Measurement of the b-jet cross section at $\sqrt{s}=1.96$ TeV
Peters, O.

Citation for published version (APA):
Peters, O. (2003). Measurement of the b-jet cross section at $\sqrt{s}=1.96$ TeV

General rights
It is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), other than for strictly personal, individual use, unless the work is under an open content license (like Creative Commons).

Disclaimer/Complaints regulations
If you believe that digital publication of certain material infringes any of your rights or (privacy) interests, please let the Library know, stating your reasons. In case of a legitimate complaint, the Library will make the material inaccessible and/or remove it from the website. Please Ask the Library: http://uba.uva.nl/en/contact, or a letter to: Library of the University of Amsterdam, Secretariat, Singel 425, 1012 WP Amsterdam, The Netherlands. You will be contacted as soon as possible.
Chapter 6

**B-jet cross section**

In $pp$ collisions, the $b$-jets are produced with a transverse momentum that follows a certain underlying distribution. In the experiment the goal is to measure this distribution. However, it cannot be measured directly due to contamination from background events and physics effects like the $b \rightarrow \mu + \nu$ decay, in which the neutrino carries energy away from the jet. Detector resolution and acceptance effects also distort the distribution. To measure the true distribution with which the $b$-jets are produced, we correct for these effects in this chapter.

### 6.1 Measurement strategy

In chapter 5, we have measured the $\mu$+jet cross section as a function of calorimeter jet $E_T$, corrected for the Jet Energy Scale, without the muon and neutrino energies but with the energy deposit of the muon in the calorimeter subtracted (see section 5.1). We call this jet energy $E_T^{jet,cal}$ (see figure 5.7). We will use this cross section for two purposes. The first is the calculation of the $\mu$+jet cross section as a function of the true particle level jet energy without the muon and neutrino energy, $E_T^{jet,pr}$. This distribution is convoluted with the calorimeter resolution and the kinematic acceptance of the detector:

$$\frac{d\sigma(\mu + jet)}{dE_T^{jet,cal}} = R^{cal}(E_T^{jet,pr}) \otimes K(E_T^{jet,pr}) \otimes \frac{d\sigma(\mu + jet)}{dE_T^{jet,pr}}$$  \hspace{1cm} (6.1)

where $R^{cal}(E_T^{jet,pr})$ represents the calorimeter smearing and $K(E_T^{jet,pr})$ is the kinematic acceptance of the detector. Correcting for these effects is done by using an unfolding procedure which corrects for the calorimeter resolution, in addition to a kinematic acceptance correction.

The second use of this initial $\mu$+jet cross section is to extract the $b$-jet cross section. The measurement of the $b$-jet content is done using a variable called $P_T^{rel}$, which is the...
momentum of the muon relative to the combined $\mu$+jet axis, and results in a fractional $b$-jet content as a function of $E_T^{jetCal}$. Applying this fractional $b$-jet content to the measured $\mu$+jet cross section (as function of $E_T^{jetCal}$) yields the $b$-jet cross section, measured as a function of $E_T^{jetCal}$, which we call $B(E_T^{jetCal})$:

$$B(E_T^{jetCal}) = \frac{d\sigma(b \rightarrow \mu)}{dE_T^{jetCal}} \quad (6.2)$$

This distribution $B(E_T^{jetCal})$ again forms the basis from which we extract two underlying distributions.

The first distribution is the $b$-jet cross section as a function of the particle level energy ($E_{TP}^{jet}$), without the energy of the muon and neutrino, and with the $b$-quark decaying to a muon. We call this distribution $G(E_T^{jetp})$:

$$G(E_T^{jetp}) = \frac{d\sigma(b \rightarrow \mu)}{dE_T^{jetp}} \quad (6.3)$$

This distribution is convoluted with the kinematic acceptance of the detector $K(E_{TP}^{jetp})$ and the resolution of the calorimeter $R^{Cal}(E_T^{jetp})$ to result in the measured $b$-jet cross section:

$$B(E_T^{jetCal}) = R^{Cal}(E_T^{jetp}) \otimes K(E_{TP}^{jetp}) \otimes G(E_T^{jetp}) \quad (6.4)$$

To correct the measured $b$-jet cross section for the calorimeter resolution $R^{Cal}(E_T^{jetp})$, we follow an unfolding procedure, which results in a correction function $C_R(E_T^{jetCal})$. We correct for the effects of the kinematic acceptance $K(E_{TP}^{jetp})$ by a correction function $C_K(E_T^{jetp})$, which is extracted from the Monte Carlo simulation.

The second distribution is the differential $b$-jet cross section $F(E_{T^{jetpL}})$, with no constraint on the decay of the $b$-quark and measured as a function of the total $b$-jet energy, $E_{T^{jetpL}}$:

$$F(E_{T^{jetpL}}) = \frac{d\sigma}{dE_{T^{jetpL}}} \quad (6.5)$$

This energy includes the muon and neutrino energy. In addition to the two corrections described above to measure $G(E_T^{jetp})$, we now have to apply two extra corrections to correct for the energy of the muon and the neutrino. First, we have the $b \rightarrow \mu + \nu$ branching ratio, $T(b \rightarrow \mu)$. This is an overall scale factor, independent of the jet energy. Then, we have to correct for the energy of the muon and neutrino, which changes the jet energy. Unlike all the previous effects, this effect, $E^{b\rightarrow\mu+\nu}(E_T^{jetp})$, directly changes the horizontal scale of the cross section. It also includes a resolution effect, due to the
varying energy of the muon and neutrino. We can express the measured cross section $B(E_T^{jet,cal})$ in terms of these effects:

$$
B(E_T^{jet,cal}) = R^{Cal}(E_T^{jet,pL}) \otimes L^{b\rightarrow\mu+\nu}(E_T^{jet,pL}) \otimes K(E_T^{jet,pL}) \otimes T(b \rightarrow \mu) \otimes F(E_T^{jet,pL})
$$

Thus, to measure $F(E_T^{jet,pL})$ we have to apply corrections to $B(E_T^{jet,cal})$ for these effects. To correct for the $b \rightarrow \mu + \nu$ branching ratio, we use the function $C_T(b \rightarrow \mu)$, which is exactly the inverse of $T(b \rightarrow \mu)$. The kinematic acceptance is corrected for by a correction function $C_K(E_T^{jet,pL})$, which is extracted from the Monte Carlo simulation. For the scale effect of the lepton correction, we calculate a correction function $S(E_T^{jet,cal})$, which scales the cross section to the appropriate energy scale, namely $E_T^{jet,pL}$. We combine the remaining resolution effect of $L^{b\rightarrow\mu+\nu}(E_T^{jet,pL})$ with the resolution effect $R^{Cal}(E_T^{jet,pL})$ of the calorimeter, and correct for both by the function $C_{RL}(E_T^{jet,pL})$. Due to the additional smearing on the jet energy, coming from the uncertainty on the energy of the muon and neutrino, the cross section measured as a function of $E_T^{jet,pL}$ will require bigger bin sizes than the ones used for the $\mu$+jet and $B(E_T^{jet,pL})$ cross sections.

Summarizing, the following distributions will be measured in this chapter:

- The $\mu$+jet cross section as a function of $E_T^{jet,pL}$;
- The $b$-jet fraction of the $\mu$+jet cross section, as a function of $E_T^{jet,cal}$;
- The $b$-jet cross section as a function of $E_T^{jet,pL}$, with the $b$-quark decaying to a muon, which can be compared to the theoretical prediction, if the latter is corrected for the energy loss of the muon and the neutrino, and the $b \rightarrow \mu + \nu$ branching ratio;
- The $b$-jet cross section as a function of the total $b$-jet energy $E_T^{jet,pL}$, which can be directly compared with the theoretical predictions.

### 6.2 Unfolding the $\mu$+jet cross section

To extract the $\mu$+jet cross section as a function of the true particle energy of the jet from the cross section $\frac{d\sigma}{dE_T^{jet,cal}}$ we have to remove the effect of the finite resolution with which the calorimeter measures the jet energy. We start with an ansatz function for the $\mu$+jet cross section with 3 free parameters, of the form:

$$
H(E_T, \alpha, \beta, \gamma) = \alpha E_T^\beta \left(1 - \frac{2}{\sqrt{s}} E_T\right)^\gamma
$$

where $E_T$ is the true energy of the jet, $\alpha, \beta$ and $\gamma$ are free parameters, and $\sqrt{s}$ is the center of mass energy. The shape of this function is based on Monte Carlo trials, and
has also been used in the Run I analysis [56]. It is schematically displayed in figure 6.1a. We fold each point on this distribution with a gaussian shape, with the width of the gaussian corresponding to the jet energy resolution, as discussed in section 4.1. The resulting "smeared ansatz" distribution is a function of the parameters $\alpha, \beta$ and $\gamma$. In a $\chi^2$ minimization procedure we vary $\alpha, \beta$ and $\gamma$ such that the resulting distribution describes the data, with the $\chi^2$ defined as:

$$\chi^2 = \sum_{i=1}^{n} \left( \frac{\Delta_i}{\epsilon_i} \right)^2 \quad (6.8)$$

where $\Delta_i$ is the difference between the data and the smeared ansatz for data point $i$, and $\epsilon_i$ is the error on data point $i$ (see figure 6.1b). The errors on the parameters of the jet energy resolution are analytically propagated to the resolution function using the full covariance matrix (see appendix A), and the error on the unfolded cross section resulting from this uncertainty is estimated by varying the resolution function within the errors. Figure 6.2 shows the result of this unfolding procedure, where the solid curve is the ansatz function, and the dotted curve the smeared ansatz function. The parameters of the ansatz function are listed in table 6.1. The smeared ansatz function shows good agreement with the data points. The fact that the ratio of the smeared ansatz function (dashed curve) and the ansatz function itself (solid curve) is greater than one over the entire $E_T^{jettcal}$ range can be understood by Gaussian smearing of each bin in the ansatz function. Since the cross section drops rapidly with increasing $E_T^{jettcal}$, more events migrate from low energy bins to higher energy bins than there are events that migrate from higher energy bins to lower energy bins. Shown in figure 6.3 is the ratio $1/C_U(E_T^{jettcal})$ between the smeared ansatz and the ansatz function itself, with the dashed lines representing the errors resulting from varying the jet energy resolution errors.

The measured cross section is now updated according to this ratio, according to:

$$\frac{d\sigma}{dE_T^{jett}} = C_U(E_T^{jettcal}) \frac{d\sigma}{dE_T^{jettcal}} \quad (6.9)$$

The relative error on $C_U(E_T^{jettcal})$ is added in quadrature to the previously calculated systematic errors of the cross section.
6.3 Kinematic acceptance corrections

We measure the cross section in the kinematic region with $p_T^\mu > 6 \text{ GeV}/c$, $|\eta_\mu| < 0.8$ and $|\eta_{\text{jet}}| < 0.6$. The reconstructed kinematic properties of the muons and jets are smeared around their true values, which cause fluctuations of events in and out of the kinematic region in which the cross section is measured. Therefore, the unfolded cross section needs an additional correction for the effects of these cuts. We investigate this effect using the QCD Monte Carlo simulation, in which we select particle jets with an associated muon with $\delta R < 0.7$. Two distributions are extracted, both as function of the transverse particle jet energy, $E_T^{\text{jet}p}$. The first distribution is the number of events in each $E_T^{\text{jet}p}$ bin that passes the kinematic acceptance cuts, using the Monte Carlo true values of the jet and the muon. For the second distribution, we smear the kinematic properties of the muons and jets, except for the jet energy, according to the measured resolutions in the data, and count the number of events in each $E_T^{\text{jet}p}$ bin that pass the kinematic acceptance cuts using the smeared kinematic properties of the

![Diagram](image)

Figure 6.1: Schematic view of the procedure used to unfold the $\mu$+jet cross section. Figure a shows the ansatz function $H$ as function of $E_T^{\text{jet}p}$, which is smeared according to the jet resolution to get the distribution as function of $E_T^{\text{jet}eal}$, which is then fitted to the measured data points.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$3.06 \cdot 10^4 \pm 1.05 \cdot 10^3$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$-3.09 \pm 1.60 \cdot 10^{-2}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$15.2 \pm 8.24 \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 6.1: Final parameters used in the ansatz function to minimize the difference between the smeared ansatz function and the measured data points.
Figure 6.2: Comparison of the smeared ansatz function, represented by the dashed line, with the data points. Also shown by the solid line is the underlying ansatz function which generates the smeared ansatz function through the jet energy resolution.

Figure 6.3: Ratio between the smeared ansatz and the ansatz function. The grey band represents the error on this ratio.
jet and the muon. Dividing the first histogram by the second results in the correction factor \( C_K(E_T^{jet,p}) \) that we have to apply to the measured cross section. This factor is independent of \( E_T^{jet,p} \) and close to one \((1.018 \pm 0.036)\), as can be seen in figure 6.4. This is clearly a small effect, and does not have much relevance for the measurement of the cross section.

Correcting the measured cross section for the unfolding and the kinematic acceptance results in the differential \( \mu+\)jet cross section as a function of particle jet \( E_T \), or \( E_T^{jet,p} \), without the muon and neutrino energy, as is shown in figure 6.5. Also shown is the Pythia prediction for \( \mu+\)jet events. Note, that for the further calculation of the \( b \)-jet cross section, we will not use this result, but rather the \( \mu+\)jet cross section as measured in chapter 5.

### 6.4 Extraction of the \( b \)-jet component

To extract the \( b \)-jet component \( Z(E_T^{jet,cal}) \) from the \( \mu+\)jet sample we use the properties of the muon with respect to the jet to identify direct \( b \rightarrow \mu \) decays. Background processes are \( c \rightarrow \mu \) decays, \( \pi/K \rightarrow \mu \) decays and \( \tau \rightarrow \mu \) decays. The cross section for \( W/Z \rightarrow \mu \) decays is so small compared to the \( b \)-jet cross section that these decays can be neglected. A good discriminant between these production processes is the quantity...
Figure 6.5: Differential $\mu+\text{jet}$ cross section as function of particle jet $E_T$, without muon and neutrino corrections. The black curve represents the Pythia prediction for this cross section.

$P_T^{\text{Rel}}$, which is explained in detail below.

### 6.4.1 $P_T^{\text{Rel}}$ Tagging variable

The $P_T^{\text{Rel}}$ tagging variable is defined as the transverse momentum of the muon with respect to the combined axis of the muon and jet (see figure 6.6). It is measured using the jet direction and energy, and the muon direction and momentum. The variable is based on the decay of the quark in the jet: in the rest system of the meson or hadron that contains the $b$-quark, the muon gets a significant momentum due to the mass difference of the $b$-quark and its decay products. This momentum is lower when a $c$- or light quark decays, because the mass difference between those quarks and their decay products is lower. If the whole system is boosted along the quark momentum axis, the $P_T^{\text{Rel}}$ variable is a measure of the transverse boost of the muon with respect to the quark momentum axis. The discriminating power of this variable is shown in figure 6.7, which shows the $P_T^{\text{Rel}}$ of the muon from different decays on the Monte Carlo true level. Already on this level, it can be seen that $\pi/K \rightarrow \mu$ decays, $c \rightarrow \mu$ decays and $b \rightarrow c \rightarrow \mu$ decays are practically indistinguishable. The detector resolution of the muon momentum and direction measurement and the jet energy and direction measurement will make the difference between the distributions even less, resulting in
6.4 Extraction of the $b$-jet component

Figure 6.6: The definition of the $P_T^{Rel}$ variable. The muon direction and momentum are indicated by $\vec{P}_\mu$, the jet direction and energy by $\vec{P}_\text{jet}$. Adding these two vectors results in the combined $\mu$+jet axis, indicated by the dashed line. The $P_T^{Rel}$ of the muon is now the transverse momentum of the muon with respect to this axis.

an inability to separate these decays.

Therefore, we divide the sample in two parts: $b \rightarrow \mu$ and non-$b \rightarrow \mu$, where the non-$b \rightarrow \mu$ includes the cascade decay $b \rightarrow c \rightarrow \mu$. The resulting $P_T^{Rel}$ distributions in the two samples are used for the extraction of the $b$-jet content, and are referred to as templates.

Figure 6.8 shows the $P_T^{Rel}$ distribution measured in the data, with the following cuts:

- $E_T^{jet Cal} > 20$ GeV;
- $|\eta^{jet Cal}| < 0.6$;
- $p_T > 6$ GeV/c;
- $|\eta^\mu| < 0.8$;
- $\delta R(jet Cal, \mu) < 0.7$.

To measure the $b$-jet content of the data, we need templates that describe the shape of the $P_T^{Rel}$ variable in background events and signal events. These templates are in part extracted from the Monte Carlo simulation, as explained in detail in the next sections.

Smearing of Monte Carlo templates

As was shown in chapter 4, the resolution with which the Monte Carlo simulation measures the kinematic properties of reconstructed muons and jets is underestimated
compared to the resolutions in data. To be able to use the $P_T^{\text{Rel}}$ shape that is extracted from the Monte Carlo simulation to measure the $b$-jet content in the data, we have to account for this deficiency, and apply smearing corrections to the kinematic properties of the muons and jets we extract from the Monte Carlo events. For the jets, we only need to smear the transverse energy since the $\eta$ and $\phi$ directions are measured with comparable resolution in the Monte Carlo simulation as in the data (see section 4.1). The transverse energy of the jets is smeared according to the results of section 4.1. For the reconstructed muons in the Monte Carlo simulation we smear the $p_T$, $\eta$ and $\phi$ of the muons, as they were derived in section 4.2.5.

**Extraction of the signal $P_T^{\text{Rel}}$ template**

The shape of the $P_T^{\text{Rel}}$ distribution for $b \to \mu$ decays is obtained with a Monte Carlo sample of 0.5 million $b\bar{b}$ events, with no initial constraints on the decay of the $b$-quarks. From this sample, events are selected that contain a reconstructed jet and a reconstructed muon, with a $\delta R$ separation between the jet and the muon of less than
0.7. Also, the muon is matched with a muon from the Monte Carlo truth information within a cone of $\delta R < 0.3$ and is required to be a direct decay product of the parent $b$-quark. The kinematic properties of the muons and jets are then smeared as explained previously, and the same cuts are applied as listed above, resulting in the $P_T^{rel}$ template illustrated by the open circles in figure 6.9.

The $bb$ events are generated in Pythia using the direct $bb$ production process, which uses the leading order matrix elements for massive quarks, but which does not include processes like flavor excitation and gluon splitting. Especially the latter might impact the shape of the $P_T^{rel}$ distribution, due to the possibility of two heavy quarks present in the same jet. This effect is investigated by running the Pythia simulator in a generic QCD mode that includes the flavor excitation and gluon splitting processes. From the generated events the $b$-producing events are then extracted. We now look at the difference in the resulting $P_T^{rel}$ shape for the $b \rightarrow \mu$ decays in both the direct $bb$ production and the $b \rightarrow \mu$ decays extracted from the QCD generation mode. We split the templates in two $E_T^{jet}$ ranges, $E_T^{jet} < 35$ GeV and $E_T^{jet} > 35$ GeV, to account for the fact that high $E_T$ jets are more likely to contain two $b$-quarks (see figure 1.8). The resulting templates are shown in figure 6.10. For both $E_T$ ranges, no significant
difference is visible between the $P_T^{\text{Rel}}$ templates from QCD and $b\bar{b}$ production. Since we have higher statistics in the sample extracted from direct $b\bar{b}$ production, we use that for further analysis\(^1\).

**Extraction of the background $P_T^{\text{Rel}}$ template**

The biggest difficulty in extracting the $P_T^{\text{Rel}}$ template for non-$b \rightarrow \mu$ decays is the contribution of $\pi/K \rightarrow \mu$ decays. Pythia treats pions and kaons as stable at the generation level, and they decay at the detector simulation step. To be able to get a template for the background with reasonable statistics, we have simulated 2.5 million QCD events, from which all non-$b \rightarrow \mu$ decays have been extracted. If we apply the same fiducial cuts as used in the data, the majority of the events is removed, resulting in low statistics in the background template. Properly applying the jet and muon

\(^1\)We choose not to combine the samples, due to the fact that one production process uses massive quarks while the other does not, and one also has to take into consideration interferences between the production diagrams.
resolutions results in the template as shown in figure 6.9 by the black circles, where the templates are normalized to one.

To circumvent this problem of low statistics, the background template is also extracted directly from the data sample. This is done by selecting tracks reconstructed in the central tracker, which satisfy the following requirements:

- The event is taken during a run which is qualified as good by the central tracking groups;
- Tracks reconstructed using only the axial fibers of the CFT are removed, since they do not contain any $\eta$ information;
- The relative error on the measurement of the $p_T$ of the central track is less than 2.5%, to ensure properly reconstructed tracks;
- $\delta R(\text{jet, central track}) < 0.7$ to make sure the track is associated with the jet.

If we assume that all tracks are muons from pion decays, we can use them in the $P_T^{\text{Rel}}$ template if we weigh them with $p_T^{-1}$ to account for the probability of a pion decaying to a muon, and smear them according to the muon resolution. We do not have to take into consideration the resolution of the central tracks, since the muon resolution is measured with respect to these central tracks, and the resolution with which the central tracks are measured is negligible with respect to the local muon resolution (see section 4.2.5). In addition to higher statistics, this method has the added benefit that
the properties of the jets in the events do not have to be smeared, since we are using data events. The data used for the extraction of this template still contains a certain amount of $b$-quarks. However, since we do not require the presence of a muon in the events, this contribution is not significant. Figure 6.11 shows the resulting template from central tracks, compared to the one using the muons from the QCD Monte Carlo simulation.

![Figure 6.11: $P_T^{Rel}$ templates for light quark jets, extracted from the Monte Carlo simulation (black circles) and from the data using central tracks (open circles). The templates are normalized to one.](image)

**6.4.2 Kinematic dependance**

Theoretically, the $P_T^{Rel}$ variable is a boost independent quantity. However, due to the finite resolution of the detector, dependencies on $p_T^\mu$ and $E_{T}^{jet_{Cal}}$ are introduced. Since we are ultimately interested in the differential cross section as a function of $E_{T}^{jet_{Cal}}$, we integrate over $p_T^\mu$ and investigate the dependance of the templates on $E_{T}^{jet_{Cal}}$. This is shown in figure 6.12 for the $b \rightarrow \mu$ and non-$b \rightarrow \mu$ templates, in the $E_{T}^{jet_{Cal}}$ bins:

a. 20 - 25 GeV
b. 25 - 35 GeV
c. 35 - 50 GeV
d. 50 - 100 GeV
where the size of the bins is constrained by statistics, especially for the non-$b \to \mu$ template. The strategy is now to divide the data in the same bins of $E_T^{jetCal}$ and fit the templates to the data to measure the contributions of signal and background in the data, as a function of $E_T^{jetCal}$.

![Graphs](image)

Figure 6.12: Dependence of the $P_T^{Rel}$ templates on $E_T^{jet}$, in the bins 20-25-35-50-100. The open circles show the $b \to \mu$ template, while the black dots show the non-$b \to \mu$ template. The errors on the data points are statistical only. The templates are normalized to one.
6.4.3 Template fit

In each of the $E_{T}^{jet_{CaT}}$ bins a fit is done of the $P_{T}^{Rel}$ templates to the data, according to:

$$\left( \frac{dN}{dP_{T}^{Rel}} \right)_{data} = C_{b \rightarrow \mu} \left( \frac{dN}{dP_{T}^{Rel}} \right)_{i}^{b \rightarrow \mu} + C_{non-b \rightarrow \mu} \left( \frac{dN}{dP_{T}^{Rel}} \right)_{i}^{non-b \rightarrow \mu}, \quad (6.10)$$

where $i$ loops over each $E_{T}^{jet_{CaT}}$ bin, and $\frac{dN}{dP_{T}^{Rel}}$ represents the $P_{T}^{Rel}$ distribution in each $E_{T}^{jet_{CaT}}$ bin. The factors $C$ are the scaling factors with which the templates need to be scaled to fit the data. The fit is done using the HMCMLL [58] program, which fits the Monte Carlo templates to the data distribution using a maximum likelihood fit that includes the effect of both the data and Monte Carlo statistics, and returns the estimate of the fraction of each Monte Carlo template which represents the data best.

This procedure is followed using both the non-$b \rightarrow \mu$ template from the QCD Monte Carlo simulation as well as with the non-$b \rightarrow \mu$ template extracted from the data, to investigate systematic effects. Figure 6.13 shows the fit of the QCD template and the $bb$ template to the data in all four bins, and the results are summarized in table 6.2. Figure 6.14 shows the fit of the background template extracted from data, and the $bb$ template to the data, and is summarized in table 6.3. Figure 6.15 illustrates these numbers, together with a fit of the form:

$$Z(E_{T}^{jet_{CaT}}) = a + \frac{b}{E_{T}^{jet_{CaT}}} \quad (6.11)$$

The shape of this function is chosen after trials using the Monte Carlo simulation, and was also used in the Run I analysis [56]. The resulting values for the parameters $a$ and $b$ are listed in table 6.4. For the final $b$-jet content measurement, we take the fit that used the background template from data, because of lower errors on the fit. The difference of that fit with the result of the fit using the template from the QCD Monte Carlo simulation is added in quadrature to the error from the fit to cover the uncertainty introduced by the method. This results in the solid grey lines in figure 6.15.

A comment about error propagation has to be made here. The correlation matrix that is provided by the fit to the four data points allows us to measure the deviation $\Delta Z(E_{T}^{jet_{CaT}})$ at each point on the function $Z(E_{T}^{jet_{CaT}})$. However, it does not tell us how these errors are correlated from each point on the function to an other point on the function, due to changes in the parameters $a$ and $b$ of the fitted function. The derivation of the correlation matrix for the points on the function uses a numerical procedure, which is explained in detail in appendix B.

As a consistency check of the $b$-content measurement method, we perform a closure test in which we simulate the data sample by taking all $\mu + jet$ events from the QCD
Monte Carlo simulation. This sample then includes well known amounts of $b \rightarrow \mu$ decays and non-$b \rightarrow \mu$ decays. Using the $bb$ template extracted from the $bb$ sample and the non-$b \rightarrow \mu$ template extracted from the data, we apply the same fit as described above to measure the amount of $b \rightarrow \mu$ decays in the sample. Integrated over the entire $E_T^{jet-c-a}$ range, this results in a measured $b$-jet content of $(26 \pm 3)\%$, compared to a true $b$-jet content in the sample of $(28 \pm 1)\%$. 
Table 6.2: The $b$-jet and background content in each bin of $E_{T}^{jetCal}$, where the background template is extracted from the QCD Monte Carlo simulation.

<table>
<thead>
<tr>
<th>$E_{T}^{jetCal}$ range</th>
<th>$b$-content</th>
<th>background content</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 - 25 GeV</td>
<td>0.39 ± 0.064</td>
<td>0.61 ± 0.073</td>
</tr>
<tr>
<td>25 - 35 GeV</td>
<td>0.22 ± 0.066</td>
<td>0.78 ± 0.077</td>
</tr>
<tr>
<td>35 - 50 GeV</td>
<td>0.24 ± 0.055</td>
<td>0.76 ± 0.068</td>
</tr>
<tr>
<td>50 - 100 GeV</td>
<td>0.14 ± 0.08</td>
<td>0.86 ± 0.10</td>
</tr>
</tbody>
</table>

Figure 6.13: Fit of the background template, extracted from the QCD Monte Carlo simulation, and the signal template, to the data in each of the four $E_{T}^{jetCal}$ bins.
### Table 6.3: The \( b \)-jet and background content in each bin of \( E_T^{jet_{cal}} \), where the background template is extracted from the data using central tracks.

<table>
<thead>
<tr>
<th>( E_T^{jet_{cal}} ) range</th>
<th>( b )-content</th>
<th>background content</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 - 25 GeV</td>
<td>0.32 ± 0.041</td>
<td>0.68 ± 0.043</td>
</tr>
<tr>
<td>25 - 35 GeV</td>
<td>0.29 ± 0.028</td>
<td>0.71 ± 0.030</td>
</tr>
<tr>
<td>35 - 50 GeV</td>
<td>0.24 ± 0.031</td>
<td>0.76 ± 0.034</td>
</tr>
<tr>
<td>50 - 100 GeV</td>
<td>0.19 ± 0.043</td>
<td>0.81 ± 0.048</td>
</tr>
</tbody>
</table>

Figure 6.14: Fit of the background template, extracted from the data using central tracks, and the signal template, to the data in each of the four \( E_T^{jet_{cal}} \) bins.
Figure 6.15: Measured b-jet content as function of $E^\text{jet}_{\text{cal}}$, using the background template extracted from the QCD Monte Carlo simulation (open circles), and the template extracted from the data (black circles)). For clarity, the black circles are offset horizontally by -1 GeV, while the open circles are offset +1 GeV. The errors from the fit are represented by the grey dotted lines. The solid grey line includes the systematic error from the method.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>$a$</th>
<th>$b$</th>
<th>$\rho_{ab}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background template, QCD</td>
<td>0.135 ± 6.4·10^{-2}</td>
<td>125 ± 58.3</td>
<td>-0.818</td>
</tr>
<tr>
<td>Background template, central tracks</td>
<td>0.198 ± 3.7·10^{-2}</td>
<td>68.8 ± 33.8</td>
<td>-0.841</td>
</tr>
</tbody>
</table>

Table 6.4: Parameters resulting from the fit to the b-content using both background templates from the QCD Monte Carlo simulation and from the central tracks in the data. The correlation between the fit parameters $a$ and $b$ is given by $\rho_{ab}$. 
6.5 $B^{b\rightarrow\mu}$-jet cross section measurement

Using the $b$-jet content measured above, we can now extract the $b$-jet cross section, with $b \rightarrow \mu$, from the measured $\mu$+jet spectrum (corrected for all efficiencies) shown in figure 5.10, according to:

$$B(E_T^{jet, Cal}) = \frac{d\sigma(b \rightarrow \mu)}{dE_T^{jet, Cal}} = Z(E_T^{jet, Cal}) \frac{d\sigma(\mu + jet)}{dE_T^{jet, Cal}}$$ (6.12)

This correction is done on a bin-by-bin basis, where every bin is weighted with the $b$-jet content that corresponds to its $E_T$. The errors on the fit parameters of equation 6.11 are taken into account by propagating the full covariance matrix (further explained in appendix A). This results in the $b$-jet cross section as a function of $E_T^{jet, Cal}$, with the $b$-quark decaying to a muon, as is shown in figure 6.16. The next step will be to correct for the detector effects on the measurement of this cross section.

![Figure 6.16: The $b$-jet cross section, with $b \rightarrow \mu$, as a function of calorimeter $E_T^{jet, Cal}$. The errors on the data points include both systematic and statistical errors.](image-url)
6.5.1 Unfolding the b-jet cross section

We now proceed to unfold this b-jet cross section, measured as function of $E_T^{jet_{cal}}$ according to equation 6.12, for the effect of the finite resolution of the calorimeter $R^{Cal}(E_T^{jet_p})$ and the kinematic acceptance of the detector $K(E_T^{jet_p})$. This does not take into account the energy carried away from the jet by the muon and neutrino which are present in the jet. Hence the distribution we extract in this section is the b-jet cross section as function of the particle jet energy $E_T^{jet_p}$, with $b \rightarrow \mu$.

To unfold the b-jet cross section we follow a similar procedure as outlined in section 6.2. Again, we start with an ansatz function $I(E_T^{jet_p})$ with three free parameters, according to:

$$I(E_T^{jet_p}, \alpha, \beta, \gamma) = \alpha \left( E_T^{jet_p} \right)^{\beta} \left( 1 - \frac{2}{\sqrt{s}} \right) E_T^{jet_p}^{\gamma}$$  \hspace{1cm} (6.13)

with $\alpha, \beta$ and $\gamma$ the free parameters. $I(E_T^{jet_p})$ represents the function $G(E_T^{jet_p})$, convoluted with the kinematic acceptance $K(E_T^{jet_p})$: $I(E_T^{jet_p}) = K(E_T^{jet_p}) \otimes G(E_T^{jet_p})$. We smear this distribution with the jet energy resolution $R^{Cal}(E_T^{jet_p})$, and fit to the b-jet cross section measured in the previous section, as shown in figure 6.16. Upon convergence of the fit, resulting in the parameters $\alpha, \beta$ and $\gamma$, as listed in table 6.5, we use the ratio $C_R(E_T^{jet_{cal}})$ of the ansatz function to the smeared ansatz function to update the data points, where $C_R(E_T^{jet_{cal}})$ is defined as:

$$\frac{1}{C_R(E_T^{jet_{cal}})} = \frac{R^{Cal}(E_T^{jet_p}) \otimes I(E_T^{jet_p})}{I(E_T^{jet_p})} \hspace{1cm} (6.14)$$

This ratio is shown in figure 6.17. This ratio is higher than the one used for unfolding the $\mu$+jet cross section, as shown in figure 6.3, which is due to the fact that the b-jet cross section is falling steeper than the $\mu$+jet cross section. We now have to make the kinematic acceptance correction as explained in section 6.3. Even though we have to make the kinematic acceptance corrections for b-jets, and not generic $\mu$+jets, the underlying distributions are the same, and we use the same factor $C_R(E_T^{jet_p})$ as shown in figure 6.4.

The b-jet cross section, measured as a function of $E_T^{jet_p}$ can then be defined as:

$$\frac{d\sigma(b \rightarrow \mu)}{dE_T^{jet_p}} = C_R(E_T^{jet_p})C_R(E_T^{jet_{cal}}) \frac{d\sigma(b \rightarrow \mu)}{dE_T^{jet_{cal}}}$$  \hspace{1cm} (6.15)

which uses equation 6.12. The b-jet cross section thus defined, still with $b \rightarrow \mu$, is shown in figure 6.18 as a function of $E_T^{jet_p}$. 


Table 6.5: Final parameters used in the ansatz function to minimize the difference between the smeared ansatz function and the measured data points.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$1.00 \times 10^5 \pm 16.9$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$-3.55 \pm 3.18 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$25.2 \pm 0.807$</td>
</tr>
</tbody>
</table>

Figure 6.17: Ratio $1/C_R(E_{T}^{jet,cut})$ between the smeared ansatz and the ansatz function. The dashed lines represent the errors on this ratio.
Figure 6.18: The $b$-jet cross section as a function of particle level transverse jet energy, $E_T^{jet_p}$, without muon and neutrino energies.
6.6 Measurement of the $b$-jet cross section

To measure the $b$-jet cross section as a function of the full $b$-jet energy $E_T^{\text{jet}pL}$, including the muon and neutrino energies and the $b \rightarrow \mu + \nu$ branching ratio, we start out again from the $b$-jet cross section measured as function of $E_T^{\text{jet}cal}$ (see figure 6.16). We again have to take into account the kinematic acceptance correction and the resolution of the calorimeter, as we did in the previous section. In addition, we now also have to deal with two other effects, resulting from the addition of the muon and neutrino energies to the jet energy. The first effect is a change of the energy scale, for which we correct with a scale factor $S(E_T^{\text{jet}p})$, which changes the scale of the cross section from the measured jet $E_T^T$, $E_T^{\text{jet}cal}$ to the full $b$-jet $E_T^T$, $E_T^{\text{jet}pL}$. The second effect is a resolution effect, caused by the varying energy carried away by the muon and neutrino. We correct for this effect by an unfolding procedure similar to the one explained in the previous section. This time, however, we correct for both the effect of the calorimeters resolution and the resolution of the lepton correction simultaneously. After this correction, we will correct for the $b \rightarrow \mu + \nu$ branching ratio and the kinematic acceptance of the detector.

First we will derive the scale factor $S(E_T^{\text{jet}p})$. For this we use the Monte Carlo simulation, by generating QCD events and extracting the $b\bar{b}$ events, with at least one $b$-quark decaying to a muon, and with both the jet and the muon in the correct fiducial volume. We require the $\delta R$ between the jet and the muon to be less than 0.7. We then plot:

$$C_L(E_T^{\text{jet}pL}) = \frac{E_T^{\text{jet}pL}}{E_T^{\text{jet}p}}$$

(6.16)

as a function of the total $b$-jet energy $E_T^{\text{jet}pL}$, where:

$$E_T^{\text{jet}pL} = E_T^{\text{jet}p} + E_T^\mu + E_T^\nu$$

(6.17)

This results in the scatter plot shown in figure 6.19. The spread in each vertical slice of this scatter plot can be interpreted as a resolution effect on the jet energy, caused by the uncertainty on the energy carried away by the muon and the neutrino. To extract the scale factor $S(E_T^{\text{jet}cal})$ we make vertical slices of this scatter plot in each bin of $E_T^{\text{jet}pL}$. An example of one slice, for $E_T^{\text{jet}pL} = 40$ GeV, is shown in figure 6.20. For each distribution in each slice we take the position of the maximum as the scale factor needed for that bin of $E_T^{\text{jet}pL}$. This scale factor then transforms each value of $E_T^{\text{jet}pL}$ into a corresponding value of $E_T^{\text{jet}p}$, and can thus be used to scale $E_T^{\text{jet}pL}$ into $E_T^{\text{jet}p}$. However, since our starting point for the measurement of the $b$-jet cross section is a measurement in terms of $E_T^{\text{jet}cal}$ we need to determine the scale factor as a function of $E_T^{\text{jet}cal}$ rather than $E_T^{\text{jet}pL}$. As such, we have to consider the smearing effects of the calorimeter on $S(E_T^{\text{jet}cal})$. If we have a distribution of $E_T^{\text{jet}pL}$, with a well defined maximum, smearing
Figure 6.19: Lepton correction factor $C^j_{L}^{jet_{PL}}$ as a function of the total b-jet transverse energy $E_T^{jet_{PL}}$.

this distribution with a gaussian shape (due to the calorimeter resolution) will not change the location of the maximum. Hence, for $S(E_T^{jet_{cal}})$ we can use the same scale factor that scales $E_T^{jet}$ into $E_T^{jet_{PL}}$. We now fit a straight line through the values of the maxima, as a function of $E_T^{jet}$, and define that as the scale factor $S(E_T^{jet_{cal}})$.

This scale factor allows us to transform a distribution, measured as a function of $E_T^{jet_{cal}}$, into a distribution of $E_T^{jet_{PL}}$. When applying this scale factor to the data we obtain the data points as given in figure 6.22. This distribution corresponds to:

$$S(E_T^{jet_{cal}}) \otimes \frac{d\sigma(b \rightarrow \mu)}{dE_T^{jet_{cal}}} \quad (6.18)$$

Having incorporated the muon and neutrino energy scale, the effects of the resolution need to be accounted for. This is done using an ansatz function, which will be corrected for the effects of the energy carried away by the leptons and the calorimeter resolution simultaneously. For this ansatz function, called $J(E_T^{jet_{PL}})$, we assume a shape according to:

$$J(E_T^{jet_{PL}}, \alpha, \beta, \gamma) = \alpha (E_T^{jet_{PL}})^{\beta} \left(1 - \left(\frac{2}{\sqrt{8}}\right) E_T^{jet_{PL}}\right)^{\gamma} \quad (6.19)$$

which is illustrated in figure 6.21a. This function $J(E_T^{jet_{PL}})$ is defined as (see equation 6.6):

$$J(E_T^{jet_{PL}}, \alpha, \beta, \gamma) = K(E_T^{jet_{PL}}) \otimes T(b \rightarrow \mu) \otimes F(E_T^{jet_{PL}}) \quad (6.20)$$
To correct for the variation of the energy of the muon and neutrino, we again use the scatter plot of figure 6.19. The jet energy is scaled following a fit to the distribution for $C_L(E_{\text{jet}}^{\text{jetPL}})$ for each slice in $E_{\text{T}}^{\text{jetPL}}$. That is, each jet energy $E_{\text{T}}^{\text{jetPL}}$ is scaled and distributed according to the fit for that particular $E_{\text{T}}^{\text{jetPL}}$ slice, with the area under this fit normalized to one. This procedure then yields the distribution shown in figure 6.21b.

The next step is to take into account the resolution of the calorimeter. We smear the distribution with the jet resolution, as described in section 6.2. This results in the distribution of figure 6.21c, which is now a function of the transverse calorimeter jet energy, $E_{\text{T}}^{\text{jetCal}}$. This distribution corresponds to the measured data points that are displayed in figure 6.16.

Finally, we have to use the scaling function $S(E_{\text{T}}^{\text{jetCal}})$ to scale this distribution, resulting in a smeared ansatz function. This function is fit to the data points by changing the parameters in the original ansatz function, and minimizing the $\chi^2$ as defined in equation 6.8.

Upon convergence of the fit, the resulting ansatz function now represents the true $b$-jet cross section as a function of lepton corrected transverse energy. Figure 6.22 shows the result of the fit, where the dotted curve is the smeared ansatz function. The data points are the same as the points in figure 6.16, scaled with the scaling factor $S(E_{\text{T}}^{\text{jetCal}})$, taking into account the changing bin sizes. The smeared ansatz function fits the data points well ($\chi^2 = 0.11$). Figure 6.23 shows the ratio of the smeared ansatz
Figure 6.21: Schematic illustration of the lepton correction and unfolding procedure.

and ansatz function. The dotted curves represent the error on the ratio, resulting from varying the measured jet resolution, taking into account the full covariance matrix.

The scaled data points are now divided by the ratio shown in figure 6.23 to arrive at the $b$-jet cross section as a function of total jet $E_T$, according to:

$$\frac{d\sigma(b \rightarrow \mu)}{dE_T^{jetPL}} = C_{RL}(E_T^{jetPL}) \left( S(E_T^{jetCal}) \otimes \frac{d\sigma(b \rightarrow \mu)}{dE_T^{jetCal}} \right)$$

(6.21)

What remains is to correct for the $b \rightarrow \mu$ branching ratio $T(b \rightarrow \mu)$ and the kinematic acceptance $K(E_T^{jetPL})$ to get the final differential $b$-jet cross section.

A final comment has to be made with regards to this algorithm. Up to some extent, the scaling function $S(E_T^{jetCal})$ can be chosen arbitrarily. This can be seen by the following: at one point in the algorithm, both the data and the smeared model are measured as a function of $E_T^{jetCal}$. One could possibly perform the fit of the smeared
### 6.6 Measurement of the b-jet cross section

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>$2.28 \times 10^6 \pm 1.52 \times 10^6$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$-4.34 \pm 0.148$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$5.06 \pm 2.49$</td>
</tr>
</tbody>
</table>

Table 6.6: Final parameters used in the ansatz function $J(\alpha, \beta, \gamma)$ to minimize the fit of the smeared ansatz function to the measured data points.

Figure 6.22: Result of fit to scaled data points. The data points are the same as the points in figure 6.16, scaled with the scaling factor $S(E_{T\text{jet}}^{jetcut})$, taking into account the changing bin sizes. The dotted curve represents the smeared ansatz function.

model to the data points at this stage, and the resulting ansatz function would represent the underlying theoretical distribution with which the $b$-jets are produced. However, for the final measurement we do not want to measure the parameters $\alpha$, $\beta$ and $\gamma$, which define the theoretical curve, but we want to measure the specific data points. Therefore we scale both the data and the model to the new energy scale $E_T^{jetpL}$, and fit the model to the data points at that scale. Regardless of what we take for the scale factor, the parameters $\alpha$, $\beta$ and $\gamma$ will remain the same. What does change with a changing scale is the unfolding correction $C_{RL}(E_T^{jetpL})$; if we use a another scale than currently used in the algorithm, the smeared ansatz function will be different and consequently the ratio will change. In fact, only the combination of the correction factors $C_{RL}(E_T^{jetpL})$
Table 6.7: Values of the data points shown in figure 6.22.

<table>
<thead>
<tr>
<th>$E_T^{jetPL}$ bin</th>
<th>$\langle E_T^{jetPL} \rangle$</th>
<th>$d\sigma/dE_T^{jetPL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>29 - 43</td>
<td>34.1</td>
<td>$4.03^{+2.70}_{-1.38} \cdot 10^{-1}$</td>
</tr>
<tr>
<td>43 - 57</td>
<td>48.5</td>
<td>$8.20^{+2.57}_{-1.91} \cdot 10^{-2}$</td>
</tr>
<tr>
<td>57 - 70</td>
<td>62.2</td>
<td>$2.56^{+0.71}_{-0.64} \cdot 10^{-2}$</td>
</tr>
<tr>
<td>70 - 83</td>
<td>75.6</td>
<td>$1.16^{+0.36}_{-0.38} \cdot 10^{-2}$</td>
</tr>
<tr>
<td>83 - 107</td>
<td>93.5</td>
<td>$4.47^{+1.66}_{-1.57} \cdot 10^{-3}$</td>
</tr>
<tr>
<td>107 - 131</td>
<td>117</td>
<td>$1.65^{+0.66}_{-0.71} \cdot 10^{-3}$</td>
</tr>
</tbody>
</table>

Figure 6.23: The ratio between the smeared ansatz function and the ansatz function. The dotted lines represent the errors on the ratio that are coming from the uncertainty on the jet energy resolution function.

and $S(E_T^{jetCal})$ has physical relevance. In the algorithm, the scale factor $S(E_T^{jetCal})$ is chosen such that it maximizes the purity of the algorithm, which is further explained below.

### 6.6.1 Purity considerations

The procedure discussed above can only be used if sufficient events in a certain measured $E_T^{jetCut}$ region, i.e. bin, originate from that particular region. If too many events
migrate into that bin from other bins, the number of events in the bin does not represent the cross section in the corresponding $E_T^{jet_{cal}}$ region. Therefore, we define the purity in a particular bin as the number of events in that bin of $E_T^{jet_{PL}}$ in the ansatz function, divided by the number of events in that same bin in the smeared ansatz function. We also define the efficiency in that bin as the percentage of the events that does not smear out of the bin during the smearing procedure. For the six data points used in the procedure above, the purities and efficiencies are listed in table 6.8. Clearly the purities are on the low side to justify the bin sizes used. We therefore choose to decrease the number of bins and increase their size such that the purity is at least 50% in each bin. Table 6.9 lists the efficiencies and purities for three bins, which are appropriate for a correct unfolding procedure.

<table>
<thead>
<tr>
<th>$E_T^{jet_{cal}}$ bin (GeV)</th>
<th>$E_T^{jet_{PL}}$ bin (GeV)</th>
<th>Purity (%)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-30</td>
<td>29-43</td>
<td>42</td>
<td>42</td>
</tr>
<tr>
<td>30-40</td>
<td>43-57</td>
<td>36</td>
<td>38</td>
</tr>
<tr>
<td>40-50</td>
<td>57-70</td>
<td>30</td>
<td>31</td>
</tr>
<tr>
<td>50-60</td>
<td>70-83</td>
<td>27</td>
<td>31</td>
</tr>
<tr>
<td>60-80</td>
<td>83-107</td>
<td>37</td>
<td>42</td>
</tr>
<tr>
<td>80-100</td>
<td>107-131</td>
<td>29</td>
<td>37</td>
</tr>
</tbody>
</table>

Table 6.8: Purities and efficiencies for 6 bins in the data.

<table>
<thead>
<tr>
<th>$E_T^{jet_{cal}}$ bin (GeV)</th>
<th>$E_T^{jet_{PL}}$ bin (GeV)</th>
<th>Purity (%)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20-38</td>
<td>29-54</td>
<td>54</td>
<td>52</td>
</tr>
<tr>
<td>38-60</td>
<td>54-83</td>
<td>50</td>
<td>53</td>
</tr>
<tr>
<td>60-100</td>
<td>83-131</td>
<td>51</td>
<td>59</td>
</tr>
</tbody>
</table>

Table 6.9: Purities and efficiencies for 3 bins in the data.

6.6.2 Kinematic acceptance and branching ratio

Ultimately, we want to measure the differential $b$-jet cross section in the range $|\eta^{jet}| < 0.6$. Up to now, we have required a muon with $p_T^\mu > 6$ GeV/c, and we need to correct for this. We also have to apply a small correction due to smearing of reconstructed jets and muons in and out of the fiducial region. Both these corrections are again extracted from the Monte Carlo simulation, where we extract two distributions:
1. The distribution of generated $b$-jets, without any constraints on the decay of the $b$-quark in the jet. The jet is required to be within $|\eta^{\text{jet}}| < 0.6$.

2. The distribution of generated $b$-jets, with the $b$-quark decaying to a muon. After smearing the jet $\eta$ according to the detector resolution, we require that $|\eta^{\text{jet}}| < 0.6$. Also, after smearing the properties of the muon, it is required to have $p_T^\mu > 6$ GeV/c, and $|\eta^\mu| < 0.8$.

Both distributions are made as a function of $E_T^{\text{jet} p_L}$. The ratio between distributions 1 and distribution 2 can now be used to scale the measured $b$-jet cross section in the data. This ratio is shown in figure 6.24. Scaling the measured $b$-jet cross section with this ratio according to:

$$
\frac{d\sigma}{dE_T^{\text{jet} p_L}} = C_K(E_T^{\text{jet} p_L}) C_T(b \rightarrow \mu) \left( \frac{d\sigma(b \rightarrow \mu)}{dE_T^{\text{jet} p_L}} \right)
$$

(6.22)

where $\frac{d\sigma(b \rightarrow \mu)}{dE_T^{\text{jet} p_L}}$ is the cross section calculated in equation 6.21. This results in the differential $b$-jet cross section as shown in figure 6.25. This $b$-jet cross section can now be compared directly with theoretical predictions, as will be done in the next chapter.

Figure 6.24: Ratio between the number of generated $b$-jets and $b$-jets that decay to a muon with $p_T^\mu > 6$ GeV/c, in the fiducial volume defined by $|\eta^{\text{jet}}| < 0.6$. 
6.7 Conclusions

In this chapter we have measured the four distributions mentioned in the first section, namely the $\mu$+jet cross section as function of the jet energy without muon and neutrino energies, the $b$-jet content as a function of $E_T^{\text{jet cut}}$, the $b$-jet cross section, with $b \to \mu$, as function of the jet energy without muon and neutrino energies and the $b$-jet cross section as function of the total $b$-jet transverse energy. The latter cross section has the complication that the bin sizes needed to be increased to retain a proper purity in the unfolding procedure. The two $b$-jet cross sections can now be compared to the theoretical predictions as outlined in Chapter 1, as will be done in the next chapter.

Figure 6.25: Differential $b$-jet cross section as function of total $b$-jet $E_T$, corrected for all detector and tagging effects.

<table>
<thead>
<tr>
<th>$E_T^{\text{jet}_PL}$ bin</th>
<th>$\langle E_T^{\text{jet}_PL} \rangle$</th>
<th>$d\sigma/dE_T^{\text{jet}_PL}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>29 – 54</td>
<td>35.9</td>
<td>$5.44^{+2.68}_{-2.44}$</td>
</tr>
<tr>
<td>54 – 83</td>
<td>64.1</td>
<td>$2.82^{+0.89}_{-0.84} \cdot 10^{-1}$</td>
</tr>
<tr>
<td>83 – 131</td>
<td>99.6</td>
<td>$3.51^{+1.41}_{-1.35} \cdot 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 6.10: Values of the data points shown in figure 6.25.