Mapping and Localization from a Panoramic Vision Sensor
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Appendix B

Belief Function Representations

The key technical difficulty in implementing the recursive localization approach presented in section 3.3 is to maintain an accurate representation of the belief function and to do so efficiently. In this section we outline some important approaches that have been proposed in literature.

Kalman filtering. A traditional approach to recursive state estimation is Kalman filtering. The Kalman filter represents posteriors by a Gaussian distributions. A Gaussian distribution is completely characterized by its mean and covariance matrix. The Gaussian distribution is closed under linear transformations. In a Kalman filter this property is exploited to derive a posterior analytically under this assumption. The Gaussian distribution is however not closed under convolution with an arbitrary motion model or multiplication with an arbitrary observation model. These happen to be the main steps in probabilistic robot localization. This means that even if at some point in time the belief is Gaussian, the posterior derived from a new measurement will generally not be Gaussian. This problem can be overcome partially if the sensor model and motion model are linearized (extended Kalman filtering, unscented Kalman filtering). The extended Kalman filter maintains a Gaussian belief which approximates the true posterior. The quality of the approximation depends on the uncertainty of the current state estimate. Poor approximations generally result when the true posterior is multi-modal. This typically occurs when the robot is very uncertain about its pose. In this case, only one of these peaks will be retained by the Kalman filter. The use of Kalman filtering is therefore restricted to pose tracking, where the robot initially knows its location. As a straightforward extension of the Kalman filter, multi-hypothesis Kalman filtering has been proposed, which use a mixture of Gaussians to represent multiple hypotheses concerning the robot pose [40].

Grid-based methods. A popular alternative are grid-based methods. In grid based methods, densities are modeled as piece-wise constant functions (histograms). The
workspace of the robot is discretized into cells. The value of each cell reflects the likelihood that the robot is located somewhere in the area covered by the cell. Grid based methods can accurately represent multi-modal distributions. The localization accuracy that can be obtained is limited by the grid resolution. Another problem of the naive grid based method is that the memory and computation requirements grow proportional in the number of cells. To overcome these issues, several enhancements such as octree decompositions and geometric hashing methods have been proposed in literature [5].

**Particle-based methods.** Particle-based methods are quickly gaining popularity (see [14] for an overview of the state of the art). Particle based methods approximate posterior densities by a weighted set of \( m \) samples (called particles). The weights are called importance factors. The basic algorithm is as follows. The initial density is represented by a uniform sample of fixed size. When the robot has moved, a new set of \( m \) samples is drawn according to the importance factors. Samples with a larger importance factor are thus more likely to be drawn. For each such drawn sample, a successor location is guessed according to the motion model. If a new observation arrives, a new importance factor is calculated for each sample according to the observation model. Finally, the importance factors are normalized such that their sum equals unity.

The particle based algorithms are popular because they are relatively easy to implement, yet can faithfully represent arbitrary densities provided that enough particles are used. There are many variants of the basic algorithm. Modifications have been proposed to adapt the size of the sample set dynamically. The rationale is that fewer particles are needed to accurately approximate a density which has a few narrow peaks. One problem of the basic algorithm is that particles are drawn from the prior \( p(x) \). This may fail to produce enough particles in the overlapping region between the prior and the likelihood \( p(o|x) \). This may cause the posterior may be poorly represented. As a result, a robot that is well localized may get lost because the posterior is poorly represented. One way to recover from such situations is to insert some random samples. More advanced methods have been proposed which do not just sample from the prior, but instead aim to optimally sample from the posterior [105]. These methods have been shown to be able to recover from situations where a prior is peaked at the wrong location (the so-called *kidnapped robot problem*).