s_w = \frac{a_c \delta_{s_w} + \mu_c (1 - \alpha_{Gc})}{1 - a_K \delta_{K_s}} p_L - \frac{\mu_w (1 - \alpha_{Gw})}{1 - a_K \delta_{K_s}} \frac{p_L N_c + p_L N_w}{N_w}$$

(2.34)

$$L_s = \frac{a_c \delta_{L_s} - p_K^{k_0} N_c + p_L N_c + p_L N_w}{p_L}$$

(2.35)

It turns out that the after-tax full incomes, $f_c$ and $f_w$, are identical under the tax-transfer system and the endogenous lump sum tax system. This result is due to the separability of the utility and political interest functions. Because of this assumption, the distribution of aggregate full income, $(p_K^{k_0} + p_L) N_c + p_L N_w$, over the after-tax full incomes $(f_c N_c$ and $f_w N_w$) and the public production costs $(p_L L_s)$ is fixed and is, consequently, independent of the tax system that is chosen; it is, if proper account is taken of the shadow prices of these outlays, which are defined here as the per unit share of aggregate full income that is allocated to these outlays. The structure of the tax system may influence these shadow prices of $f_c N_c$, $f_w N_w$ and $p_L L_s$. If the tax system switches from the tax-transfer system to the lump sum system, the shadow prices of $f_c N_c$ and $f_w N_w$ are unaffected and equal to one, because the transfers $\sigma_c$ and $\sigma_w$ in the tax-transfer system operate like $s_c$ and $s_w$ in the lump sum system. The shadow price of government expenditures differs, however. In case of the tax-transfer system the shadow price of $L_s$ can be written as

$$p_L = 1 + \frac{p_L (f_c N_c + f_w N_w)}{p_K^{k_0} N_c + p_L (N_c + N_w) - p_L (f_c N_c + f_w N_w)}$$

(2.36)

14 These fixed fractions are $\mu_c (1 - \alpha_{Gc}) / [1 - a_K \delta_{K_s}]$, $\mu_w (1 - \alpha_{Gw}) / [1 - a_K \delta_{K_s}]$ and $a_K \delta_{K_s} / [1 - a_K \delta_{K_s}]$ for $f_c N_c$, $f_w N_w$ and $p_L L_s$, respectively.
where the second term on the right-hand side is due to the fact that only gross income can be taxed. Under a lump sum tax system, the second term vanishes.

As can now be calculated from eqs. (2.9) and (2.12), expenditures for commodities \((p_c c_c \text{ and } p_c c_w)\) and thus total returns from private production \((p_c X_c)\) will also be equal under these two tax systems. Furthermore, because the given private capital endowment will again be fully employed, it immediately follows from the linear homogeneity of the production function that the price of capital will be identical under the two tax systems as well [see eq. (2.3)]. The same holds, by implication, for private labor input and the private commodity price. The result that the private sector is not affected by the switch from the tax-transfer system to the lump sum system is related to the fact that the tax-transfer system does not affect the price ratio of the labor and capital input in the private sector.

What changes, though, is total labor supply (unless, of course, \(\alpha_{tc} = \alpha_{tw} = 0\)). If taxes are levied as lump sums, the price of leisure increases and becomes equal to the wage rate. The following relationship between the demand for leisure under the endogenous lump sum tax system \((\ell_t)\) and the demand for leisure under the tax-transfer system \((\ell_t)\) is obtained

\[
\ell_t^e = \frac{1 - a_G}{1 - a_c} \ell_t, \quad i = c, w
\]

Because \(a_G\) and \(\delta_{ki}\) are between zero and one, individuals of both social groups will take less leisure under the endogenous lump sum tax system. As a consequence, total labor supply will increase, leading in equilibrium to a higher total labor input. Since private labor input does not change this means that labor input in the public sector is higher in case of the endogenous lump sum tax system. Denoting the latter input by \(L_s\), the following relationship can be established

\[
L_s^e = \frac{1 - a_G}{a_c} L_s
\]

Because \(1 - a_G = a_c + a_t\), it follows that \(L_s^e > L_s\), unless \(a_t = 0\) (in which case \(\alpha_{tc} = \alpha_{tw} = 0\)). Consequently, public output \(G_s\) will be higher as well under this tax system, if \(a_t > 0\). The change in demand for labor in the public sector \((L_t^e - L_t)\) exactly equals the increase in labor supply, which is not surprising because labor input in the private sector does not change. The change in labor supply can be written as \(\ell_c + \ell_w - \ell_c^e - \ell_w^e = \tau_h (\ell_c + \ell_w)\), which corresponds exactly with the second term
in the shadow price of labor input in the public sector under the tax-transfer system, as can be checked with eqs. (2.A.1) and (2.A.5). The distortion, represented by the second term in the shadow price, is due to the fact that leisure is taxed under the tax-transfer system. Consequently, the Samuelson condition for an efficient public output is fulfilled under the lump sum tax system (recall that under the tax-transfer system this condition is only fulfilled if there is no demand for leisure, i.e. \( a_t = 0 \)). The increase in labor input in the public sector \( (L^c_T - L_T) \) exactly offsets the distortion.

Summarizing, the main effect of a switch in tax regime from a tax-transfer system to an endogenous lump sum system in the model is the increase in the public output level \( G_s \), which goes to the efficient level, whereas the market sector is not affected.\(^{15}\) All in all, the change in the tax system influences the utility of individuals in two ways: through the increase of the price of leisure and through the increase in the production of public goods, where the first has a negative effect and the second a positive effect on utility. Comparing the (indirect) utilities under the two tax systems gives

\[
V'_i = \left[ \frac{1 - a_G}{1 - a_G \delta_{Ki}} \right]^{\alpha_c} \left[ \frac{1 - a_G}{a_c} \right]^{\alpha_c \delta_{Ki}} V_i, \quad i = c, w
\]

Since \( 1 - a_G = a_c + a_t \), it follows immediately that the utility of the individuals of both social groups increases if the tax system switches from the tax-transfer system to the endogenous lump sum tax system (unless, again, \( a_t = 0 \)). As a consequence, the lump sum tax system is Pareto superior to the tax-transfer system.\(^{16}\)

Finally, it is noted that the endogenous lump sum tax system is not Pareto superior to all other tax systems. The lump sum tax system is, for example, not Pareto superior to an endogenous tax system consisting only of a uniform tax rate, and no group-specific transfers. The results for this tax system follow immediately if in Sections 2.2 and 2.3 the group-specific transfers are set equal to zero \( (a_c = \sigma_c = 0) \). The simple

\(^{15}\) The assumption that utility and political interest functions are separable with respect to the public good is important in this respect.

\(^{16}\) It is noticed here that a change to an equal yield lump sum tax system need not be Pareto improving. Under such a system the lump sums are exogenously imposed and set equal to the payments under the tax-transfer system for every individual. Since by definition tax revenues are constant, public production is not affected if the tax system changes. Now private production and prices change, however. It can be shown that the change in the tax regime may have a negative effect on the utility of individuals of a social group. This occurs if the negative price effects dominate the positive income effects.
intuition of this result is that in a tax system with a group-specific tax (segment) total full income of the members of a social group can be taxed away if this social group has no political influence. If, however, a uniform tax system obtains, a politically dominant social group cannot tax away the total full income of the other social group without taxing away its own full income. As long as the dominant group attaches a positive preference weight to private commodities and/or leisure, the full income of the other social group will not be taxed away under a uniform tax system.\textsuperscript{17} Therefore, the latter group will obtain a higher utility under a uniform tax system than in case of a tax system with group-specific taxation. If (exogenous) compensations were feasible, lump sums could be found that Pareto improve on the uniform tax system. Politically, these lump sum taxes would not be optimal, however, as shown by the endogenous lump sum tax system above.

2.6.3. On tax reform

In Subsection 2.6.2 the relative efficiency of some alternative tax systems were investigated, to wit: an endogenous lump sum tax system, and an endogenous uniform income tax system without group-specific transfers. The endogenous lump sum tax system appeared to be Pareto superior to the tax-transfer system of the model.

This raises the question of the viability of the tax-transfer system. For, within the framework of the political economic model, any alternative tax system that leads to a higher value $P$ of the political interest function would be preferable to the political decisionmakers, even if the alternative is not Pareto superior. In answering this question, the following two issues should be pointed at: the cost of tax reform, and the interpretations of the tax system referred to in Section 2.3.

As pointed out by \textsc{Van Velthoven and Van Winden} (1991), a change in the tax system demands that the expected change in $P$ makes up for the costs involved in a tax reform. These costs are (at least) of two types: first, the set-up costs to the government, due to changes in legislation and administration, which affects the government budget constraint; and second, the adaptation costs to the taxpayers, affecting the individual budget constraint, and thereby the (indirect) utilities $V_L$. One

\textsuperscript{17} Note, furthermore, from eqs. (2.7), (2.8) and (2.10) that under a uniform tax system individuals cannot be forced to supply their full labor endowment.
need not go into an explicit analysis to conclude that these costs may thwart a switch to a preferred tax system, such as in our case the endogenous lump sum tax system.

The other issue concerns the interpretation of the ruling tax regime. In Section 2.3 it was noticed that the uniform tax segment of the tax-transfer system could be interpreted as the official part of the system. This observation was supported by a (quasi-)constitutional interpretation and an ideological (democratic value system) interpretation. Apart from the cost issue, these two interpretations would seem to rule out the more efficient lump sum tax system as a viable option. In this context, and particularly in view of the ideological interpretation, the following quote from Stiglitz (1989, p. 29) is of interest: "Thus the equity constraint forces, as it were, an inefficient outcome. The seeming public hypocrisy entailed in this kind of behavior may be difficult to understand, but is prevalent in virtually all western democracies."

For the tax-transfer system it is straightforward to derive a necessary and sufficient condition for this 'hypocritical' aspect of the tax system to vanish. This requires that $\sigma_c = \sigma_w = 0$, turning the tax-transfer system into the simple uniform tax system referred to at the end of Section 2.6.2. As can be checked from eq. (2.40), this condition reads

$$\frac{N_w}{N_c + N_w} = \frac{\mu_w (1 - \alpha_c) \delta_{kc}}{1 - \alpha_c \delta_{kc}} \frac{1 - \alpha_G \delta_{ks}}{1 - \alpha_G \delta_{ks}}$$

From eq. (2.40) it can be inferred that, if the social groups have the same preference weights ($\alpha_{jc} = \alpha_{jw}$, $j = c, i, G$), reducing the first term on the right-hand side to $\mu_w$ the political influence of the workers should be smaller than their relative numerical strength, and the more so the higher the capital elasticities of private and/or public production ($\delta_{kc}$, $\delta_{ks}$). If the political influence weights $\mu_c$ and $\mu_w$ are equal to the relative numerical strengths of the social groups, as would be the case under a probabilistic voting model as in COUGHLIN AND NITZAN (1981), then $\sigma_c < 0$ and

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18 Note that this interpretation is related to the aforementioned costs involved in tax reforms.
Public consumption and redistribution

\( \sigma_w > 0 \), implying a redistribution of income from capital owners to workers.\(^{19}\) The intuition behind this result has been discussed in Section 2.4.

2.7. Concluding remarks

The model presented in this chapter captures both the influence of government policies on private sector decisionmaking and the influence of private sector agents on government decisionmaking. For this purpose a positive model of government behavior, rooted in the interest function approach, was linked to a traditional general equilibrium model. As tax structure we employed a tax-transfer system consisting of a uniform income tax rate and a group-specific transfer. Various arguments were provided for the use of this system, which differs from the usual uniform linear income tax system considered in the literature. Another feature of the model concerns the supply by the government sector of a public good, which is an argument in the individuals' utility functions, influencing both individuals' and government's decisionmaking. By using relatively simple functional forms analytical results could be obtained.

A comparative static analysis, produced some counterintuitive results. It was shown, for example, that an increase in political influence of a social group does not necessarily lead to an increase in the utility of the members of this social group. Furthermore, an increase in the preference weight of a social group for the public good may have a negative effect on public production. Also, it turned out that the effects of policy changes determined by traditional general equilibrium models, where government policies are exogenously imposed, may be seriously misleading when used as a prediction of what will actually happen in case of such changes.

The tax-transfer system was compared with an endogenous lump sum tax system from an efficiency point of view. The lump sum tax system entails an increase in the production of the public good, while private production does not change. The elimination of the distortion in the labor-leisure choice, which occurs under the tax-transfer system, leads to a Pareto improvement. However, it also appeared that the

\[^{19}\] More generally, denoting the right-hand side of eq. (2.40) by RHS,

\[
S_v \lesssim 0, \quad (\text{and thus } S_c \gtrsim 0) \quad \text{if} \quad \frac{N_n}{N_c + N_n} \lesssim RHS
\]
endogenous lump sum tax system is not Pareto superior to a simple uniform linear income tax system, without group specific lump sum transfers. The uniformity of the latter system protects tax payers against taxing away total full income. In view of the Pareto superiority of the endogenous lump sum tax system we, finally, discussed the viability of the tax-transfer system. It was argued that (quasi-)constitutional constraints, having to do with the costs of tax reform, as well as ideological constraints would hamper the introduction of a full-fledged lump sum tax system.
Appendix 2.A The general equilibrium solution

In this Appendix the general (Nash-Cournot) equilibrium of the political economic model is presented. First the prices are determined. Recall that the wage rate is used as numéraire, thus $p_i = 1$. With eqs. (2.15), (2.16), (2.23), (2.25) and (2.26), the price of capital in the private sector, $p_K$, and the private commodity price, $p_c$, can be calculated. This gives

$$p_K = \frac{a_c \delta_{Kc} (N_c + N_w)}{(1 - a_c \delta_{Kc} - a_G \delta_{Ks}) k_0^c N_c} \tag{2.A.1}$$

$$p_c = \frac{1}{\Omega_c} \left[ \frac{a_c (N_c + N_w)}{(1 - a_c \delta_{Kc} - a_G \delta_{Ks}) k_0^c N_c} \right]^{\delta_{Kc}} \tag{2.A.2}$$

with $a_j = \mu_c \alpha_{jc} + \mu_w \alpha_{jw}$, $j = c, G$.

The private production level $X_c$ can now be obtained, with eqs. (2.2)-(2.4), (2.13) and (2.A.1)

$$X_c = \Omega_c \left[ \frac{a_c \delta_{Lc} (N_c + N_w)}{1 - a_c \delta_{Kc} - a_G \delta_{Ks}} \right]^{\delta_{Lc}} (k_0^c N_c)^{\delta_{Kc}} \tag{2.A.3}$$

From eqs. (2.7)-(2.9), (2.23), (2.25), (2.26), (2.A.1) and (2.A.2), the demand for private commodities, $c_i$, is determined

$$c_i = \frac{\mu_i \alpha_{ici} \Omega_i}{a_i N_i} \left[ \frac{a_c \delta_{Lc} (N_c + N_w)}{1 - a_c \delta_{Kc} - a_G \delta_{Ks}} \right]^{\delta_{Lc}} (k_0^c N_c)^{\delta_{Kc}} \tag{2.A.4}$$

for $i = c, w$. In this appendix, $i = c, w$, will be suppressed for notational convenience. The equilibrium condition for the product market, $X_c = c_c N_c + c_w N_w$, can easily be checked from (2.A.3) and (2.A.4).

With respect to the labor market, the expressions for leisure $\ell_i$ are first determined. Using eqs. (2.7), (2.8), (2.10), (2.23), (2.25) and (2.26), it follows that

$$\ell_i = \frac{\mu_i \alpha_{ili} (1 - a_G \delta_{Ks})(N_c + N_w)}{(1 - a_c \delta_{Kc} - a_G \delta_{Ks}) N_i} \tag{2.A.5}$$
Using eqs. (2.4), (2.A.2) and (2.A.3) for private labor demand, \( L_c \), and eqs. (2.24), (2.A.1) and (2.A.5) for public labor demand, \( L_s \), one obtains

\[
L_c = \frac{a_c \delta_{ls} (N_c + N_w)}{1 - a_c \delta_{kc} - a_G \delta_{ks}} \tag{2.A.6}
\]

\[
L_s = \frac{a_c a_G \delta_{ls} (N_c + N_w)}{(1 - a_G)(1 - a_c \delta_{kc} - a_G \delta_{ks})} \tag{2.A.7}
\]

From eqs. (2.A.5), (2.A.6) and (2.A.7) it can be checked that the equilibrium condition for the labor market, \((I - \ell_c)N_c + (I - \ell_w)N_w = L_c + L_s\), is fulfilled.

The public consumption level \( G_s \) follows from eqs. (2.19), (2.21), and (2.A.7)

\[
G_s = \Omega_s \left[ \frac{a_c a_G \delta_{ls} (N_c + N_w)}{(1 - a_G)(1 - a_c \delta_{kc} - a_G \delta_{ks})} \right]^{\frac{1}{\delta_G}} \tag{2.A.8}
\]

For completeness, the income tax rate is here reproduced

\[
\tau_n = \frac{a_G \delta_{ls}}{1 - a_G \delta_{ks}} \tag{2.A.9}
\]

Using eqs. (2.25), (2.26) and (2.A.1), the lump sum transfers to capital owners and workers are, respectively,

\[
\sigma_c = \left[ \frac{\mu_c (1 - \alpha_G)(1 - a_G \delta_{ks}) - a_c (1 - a_G \delta_{kc})}{(1 - a_G)(1 - a_c \delta_{kc} - a_G \delta_{ks})} \right] (N_c + N_w) \frac{1}{N_c} - 1 \tag{2.A.10}
\]

\[
\sigma_w = -\frac{\sigma_c N_c}{N_w} \tag{2.A.11}
\]

Finally, from equilibrium prices, disposable full income, and the amount of the public good, the indirect utilities can be calculated.
Public consumption and redistribution

\[ V_i = \Omega C \Omega S \delta K \delta V \]

\[ N_i = \frac{\left( N_c + N_b \right)^{1-a_c x_b}}{a_c x_b (1-a_c \delta K_t)^{a_c x_b}} \]

\[ \mu_i = \frac{1-a_c x_b}{(1-a_c \delta K_t)^{a_c x_b}} \]

\[ (1-a_c)^{a_c x_b} (1-a_c \delta K_t - a_c \delta K_s)^{1-a_c x_b} \]

(2.12)
Using eqs. (2.4), (2.5) and (2.7) for the derived labor demand, \( L_p \), and eqs. (2.8) and (2.9) for wage labor demand, \( L_u \), we arrive at
\[
\hat{w} = \frac{\hat{p} \hat{t}_w - \hat{p} \hat{t}_w \phi}{\hat{p} \hat{t}_w - \hat{p} \hat{t}_w \phi}
\]
and
\[
\hat{L}_u = \frac{\hat{w} \hat{t}_w - \hat{w} \hat{t}_w \phi}{\hat{w} \hat{t}_w - \hat{w} \hat{t}_w \phi}
\]
From eqs. (2.5), (2.6) and (2.7) it can be checked that the equilibrium condition for the labor market, \( I = \hat{P} M \), \( L_p = T + L_u \hat{t}_w \), \( L_u = L_p - L_u \), is fulfilled.

The phase consumption levels \( \hat{\omega} \) (derived from eqs. (2.19), (2.21) and (2.22))
\[
\hat{\omega} = \left( \frac{\hat{A} \hat{t}_w - \hat{A} \hat{t}_w \phi}{\hat{A} \hat{t}_w - \hat{A} \hat{t}_w \phi} \right)^{\frac{1}{\alpha}}
\]
For completeness, the income tax rate is here reproduced
\[
\hat{t} = \frac{\hat{A}}{1 - \hat{A}}
\]
Using eqs. (2.25), (2.26) and (2.27), the lump-sum tax, the marginal cost of labor, and the lump-sum tax, respectively,
\[
\hat{t}_w = \frac{\hat{A} \hat{t}_w - \hat{A} \hat{t}_w \phi}{\hat{A} \hat{t}_w - \hat{A} \hat{t}_w \phi}
\]
Finally, from equilibrium prices, disposable real income, and the amount of the good, the laborer's utility can be calculated.