Government decisions on income redistribution and public production
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4 Dynamic decisionmaking on private and public production

4.1 Introduction

In Part I, the government provided only one public good, with the characteristics of a public consumption good. The production of private commodities did not depend on this good. Empirical studies show, however, that production in the private sector does depend on (specific) public goods. In particular physical infrastructure and research and development expenditures are mentioned [see ASCHAUER (1989) and NADIRI AND MAMUNEAS (1994), respectively]. The importance of infrastructure for private economic performance will be discussed in Chapter 6. For this chapter, where the nature of the model is presented, it suffices to say that production in the private sector and in the public consumption sector, not only depends on labor input and the magnitude of the sector-specific capital stock, but also on a public production good, (referred to as infrastructure). For the production of this good the government needs again labor and capital as inputs. Private production, thus, depends on a public good. This is in contrast with most of the literature, where (a change in) private production depends on a public capital stock (public investments) (see Chapter 6 for a further discussion).

In order to investigate the impact of the political influence of different social groups on infrastructure and of infrastructure on private production, two private commodities are distinguished, that are produced in two different production sectors. The sectors not only produce a different commodity, but differ also in the production technology. One sector is more capital intensive, while the other sector is more labor intensive. The production in the more capital intensive sector is more dependent on infrastructure than the production in the labor intensive sector. The difference in technology between the private sectors implies that the capital owners of the different sectors differently value infrastructure. An increase in the political influence of capital of the one sector at the cost of the political influence of the capital owners of the other sector may, therefore, affect the size of infrastructure. These aspects will be further discussed in Chapter 6.

For a better understanding of the relevance of the public production good for production, it is important to know how this good influences decisions with respect to labor and capital input. Decisionmaking with respect to capital has a dynamic character. Therefore, a dynamic theory is adopted to describe the decision process regarding private and public production. According to this theory, the decisions of
private firms follow from the maximization of the value of the firm, which consists of the sum of current and discounted future dividend payments. In this way, investment decisions are endogenized, and a relation between investments and the shadowprice of capital is derived. It can be shown that the latter equals the empirically observable asset price under some homogeneity conditions [cf. HAYASHI (1982)]. In the model presented in this chapter, private production depends on infrastructure. It will be shown that this dependence requires an adjustment of the relation found by Hayashi.

The underlying postulate of the above theory is that investments are subject to installation costs on new capital. Our model assumes that this also holds for government investments. Apart from the public production good (infrastructure), the government produces a public consumption good. Labor and investment costs are financed by distortionary taxes. Different tax systems, distortionary as well as non-distortionary, are compared with each other in Chapter 7. The model that is presented in this chapter, contains most of these taxes. Furthermore, the government can redistribute income through a self-financing, lump-sum transfer system. The decisionmaking of the government with respect to production, taxation and redistribution is in accordance with the maximization of the political interest function.

The focus of our analysis concerns the decisionmaking on infrastructure and its impact on private production. For simplicity, intertemporal decisionmaking by consumers is not included in the model. This keeps the model tractable, for intertemporal consumer decisions reverberates in the decisionmaking of the government. However, it is noted that the numerical approach that is adopted for the solution of the model, allows for the use of more flexible consumer behavior.

Decisions on investments depend on expectations with respect to future economic outcomes. As a consequence, a rule for expectation formation must be determined. In specifying this rule, expectations are often assumed to be rational [for rational expectation in general equilibrium models, see, e.g., AUERBACH AND KOTLIKOFF (1987) and GOULDER AND SUMMERS (1989)]. In this chapter, the results for rational expectations (or perfect foresight) are compared with the results for other expectation rules, where individuals are myopic. In that case, individuals suppose that the economic situation does not change (lagged or static expectations), or adapt their

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1 See also the discussion in Chapters 1 and 2, and the introduction of Part II on modeling intertemporal decisionmaking of consumers.
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expectations if changes in the economic situation have occurred in the past (adaptive expectations). The expectation rule has no effect on the new steady state that occurs after a policy shock, but only on the transition path of the economy towards the new steady state. The transition effects of the different expectation rules will be discussed.

The organization of this chapter is as follows. Section 4.2 deals with private sector behavior. Government behavior is the subject of Section 4.3. In order to solve the model, a numerical computation method is chosen. The motivation for this choice is given in Section 4.4. The benchmark parameter configuration for the computation is presented in Section 4.5. Section 4.6 discusses the results for the initial steady state. The influence of the public production good (infrastructure) on the asset price and in private investment is examined in Section 4.7. Section 4.8 analyzes different expectations rules and the importance of expectation formation for the transition path to a new steady state. Section 4.9 concludes.

4.2 Private sector behavior

4.2.1 Introduction

In this section the private sector is modeled. First, the decisionmaking of capital owners (or entrepreneurs) on production and factor inputs is presented. Then, consumer behavior is considered. The section closes with the market equilibrium conditions. On the production side two private production sectors are distinguished. Producers maximize the value of the firm, consisting of the sum of current dividend payments and discounted future dividends. The goods produced in sector 1 are used for private consumption as well as for investment, while the goods produced in sector 2 are pure consumption goods. Both goods may be exported. Production does not only rely on the size of the capital stock and the number of people that are employed, but also on the level of a public production good (infrastructure). On the consumption side, consumers are split up in three groups. Two groups are formed by the capital owners of the different production sectors, while the third group embodies workers. Workers' income consists of after-tax labor income, while entrepreneurs receive, in addition, income from the shares they hold. Consumers maximize their utility subject to their budget restriction. Utility is determined by the consumption of the two domestic goods, two foreign goods, leisure and the public consumption good. Goods and labor markets are assumed to clear.
4.2.2 Production

In modeling the behavior of entrepreneurs, the q-theory of investments, as exposed in SUMMERS (1981a) and HAYASHI (1982), is adopted. This theory is based on earlier work of LUCAS (1967) and TOBIN (1969). The theory describes how decisions with respect to investments are made. It starts from the view that the installation of new capital is not costless. Expansion of the capital stock leads, therefore, to adjustment costs that are included in total investment costs. The presence of adjustments costs makes that capital is neither perfectly fixed nor perfectly variable. Another feature of the q-theory of investments is that it assumes dynamic decisionmaking. Entrepreneurs do not only take account of the current benefits of their decisions, but also of the future benefits. Whereas standard neoclassical production theory only generates cost shares of input factors per unity of production, in case of a constant returns to scale production function and endogenously determined levels of all input factors, the q-theory generates endogenously determined production and input levels and allows, in addition, for profits that are not necessarily equal to zero. The latter result is due to the adjustment costs. These costs are typically assumed to be a function of the investment-capital ratio \((I_j(t)/K_j(t))\). Total production costs are equal to labor costs plus the adjustments costs of capital. After-tax profits are equal to

\[
(1-\tau_c(t))\Pi_j(t) = (1-\tau_c(t))\left[ \frac{p_j(t)}{1+\tau_s(t)}F_j(L_j(t), K_j(t), G_p(t)) - p_L(t)L_j(t)
\right.

- \left. p_J(t)\Phi_j\left(\frac{I_j(t)}{K_j(t)}\right)I_j(t) \right], \quad j = 1, 2 \tag{4.1}
\]

where \(\Pi_j(t)\) are the profits of a representative firm in sector \(j\) in period \(t\), \(p_j(t), p_L(t)\) and \(p_I(t)\) are the prices of product \(j\), labor and investment goods, respectively, \(\tau_s(t)\) and \(\tau_c(t)\) denote the sales tax rate and the corporate tax rate, and \(F_j\) and \(\Phi_j\) stand for the production function and the adjustment cost function. Note that \(p_j(t)\) is the consumption price of good \(j\). The production price of this good then equals \(p_j(t)/(1+\tau_s(t))\), while the sales tax is equal to \(\tau_s(t)p_j(t)/(1+\tau_s(t))\). Note, furthermore, that production not only depends on capital \(K_j\) and labor input \(L_j\), but also on the level of a public production good \((G_p)\), as will be further discussed in Chapter 6. To ease the notation, the index \(j = 1, 2\) will be omitted in the remainder of this section.

Firms can use profits for the finance of new investments or for dividend payments. Capital owners are willing to hold shares as long as the risk-adjusted expected revenues from holding these shares are at least as high as revenues from holding
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(save) bonds. Equilibrium on the (international) asset market gives the following arbitrage condition between the returns on shares and on bonds:

\[ r(t) + \eta(t) = \frac{V_j(t+1) - V_j(t)}{V_j(t)} + \frac{(1-\tau_h(t))D_j(t)}{V_j(t)} \]  

(4.2)

where \( V_j(t) \) is the value of the firm in period \( t \); \( D_j(t) \) is the dividend in period \( t \); \( r(t) \) the interest rate; \( \tau_h(t) \) the income tax and \( \eta(t) \) the risk premium for shares. For convenience, it is assumed that interest income is not taxed. The left-hand side of eq. (4.2) gives the return on bonds, added with a risk-premium. The return on shares is given by the right-hand side of eq. (4.2), where the first term denotes capital gains and the second term after-tax dividend income.\(^2\) The value of the firm can now be determined by solving the arbitrage condition:

\[ V_j(t) = \sum_{s=t}^{\infty} R(t,s) (1-\tau_h(s)) D_j(s) \]  

(4.3)

where

\[ R(t,s) = \prod_{u=t}^{s} \left[ \frac{1}{1+r(u)+\eta(u)} \right] \]  

(4.4)

and where dividend payments are equal to the after-tax profits that are not invested:

\[ D_j(t) = (1-\tau_i(t))\Pi(t) - p_j(t)I_j(t) \]  

(4.5)

The entrepreneurial objective, as given in eq. (4.3), contains not only dividend payments in the current period, but also dividends that are paid in all future periods. If entrepreneurs decide to pay less dividend in the current period and invest more in capital, the increased capital stock may generate higher profits in the future, which has a positive effect on future dividend payments. The trade-off between current dividend payments and investments is, thus, comparable to the trade-off between current

\(^2\) From eqs. (4.1) and (4.2) it can be read that the model does not allow for new share issues and debt maintenance by firms. If these aspects are included, the firm must make decisions with respect to dividend payments, new share issues and new debt issue. In the model presented here, only one of these decisions can be made endogenously. The decisions with respect to the two other variables are determined by exogenously given rules, as is done, e.g., in Auerbach (1979) and Goulder and Summers (1989). Inserting debt and new share issues in such a way, may be relevant for an empirical model, but adds little to a more theoretical model as presented here. Therefore, debt and new share issues are omitted. They can, however, easily be incorporated in the model.
dividend payments and (discounted) future dividend payments. This trade-off is resolved by maximizing the value of the firm, as represented by eq. (4.3). The impact of investments on future dividends depends on the development of the capital stock. This development is given by the following accumulation function:

$$K_j(t+1) = I_j(t) + (1-dep_j)K_j(t)$$  \hspace{1cm} (4.6)

where $dep_j$ is the fixed depreciation rate.

The dynamic maximization problem that is faced by the entrepreneurs can now be summarized by the following Hamiltonian:

$$H_j(t) = V_j(t) + \sum_{s=t}^{s=\infty} R(t,s)q_j(s+1)[ I_j(s) + (1-dep_j)K_j(s) - K_j(s+1) ]$$  \hspace{1cm} (4.7)

where the current value multiplier $q_j(t+1)$ indicates the marginal benefit of an additional unit of capital. In the sequel, it will be called the shadowprice of capital in sector $j$. The relation between this shadowprice and investments follows from the first-order condition with respect to investment $I_j(t)$. Inserting the equations for dividend payments and profits into eq. (4.2), and rearranging the first derivative of the Hamiltonian with respect to investment, the relation between investment and the shadowprice of capital becomes:

$$q_j(t+1) = (1-\tau_h(t))p_j(t)[ 1 - (1-\tau_c(t))( \frac{I_j(t)}{K_j(t)} ) + I_j(t)(\frac{\partial \Phi_j}{\partial I_j} ) ]$$ \hspace{1cm} (4.8)

where the right-hand side represents the marginal cost of investment. This relation thus says that firms will invest till the marginal cost of new capital is balanced with the marginal benefit. The shadowprice of capital is often referred to as the marginal $q$, although the latter is somewhat differently defined as $q_j(t+1)/p_j(t)$. The shadowprice of capital $q_j(t+1)$ can be derived from the first-order condition involving the first derivative of the Hamiltonian with respect to the capital stock $K_j(t)$:

$$R(t,s-1)q_j(t) = R(t,s)(1-dep_j)q_j(t+1)$$

$$+ R(t,s)(1-\tau_h(t))(1-\tau_c(t))[ \frac{p_j(t)}{1+\tau_j(t)} \frac{\partial F_j}{\partial K_j} - p_j(t)I_j(t)(\frac{\partial \Phi_j}{\partial K_j} ) ]$$ \hspace{1cm} (4.9)
Solving this equation gives:

$$q_j(t) = \sum_{s=t}^{\infty} (1-dep)^{s-t} (1-\tau_h(s))(1-\tau_s(s))R(t,s).$$

$$[p_j(s) \frac{\partial F_j}{1+\tau_j(s)} - p_j(s)\frac{\partial \Phi_j}{\partial K_j}]$$

(4.10)

Thus, the marginal benefits of extra capital does not only depend on the extra revenues that the firm can acquire through production if it extends the capital stock, but also on the reduction of the installation costs that follow from the marginal extension of capital. The relation between investments and the shadowprice of capital as obtained in (4.8) has little empirical relevance because the latter cannot be observed. Hayashi (1982) demonstrates, however, that the shadowprice of capital is equal to the asset price (or: average q) if the production function and the adjustment cost function are linearly homogeneous in $K$ and $L$, and in $I$ and $K$, respectively. The asset price for sector j is defined as $V_j(t)/[p_j(t)K_j(t)]$ and is observable. The production function in the model presented here is assumed to be linearly homogeneous in $K_j$, $L_j$ and $G_p$. The identity that was derived by Hayashi must therefore be adjusted for the public production good. The relation between the asset price (average q) and the shadowprice of capital (marginal q) can be obtained from the following relation:

$$V_j(t) = \sum_{s=t}^{\infty} R(t,s)(1-\tau_h(s)) \frac{p_j(s) \frac{\partial F_j}{1+\tau_j(s)} G_p(s)}{q_j(t)K_j(t)} + q_j(t)K_j(t)$$

(4.11)

This relation says that the asset price [which follows from dividing the left-hand side by the value of the capital stock $p_j(t)K_j(t)]$ does not only depend on the marginal benefit of own, private, capital (given by $q_j(t)$), but also on the marginal benefit the

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3 The terminology of marginal q and average q follows from the fact that the shadowprice of capital gives the marginal benefits of capital $[\partial V_j(t)/\partial K_j(t)]$, which is equal to $\partial V_j(t)/\partial I_j(t)]$, while the asset price refers to the average benefits of capital $[V_j(t)/K_j(t)]$.

4 In the literature, some other factors are mentioned that can violate the equality between the asset price and the shadowprice of capital, such as tax deduction of past investment, price making firms that can generate monopoly rents, and the allowance of debt financing [see Hayashi (1982) for the first and second issue, and Summers (1981a) for the last]. Taxes, and investment tax credits, do not influence the relation between the shadowprice of capital and the asset price, whereas they do influence the relation between the shadowprice of capital and investments [as given in eq. (4.8)].
firm obtains from infrastructure or the use of the public production good (the first term on the right-hand side). Note that the assumption of linear homogeneity in $K_j$, $L_j$ and $G_p$ is crucial for this result. The impact of the marginal benefit of infrastructure on the asset price will be further discussed in Section 4.7.

4.2.3 Consumption

Consumers are split up in three different social groups. Two groups consist of the capital owners of production sectors 1 and 2, while the third group includes the workers that are employed in the private and the public sector. The sizes of these groups are assumed to be fixed and are denoted by $N_{cl}$, $N_{c2}$, and $N_w$, respectively. For simplicity, it is assumed that individuals within a group have identical preferences and income, and that income is fully consumed. The decision problem that individuals as consumers face is, therefore, essentially static.

Consumers have preferences with respect to two privately produced domestic goods, two privately produced foreign goods, leisure, and a public consumption good. These preferences are represented by a nested constant elasticity of substitution utility function [cf. KELLER (1976)]. A feature of the nested structure of the utility function is that differences in substitutability are possible between goods that are at different levels on the utility trees. On the highest level of the utility tree the consumption of privately produced goods, leisure and the public consumption good are distinguished:

$$U_i(t) = \left[ \alpha_i^{1-\gamma} c_i(t)^{\gamma} + \alpha_{it}^{1-\gamma} (\ell_i(t) - \bar{\ell})^{\gamma} + \alpha_{it}^{1-\gamma} (G_s(t) - \bar{G})^{\gamma} \right]^{\frac{1}{1-\gamma}}$$

(4.12)

where $U_i(t)$ is the utility index of the representative individual of social group $i$ ($i = c_1, c_2, w$) in period $t$, $c_i(t)$ the value of private goods consumed by this individual, $\ell_i(t)$ the demand for leisure, and $G_s(t)$ the level of the public consumption good produced by the government in period $t$. Barred variables refer to subsistence quantities. With respect to the parameters it is noticed that the $\alpha$'s refer to distribution parameters and the $\gamma$'s to substitution parameters, where the (constant) elasticity of substitution between goods on the same level of the nested utility function equals $1/(1-\gamma)$. The nonnegativity of the elasticity of substitution requires that substitution parameters are not larger than 1 ($\gamma \leq 1$). For notational convenience, the index $i = c_1, c_2, w$, will be omitted in the rest of this section.
On the second level of the nested utility function, the consumption of privately produced goods is split up in a bundle of type 1 and one of type 2, where the types refer to the kind of goods produced by the sectors 1 and 2:

$$c_i(t) = [\omega_1^{1-\gamma^o_i} c_{1i}(t)^{\gamma^o_i} + \omega_2^{1-\gamma^o_i} c_{2i}(t)^{\gamma^o_i}]^{\frac{1}{\gamma^o_i}}$$  \hspace{1cm} (4.13)

with $c_{1i}(t)$ and $c_{2i}(t)$ denoting the value of the bundle of type 1 and of type 2, that is consumed by the representative individual of social group $i$ in period $t$.

On the third level these bundles are further subdivided in goods that are domestically produced and goods that are imported from abroad, where the imported goods are assumed to be imperfect substitutes for domestic goods of a given type (known as the Armington condition):

$$c_{ji}(t) = [\omega_1^{1-\gamma^o_{di}} (c_{di}(t) - c_{di}^{\gamma^o_i} + \omega_2^{1-\gamma^o_{di}} (c_{fi}(t) - c_{fi}^{\gamma^o_i})^{\gamma^o_i}]$$  \hspace{1cm} (4.14)

with $c_{di}(t)$ denoting the consumption of domestically produced goods and $c_{fi}(t)$ the consumption of foreign goods ($j = 1, 2$).

Consumers maximize utility subject to the budget constraint. The absence of savings implies that consumers' income is in every period equal to the after-tax earnings in that period. Disposable full income of workers consists of the (after income-tax) sum of the value of their labor endowments and a lump sum transfer (special provisions) from the government. The latter follows from the redistribution system of the government, as will be explained in Section 4.2. Capital owners have, in addition, capital endowments, consisting of (sector specific) shares. For simplicity, it is assumed that capital gains are not capitalized, so that capital earnings only consist of dividend income. The budget constraint can be written as:

$$p_f(t)c_{df}(t) + p_g(t)c_{dg}(t) + (1+\tau_h(t))p_e(t)[p_f(t)c_{df}(t) + p_g(t)c_{dg}(t)] + (1+\tau_h(t))p_e(t)[p_f(t) + \eta(t) + d(t)]$$  \hspace{1cm} (4.15)

where $p_f(t)$ and $p_g(t)$ are the prices of the foreign goods, $p_e(t)$ is the exchange rate, $\tau_h(t)$ the income tax rate, and $\eta(t)$ the group-specific lump-sum transfer. Imports are charged with the sales tax $\tau_f(t)$. Individual labor endowments are set equal to unity and $d_f(t)$ denotes individual dividend income, with $d_f(t) = D_f(t)/N_f$ and $d(t) = 0$. 


4.2.4 Foreign sector

As noticed above, consumers have preferences with respect to two foreign goods that are imperfect substitutes of the domestic goods of the same type. The demand for these foreign goods is described above. The assumption of imperfect substitutability between domestic and foreign goods is called the Armington condition. This condition is also adopted for foreigners. The demand foreigners have for the domestic goods 1 and 2 is assumed to depend on the price ratio between the domestic price (which is equal to the production price) and the price of the foreign good of the same type [see Whalley and Yeung (1984)]. More precisely, the export demand for domestic good \( j \), \( E_j(t) \), is assumed to equal:

\[
E_j(t) = \eta_j \left( \frac{p_j(t)}{(1+\tau_j(t))p_e(t)p_p(t)} \right)^{\eta_j},
\]

where \( p_e(t) \) is the exchange rate, \( \eta_0 \) is a scaling parameter, and \( \eta_j \) is the (constant) price elasticity of foreign demand for good \( j \).

4.2.5 Private sector equilibrium

The market clearing conditions for an equilibrium determine the prices for good 1 \([p_1(t)]\), good 2 \([p_2(t)]\) and the wage rate \([w(t)]\). Recall that good 1 is a multipurpose good that can be used for consumption as well as for investment. Demand for this good consists of domestic consumer demand, export demand and investment demand. In addition to these market clearing conditions, it is required that the current account is balanced. The equilibrium conditions are given by

\[
X_1(t) = c_{dcl}(t)N_{cl} + c_{dcl}(t)N_{c2} + c_{dwl}(t)N_w + E_1(t)
+ (1+\Phi_1(\frac{I_1(t)}{K_1(t)}))I_1(t) + (1+\Phi_2(\frac{I_2(t)}{K_2(t)}))I_2(t)
+ (1+\Phi_3(\frac{I_3(t)}{K_3(t)}))I_1(t) + (1+\Phi_4(\frac{I_4(t)}{K_4(t)}))I_4(t)
\]

\[
X_2(t) = c_{dcl}(t)N_{cl} + c_{dcl}(t)N_{c2} + c_{dwl}(t)N_w + E_2(t)
\]
(1 - \ell_{el}(t))N_{el} + (1 - \ell_{e2}(t))N_{e2} + (1 - \ell_{w}(t))N_{w} \\
= L_1(t) + L_2(t) + L_3(t) + L_4(t) \quad (4.19)

\begin{align*}
&\rho_1(t)E_1(t) + \rho_2(t)E_2(t) = (1+\tau(t))\rho_e(t) \left[ \rho_{1e}(t)c^{c_1e}(t)N_{el} + c^{fle_2}(t)N_{e2} \\
&+ c^{fe_1w}(t)N_{el} + \rho_{2e}(t)c^{c_2e_2}(t)N_{e2} + c^{fpe_2}(t)N_{w} \right] \quad (4.20)
\end{align*}

where \( I_s(t), K_s(t) \) and \( L_s(t) \), and \( I_p(t), K_p(t) \) and \( L_p(t) \) refer to investment, capital and labor used for the production of the public consumption good and public production good, respectively. These variables will be discussed in the next section.

The four prices \( \rho_1(t), \rho_2(t), \rho_L(t) \) and \( \rho_e(t) \) follow from the above conditions. Because of Walras' Law, one of the equations is redundant, and one of the prices can be chosen as numéraire. Here \( \rho_e(t) \) is assumed to be the numéraire. Because the investment goods are produced in sector \( I \), the price of investment goods equals the production price of good \( I \), \( \rho_1(t)/(1+\tau_{st}(t)) \). Furthermore, the foreign prices \( \rho_{jt}(t) \) and \( \rho_{jt}(t) \) are assumed to be fixed and equal to one. In contrast with private commodities, domestic and foreign financial assets are assumed to be perfect substitutes. The interest rate follows from the equilibrium condition on the international financial market. The influence of a small country on the interest rate is, in that case, negligible. Therefore, the small country can regard the interest rate as given. This small country assumption is adopted here. Moreover, the interest rate is assumed to be constant over time.

4.3 Public sector behavior

As in the previous chapters, we use the interest function approach for the description of government behavior. Government produces a pure public consumption good and a public production good, that can be interpreted as infrastructure. The costs of production are financed with distortionary taxes. Furthermore, the government redistributes income. The decisions of the government with respect to public production, redistribution and taxation are assumed to be in accordance with the maximization of the following dynamic interest function:
\[ P(t) = \sum_{s=1}^{\infty} \frac{1}{(1+\phi)^s} \left[ U_{c_d}(s)^{\alpha_d} \ U_{c_d}(s)^{\alpha_d} \ U_{w}(s)^{\alpha_w} \right] \]  

(4.21)

where \( P(t) \) is the value of the interest function in period \( t \), \( \rho \) is the public discount rate and \( \mu_i \) is the political influence weight of group \( i \).

The decisions of the government with respect to production show some analogy with production decisions of the firms. While decisions of the latter are in accordance with the maximization of the value of the firm, the decisions of the government are in line with the maximization of the value of the interest function. Firms have to decide every period whether they will spend current profits on dividend payments or on investment in private capital, where the latter has a positive effect on future dividend payments. The government has to decide every period whether current (tax) income is spent on private and public consumption, that is positively correlated with present utilities, or on investment in public capital that has a positive effect on future utilities. The production of the public consumption good \( G_s(t) \) depends on the input of capital and labor, as well as on the level of the public production good, whereas the production of the public production good \( G_p(t) \) only requires capital and labor:

\[ G_s(t) = F_s(L_s(t), K_s(t), G_p(t)) \]  

(4.22)

\[ G_p(t) = F_p(L_p(t), K_p(t)) \]  

(4.23)

The accumulation function of the two public capital stocks is given by:

\[ K_g(t+1) = I_g(t) + (1 - dep_g)K_g(t), \quad g = p, s \]  

(4.24)

The index \( g = p, s \) will be omitted in the remainder, for notational convenience.

Production costs of the government consist of labor costs, investment costs and adjustment costs, due to the installation of the new capital goods. The government levies taxes to finance these costs. Assuming a balanced budget restriction, for simplicity, the budget constraint is given by:

\[ T(t) = p_s(t)[(1 + \Phi_s(\frac{I_s(t)}{K_s(t)}))I_s(t) + (1 + \Phi_p(\frac{I_p(t)}{K_p(t)}))I_p(t)] \]

\[ + p_s(t)(L_s(t) + L_p(t)) \]  

(4.25)
where $T(t)$ refers to total tax revenues, which are specified below.

The optimal investment in the public capital stocks follows from the maximization of eq. (4.21) subject to eqs. (4.22)-(4.25). This optimization gives the following relation between the shadowprice of capital $q_g(t)$ and investment:

$$\frac{q_g(t+1)}{1+\rho} = \xi(t)\rho(t)\left[ 1 + \Phi_g(t) \frac{I_g(t)}{K_g(t)} + I_g(t) \frac{\delta g}{\delta I_g} \right]$$

(4.26)

where $\xi(t)$ refers to the marginal social costs of public revenues. Note the difference with the shadowprice of private capital [eq. (4.8)]. The shadowprice of public capital depends on the extra public spending, required for extra public investment, and the social costs of the extra spending. Because the shadowprice in period $t+1$ is determined by the social costs in period $t$, the relation between these variables is adjusted for the discount rate $1+\rho$.

In the basic model the government has only the income tax at its disposal to collect the revenues it needs for the finance of the production of the public goods. In Chapter 7 some alternative tax systems are discussed that also contain indirect, commodity, taxes or a corporate tax. To avoid repetition, all these taxes will be presented in the budget restriction presented below:

$$T(t) = \tau_i(t) [ \Pi_i(t) + \Pi_j(t) ]$$

$$+ \tau_c(t) \left[ p_c(t) (X_c(t) - E_c(t)) + \frac{p_c(t)}{1+\tau_c(t)} (X_c(t) - E_c(t)) \right.$$ 

$$+ \sigma_{c1}(t)N_{c1} + \sigma_{c2}(t)N_{c2} + \sigma_{c3}(t)N_{c3} + \sigma_{c4}(t)N_{c4} \right.$$ 

$$+ p_c(t) \rho_{c1}(t) (c_{j1c1}(t)N_{c1} + c_{j1c2}(t)N_{c2} + c_{j1c3}(t)N_{c3} + c_{j1c4}(t)N_{c4})$$

$$+ p_c(t) \rho_{c2}(t) (c_{j2c1}(t)N_{c1} + c_{j2c2}(t)N_{c2} + c_{j2c3}(t)N_{c3} + c_{j2c4}(t)N_{c4}) ]$$

$$+ \tau_h(t) [ (p_h(t)(1 - l_h(t)) + \sigma_h(t) + d_h(t))N_h + (p_h(t)(1 - l_h(t))$$

$$+ \sigma_{h1}(t) + d_{h1}(t))N_{h1} + (p_h(t)(1 - l_h(t)) + \sigma_{h2}(t))N_{h2} ]$$

(4.27)

As in the previous chapters, the government uses special provisions of a lump-sum character to redistribute income. This transfer system is self-financing:

$$\sigma_{c1}(t)N_{c1} + \sigma_{c2}(t)N_{c2} + \sigma_{c3}(t)N_{c3} = 0$$

(4.28)
4.4 Solution method

The model presented above is too complicated to solve analytically. For analytical results, the linearization method as developed by Johansen might be used [see JOHANSEN (1960) and DIXON ET AL. (1992)].\(^5\) The Johansen linearization method specifies a linear version of the original model around a given point (e.g., a static or steady state equilibrium). The method can be described as follows. Suppose, that the original model (that is made up of the first order conditions of the maximization problems and the appropriate restrictions) can be written as a system of equations \(F(x) = 0\), where \(x\) denotes the vector of variables and \(F\) is a vector-valued function. If the system of equations is totally differentiated one obtains \(F'(x).dx = 0\), where \(F'(x)\) is a matrix of first derivatives and \(dx\) is a vector of changes in the variables of \(x\). The system is linearized by evaluating the matrix \(F'(x)\) at the equilibrium \(x'\).\(^6\) This system is, thus, linear in changes in the variables of \(x\). The system can, however, easily be rearranged into a linear system in relative changes or in changes in logarithms.

If the vector of variables \(x\) is split up in endogenous and exogenous variables, the impact of a change in the subvector of exogenous variables on endogenous variables can be determined. If, for example, fiscal policy is under investigation, taxes can be chosen as exogenous variables and the impact of marginal changes in tax rates can be analyzed. The analysis must, however, be restricted to the study of the effects of marginal changes, because the linearized model approximates the original model rather well close to the initial equilibrium, but may become grossly inaccurate if one moves away from the initial equilibrium. Therefore, the linear model is not well suited for the analysis of substantial changes in tax rates or for the study of tax reforms, one of the issues that we are interested in.

A second caution regards the use of linearized methods for models with endogenous government behavior. The usual set up of the linear model, where (relative) changes...
in endogenous variables are written as linear functions of (relative) changes in exogenous variables [cf. KELLER (1980)], is not very useful if government behavior is endogenized. Fiscal models typically choose tax rates and government expenditures as exogenous variables. Endogenization of these variables implies that the investigation of the effects of changes in exogenous variables on endogenous variables must be replaced by an analysis of the effects of changes in parameters on endogenous variables, making the investigation a type of 'linearized' sensitivity analysis [as in, e.g., BOVENBERG AND VAN DER PLOEG (1994)]. Furthermore, introducing endogenous government behavior strongly increases the complexity of the model. Linearization of the model is not only extremely complicated in that case, it also burdens the flexibility of the model for extensions and applications. Solving the model analytically by using a linearization method is, all in all, an unattractive option. Therefore, a numerical solution method is adopted to solve the model.

The numerical solution procedure starts with the calculation of the initial steady state. Because there is neither exogenous nor endogenous growth in the model, expectation variables and current variables have the same value in the steady state. The steady state problem then boils down to a static problem and can be solved by using the Newton method [for a discussion of numerical methods, see JACOBS (1977) and PRESS ET AL. (1986)]. By using the advanced mathematical software system Mathematica [cf. WOLFRAM (1991)] we were able to derive the Hessian (the matrix that consists of second order derivatives) analytically and to recalculate the Hessian numerically for every iteration.

This procedure turned out to be fortunate, because other methods, that require less computation time than the Newton method, did not guarantee convergence to the solution. The Davidon-Fletcher-Powell method, with the identity matrix as a first approximation for the Hessian matrix, gave very poor results. In fact, convergence never occurred while using this method. A second method that is recommended to speed up the Newton method, is the Gauss-Seidel method, which is used by, e.g., AUERBACH AND KOTLIKOFF (1987). To test this method, the model was split up in five submodels, consisting of the first order equations concerning consumption variables, private production variables, prices, public production variables and taxes. With this solution method, convergence was not guaranteed. Even if the number of blocks was reduced to three by combining the first three blocks, the method gave unsatisfactory results. Apart from the fact that convergence was not guaranteed, the method was very slow in cases it converged, which was due to the strong increase in the number of iterations that were required for a sufficiently small error statistic.
A third procedure, that was examined to speed up the calculations, aimed at a reduction of the time required for the recalculation of the Hessian. Computation time can be reduced if not all the elements of the Hessian are updated for every iteration. As with the Davidon-Fletcher-Powell method, a first approximation of the Hessian must be chosen. Because the identity matrix as a first guess did not work (see above), a more elaborate first approximation was chosen. First, the diagonal matrix with the elements of the Hessian on the diagonal, was used as a first approximation. This method did not guarantee convergence, even if the diagonal elements were updated for every iteration. Moreover, calculating the diagonal elements and setting the nondiagonal elements to zero was more time-consuming while using the Mathematica software, than the calculation of the full Hessian. Despite this slackness, a second approach was tested. In this approach the diagonal as well as the subdiagonal elements of the Hessian were calculated and the other elements in the matrix were set to zero. Again, the method did not always converge.

For sensitivity analyses, the Hessian that belongs to the initial steady state can be chosen as a first approximation. In the most extreme case the Hessian is held constant for all further iterations. This procedure was first followed. Refinements of this procedure were necessary, however, to achieve satisfactory convergency results. The use of a fixed Hessian frequently led to no convergence. Refinements of the fixed Hessian method focused on the diagonal and lower subdiagonal of the Hessian. If the diagonal elements of the Hessian were updated for every iteration, while the other elements were held fixed, the method became satisfactory if the changes for the sensitivity analysis were not too large. A further refinement that allows for an additional updating of the elements on the lower subdiagonal added little. Though these refinements methods sped up the computing time that was necessary for one iteration, they had the disadvantage that they demand extra computing time to design the Hessian with fixed nondiagonal elements and flexible diagonal elements. The refinement methods are, therefore, only efficient if many iterations are required before convergence occurs. Consequently, in case the refinement methods did converge, it required more computation time than the Newton method with an every-iteration-

\[ H(x')\text{diag}(x')x = 0, \]

where \( x \) is the vector of relative changes, \( x = (dx_1/x_i, \ldots, dx_n/x_n) \) and \( \text{diag}(x') \) is a diagonal matrix with the elements of \( x' \), the vector with outcomes for the initial state, on the diagonal.

---

7 In fact, this approach is familiar to the numerical linearization method, that is used in KELLER (1980) and DIXON ET AL. (1992). The numerical linearization method uses a fixed Hessian to linearize the original model. The model in relative changes, as developed in KELLER (1980), is represented by \( H(x')\text{diag}(x')x = 0 \), where \( x \) is the vector of relative changes, \( x = (dx_1/x_1, \ldots, dx_n/x_n) \) and \( \text{diag}(x') \) is a diagonal matrix with the elements of \( x' \), the vector with outcomes for the initial state, on the diagonal.
update of the Hessian. On the other hand, the refinement methods did not converge in the relatively favorable case that the Newton method demanded for a large number of iterations before obtaining convergence. Therefore, the unconditioned Newton method was used to compute the initial steady state.

Now, we turn to the computation of dynamic paths. Before the dynamic paths can be calculated, an expectation rule must be determined. If expectations depend only on values of past and present variables, the expectation rule can be inserted in the model and the dynamic path can be calculated sequentially, from the initial period to the period in which the new steady state is reached. The sequential calculation problem can then be solved by simply applying the Newton method for every period. However, if expectations also depend on future values of variables, as in case of rational expectations (perfect foresight), the computation of the dynamic path becomes much more complicated. In addition to the dynamic path, expectations for every period must be calculated. The calculation of these expectations starts with a first guess for expectations over the entire path. After the values of expected variables are determined, the dynamic path can again be calculated sequentially with the Newton method. The values of the expectation variables are now updated while using the values that were calculated for these variables on the dynamic path. The updating of expectations, is in line with the Fair-Taylor algorithm [see Fair and Taylor (1983) and Keuschnigg (1991)]. This algorithm uses the calculated values for the actual path of a variable for the updating of the values of the expectation variable. The value of a variable in period \( t+1 \) as expected by an agent in period \( t \) is, thus, updated for the next iteration \( i+1 \) with the calculated value of this variable for period \( t+1 \) in iteration \( i \):

\[
E[x^{i+1}(t+1)|t] = \lambda x'(t+1) + (1-\lambda)E[x'(t+1)|t] \tag{4.29}
\]

where \( \lambda \) is a parameter with a value between 0 and 1. If \( \lambda = 1 \), the update of the values of the expectation variables uses in iteration \( i+1 \) only information about the calculated value of that variable in iteration \( i \) and neglects information about the

---

8 The initial and new steady state are easily determined with the Newton method, because the values of expected variables are then equal to the current values of these variables. Suppose that expectations are rational, agents have only expectations with respect to next-period-variables and the new steady state is reached in \( T \) periods. An obvious first guess of the expectations is, in this case, the straight line between the steady state values. If \( x_0^* \) is the initial steady state value and \( x_T^* \) is the value of the same variable in the new steady state, this first guess entails that an agent's expectation in period \( t-I \) with respect to the value of variable \( x \) in period \( t \), will be equal to \( (T-t).x_0^* + T.x_T^*/T \).
expectation variable in iteration $i$. The procedure is used for the update of all expectation variables over the entire transition path. The calculation terminates if the values of all variables over the entire transition path as well as the values of all expectation variables satisfy the convergence conditions. Application of this method for the calculation of a perfect foresight equilibrium showed rather slow convergence.\(^9\)

### 4.5 Parameter choice

Before the model can be solved numerically, the functional forms for the production functions and the adjustment cost functions as well as the values of the parameters must be specified. To start with the functional forms, the production functions of the private commodities, the public consumption good and the public production good are assumed to be of the Cobb-Douglas type:

\[
X_j(t) = e^{\alpha}_j L_j(t)^{\delta_{L_j}} K_j(t)^{\delta_{K_j}} G_p(t)^{\delta_{G_p}}, \quad \delta_{L_j} + \delta_{K_j} + \delta_{G_p} = 1 \quad (4.30)
\]

\[
G_s(t) = e^{\alpha}_s L_s(t)^{\delta_{L_s}} K_s(t)^{\delta_{K_s}} G_p(t)^{\delta_{G_p}}, \quad \delta_{L_s} + \delta_{K_s} + \delta_{G_p} = 1 \quad (4.31)
\]

\[
G_p(t) = e^{\alpha}_p L_p(t)^{\delta_{L_p}} K_p(t)^{\delta_{K_p}}, \quad \delta_{L_p} + \delta_{K_p} = 1 \quad (4.32)
\]

where the $e^{\alpha}_j$'s are scaling parameters and $\delta$'s are production elasticities of input factors. Linear homogeneity requires that the distribution parameters sum up to one.

Following HAYASHI (1982) and GOULDER AND SUMMERS (1989), the adjustment cost functions are specified as quadratic functions:

\[
\Phi_j(t) = \frac{\phi_{ij} \left( \frac{I_j(t)}{K_j(t)} - \phi_{ij} \right)^2}{I_j(t) / K_j(t)}, \quad j = 1, 2, p, s \quad (4.33)
\]

---

\(^9\) To speed up the calculations, multiple shooting methods were tested [cf. BLANCHARD AND KAHN (1980), LIPTON ET AL. (1982) and PRESS ET AL. (1986)]. These methods did not lead to significant faster convergence, however.
Table 4.1 Benchmark values of production parameters

<table>
<thead>
<tr>
<th></th>
<th>Private good 1</th>
<th>Private good 2</th>
<th>Public good p</th>
<th>Public good s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scaling parameter production $\Omega_j$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Production elasticity labor $\delta_{lj}$</td>
<td>0.65</td>
<td>0.75</td>
<td>0.65</td>
<td>0.80</td>
</tr>
<tr>
<td>Production elasticity capital $\delta_{kj}$</td>
<td>0.25</td>
<td>0.15</td>
<td>0.35</td>
<td>0.10</td>
</tr>
<tr>
<td>Production elasticity public good $\delta_{qj}$</td>
<td>0.10</td>
<td>0.10</td>
<td>----</td>
<td>0.10</td>
</tr>
<tr>
<td>Depreciation rate $\text{dep}_j$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Scaling parameter adjustment costs $\varphi_{lj}$</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Adjustment cost parameter $\varphi_{qj}$</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>Scaling parameter export $\eta_{lj}$</td>
<td>50</td>
<td>50</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>Export elasticity $\eta_{qj}$</td>
<td>-2.0</td>
<td>-2.0</td>
<td>----</td>
<td>----</td>
</tr>
</tbody>
</table>

The benchmark values of the parameters are now determined. The specification of the initial values of the parameters is based on a qualitative calibration, as in, e.g., Bovenberg (1986, 1989), Auerbach and Kotlikoff (1987), Goulder and Summers (1989) and Broer and Westerhout (1992). Qualitative calibration implies that the structural parameters are chosen such that the results for the initial steady state globally reproduce important economic relations of a small open economy, as the Dutch. Parameters that refer to production in the private and the public sector are summarized in Table 4.1. Production in these sectors differs in the benchmark case only with respect to labor and capital intensity. The influence of the public good on production, the adjustment costs parameters, and the depreciation rate are assumed to be similar for all production sectors. This similarity allows us to investigate the impact of infrastructure and adjustment costs carefully by (steady state) sensitivity analysis. Note that a modest influence of infrastructure on production is assumed [cf. Aschauer (1989)]. For the private sectors it is, furthermore, assumed that the export demand functions are similar for both sectors; the export parameters are identical for the two sectors. With respect to labor input, production in the public consumption sector is taken to be the most labor intensive. Compared to private sector 1, private sector 2 is supposed to have a higher labor intensity. This sector can be interpreted as a service sector, whereas sector 1, that produces goods that can be used for consumption as well as investment, can be loosely interpreted as a manufacturing sector. See for a more accurate calibration method Mansur and Whalley (1984) and Ballard et al. (1985), and, for an econometric method to specify the values of the parameters, Jorgenson (1984).
The labor intensity of infrastructure (public production good) is assumed to be identical to the labor intensity in private sector 1.

The preference weights and the political influence weights are presented in Table 4.2. For the benchmark case it is assumed that the representative individuals of the different social groups have identical preferences. Subsistence levels are assumed to be zero. The preferences are then homothetic. Furthermore, the political influence weights are assumed to be in accordance with the numerical strength of the social groups.

Finally, some exogenous prices and the discount rate must be specified. Foreign prices are assumed to be fixed and equal to one \( p_{fj}(t) = 1 \) and \( p_{gj}(t) = 1 \). With respect to the interest rate, the small country assumption that domestic behavior does not influence the (world) interest rate, implying a given rate, which is taken to be constant over time \( r(t) = 0.05 \), for all \( t \). The share risk premium \( \eta(t) \) is also assumed to be constant over time and set equal to 0.01. Finally, we assume that the discount rate of the government equals the interest rate (\( \rho = 0.05 \)).
4.6 The initial steady state

The results for the initial steady state are presented in Tables 4.3 - 4.5. From Table 4.3 it can be read that the investment-capital ratio \( (I_j/K_j) \) in the steady state equals the depreciation rate, which equals 0.1. Furthermore, Table 4.3 shows that in the steady state the shadow prices of capital are identical for the two private sectors and for the two public production sectors. This identity follows from the fact that the sectors have the same depreciation rate, which leads to an equal investment-capital ratio in the steady state, and equal adjustment costs. The interrelations between adjustment costs, shadow prices of capital, and investment decisions will be discussed in Section 4.7. Profits are higher in sector 1 than in sector 2, but sector 1 has to spend a larger share of these profits on investments than sector 2. As a consequence, dividend payments in sector 1 fall below the payments shareholders in sector 2 receive. It follows that the value of firm 2 is also higher than the value of firm 1. The relation between dividends and the value of the firm is further discussed in Section 4.7.

As can be read from Table 4.4, the utility and consumption levels are equal across social groups. This result can be explained as follows. In the initial situation it is assumed that political influence weights are in accordance with the numerical strength of the social groups. In that case, the government is interested in equating the utility levels of the representative individuals of the different social groups. The government is able to achieve this objective if it can use a redistribution system that generates no efficiency losses, as is the case with lump sum transfers, and individuals have identical preferences.\(^1\) These conditions are fulfilled in the initial situation. In that case, the government policy not only leads to utilities that have the same level for all individuals, but also to similar consumption bundles, as can be read from Table 4.4. Because all individuals supply the same amount of labor, they receive a similar wage income. To obtain equal utility levels, the government redistributes the dividend income of the entrepreneurs through the lump sum special provisions. The equal utility levels imply that the value of the interest function is also equal to this level, as can be checked from Table 4.5.

\(^1\) If preferences were not identical the government would fail in equating the utility levels, because individuals have different preferences with respect to the (pure) public consumption good. Though individuals differ in preferences for the public good, they hand over a similar proportion of their income for the finance of this good. If the production of the public good were not financed with a uniform income tax, but with a group-specific lump sum tax, the government would be able to equate utility levels, even if individuals differ in preferences.
### Table 4.3 Initial steady state values of the production variables

<table>
<thead>
<tr>
<th></th>
<th>Sector 1</th>
<th>Sector 2</th>
<th>Sector p</th>
<th>Sector s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production level $X_j$, $G_j$</td>
<td>286.55</td>
<td>323.16</td>
<td>87.102</td>
<td>125.53</td>
</tr>
<tr>
<td>Capital stock $K_j$</td>
<td>511.69</td>
<td>369.57</td>
<td>166.84</td>
<td>89.075</td>
</tr>
<tr>
<td>Labor demand $L_j$</td>
<td>236.10</td>
<td>327.93</td>
<td>52.629</td>
<td>121.04</td>
</tr>
<tr>
<td>Investment $I_j$</td>
<td>51.169</td>
<td>36.957</td>
<td>16.684</td>
<td>8.9075</td>
</tr>
<tr>
<td>Shadowprice capital $q_j$</td>
<td>0.3696</td>
<td>0.3696</td>
<td>0.00219</td>
<td>0.00219</td>
</tr>
<tr>
<td>Profits $\Pi_j$</td>
<td>79.816</td>
<td>68.856</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>Dividends $D_j$</td>
<td>38.245</td>
<td>38.832</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>Value of the firm $V_j$</td>
<td>483.32</td>
<td>490.74</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>Exports $E_j$</td>
<td>75.755</td>
<td>66.495</td>
<td>-----</td>
<td>-----</td>
</tr>
</tbody>
</table>

### Table 4.4 Initial steady state values of the consumer variables

<table>
<thead>
<tr>
<th></th>
<th>Entrepreneurs $c1$</th>
<th>Entrepreneurs $c2$</th>
<th>Workers $w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic good $1$ $c_{d1}$</td>
<td>0.095253</td>
<td>0.095253</td>
<td>0.095253</td>
</tr>
<tr>
<td>Foreign good $1$ $c_{f1}$</td>
<td>0.029472</td>
<td>0.029472</td>
<td>0.029472</td>
</tr>
<tr>
<td>Domestic good $2$ $c_{d2}$</td>
<td>0.25667</td>
<td>0.25667</td>
<td>0.25667</td>
</tr>
<tr>
<td>Foreign good $2$ $c_{f2}$</td>
<td>0.089734</td>
<td>0.089734</td>
<td>0.089734</td>
</tr>
<tr>
<td>Leisure $\ell_i$</td>
<td>0.26230</td>
<td>0.26230</td>
<td>0.26230</td>
</tr>
<tr>
<td>Special provisions $\sigma_i$</td>
<td>-0.30537</td>
<td>-0.31124</td>
<td>0.077077</td>
</tr>
<tr>
<td>Utility level $U_i$</td>
<td>2.19475</td>
<td>2.19475</td>
<td>2.19475</td>
</tr>
</tbody>
</table>

### Table 4.5 Initial steady state values of prices and some other variables

<table>
<thead>
<tr>
<th></th>
<th>Prices</th>
<th>Taxes et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price good $1$ $p_1$</td>
<td>0.81242</td>
<td>Income tax $\tau_h$</td>
</tr>
<tr>
<td>Price good $2$ $p_2$</td>
<td>0.86714</td>
<td>Tax revenues $T$</td>
</tr>
<tr>
<td>Wage rate $p_L$</td>
<td>0.64091</td>
<td>Marginal social cost $\xi$</td>
</tr>
<tr>
<td>Price investment good $p_L$</td>
<td>0.81242</td>
<td>Value interest function $P$</td>
</tr>
</tbody>
</table>
4.7 Adjustment costs, investment, infrastructure and the asset price

In Section 4.2 it was concluded that the dependence of private production on infrastructure influences the relation between the shadowprice of capital (the marginal q) and the asset price (the average q). In this section the impact of infrastructure on the asset price is further analyzed. First the relations between the shadowprice of capital and investment, as given by eqs. (4.8) and (4.26), are re-examined. Using the functional forms for the adjustment cost functions as specified in eq. (4.33), the shadowprices of capital for the private sectors and the public production sectors are, respectively:

\[ q_j(t+1) = (1-\tau_h(t))p_j(t)\left[1 + 2\phi_{ij}(1-\tau_c(t))(I_j(t) - \phi_{2j})\right] \]  
(4.34)

\[ \frac{q_s(t+1)}{1+\rho} = \xi(t)p_s(t)\left[1 + 2\phi_{1s}(I_s(t) - \phi_{2s})\right] \]  
(4.35)

It can be easily seen from these equations that the two private sectors face an identical shadowprice for capital if these sectors have the same adjustment costs \(\phi_{1i} = \phi_{12}\) and \(\phi_{2i} = \phi_{22}\). Analogously, identical adjustment costs for the two public capital stocks \(\phi_{1p} = \phi_{1s}\) and \(\phi_{2p} = \phi_{2s}\) equate the shadowprices for the two public sectors. In the initial steady state the adjustment costs for the two private capital stocks and the two public capital stocks are assumed to be the same, which explains the equal values of the shadowprices of capital for private sectors 1 and 2, and for public sectors p and s, in Table 4.3. Furthermore, as can be checked from Table 4.1, adjustment costs for the private capital stocks are assumed to be identical to the adjustment costs for the public capital stocks. Finally, it is assumed that the tax system does not contain a corporate income tax \(\tau_c\) in the initial steady state. If these three conditions hold, the following relation between the shadowprice of a private capital stock and a public capital stock can be obtained from eqs. (4.34) and (4.35):

\[ q_j(t+1) = \frac{(1-\tau_h(t))}{(1+\rho)\xi(t)} q_s(t+1) \]  
(4.36)

This relation can be checked for the initial steady state with the results presented in Tables 4.3 - 4.5.
The value of the firm \( (V_j) \), as determined in Section 4.2, depends on the after-tax dividends paid in the current period and in all future periods. If firms expect that the steady state equilibrium perpetuates they expect that dividends paid in all future periods equal the current dividend payments. Eq. (4.3) then boils down to:

\[
V_j^* = \frac{1-\tau_h^*}{r+\eta} D_j^*
\]

where starred variables refer to steady state values (note that the interest rate \( r \) and the risk premium \( \eta \) are constants, as discussed in Section 4.4).

To investigate the relation between the price of shares (the average \( q \)) and the shadowprice of capital (the marginal \( q \)) in the steady state, eq. (4.11) must be solved for the steady state. For the Cobb-Douglas specification of the production function, this equation becomes

\[
V_j(t) = \sum_{s=0}^{\infty} R(t,s)(1-\tau_d(s)) \frac{p_j(s)}{1+\tau_j(s)} \delta_{ij} X_j(s) + q_j(t)K_j(t)
\]

The value of the firm \( (V_j) \) thus depends on the discounted current and future contribution of infrastructure (the public production good) to the sector's sales revenues, which is given by the first term on the right-hand side, and on the value of the capital stock, which is given by the second term on the right-hand side.

Using eq. (4.38), the relation between the asset price and the shadowprice of capital in the steady state can now be determined as:

\[
\frac{V_j^*}{p_t^* K_j^*} = \frac{1-\tau_h^*}{r+\eta_j^*} \frac{p_j^*}{1+\tau_j^*} \frac{\delta_{ij}}{p_t^*} X_j^* + \frac{q_j^*}{p_t^*}
\]

where the left-hand side is equal to the asset price (average \( q \)) and the second term on the right-hand side is equal to the shadowprice of capital, relative to the investment price (marginal \( q \)). As can be calculated with Tables 4.3 - 4.5, the asset price of sector 1 is equal to 1.1626, while the asset price of sector 2 is 1.6344. The shadowprice of capital, relative to the investment price, is for both sectors equal to 0.4549. The difference between the asset price and the shadowprice of capital is due to the fact that production depends on a public good (given by the first term on the right-hand side of eq. (4.39) for the steady state). The rather substantial gap between
the asset price and the shadowprice of capital in both sectors indicates that the impact on the asset price of the dependency of private production on infrastructure is very large. This is particularly so, since the dependency of private production on the public production good is assumed to be modest \( \delta_G = 0.10 \); compare the values of the production elasticities of infrastructure in ASCHAUER (1989) and MUNNELL (1992). This result suggests that the tax-adjusted asset price is a poor indicator of the shadowprice of capital. Investment decisions are not only based on the tax-adjusted asset price, but also on the marginal benefits of the use of the public production good, relative to the value of the private capital stock \( b \). Rearranging eq. (4.34) and inserting the expression for the shadowprice of capital, that can be obtained from eq. (4.39), in this equation gives the following investment function:

\[
\frac{I_j^*}{K_j^*} = \phi_j + \frac{1}{2\phi_j(1-\tau_j^* \eta_j^*)} \left( \frac{V_j^*}{1+\tau_j^*} - \frac{1}{1+\tau_j^*} \frac{\delta_G X_j^*}{p_j^*} - 1 \right)
\]  

(4.40)

It follows from this equation that, ceteris paribus, an increase in the marginal benefits from infrastructure (the public production good) negatively affects investment in private capital.

As an extra check on the computed outcomes, an alternative equation for dividends can be calculated. Because investment decisions and the asset price depend on the marginal benefits of the public production good, dividends also depend on the marginal benefits of this good. This follows immediately by combining eqs. (4.37) and (4.39), which gives:

\[
D_j^* = \frac{p_j^*}{1+\tau_j^*} \delta_G X_j^* + \frac{r+\eta_j^*}{1-\tau_h^*} q_j^* K_j^*
\]  

(4.41)

4.8 Transition effects under various expectation rules

In Section 4.5, the steady state results for the benchmark parameters were discussed. If this parameter set changes, the economy moves away from the initial steady state. If the steady state is reached, the values of the endogenous variables will be constant in every period. Therefore, the values of the variables in the new steady state do not depend on the way individuals form expectations. Whether expectations depend on lagged, current or future values is not relevant for the steady state, because lagged,
current and future values are equal to each other in the steady state. However, the transition path of the economy from the initial steady state to the new steady state does depend on the formation of expectations. In the model presented in this chapter, the only variables of future periods that influence decisions in the current period are the shadow prices of capital for the next period \([q_j(t+1), j = 1, 2, p, s]\). In this section the transition path is studied for four different expectation rules: lagged expectations, static expectations, adaptive expectations and rational expectations.\(^{12}\) For the shadow prices of capital, these expectation rules can be defined as follows:

<table>
<thead>
<tr>
<th>Expectation Rule</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged Expectations</td>
<td>(E[q_j(t+1), t] = q_j(t-1))</td>
</tr>
<tr>
<td>Static Expectations</td>
<td>(E[q_j(t+1), t] = q_j(t))</td>
</tr>
<tr>
<td>Adaptive Expectations</td>
<td>(E[q_j(t+1), t] = (1-\beta_j)q_j(t-1) + \beta_j q_j(t), \ 0 &lt; \beta_j &lt; 1)</td>
</tr>
<tr>
<td>Rational Expectations</td>
<td>(E[q_j(t+1), t] = q_j(t+1))</td>
</tr>
</tbody>
</table>

If expectations follow the lagged, static or adaptive rule, agents do not need information about values of future variables while making decisions in the current period. For these expectation rules the calculation of the transition path is straightforward. Starting with the initial steady state, the transition path can be calculated period by period until the new steady state is reached. The computation procedure is much more complex and time consuming if expectations are assumed to be rational. Under this rule present decisions depend on future decisions. The procedure starts with a first guess for every period with respect to the values of the expectation variables. For every variable a vector with expectations is constructed with a length that is similar to the (expected) length of the transition path. This vector gives, for every period on the transition path, the value of a variable that agents expect for the next period in the current period \([E[q_j(t+1), t]\), which is element \(t\) in the vector]. The initial expectation vectors give the initial guesses of the expectation vectors for the entire transition path. After the first calculation of the whole transition path, the expectation vectors are updated. This is done by updating the elements of the initial vector with the calculated values in the first iteration of the realization of this variable in the next period \([q_j(t+1), t+1\) in the calculated vector]. The new value then yields:

---

\(^{12}\) Although ‘expectation’ is, in fact, not the correct expression in a model without uncertainty, this expression is often used in such models. Other expressions in case of deterministic models are myopic behavior instead of static expectations and perfect foresight instead of rational expectations.
\[ E[q_j^{t+1}(t+1), t] = \lambda E[q_j^t(t+1), t] + (1-\lambda)q_j^t(t+1), \quad 0 < \lambda < 1 \]  

(4.42)

where the superscripts refer to iteration numbers. The procedure terminates if two succeeding iterations give sufficiently similar results for all variables on the entire transition path and for the expectation vectors. The calculation procedures were discussed in more detail in Section 4.4. In order to speed up the time consuming calculations we excluded the foreign sector for this analysis. The changes in the model that are necessary for this adaptation are straightforward. The parameter configuration that is used for the analysis of the expectation rules is presented in Appendix 4.A. In this Appendix one can also find the outcomes for the initial and the end steady state. The transition effects are studied for a technological change. It is assumed that production in the private sectors becomes more dependent on infrastructure (the public production good), under a simultaneous decrease of their dependence on private capital. The technological shock that occurs after the initial period is specified by a change in the production elasticities of capital and infrastructure, \( \delta_{K1} \) and \( \delta_{G1} \), for the private sectors. \[^{13}\]


The figures show that the lagged expectation rule gives a rather different transition path compared to the other expectation rules. Under lagged expectations, production levels and levels of capital stocks cycle to the new steady state, whereas they follow a monotonic path to the new steady state if another expectation rule holds (cf. Figures 4.B.1 - 4.B.8). The lagged expectation rule requires, therefore, a larger number of periods before the new steady state is reached. The variables approach the new steady state values after approximately 240 periods under the lagged expectation rule, while this state is reached within 120 periods if rational expectations are assumed.

Although the exact transition path depends on the expectation rule, the following interpretation of the economic development during the transition holds more or less for all expectation rules. The technological change that occurs in both private sectors, making production more dependent on infrastructure, causes the level of the public production good to increase (cf. Figure 4.B.8). The higher production of the public production good requires higher labor and capital input in this sector. In order to increase the capital stock for the public production good the government is willing to invest more in this stock. As a result, the shadowprice of capital used for infrastructure \( (q_p) \) increases strongly (cf. Figure 4.B.12). Because the adaptation of the

\[^{13}\] To be precise, \( \delta_{K1} \) changes from 0.25 to 0.20, \( \delta_{G1} \) from 0.10 to 0.15, \( \delta_{K2} \) from 0.15 to 0.10, and \( \delta_{G2} \) from 0.10 to 0.15.
capital stock takes time, the extension of infrastructure depends strongly on extra labor input in the first periods. This extra labor input pushes up the wage rate, which decreases labor input in the other sectors. This effect is reinforced by the increase in the income tax, which is necessary to finance the expansion of infrastructure (cf. Figure 4.B.13). Consequently, the production levels in the private sectors and the public consumption sector decrease in the first periods. Although there is no technological change in the public consumption sector, the capital stock in this sector decreases because production in this sector profits from the increased production of the public production good (cf. Figure 4.B.7).

The technological change induces entrepreneurs in the private sectors to decrease their capital stock. The concomitant cut in investment leads to a strong decrease in the shadowprices of the capital stocks (cf. Figures 4.B.9 and 4.B.10). Note that these shadowprices may differ between the two private sectors, as well as between the two public sectors, on the transition path, because the investment-capital ratios may differ along the path, leading to differences in the adjustment costs. Although the investment-capital ratio is smaller than the depreciation rate in the first periods and capital stocks, thus, decline, there is still an overcapacity of capital, which increases the costs per unit of production. These costs are further increased by the higher adjustment costs per unit of investment, due to the concavity of the adjustment cost function. The higher costs per unit of production lead to a decrease in profits and, as a consequence, in dividend payments. Since individuals of the different social groups have identical preferences, and political influence is based on the numerical strength of these groups, the utilities of individuals are equalized through redistribution in every period. The lower dividend payments lead to a reduction in the difference in income between individuals of different social groups. Therefore, less income is redistributed in the first periods after the shock than in the initial steady state, which leads to smaller transfers (cf. Figures 4.B.14 - 4.B.16).

After the first periods where the changes in labor input in the different sectors play a dominant role, the transition path is dominated by the development of capital stocks. The private capital stocks continue their decreasing paths until they have reached the new steady state. The exact path depends on the expectation rule (cf. Figures 4.B.1 and 4.B.2). The overcapacity of capital disappears which makes a recovery of shadowprices of the private capital stocks possible (cf. Figures 4.B.9 and 4.B.10). The

---

14 Not that, because of their identical preferences, all individuals have the same labor income in every period.
investment-capital ratio increases, leading to lower adjustment costs per unit of production. Profits increase and investment costs decrease, giving dividends the opportunity to recover. Dividend payments in sector 2 even exceed the initial payments, because this sector benefits more from the technological change than sector 1. This is due to the higher relative change in the capital share of this sector.\(^\text{15}\) In sector 1, dividend payments are higher in the medium term (that is, after about ten till fifty periods) than in the new steady state, because production in this sector profits from the size of infrastructure that is higher in the medium term than in the new steady state, as will be explained hereafter. The higher dividend payments cause redistributions to increase, as shown by the higher special provisions that are received by workers and paid by entrepreneurs. Entrepreneurs in sector 2 end up with paying larger transfers in the new steady state, compared with the initial steady state, while entrepreneurs in sectors 1 pay more or less a similar amount (cf. Figures 4.B.14 and 4.B.15). The path of the transfers paid by entrepreneurs in sector 1 follows the path of the dividends payments for this sector, which implies that these transfers are higher in the medium run than in the new steady state.

Although private production profits from the increase in the level of production of the public production good, private production follows the motion of the capital stock and decreases over the entire transition path. This decrease is stronger in sector 1 than in sector 2 after the first period (cf. Figures 4.B.5 and 4.B.6). The first period showed the strongest decrease in the production of sector 2. These differences in the development of production can be explained from the fact that production in sector 2 relies relatively more on labor and less on capital than the production in sector 1, which makes production in sector 2 more flexible.

In the meantime, the transition path of public capital that is used for infrastructure further increases (cf. Figure 4.B.4). The government’s interest in increasing this capital stock diminishes, which is reflected in a lower shadowprice (cf. Figure 4.B.12). It appears that an overshooting in investment for infrastructure arises if expectations are lagged, adaptive or static, which leads to a capital stock in the medium run that is higher than its size in the new steady state. If expectations are rational, this overshooting does not take place. In that case, the dynamic paths of the capital stock in the public production sector and the expansion of infrastructure are monotonic paths to the new steady state levels. In the public consumption sector a

\(^{15}\) That is, \(\delta_{k2}\) decreases from 0.15 to 0.10, implying a decrease of one third, while \(\delta_{k1}\) decreases from 0.25 to 0.20, implying a decrease of one fifth.
further decrease in the capital stock can be observed (cf. Figure 4.B.3). Production in this sector profits from the expansion of infrastructure causing the initial decline in production to convert into an increase. Though the capital stock for the public consumption sector is lower in the new steady state than it was before, the level of the public consumption good is higher, due to the larger size of infrastructure in the new steady state (cf. Figures 4.B.7 and 4.B.8). The larger size of infrastructure not only reduces the capital stock that is used for the production of the public consumption good, it also cuts the demand for labor in this sector, which is the most labor intensive production sector. This cut is accompanied by a substantial increase in the consumption of leisure (cf. Figure 4.B.19; note that consumption does not differ between individuals of different social groups, for similar reasons as put forward in Section 4.6).

All the changes in costs that arise in the public sector after the first period hardly affect the distribution of total factor income over private and public spending after this period, as can be observed from the development of the income tax rate during the transition (cf. Figure 4.B.13). Stationarity of the income tax particularly occurs under rational expectations. Other expectation rules lead to an overshooting of investment in infrastructure, requiring a higher income tax rate for some periods.

With respect to consumption after the first period, we noticed already a higher level of leisure and public consumption. Private consumption decreases, however (cf. Figures 4.B.17 and 4.B.18). The negative effect of the decrease in private consumption on utility is not offset by the positive effect of extra leisure and public consumption.

4.9 Concluding remarks

With the model presented in this chapter some new elements are introduced in a dynamic general equilibrium model. The first extension entails the introduction of a public good that influences private production. Only a few macro models focusing on endogenous growth include a public good (or public capital) in the production function [see BARRO (1990) and ALOGOSKOUFIS AND VAN DER PLOEG (1990)]. The second new element is the dynamic decisionmaking by the government concerning public production. The production process in the public sector is a neglected issue in general equilibrium models. Exceptions are BALLARD ET AL. (1985), where the production of public enterprises (providing goods and services subject to a user charge) is explicitly
modeled, and the model of Pereira (1994) which incorporates the production of a publicly provided consumption good.

The appearance of a public production good (or infrastructure) in the production function leads to an adjustment of the relation between the asset price and the shadow price of capital. The asset price is not only based on the marginal benefits of the private capital stock, but also on the marginal benefits of infrastructure. The latter appears to have a substantial impact on the asset price. As empirically observed in Lynde and Richmond (1993) the dependence of private production on infrastructure has a positive effect on the return on capital, whereas Morrison and Schwartz (1996) find a positive effect of infrastructure on the productivity of input factors. This chapter gives a theoretical underpinning of this observation. Important for this result is that there are constant returns to scale in private capital input, labor input and infrastructure, which leads to decreasing returns to scale in the private inputs. The absence of constant returns to scale in private inputs leads to a discrepancy between total factor costs and production revenues, which can be sustained in the model of this chapter, because adjustments costs are a barrier for entry. The substantial impact of infrastructure on the asset prices occurs in spite of the modest production elasticity of infrastructure that is assumed in the private sectors [see, e.g., Aschauer (1989)].

Two aspects that are not considered in the model may even reinforce this effect. First, the model abstains from congestion in infrastructure. If congestion is allowed for, infrastructure may be further expanded, because it reduces congestion, which implies an extra positive benefit. Second, infrastructure is assumed to be a pure production good that has no consumption value. If infrastructure is considered as a mixed good that is used for both production and consumption, the production of infrastructure will further increase. An expansion of infrastructure leads to an increase in the influence of infrastructure on the asset price, as can be read from eqs. (4.11) and (4.39). Therefore, infrastructure may even have a stronger effect on the asset price than is suggested by the model in this chapter.

With respect to dynamics, the model concentrates on production decisions. It abstracts from intertemporal consumer decisions. The absence of consumer dynamics may lead to an underinvestment in infrastructure in the beginning of a transition path. Myopic consumers are only interested in today's consumption, while consumers that take account of the future may shift some income to the future, unless, of course, they expect their income to increase continuously in the future.
The dynamics in the model require the specification of an expectation rule. For a technological change that makes production in both private sectors more dependent on infrastructure we analyzed the transition path for four different expectations rules. If expectations are rational, individuals foresee the long term effects of these technological changes. In that case, the transition paths of all economic variables show a monotonic development towards the new steady state. If expectations are static or adaptive, the government overestimates the marginal benefits of extra investments in public capital. Consequently, the production level and the capital stock of both the public consumption and public production good are too high (that is, higher than the new steady state level) for a short period, which is corrected by a cut in investment. Compared to rational expectations, investment in the private sectors is more rapidly reduced if expectations are static or adaptive, but these underinvestments are not so strong that the production levels and the capital stocks in the private levels become too low. Lagged expectations, on the other hand, lead to overinvestment in the public sectors and to underinvestment in the private sectors. The transition paths show a cyclical pattern towards the new steady state. Under lagged expectations much more periods are required before the new steady state is reached than for the other expectation rules.
Appendix 4.A Parameter set for the analysis of expectations

The analysis concerning expectations is discussed in Section 4.8. The calculations for this analysis were very time consuming, especially when rational expectations were assumed. To reduce calculation time, the model was reduced to a closed economy. This was done by omitting the export demand equations and the balanced current account condition [eqs. (4.16) and (4.19), respectively]. Furthermore, consumers’ utility trees were stripped of the third nesting [eq. (4.14)]. The required adaptation of the second nesting is straightforward [eq. (4.13)].

Compared to the benchmark parameters of Section 4.5 there is only a difference in the production parameters of private sector 1. To avoid confusion due to the elimination of the third nesting in the utility function, the consumption parameters are reproduced here. Production and consumption parameters can be found in Tables 4.A.1 and 4.A.2, respectively.

The shock that underlies the analysis of expectations is a technology shock in private sectors 1 and 2. Production in these sectors is more dependent on infrastructure and less dependent on private capital after the shock has occurred. To be more specific, the capital share $\delta_{K1}$ reduces to 0.20, while the infrastructure share $\delta_{G1}$ goes up to 0.15, while in private sector 2 the capital share $\delta_{K2}$ becomes 0.10 and the infrastructure share $\delta_{G2}$ increases to 0.15. The initial and end steady states are presented in Tables 4.A.3 - 4.A.8.

Table 4.A.1 Values of production parameters for the analysis of expectations

<table>
<thead>
<tr>
<th></th>
<th>Private good 1</th>
<th>Private good 2</th>
<th>Public good p</th>
<th>Public good s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scaling parameter $\Omega_j$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Production elasticity labor $\delta_{l_j}$</td>
<td>0.65</td>
<td>0.75</td>
<td>0.65</td>
<td>0.80</td>
</tr>
<tr>
<td>Production elasticity capital $\delta_{K_j}$</td>
<td>0.25</td>
<td>0.15</td>
<td>0.35</td>
<td>0.10</td>
</tr>
<tr>
<td>Production elasticity public good $\delta_{G_j}$</td>
<td>0.10</td>
<td>0.10</td>
<td>----</td>
<td>0.10</td>
</tr>
<tr>
<td>Depreciation rate $dep_j$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Scaling parameter $\varphi_{ij}$</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Adjustment cost parameter $\varphi_{ij}$</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
<td>0.12</td>
</tr>
</tbody>
</table>
Table 4.A.2 Values of consumption parameters for the analysis of expectations

<table>
<thead>
<tr>
<th></th>
<th>Entrepr. c1</th>
<th>Entrepr. c2</th>
<th>Workers w</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substitution parameter $\gamma_i$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Private goods weight $\alpha_{ci}$</td>
<td>0.75</td>
<td>0.75</td>
<td>0.50</td>
</tr>
<tr>
<td>Leisure weight $\alpha_{b}$</td>
<td>0.15</td>
<td>0.15</td>
<td>0.25</td>
</tr>
<tr>
<td>Public good weight $\alpha_{oi}$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.25</td>
</tr>
<tr>
<td>Substitution parameter $\gamma_{ei}$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>Weight private good 1 $\alpha_{ii}$</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Weight private good 2 $\alpha_{ii}$</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>Numerical strength $N_i$</td>
<td>100</td>
<td>100</td>
<td>800</td>
</tr>
<tr>
<td>Political influence weight $\mu_i$</td>
<td>0.10</td>
<td>0.10</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Table 4.A.3 Initial steady state values for production variables

<table>
<thead>
<tr>
<th></th>
<th>Sector 1</th>
<th>Sector 2</th>
<th>Sector p</th>
<th>Sector s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production level $X_p, G_j$</td>
<td>249.24</td>
<td>357.15</td>
<td>86.221</td>
<td>125.47</td>
</tr>
<tr>
<td>Capital stock $K_j$</td>
<td>445.07</td>
<td>420.38</td>
<td>169.63</td>
<td>90.715</td>
</tr>
<tr>
<td>Labor demand $L_j$</td>
<td>201.32</td>
<td>365.67</td>
<td>51.351</td>
<td>120.84</td>
</tr>
<tr>
<td>Investments $I_j$</td>
<td>44.507</td>
<td>42.038</td>
<td>16.963</td>
<td>9.0715</td>
</tr>
<tr>
<td>Shadowprice capital $q_j$</td>
<td>0.4151</td>
<td>0.4151</td>
<td>0.00073</td>
<td>0.00073</td>
</tr>
<tr>
<td>Profits $\Pi_j$</td>
<td>77.787</td>
<td>87.757</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>Dividends $D_j$</td>
<td>37.273</td>
<td>49.491</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>Value of the firm $V_j$</td>
<td>472.11</td>
<td>626.89</td>
<td>-----</td>
<td>-----</td>
</tr>
</tbody>
</table>

Table 4.A.4 Initial steady state values for consumer variables

<table>
<thead>
<tr>
<th></th>
<th>Entrepr. c1</th>
<th>Entrepr. c2</th>
<th>Workers w</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic good 1 $c_{ail}$</td>
<td>0.13216</td>
<td>0.13216</td>
<td>0.13216</td>
</tr>
<tr>
<td>Domestic good 2 $c_{a2i}$</td>
<td>0.35715</td>
<td>0.35715</td>
<td>0.35715</td>
</tr>
<tr>
<td>Leisure $l_i$</td>
<td>0.26083</td>
<td>0.26083</td>
<td>0.26083</td>
</tr>
<tr>
<td>Special provisions $\alpha_i$</td>
<td>-0.28596</td>
<td>-0.40814</td>
<td>0.086763</td>
</tr>
<tr>
<td>Utility level $U_i$</td>
<td>2.25683</td>
<td>2.25683</td>
<td>2.25683</td>
</tr>
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Table 4.A.5 Initial steady state values for prices and miscellaneous

<table>
<thead>
<tr>
<th></th>
<th>Prices</th>
<th>Taxes et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price good 1 $p_1$</td>
<td>0.91027</td>
<td>0.24001</td>
</tr>
<tr>
<td>Price good 2 $p_2$</td>
<td>1.00000</td>
<td>150.78</td>
</tr>
<tr>
<td>Wage rate $p_L$</td>
<td>0.73253</td>
<td>0.00128</td>
</tr>
<tr>
<td>Price investment good $p_i$</td>
<td>0.91027</td>
<td>2.25683</td>
</tr>
</tbody>
</table>

Table 4.A.6 End steady state values for production variables

<table>
<thead>
<tr>
<th></th>
<th>Sector 1</th>
<th>Sector 2</th>
<th>Sector p</th>
<th>Sector s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Production level $X_j, G_j$</td>
<td>206.29</td>
<td>322.66</td>
<td>113.14</td>
<td>126.88</td>
</tr>
<tr>
<td>Capital stock $K_j$</td>
<td>294.70</td>
<td>252.36</td>
<td>210.11</td>
<td>82.368</td>
</tr>
<tr>
<td>Labor demand $L_j$</td>
<td>182.06</td>
<td>359.82</td>
<td>69.513</td>
<td>119.88</td>
</tr>
<tr>
<td>Investments $I_j$</td>
<td>29.470</td>
<td>25.236</td>
<td>21.011</td>
<td>8.2368</td>
</tr>
<tr>
<td>Shadowprice capital $q_j$</td>
<td>0.4029</td>
<td>0.4029</td>
<td>0.00115</td>
<td>0.00115</td>
</tr>
<tr>
<td>Profits $\Pi_j$</td>
<td>64.853</td>
<td>79.742</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>Dividends $D_j$</td>
<td>37.943</td>
<td>56.699</td>
<td>----</td>
<td>----</td>
</tr>
<tr>
<td>Value of the firm $V_j$</td>
<td>465.08</td>
<td>694.98</td>
<td>----</td>
<td>----</td>
</tr>
</tbody>
</table>

Table 4.A.7 End steady state values for consumer variables

<table>
<thead>
<tr>
<th></th>
<th>Entrepreneurs $c_1$</th>
<th>Entrepreneurs $c_2$</th>
<th>Workers $w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic good 1 $c_{d1i}$</td>
<td>0.11898</td>
<td>0.11898</td>
<td>0.11898</td>
</tr>
<tr>
<td>Domestic good 2 $c_{d2i}$</td>
<td>0.32266</td>
<td>0.32266</td>
<td>0.32266</td>
</tr>
<tr>
<td>Leisure $\xi_i$</td>
<td>0.26873</td>
<td>0.26873</td>
<td>0.26873</td>
</tr>
<tr>
<td>Special provisions $a_i$</td>
<td>-0.28479</td>
<td>-0.47235</td>
<td>0.094642</td>
</tr>
<tr>
<td>Utility level $U_i$</td>
<td>2.13426</td>
<td>2.13426</td>
<td>2.13426</td>
</tr>
</tbody>
</table>

Table 4.A.8 End steady state values for prices and miscellaneous

<table>
<thead>
<tr>
<th></th>
<th>Prices</th>
<th>Taxes et al.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price good 1 $p_1$</td>
<td>0.91312</td>
<td>0.26456</td>
</tr>
<tr>
<td>Price good 2 $p_2$</td>
<td>1.00000</td>
<td>155.15</td>
</tr>
<tr>
<td>Wage rate $p_L$</td>
<td>0.67254</td>
<td>0.00201</td>
</tr>
<tr>
<td>Price investment good $p_i$</td>
<td>0.91312</td>
<td>2.13426</td>
</tr>
</tbody>
</table>
Appendix 4.B Transition effects under different expectation rules

Figure 4.B.1 Effects of an infrastructural technology shock on the capital stock of sector 1 under different expectation rules

Figure 4.B.2 Effects of an infrastructural technology shock on the capital stock of sector 2 under different expectation rules
Figure 4.B.3 Effects of an infrastructural technology shock on the capital stock of sector $s$ under different expectation rules.

Figure 4.B.4 Effects of an infrastructural technology shock on the capital stock of sector $p$ under different expectation rules.
Figure 4.B.5 Effects of an infrastructural technology shock on the production of sector 1 under different expectation rules

Figure 4.B.6 Effects of an infrastructural technology shock on the production of sector 2 under different expectation rules
Figure 4.B.7 Effects of an infrastructural technology shock on the level of the public consumption good under different expectation rules

Figure 4.B.8 Effects of an infrastructural technology shock on the level of the public production good under different expectation rules
Figure 4.B.9 Effects of an infrastructural technology shock on the shadow price of capital of sector 1 under different expectation rules

Figure 4.B.10 Effects of an infrastructural technology shock on the shadow price of capital of sector 2 under different expectation rules
Dynamic decisionmaking on private and public production

Figure 4.8.11 Effects of an infrastructural technology shock on the shadowprice of capital of sector $s$ under different expectation rules

Figure 4.8.12 Effects of an infrastructural technology shock on the shadowprice of capital of sector $p$ under different expectation rules
Figure 4.B.13 Effects of an infrastructural technology shock on the income tax rate under different expectation rules

Figure 4.B.14 Effects of an infrastructural technology shock on special provisions to capital owners of sector 1 under different expectation rules
Figure 4.B.15 Effects of an infrastructural technology shock on special provisions to capital owners of sector 2 under different expectation rules.

Figure 4.B.16 Effects of an infrastructural technology shock on special provisions to workers under different expectation rules.
Figure 4.B.17 Effects of an infrastructural technology shock on the consumption of private commodity 1 under different expectation rules

Figure 4.B.18 Effects of an infrastructural technology shock on the consumption of private commodity 2 under different expectation rules
Figure 4.B.19 Effects of an infrastructural technology shock on leisure under different expectation rules

![Graph showing the effects of an infrastructural technology shock on leisure under different expectation rules. The graph compares Lagged, Static, Adaptive, and Rational expectation rules over 250 periods, with leisure levels indicated by lines of different patterns and colors.](image)

An argument for the result in Chapter 4 shows that changes in the political influence structure do not affect private and public production decisions. If both types have identical preferences, they can be seen in the utility function that are used in this chapter. The Cobb-Douglas specification that was used in Chapter 4 is a special case of the CES utility function in the sense that the Cobb-Douglas function yields the same elasticity of substitution parameter as the CES function when the CES type is more flexible than the Cobb-Douglas function. It still satisfies homothetic preferences. A further advantage of the CES type is that it can be obtained by specifying a mean-behavioral function, such as constant elasticity of substitution. If the Cobb-Douglas utility function is used, the elasticity of substitution is the same across all types, as presented in Chapter 4. If the Cobb-Douglas function is used, the elasticity of substitution between types will be equal across all types. The analysis of changes in production is limited.