Quantum optics and multiple scattering in dielectrics

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Chapter 3

Spontaneous emission of vector waves in crystals of plane scatterers

The concept of a plane scatterer that was developed in the previous chapter for scalar waves is generalized to Maxwell vector waves. $T$-matrices can be defined after a Green function regularization. Optical modes and Green functions are determined and differences with scalar waves are stressed. The theory is used to calculate position- and orientation-dependent spontaneous-emission rates and radiative line shifts.

3.1 Multiple-scattering theory for vector waves

In section 2.1 a general introduction and motivation to study optical properties of crystals of plane scatterers was given. Since chapter 2 dealt with scalar waves only and since light is a vector wave, it is essential to study how the concept of a plane scatterer can be generalized to vector waves. This is the subject of the present chapter.

The general multiple-scattering theory as introduced in section 2.2 for scalar waves described by the Helmholtz equation, can be generalized to vector waves that satisfy Maxwell’s equations. The notation inevitably becomes more involved: scalar quantities such as frequency $\omega$ or components of vectors or matrices such as the wave vector component $k_z$ will be written in standard font; vectors will be written in bold standard font, for example the electric field “$E_{k\sigma}$”; finally, matrices or tensors such as the free-space Green function “$G_0$” will also be written bold but in a sans serif font.

The wave equation for the electric field $E_0(\mathbf{r}, \omega)$ in vacuum is

$$\left\{ \frac{(\omega/c)^2}{k} \mathbf{I} - \nabla \times \nabla \times \right\} E_0(\mathbf{r}, \omega) = 0.$$  

(3.1)

The symbol I denotes the identity operator in three-dimensional space. The solutions of Eq. (3.1) are plane waves with wave vector $\mathbf{k}$ and polarization direction normal to $\mathbf{k}$. 

With the wave equation (3.1) the free-space Green tensor (or dyadic Green function) is associated that satisfies
\[
\{(\omega/c)^2\mathbf{l} \cdot -\nabla \times \nabla \times \} \mathbf{G}_0(r, r', \omega) = \delta^3(r - r') \mathbf{l}.
\]
(3.2)

Now it will be convenient to consider Eq. (3.1) as the real-space representation of an abstract tensor operator \( \mathbf{L}(\omega) \) acting on the vector field \( \mathbf{E}_0(\omega) \), so that (3.1) in abstract notation becomes
\[
\mathbf{L}(\omega) \cdot \mathbf{E}_0(\omega) = 0.
\]
(3.3)

In the same notation the dyadic Green function \( \mathbf{G}_0(\omega) \) satisfies
\[
\mathbf{L}(\omega) \cdot \mathbf{G}_0(\omega) = \mathbf{1} \otimes \mathbf{l}.
\]
(3.4)

The identity operator in real space we denote by \( \mathbf{l} \) and it has the property \( \langle r | \mathbf{l} | r' \rangle = \delta^3(r - r') \); confusion with the unit tensor \( \mathbf{l} \) should not arise; the \( \otimes \) denotes the tensor product.

In the presence of an inhomogeneous dispersive linear dielectric, the wave equation for the electric field is modified as follows:
\[
\{(\omega/c)^2\mathbf{l} \cdot -\nabla \times \nabla \times \} \mathbf{E}(r, \omega) = - \left[ (\varepsilon(r, \omega) - 1)(\omega/c)^2 \right] \mathbf{E}(r, \omega)
\equiv \mathbf{V}(r, \omega) \cdot \mathbf{E}(r, \omega),
\]
(3.5)

where in the last equality the frequency-dependent optical potential \( \mathbf{V} \) was defined in terms of the dielectric function \( \varepsilon(r, \omega) \). The electric field \( \mathbf{E}_{\text{in}}(\omega) \) is modified into \( \mathbf{E}(\omega) \), and the two fields are related through the Lippmann-Schwinger equation
\[
\mathbf{E}(\omega) = \mathbf{E}_{\text{in}}(\omega) + \mathbf{G}_0(\omega) \cdot \mathbf{V}(\omega) \cdot \mathbf{E}(\omega).
\]
(3.6)

The field \( \mathbf{E}(\omega) \) that satisfies Eq. (3.6) is also a solution of Eq. (3.5). The solution of Eq. (3.6) can be found iteratively in higher and higher orders of the optical potential \( \mathbf{V} \):
\[
\mathbf{E} = \mathbf{E}_{\text{in}} + \mathbf{G}_0 \cdot \mathbf{V} \cdot \mathbf{E}_{\text{in}} + \mathbf{G}_0 \cdot \mathbf{G}_0 \cdot \mathbf{V} \cdot \mathbf{E}_{\text{in}} + \ldots.
\]
(3.7)

So we have a multiple-scattering series for the electric field in terms of the potential, both when the potential consists of many scatterers and when it models just a single scatterer. A (dyadic) T-matrix can be defined that sums up how the electric field depends on incoming field and scatterer:
\[
\mathbf{E}(\omega) = \mathbf{E}_{\text{in}}(\omega) + \mathbf{G}_0(\omega) \cdot \mathbf{T}(\omega) \cdot \mathbf{E}_{\text{in}}(\omega).
\]
(3.8)

This is a formal definition of the T-matrix as a 3 \( \times \) 3 tensor, and by combining Eqs. (3.7) and (3.8), the formal solution for the T-matrix is
\[
\mathbf{T}(\omega) = \mathbf{V}(\omega) \cdot [\mathbf{l} - \mathbf{G}_0(\omega) \cdot \mathbf{V}(\omega)]^{-1}.
\]
(3.9)

The scattering problem is solved exactly once the T-matrix is known.

As we saw for scalar waves, there may exist electric field modes that are bound to the scatterer. Such bound modes are solutions of the Lippmann-Schwinger equation in the absence of an incident field:
\[
\mathbf{E}(\omega) = \mathbf{G}_0(\omega) \cdot \mathbf{V}(\omega) \cdot \mathbf{E}(\omega).
\]
(3.10)
With the help of the formal definition (3.9) of the T matrix, we can rewrite this homogeneous equation as

\[ \mathbf{V}(\omega) \cdot T^{-1}(\omega) \cdot \mathbf{E}(\omega) = 0. \]  

(3.11)

This equation shows that nontrivial bound solutions of the electric field will correspond to the poles of the T-matrix. So the T-matrix not only solves the scattering problem for incident fields but also contains all information about bound modes.

In the presence of the dielectric the Green function also changes from \( G_0 \) to \( G \) and the latter satisfies the following equation

\[ [L(\omega) - V(\omega)] \cdot G(\omega) = \mathbb{1} \otimes \mathbb{1}. \]  

(3.12)

The solution for the Green function analogous to Eq. (3.6) for the electric field is the three-dimensional Dyson-Schwinger equation

\[ \mathbf{G}(\omega) = \mathbf{G}_0(\omega) + \mathbf{G}_0(\omega) \cdot \mathbf{V}(\omega) \cdot \mathbf{G}(\omega). \]  

(3.13)

It can be verified that a solution of (3.13) also is a solution of equation (3.12). The problem how to find such a solution is solved once the T-matrix (3.9) is determined, because an iteration of Eq. (3.13) analogous to the series expansion (3.7) for the electric field shows that the Green function can also be expressed in terms of the T-matrix:

\[ \mathbf{G}(\omega) = \mathbf{G}_0(\omega) + \mathbf{G}_0(\omega) \cdot T(\omega) \cdot \mathbf{G}_0(\omega). \]  

(3.14)

The equations of multiple-scattering theory for vector waves were introduced, in the notation that will be used throughout this chapter. The analogies with and differences from scalar waves can be found when comparing with section 2.2.

### 3.2 Plane scatterers for vector waves

The elements of the scattering theory are the potential \( V \), the free-space Green function \( G_0 \), the T-matrix \( T \), and the incoming electric field \( \mathbf{E}_i \). Here we will determine the specific form that these elements take for dielectrics that can be described as a collection of parallel planes. First the free-space Green function is determined, followed by the single-plane T-matrix and finally the T-matrix for an arbitrary number of planes.
3.2.1 Dyadic Green function in plane representation

A solution for the dyadic Green function can be found in three-dimensional Fourier space and by translational invariance of free space we have \( \langle k|G_0(\omega)|k' \rangle = (2\pi)^3 \delta^3(k-k')G_0(k,\omega) \). The Green function \( G_0(k,\omega) \) that we thus defined satisfies

\[
\left\{ \left[ (\omega/c)^2 - k^2 \right] I + k^2 \hat{k} \hat{k} \right\} G_0(k,\omega) = I. \tag{3.17}
\]

Here, \( \hat{k} \) denotes a unit vector in the direction of the wave vector \( k \). Equation (3.17) is a 3 x 3 matrix equation whose representation diagonalizes in the polarization basis \( \{ \hat{k}, \hat{\sigma}_1, \hat{\sigma}_2 \} \) with the longitudinal direction \( k \) and two orthogonal transverse directions \( \hat{\sigma}_1, \hat{\sigma}_2 \). Note that this representation is co-rotating with the wave vector. The solution of (3.17) is

\[
G_{ij}^0(k,\omega) = \frac{1}{(\omega/c)^2 - k^2} \left[ (\omega/c)^2 + \partial_z^2 \right] \delta(z-z')L(k|\hat{k},\hat{z},\omega),
\]

\[
G_{kk}^0(k,\omega) = (c/\omega)^2, \tag{3.18}
\]

where \( j \) denotes \( \sigma_1 \) or \( \sigma_2 \). All six non-diagonal elements of the Green tensor are zero in this representation.

The spatial Fourier representation is not what we need. As in chapter 2, it is convenient to work in the "plane representation": in two-dimensional Fourier space in the directions parallel to the planes and in real space in the \( z \)-direction perpendicular to the planes. For the polarization representation we choose the orthonormal basis \( \{ \hat{s}_k, \hat{v}_k, \hat{z} \} \). Here, \( \hat{z} \) is the unit vector in the \( z \)-direction; \( \hat{v}_k \) is the unit vector in the direction of the projection of the wave vector \( k \) on the plane, so that the wave vector \( k \) has a wave vector component \( k_{||} \) in the \( \hat{v}_k \)-direction and its full representation is \( (0, k_{||}, k_z) \); the \( s_k \)-polarization direction is orthogonal to the optical plane that is spanned by the other two basis vectors. Then the operator \( L(\omega) \) has the form \( \langle k_{||}, z|L(\omega)|k'_{||}, z' \rangle = (2\pi)^2 \delta^2(k_{||} - k'_{||})\delta(z-z')L(k_{||}, z, \omega) \) and the operator \( L(k_{||}, z, \omega) \) has the matrix representation

\[
L(k_{||}, z, \omega) = \begin{pmatrix}
(\omega/c)^2 - k^2_{||} + \partial_z^2 & 0 & 0 \\
0 & (\omega/c)^2 + \partial_z^2 & -ik_{||}\partial_z \\
0 & -ik_{||}\partial_z & (\omega/c)^2 - k^2_{||}
\end{pmatrix}. \tag{3.19}
\]

The wave vector \( k_{||} \) is a two-dimensional in-plane wave vector. The Green function in the same representation becomes \( \langle k_{||}, z|G_0(\omega)|k'_{||}, z' \rangle = (2\pi)^2 \delta^2(k_{||} - k'_{||})G_0(k_{||}, z, z', \omega) \), and the matrix equation (3.17) becomes a system of differential equations in the plane representation:

\[
L(k_{||}, z, \omega) \begin{pmatrix}
G_{ss}^0 \\
G_{sv}^0 \\
G_{zs}^0
\end{pmatrix} = \delta(z-z') \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
G_{ss}^0 \\
G_{sv}^0 \\
G_{zs}^0
\end{pmatrix}. \tag{3.20}
\]

The \( G_{pq}^0 \) are the components of \( G_0 \) and the arguments \( (k_{||}, z, z', \omega) \) were dropped for brevity. By choosing this representation, the matrix elements of \( G_0 \) only depend on the magnitude and not on the orientation of \( k_{||} \). Evidently, all components involving an \( s \)-label
are zero except for the $ss$-component that has an equation that is uncoupled from the rest. Actually, $G_0^{ss}$ satisfies the differential equation for the Green function of scalar waves that we encountered in the previous chapter. So we have

$$G_0^{ss}(k_\parallel, z, z', \omega) = \frac{e^{ik_z|z-z'|}}{2ik_z}.$$  \hspace{1cm} (3.21)

where $k_z$ is not a new independent variable different from $k_\parallel$, but rather an abbreviation for $\sqrt{(\omega/c)^2 - k_\parallel^2}$. The remaining differential equations are coupled and can also be solved. It turns out that $G_0^{uv}$ is proportional to $G_0^{ss}$ that is already known. Once $G_0^{uv}$ is known, $G_0^{zu}$ immediately follows. When solving for $G_0^{uu}$, the equation for $G_0^{zz}$ is satisfied by the solution for component $G_0^{zu}$ that we just found, so that the two these components are equal. The last nonzero component $G_0^{zz}$ can then be expressed in terms of the others and the results are

$$G_0^{uu}(k_\parallel, z, z', \omega) = \frac{k_z^2}{(\omega/c)^2} \frac{e^{ik_z|z-z'|}}{2ik_z}$$

$$G_0^{uz}(k_\parallel, z, z', \omega) = \frac{k_\parallel k_z}{(\omega/c)^2} \frac{e^{ik_z|z-z'|}}{2ik_z} \text{sign}(z - z')$$

$$G_0^{zu}(k_\parallel, z, z', \omega) = G_0^{zu}(k_\parallel, z, z', \omega)$$

$$G_0^{zz}(k_\parallel, z, z', \omega) = \frac{k_z^2}{(\omega/c)^2} \frac{e^{ik_z|z-z'|}}{2ik_z} + (c/\omega)^2 \delta(z - z').$$  \hspace{1cm} (3.22)

The above method of solving differential equations does not give a clue as to what the value of the sign-function for $z$ equal to $z'$ should be. It appears that there is another way to obtain the results above, one that does give the value of $\text{sign}(0)$, namely by using the Fourier relation between the Green functions in the two representations:

$$G_0(k_\parallel, z, z', \omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk_z G_0(k_\parallel, k_z, \omega) e^{ik_z(z-z')}.$$  \hspace{1cm} (3.23)

The integration can only be performed in a representation that does not co-rotate with $k_z$. The basis of Eq. (3.18) is not adequate, but again the basis $\{\mathbf{s}_k, \mathbf{v}_k, \mathbf{z}\}$ suits well. Again we find the Green function components of Eq. (3.22), with the additional information that integrands asymmetric in the variable $k_z$ lead to a sign-function that is zero when $z$ equals $z'$. The free-space dyadic Green function in the plane representation is now determined.

### 3.2.2 Attempt to define a T-matrix

The T-matrix of a plane scatterer for vector waves can be found by solving the appropriate Lippmann-Schwinger (LS) equation (3.6), just as was done in the previous chapter for scalar waves. A plane wave incident from $z = -\infty$ with wave vector $k$ and arbitrary amplitude $E_0$ and transverse polarization vector $\sigma_k = (\sigma_s, \sigma_v, \sigma_z)$ is scattered by a plane at $z = z_\alpha$. Because of the symmetry in the in-plane directions, it is convenient to choose the plane representation for the LS equation. In terms of the Dirac-notation, the electric
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Field is a "ket" and the plane representation is found by taking the inner product of equation (3.6) for the electric field with the "bra" \( \langle k'\|, z' \rangle \), and by inserting the unit operator

\[
\frac{1}{(2\pi)^2} \int d^2k' d\omega \ |k'\|, z'\rangle \langle k'\|, z'\|
\]

at the positions of the dots in the representation-independent equation (3.6). The incident field takes the form \( E_{k,\sigma,\text{in}}(k\|, z, \omega) = E_0 \sigma_k \exp(ikz) \). The solution of the LS equation corresponding to this incident field is the field \( E_{k,\sigma}(\omega) \) and note that it is labelled by the wave vector and polarization of the incoming field from which it originates. The plane itself is described by the optical potential \( V(z, \omega) = -[\varepsilon(z, \omega) - 1](\omega/c)^2 I \equiv V(\omega)\delta(z - z_0)I \). Then the Lippmann-Schwinger equation in the mixed representation becomes

\[
E_{k,\sigma}(k\|, z, \omega) = E_0 \sigma_k e^{ikz} + \int_{-\infty}^{+\infty} dz' G_0(k\|, z, z', \omega) \cdot V(z', \omega) \cdot E_{k,\sigma}(k\|, z', \omega).
\]

The integral is very simple because the planes are assumed to be infinitely thin, and we get

\[
E_{k,\sigma}(k\|, z, \omega) = V(\omega)G_0(k\|, z, z_0, \omega) \cdot E_{k,\sigma}(k\|, z_0, \omega) = E_0 \sigma_k e^{ikz}.
\]

The analogous equation for scalar waves could be solved at this point by putting the position \( z \) in this equation equal to \( z_0 \), solving for \( E_{k,\sigma}(k\|, z_0, \omega) \) and putting back this result in the above equation to obtain an expression for \( E_{k,\sigma}(k\|, z_0, \omega) \). However, unlike the scalar Green function \( g_0 \), the Green tensor \( G_0 \) is not defined when the positions \( z \) and \( z_0 \) are identical, because of the delta function in the component \( G_0^{zz} \) [Eq. (3.22)]. Therefore it is impossible with this Green tensor to set up a scattering theory for vector waves scattered by infinitely thin planes. In the next section we propose a regularization of the Green function in order to overcome the problem encountered above. We could just neglect the delta function, as is sometimes done in other calculations [77], but as we shall see later in subsection 3.2.4, this procedure does not give the correct results in our case.

### 3.2.3 Regularization of the Green function

When studying multiple scattering of a wave by objects much smaller than the wavelength, the assumption that the scatterers are infinitely thin or small can simplify the analysis considerably. Often it is the only route to analytical results for a truly multiply-scattering system. Thus, spatially extended physical objects are modelled as mathematical objects with zero volume. But this simplification comes at a price. One may run into the kind of problem encountered in the previous section, that a Green function does not have a finite value precisely at the position of a zero-volume-scatterer. This technical complication can be remedied by a procedure called "regularization".

When modelling scatterers not as infinitely thin planes but as mathematical points, the above problem of unphysical infinities shows up both for scalar and for vector waves. For point scatterers the problem has been studied extensively and several regularization schemes have been proposed (see [69] and references therein). In a regularization procedure usually some cutoff parameter is introduced that modifies the behavior of Green
functions at distances much smaller than optical wavelengths and infinities are thus removed.

After a successful regularization, two situations can present themselves. In the first situation, the optical properties are not affected by the procedure, in the sense that the value of the cutoff parameter can be chosen infinitely large in the final stage. In this first case, the mathematical problem of an infinity could be solved with mathematics and the scattering theory is called “renormalizable”.

In the second situation, the cutoff parameter can not be sent to infinity. It should then be possible to express the finite cutoff parameter in terms of physical quantities. But then it also works the other way around: physical properties depend on the cutoff parameter. In this second situation, the theory is not renormalizable and the physical parameters together with the cutoff parameter form an overdetermined set. The latter situation was found for point scatterers for vector waves [69]. A T-matrix for a point scatterer could only be defined when two regularization parameters were introduced, namely a longitudinal and a transverse cutoff parameter. The longitudinal cutoff is related to a static quantity, namely the natural size of the scatterer; the transverse cutoff can be expressed either in terms of dynamic quantities (for example: resonance frequencies) or static quantities, the latter choice depending on the physical scatterer that is modelled as a point. So much for general remarks about Green function regularization.

In the rest of this subsection, we regularize the dyadic Green function with components as in Eq. (3.22). We propose the same regularization procedure as was done for point scattering, namely a high-momentum cutoff in three-dimensional Fourier space: instead of the free-space Green function $G_0(k, \omega)$ of Eq. (3.18) we use the regularized free-space Green function $\tilde{G}_0(k, \omega)$, and define the latter in terms of the former as

$$
\tilde{G}_0(k, \omega) = \frac{\Lambda^2}{\Lambda^2 + k^2} G_0(k, \omega).
$$

Here, the cutoff momentum $\Lambda$ is assumed to be much larger than any optical momentum, so that at optical wavelengths $\tilde{G}_0 \simeq G_0$. Now we are interested what the effect of this cutoff will be in the plane representation. For the unregularized Green function, we showed that the plane representation could be found either by solving differential equations (3.20) or by an inverse Fourier transform (3.23). The latter method is the simplest for finding the regularized Green function in the plane representation, and we find:

$$
\tilde{G}_0^{ss}(k, z, z_1, \omega) = \frac{\Lambda^2}{\Lambda^2 + (\omega/c)^2} \left( \frac{e^{ik_z|z-z_1|}}{2ik_z} + \frac{e^{-\Lambda||z-z_1||}}{2\Lambda} \right),
$$

$$
\tilde{G}_0^{nn}(k, z, z_1, \omega) = \frac{k_z^2}{\omega^2} \tilde{G}_0^{ss}(k, z, z_1, \omega),
$$

$$
\tilde{G}_0^{nn}(k, z, z_1, \omega) = -\frac{k_z^2}{\omega^2} \text{sign}(z - z_1) \tilde{G}_0^{ss}(k, z, z_1, \omega),
$$

$$
\tilde{G}_0^{zz}(k, z, z_1, \omega) = \tilde{G}_0^{zz}(k, z, z_1, \omega),
$$

$$
\tilde{G}_0^{zz}(k, z, z_1, \omega) = \frac{k_z^2}{\omega^2} \tilde{G}_0^{ss}(k, z, z_1, \omega) + \frac{\Lambda^2}{2(\omega/c)^2} \frac{e^{-\Lambda||z-z_1||}}{\Lambda}. \quad (3.28)
$$
In these equations, $\Lambda_{\parallel}$ is short-hand notation for $\sqrt{\Lambda^2 + k^2_{\parallel}}$. (Again, the sign-function is zero when its argument is.) All components of the regularized Green tensor consist of two parts: an oscillating and a decaying part, as a function of $|z - z'|$. The decay is very fast because the decay length $\Lambda^{-1}$ is much smaller than the optical wavelength $\lambda = 2\pi c/\omega$. For $\lambda |z - z'| \gtrsim 1$ and $\Lambda \gg \lambda$, the regularized Green function approaches the unregularized one. If one would take the limit of $\Lambda \rightarrow \infty$, then all the components in (3.28) approach the unregularized components of equation (3.22), and in particular the limit of the last term in $\mathbf{G}_0^{zz}$ gives the delta function that made the regularization procedure necessary. However, we keep $\Lambda$ finite for the moment. Unlike $\mathbf{G}_0^z$, the component $\mathbf{G}_0^{zz}$ does not have a term proportional to $\delta(z - z')$, but it has a term that grows with $\Lambda$ instead. This enables us to set up a theory of scattering by vector waves from plane scatterers.

### 3.2.4 T-matrix of a plane for vector waves

In subsection 3.2.2, we stumbled on a problem to define the T-matrix of a plane for vector waves. After completing the regularization of the Green function in subsection 3.2.3, we now go back to the Lippmann-Schwinger equation (3.25) and replace the Green function by the regularized one:

$$
\mathbf{E}_{k\sigma}(k_{||}, z, \omega) = \mathbf{E}_0 \sigma_k e^{ik_{\parallel} z} + \int_{-\infty}^{+\infty} dz' \mathbf{G}_0(k_{||}, z, z', \omega) \cdot \mathbf{V}(z', \omega) \cdot \mathbf{E}_{k\sigma}(k_{||}, z', \omega).
$$

(3.29)

Now take $z = z_{\alpha}$ in equation (3.29) and write it out in components:

$$
\begin{pmatrix}
E_{k\sigma}^s \\
E_{k\sigma}^v \\
E_{k\sigma}^{zz}
\end{pmatrix} = \mathbf{E}_0 \begin{pmatrix}
\sigma_s \\
\sigma_v \\
\sigma_z
\end{pmatrix} e^{ik_{\parallel} z_{\alpha}} + V(\omega) \begin{pmatrix}
\mathbf{G}_0^{ss} & 0 & 0 \\
0 & \mathbf{G}_0^{vv} & 0 \\
0 & 0 & \mathbf{G}_0^{zz}
\end{pmatrix} \begin{pmatrix}
E_{k\sigma}^s \\
E_{k\sigma}^v \\
E_{k\sigma}^{zz}
\end{pmatrix}.
$$

(3.30)

Here, $\mathbf{G}_0^{ss} = \mathbf{G}_0^{vv}(k_{||}, z_{\alpha}, z_{\alpha}, \omega)$ and similarly for the other components. The off-diagonal elements of the Green tensor are all zero when the position $z$ is equal to $z_{\alpha}$. Hence the equation can be solved for every component separately. If we insert this result into the Lippmann-Schwinger equation for general $z$, we find

$$
\mathbf{E}_{k\sigma}(k_{||}, z, \omega) = \mathbf{E}_{k\sigma, in}(k_{||}, z, \omega) + \mathbf{G}_0(k_{||}, z, z_{\alpha}, \omega) \cdot \mathbf{T}(k_{||}, \omega) \cdot \mathbf{E}_{k\sigma, in}(k_{||}, z_{\alpha}, \omega),
$$

(3.31)

where the T-matrix for scattering from a plane by arbitrarily polarized light is given by

$$
\mathbf{T}(k_{||}, \omega) = \begin{pmatrix}
\frac{V(\omega)}{1 - V(\omega)\mathbf{G}_0^{ss}} & 0 & 0 \\
0 & \frac{V(\omega)}{1 - V(\omega)\mathbf{G}_0^{vv}} & 0 \\
0 & 0 & \frac{V(\omega)}{1 - V(\omega)\mathbf{G}_0^{zz}}
\end{pmatrix}.
$$

(3.32)

This T-matrix is what we were looking for. Now that it is known we can calculate transmission and reflection and local densities of states as well as other physical quantities.

The scattering of the $s$-polarization component of the light can be considered independently from the $\mathbf{v}$ and $\mathbf{z}$ directions, according to Eq. (3.31). It can be verified with
Eqs. (3.28, 3.32) that since $\Lambda \gg (\omega/c)$, the matrix component $\hat{T}^{ss}$ for all practical purposes is equal to the T-matrix for scalar waves, and the same holds for the Green tensor component $G_0^{zz}$: the regularization was not necessary for $s$-polarized light and fortunately it does not affect the scattering properties of $s$-polarized light either.

The need for regularization did show up in the description of scattering of $p$-polarized light and it is interesting to analyze how the cutoff influences the scattering properties. Assume that $p$-polarized light is impinging on a plane, with amplitude $E_0$, wave vector $k$, and the polarization state

$$\sigma = \hat{p} \equiv (k_z/k)\hat{\nu}_k - (k_{\|}/k)\hat{z}.\quad (3.33)$$

Written out explicitly, the incoming field is

$$E_{k\sigma, \text{in}}(k_{\|}, z, \omega) = \begin{pmatrix} E^v_{k\sigma, \text{in}}(k_{\|}, z, \omega) \\ E^z_{k\sigma, \text{in}}(k_{\|}, z, \omega) \end{pmatrix} = E_0 \begin{pmatrix} 0 \\ k_z/k \end{pmatrix} e^{ik_z z}.\quad (3.34)$$

The planes do not mix the $s$ and $p$-polarization so that the incoming-plus-scattered field does not have an $s$-component either. For the nonzero components we find:

$$\begin{pmatrix} E^v(z) \\ E^z(z) \end{pmatrix} = \begin{pmatrix} E^v_{\text{in}}(z) + \hat{G}_0^{vv}(z, z_\alpha)\hat{T}^{vv}E^v_{\text{in}}(z_\alpha) + \hat{G}_0^{vz}\hat{T}^{zz}E^z_{\text{in}}(z_\alpha) \\ E^z_{\text{in}}(z) + \hat{G}_0^{vv}(z, z_\alpha)\hat{T}^{vv}E^v_{\text{in}}(z_\alpha) + \hat{G}_0^{zz}(z, z_\alpha)\hat{T}^{zz}E^z_{\text{in}}(z_\alpha) \end{pmatrix},\quad (3.35)$$

where the labels $(k_{\|}, \sigma)$ were dropped for readability. Now for distances far enough from the plane so that $\Lambda|z - z_\alpha| \gg 1$, the term $\hat{G}_0^{zz}\hat{T}^{zz}$ falls off as $\Lambda^{-1} \exp(-\Lambda|z - z_\alpha|)$ and $\hat{G}_0^{zz}\hat{T}^{zz}$ as $\exp(-\Lambda|z - z_\alpha|)$, so that for optical purposes these terms can be neglected.

For finite very large $\Lambda$ we arrive at the following effective description:

$$\begin{pmatrix} E^s(z) - E^s_{\text{in}}(z) \\ E^v(z) - E^v_{\text{in}}(z) \\ E^z(z) - E^z_{\text{in}}(z) \end{pmatrix} = \begin{pmatrix} G_0^{ss} & 0 & 0 \\ 0 & G_0^{vv} & G_0^{vz} \\ 0 & 0 & G_0^{zz} \end{pmatrix} \begin{pmatrix} T^{ss} & 0 & 0 \\ 0 & T^{vv} & 0 \\ 0 & 0 & T^{zz} \end{pmatrix} \begin{pmatrix} E^s_{\text{in}}(z_\alpha) \\ E^v_{\text{in}}(z_\alpha) \\ E^z_{\text{in}}(z_\alpha) \end{pmatrix},\quad (3.36)$$

where the $ss$-component of the T-matrix is equal to $V(\omega)[1 - V(\omega)G_0^{ss}]^{-1}$, and analogously for the $vv$-component. The Green functions have arguments $(k_{\|}, z, z_\alpha, \omega)$. In this effective description, - where the T-matrix is denoted by $\hat{T}$ rather than $\hat{T}$ - the cutoff parameter $\Lambda$ does not occur anymore. The cutoff was necessary in order to set up a scattering theory and it shows up in the elements of the scattering theory such as the T-matrix (3.32) and the regularized Green function (3.28). But it does not show up in the electric field and precisely this enables us to arrive at the effective description. Note also that in the effective description the value of $G_0^{zz}$ has become irrelevant.

Can the delta function in $G_0^{zz}$ (see Eq. (3.22)) just be left out of the theory, as is sometimes done in other work [77], and produce the same T-matrix? The answer is No. Leaving out the delta function in the Lippmann-Schwinger equation (3.25) will result in a nonzero $T^{zz}$, in contrast with Eq. (3.36). Furthermore, the $T$ matrix would be such that the transmitted part of an incoming wave would not be parallel to this incoming wave, which is unphysical. The conclusion is that a regularization of the Green function was necessary, even when in the end the scattering theory is renormalizable, in the sense described in section 3.2.3.
3.2.5 Transmission and energy conservation

The transmission of light through the plane can be found by choosing $z > z_\alpha$ in Eq. (3.36). The transmitted wave can be expressed in terms of the incoming wave as $E_{k\sigma}(k_{||}, z, \omega) = \tau(k_{||}, \omega) \cdot E_{k\sigma, in}(k_{||}, z, \omega)$, with the transmission matrix

$$
\tau(k_{||}, \omega) = \begin{pmatrix}
\tau_{ss}(k_{||}, \omega) & 0 & 0 \\
0 & \tau_{sr}(k_{||}, \omega) & 0 \\
0 & 0 & \tau_{zz}(k_{||}, \omega)
\end{pmatrix}
$$

(3.37)

which has nonzero elements $\tau_{jj}(k_{||}, \omega) = \left[1 - V(\omega)G_{0}^{ij}(k_{||}, z_\alpha, z_\alpha, \omega)\right]^{-1}$ for $j = s, r$. Furthermore, $\tau_{zz}(k_{||}, \omega) = G^{zz}(k_{||}, z, z_\alpha, \omega)T^{zz}(k_{||}, \omega)$. With the help of Eqs. (3.22, 3.34, 3.36) one can find that both for $s$-polarized and for $p$-polarized light, the transmitted electric field is a polarization-dependent scalar times the incoming electric field vector. In other words, after transmission the direction is unchanged but the amplitude can be different from the incoming wave.

Now we can ask which forms the T-matrix can take such that energy is conserved in the scattering process. This has been analyzed for scalar waves before and since $s$-waves map on scalar waves, we know the optical theorem for the $ss$-component of the T matrix:

$$
\text{Im } T^{ss}(k_{||}, \omega) = -\frac{1}{2} \frac{|T^{ss}(k_{||}, \omega)|^2}{k_z}.
$$

(3.38)

The most general T-matrix satisfying this requirement has the form

$$
T^{ss}(k_{||}, \omega) = -\left[F_{s}^{-1}(k_{||}, \omega) - i/(2k_z)\right]^{-1},
$$

(3.39)

with the optical potential $F_{s}(k_{||}, \omega)$ being a real-valued function.

Energy conservation of light implies a conservation of the $z$-component of the Poynting vector before and after the plane. For reflection and transmission of $p$-polarized light, only the matrix element $T^{uv}$ is important and again we are interested in the form that this matrix element can take. An incoming plane $p$-wave with polarization (3.33) gives the electric field (3.36) and with a Maxwell equation the accompanying magnetic field can also be found. The Poynting vector is then [45]

$$
S(r, t) = \frac{c}{2\pi\mu_0} \text{Re} \left[ E^*(r, t) \times B(r, t) \right].
$$

(3.40)

First transform the equations to frequency space. At the side of the plane where light is reflected ($z < z_\alpha$), the Poynting vector is then proportional to $1 - (k_z c/\omega)^2 |T^{uv}|^2/4$. This should be equal to $|1 - ik_z (c/\omega)^2 T^{uv}|^2/2^2$, the expression that one finds for the other side. From energy conservation one can obtain the optical theorem for the scattering of $p$-polarized light by a plane:

$$
\text{Im } T^{uv}(k_{||}, \omega) = \frac{-k_z}{2(\omega/c)^2} |T^{uv}(k_{||}, \omega)|^2.
$$

(3.41)
This expression differs from the optical theorem for \( s \)-polarized light. Also, the most general solution of the optical theorem is different:

\[
T^{\nu \nu}(k_\parallel, \omega) = - \left[ F_{p}^{-1}(k_\parallel, \omega) - \frac{i k_\parallel}{2(\omega/c)^2} \right]^{-1}.
\]

(3.42)

where the optical potential \( F_{p}(k_\parallel, \omega) \) is real.

### 3.2.6 T-matrix for \( N \) planes

The Green function and the T-matrix of a single plane are known now, and with this a multiple-scattering theory can be set up. Assume now that there are \( N \) plane scatterers placed at arbitrary positions. Assume them to be parallel, so that \( s \)- and \( p \)-polarized light do not mix in the scattering process. Consequently, an \( N \)-plane T-matrix can be determined for \( s \)- and \( p \)-polarized light separately. The \( N \)-plane T-matrix for \( s \)-waves is equal to the \( N \)-plane T-matrix for scalar waves, which was derived in the previous chapter. Here, we can concentrate on the \( N \)-plane T-matrix of \( p \)-polarized light. We repeat the formal sum (3.16) for a T matrix of \( N \) scatterers

\[
T^{(N)} = \sum_{\alpha} T_{\alpha} + \sum_{\beta \neq \alpha} T_{\beta} \cdot G_{0} \cdot T_{\alpha} + \sum_{\gamma \neq \beta, \beta \neq \alpha} T_{\gamma} \cdot G_{0} \cdot T_{\beta} \cdot G_{0} \cdot T_{\alpha} + \ldots
\]

(3.43)

In Eq. (3.43) the Green functions are always sandwiched between T-matrices of planes at different positions. The values of \( G_{0}(k_\parallel, z_\beta, z_\alpha, \omega) \) with \( z_\beta \neq z_\alpha \) are finite. Therefore, once the single-plane \( T \) matrix is defined with the help of a regularization, it is not necessary to perform another regularization in order to find the \( N \)-plane T-matrix. One could argue that for consistency all Green functions in (3.43) should be replaced by the regularized ones, for without the regularization the T-matrices would not even be defined. In practice, however, the differences between \( G_{0} \) and \( \hat{G}_{0} \) can be neglected, because the planes are assumed to be at optical distances apart such that \( \Delta|z_\alpha - z_\beta| \gg 1 \) even for the planes \( \alpha \) and \( \beta \) closest to each other.

With the explicit form for the Green tensor and the T-matrices known, the series (3.43) can be summed exactly, each component separately and analogously to the scalar case in the previous chapter. This gives the central result of this section, the \( N \)-plane T-matrix for scattering by vector waves:

\[
T^{(N)}(\omega) = \frac{1}{(2\pi)^2} \int d^2k_\parallel \sum_{\alpha, \beta = 1}^{N} |k_\parallel, z_\alpha \rangle \langle k_\parallel, z_\beta | T^{(N)}_{\alpha \beta}(k_\parallel, \omega)
\]

(3.44)

The only two nonzero spatial components of the \( 3 \times 3 \) \( T \) matrices \( T^{(N)}_{\alpha \beta} \) are

\[
T^{ss,(N)}_{\alpha \beta}(k_\parallel, \omega) = T^{ss}(k_\parallel, \omega) \left[ \mathbb{1} - D_{\alpha}(k_\parallel, \omega) T^{ss}(k_\parallel, \omega) \right]^{-1}_{\alpha \beta}
\]

\[
T^{vv,(N)}_{\alpha \beta}(k_\parallel, \omega) = T^{vv}(k_\parallel, \omega) \left[ \mathbb{1} - D_{\alpha}(k_\parallel, \omega) T^{vv}(k_\parallel, \omega) \right]^{-1}_{\alpha \beta}
\]

(3.45)
The $N^2$ matrix elements $(D_j)_{ij}$ are defined as $(1 - \delta_{ij})G^j_{ij}(k_z, z_a, z_t, \omega)$, for $j = s, p$. The calculation $T^{(N)}$ boils down to the inversion of an $N \times N$ matrix for the two transverse polarization directions separately. The $ss$-component is identical to the T-matrix $T^{(N)}$ for scalar waves.

The trick of the analytical inversion of $T^{ss,(N)}$ that was presented in appendix A and used in the previous chapter can be accomplished for $T^{uv,(N)}$ as well: $\nu = \nu_s \equiv -2ik_z/T^{ss}(k_z, \omega)$ should be replaced by $\nu_p \equiv -2i(\omega/c)^2/[k_z T^{uv}(k_z, \omega)]$. The derivation will not be repeated here, but we will give some results. In the first place, the Bloch wave vector $K_p$ for $p$-polarized light reflected by planes separated by a distance $a$ is given by arccos($C_p$)/$a$, with

$$C_p = \cos(k_z a) + \left( \frac{ik_z [k_z |T^{uv}|^2 + 2(\omega/c)^2 \text{Im}T^{uv} + 2k_z (\omega/c)^2 \text{Re}T^{uv}}{k_z^2 (\text{Re}T^{uv})^2 + [2(\omega/c) + k_z \text{Im}T^{uv}]^2} \right) \sin(k_z a).$$

(3.46)

In general, $C_p$ is a complex constant. However, if there is no light absorption in the planes so that the optical theorem (3.41) holds, then the imaginary part of $C_p$ becomes identically zero. (For $s$-polarization we saw the same thing happening: without absorption, $C_s$ is real.) For $p$-polarized light with wave vector and frequency such that $|C_p| > 1$, the Bloch wave vector is complex. The system of planes has a stop band for this light, meaning that the light will be 100% reflected when falling on a semi-infinite system of planes. If the optical theorem indeed holds, then the single-plane T-matrix is of the form (3.42) and the expression (3.46) can be simplified to give

$$C_p = \cos(k_z a) - \frac{k_z c^2 F_p(k_z, \omega)}{2\omega^2} \sin(k_z a).$$

(3.47)

### 3.2.7 A model for the optical potential

The most general T-matrix for $p$-waves (3.42) features an as yet unspecified optical potential that should be real when energy is conserved but for the rest it can be an arbitrary function of the in-plane wave vector and frequency. Until now we assumed that the optical potential $F(k_z, \omega)$ was only frequency-dependent and we called it $-V(\omega)$, thereby neglecting spatial dispersion and anisotropy that would show up as a $k_z$- and $k_\parallel$-dependence, respectively. Both were neglected as early as in the wave equation (3.5).

In chapter 2 plane scatterers were introduced as a simplified model for dielectric slabs of finite thickness $d$ and nondispersive dielectric function $\varepsilon(\omega) = \varepsilon$. Recall that the optical potential for the plane scatterer in this model is obtained via the limiting process of making the dielectric slab thinner and increasing the polarizability $\varepsilon - 1$ while keeping their product constant and equal to an "effective thickness". Now for vector waves we use the same procedure and for both $s$- and $p$-polarized light we find the optical potential $F(k_z, \omega) = -V(\omega) = D_{\text{eff}}(\omega/c)^2$, with the effective thickness $D_{\text{eff}}$ equal to $d[\varepsilon(\omega) - 1]$.

One can hope that transmission and reflection properties of a plane scatterer and a finite dielectric slab do not differ much. In this respect, $p$-polarized light is different from $s$-polarized light. When light in a medium with refractive index $n_1$ meets an interface with a second medium with refractive index $n_2 > n_1$, then there is a typical angle of incidence,
3.3 Optical modes and omnidirectional mirrors

3.3.1 Propagating modes

The optical modes are the harmonic solutions of the wave equation (3.5). These modes are also the solutions of the Lippmann-Schwinger equation, which is solved once the T-matrix of the total scattering system is found:

$$E_{k\sigma}(k_{||}, z, \omega) = E_0\sigma_k e^{ik_z z} + \sum_{\alpha, \beta} G_0(k_{||}, z, z_\alpha, \omega) \cdot T^{(N)}_{\alpha, \beta}(k_{||}, \omega) \cdot \sigma_k E_0 e^{ik_z z}. \quad (3.49)$$

The modes that correspond to an incoming plane wave ($E_0 \neq 0$) are the propagating modes (or: radiative modes) and they are labelled by the incoming wave vector $k$ and polarization $\sigma_k$. The $s$-polarized modes [with $\sigma_k = (1, 0, 0)$] are identical to the modes for scalar waves derived in the previous chapter. The $p$-polarized modes [\(\sigma_k = (0, k_z / k, -k_{||} / k)\)] are the new ones.

Mode functions are complex functions of position and mode profiles are their absolute values squared. Mode profiles for $s$-polarized light are identical to the mode profiles of scalar waves that were discussed in chapter 2. We choose the light as incoming from the left. If in particular we choose light of frequency such that $a / \lambda = 0.5$, then the mode profiles of $s$-polarized light will correspond to the scalar mode profile of the same frequency in figure 2.2. For perpendicularly incident light, there is no difference between $s$- and $p$-polarization. In figure 3.1 the mode profiles for $s$- and $p$-polarized light inside a ten-plane crystal are compared both for an incoming angle of 30° and for 60°. In figure 3.1(a) we see that at an angle of 30° the mode profiles corresponding to both polarizations do not differ much yet. Both modes decay rapidly inside the crystal structure and are reflected (almost) completely. Only for the $s$-wave the polarization directions of the incoming and the reflected wave are equal, so that the amplitude of its mode profile at the left side of the crystal is four times the amplitude of the incoming electric field. The Bloch wave vectors are imaginary for both polarizations.

The situation is different at an incoming angle of 60° as shown in figure 3.1(b): there the mode profile of the $s$-polarized light again rapidly decays inside the crystal (and the corresponding Bloch wave vector will again be imaginary), whereas the $p$-polarized light...
Spontaneous emission of vector waves in crystals of plane scatterers

Figure 3.1:
Squares of absolute values of mode functions for $s$-polarized (solid lines) and $p$-polarized light (dashed lines), as a function of position. The light is scattered by a crystal of ten planes with $D_{\text{eff}} = 0.46a$, separated by a distance $a$. Both modes correspond to light incoming from the left with $a/\lambda = 0.5$. Figure (a): $\theta_{\text{in}} = 30^\circ$; figure (b): $\theta_{\text{in}} = 60^\circ$.

can propagate inside the crystal (so the Bloch wave vector is real) and the light is transmitted almost completely. For this frequency and incoming angle, the crystal is a good polarization filter.

Note in figures 3.1(a,b) that the mode profiles of the $s$-polarized waves are continuous whereas $p$-polarized waves form more irregular patterns and are discontinuous at the positions of the planes. This reflects the boundary conditions that are automatically satisfied by the modes that we find by multiple-scattering theory: the tangential components of the electric fields must be continuous and the normal components show a jump at a dielectric interface. Now the electric field of $s$-polarized light only has a tangential component and $p$-polarized light consists of both a tangential and a normal component. This explains the differences in the mode profiles for $s$- and $p$-waves.

One can also compare reflection by the ten-plane Bragg mirror as a function of frequency for the two polarization directions. This is plotted in figure 3.2. For light incident perpendicularly to the planes, both transverse polarization vectors are equivalent and accordingly in figure 3.2(a), $s$- and $p$-polarized light show identical reflection characteristics as a function of frequency. This reflection plot is identical to the one for scalar waves in the previous chapter. Differences between the two polarizations do appear for non-normal incidence. In figure 3.2(b) the incident angle is $30^\circ$ and in figures (c)-(d) the incident angle is increased further. The red edges of the stop bands for $s$-polarized light move to higher frequencies and the widths of the stop bands become larger. For $p$-polarized light on the other hand, the red edges of the stop bands also shift to the blue, and faster so for larger incident angles. In contrast, the stop bands for $p$-light become narrower while moving to higher frequencies. In figure (d) for an incident angle of $80^\circ$, the first stop band for $p$-polarized light has even moved out of the picture, while the $s$-stop band has become very broad. It is harder for the ten-plane crystal to stop $p$-polarized light than to stop $s$-polarized...
In the previous chapter we showed that a ten-plane crystal can act as an omnidirectional mirror for scalar waves. But is this crystal also an omnidirectional mirror for vector waves? For s-polarized light we know that there are frequency intervals in which light coming from all directions is reflected, because the s-waves map on the scalar waves. So here we need to analyze whether there are also such frequency intervals for p-waves and if so, whether any s- and p-frequency intervals have an overlap. Only in the latter case can we speak of an omnidirectional mirror for light.

It is the Bloch wave vector that distinguishes between light that can propagate inside a crystal and light that feels a stop band: a real Bloch wave vector corresponds to propagating light and a complex Bloch wave vector to light that is reflected off the crystal. In our formalism, the Bloch wave vectors are the arc cosines of the constants $C_s$ and $C_p$. Now for light of a frequency corresponding to $a/\lambda = 0.5$ and planes with $D_{\text{eff}} = 0.46a$ we know that s-waves are reflected omnidirectionally. In figure 3.3 we plot both constants for

**Figure 3.2:**
Reflection off a ten-plane crystal, as a function of $a/\lambda$, for s-polarized light (solid lines) and p-polarized light (dashed lines). Incoming angles are $0^\circ$, $30^\circ$, $60^\circ$ and $80^\circ$. The planes have effective thickness $D_{\text{eff}} = 0.46a$ and they are separated by a distance $a$. 
this frequency, as a function of angle of the incident light. Unlike for s-waves, for p-waves there are incident angles larger than the critical angle $\theta_c \gtrsim 55^\circ$ for which the values of $C_p$ are between -1 and 1. Light incident with these large angles can propagate inside the crystal and therefore the crystal is not an omnidirectional mirror for this frequency. Actually, this information could already be read off from the mode profile in figure 3.1(b). However, the conclusion does not only hold for the specific parameter values that we chose: if we choose $D_{\text{eff}}$ larger then $\theta_c$ moves up to larger angles, but it can be shown by expanding Eq. (3.47) around $\theta_{\text{in}} = 90^\circ$ that for every finite $D_{\text{eff}}$ and $a/\lambda$ there always is a finite interval of angles corresponding to propagating $p$-polarized light. So strictly speaking, omnidirectional mirrors for vector waves can not be made with equidistant identical plane scatterers, but it is possible to block incident light from almost all directions.

### 3.3.2 Guided modes

We have found the propagating modes of vector waves near plane scatterers but these do not necessarily form the complete set of modes. As pointed out in section 3.1, a scattering problem can also have bound modes that do not correspond to incoming light. By solving the Lippmann-Schwinger equation without an incoming field, modes of scalar waves bound or guided by one or more plane scatterers were found in section 2.4.2. Later on, in section 2.5, it was shown that with each guided mode a nonzero local density of states is associated. One can also turn it around and try and find guided modes by looking for values of $k_\parallel > \omega/c$ corresponding to a nonzero density of states. It is the latter approach that we follow here for vector waves, because it directly allows us to use the effective description (3.36) for the T-matrix.

For vector waves, the local density of states is a tensor proportional to the imaginary part of the Green tensor $G(r, r, \omega)$. In planar geometries the latter can best be found as an integral over the Green tensor in the plane representation:

$$G(r, r, \omega) = \frac{1}{(2\pi)^2} \int d^2k_\parallel G(k_\parallel, z, z, \omega). \tag{3.50}$$

Clearly, the local density of states can only be nonzero if the imaginary part of the integrand in (3.50) is nonzero. A guided mode manifests itself when this integrand has a
nonzero imaginary part for a certain \( k_\parallel > \omega /c \). For the crystals of plane scatterers the Green tensor directly follows from the Dyson-Schwinger equation:

\[
G(k_\parallel, z, z', \omega) = G_0(k_\parallel, z, z', \omega) + \sum_{\alpha, \beta = 1}^{N} G_0(k_\parallel, z, z_\alpha, \omega) T^{(N)}_{\alpha \beta}(k_\parallel, \omega) G_0(k_\parallel, z_\beta, z', \omega).
\]

(3.51)

All the components of \( G_0(k_\parallel, z, z', \omega) \) become real quantities for \( k_\parallel > \omega /c \). This means that there is no density of states corresponding to guided modes in free space, and this comes as no surprise. Therefore, equation (3.51) tells us that the imaginary part of the Green tensor \( G \) can only be nonzero when at least one of the two spatial components of the \( N \)-plane \( T \) matrix \( T^{(N)} \) has a pole. These components \( T^{ss,(N)}_{\alpha \beta}(k_\parallel, \omega) \) and \( T^{uv,(N)}_{\alpha \beta}(k_\parallel, \omega) \) are given in equation (3.45).

First look for the poles of the component \( T^{ss,(N)}_{\alpha \beta}(k_\parallel, \omega) \). This is easy, because this component is identical to the \( N \)-plane \( T \) matrix \( T^{(N)} \) for scalar waves, as presented in the previous chapter. So we find that there are at most \( N \) guided modes corresponding to \( s \)-polarized light in a crystal of \( N \) planes. The dispersion relations of these guided modes are given in figure 2.4.

Are there also guided modes corresponding to \( p \)-polarized light in a crystal of \( N \) planes? This question will be addressed in some detail now. We need to find the poles of the component \( T^{uv,(N)}_{\alpha \beta}(k_\parallel, \omega) \). They occur when the determinant \( \det[(T^{uv,(N)}_{\alpha \beta})^{-1}] \) is equal to zero. An expression for this determinant can be found if one replaces \( \nu_s \) by \( \nu_p \) (as defined in section 3.2.6) in the expression A.14 for the corresponding determinant for \( s \)-polarized light. The result is that for \( p \)-polarized light a guided mode exists when the following equation is satisfied:

\[
2(\omega /c)^2 \sin[(N + 1)K_p a] + [\kappa F - 2(\omega /c)^2]e^{-\kappa a} \sin(NK_p a) = 0.
\]

(3.52)

where \( \kappa \) is defined as \(-i \sqrt{(\omega /c)^2 - k_\parallel^2}\). The Bloch wave vector \( K_p \) is still defined as \( a^{-1} \) times the arc cosine of the constant \( C_p \) (Eq. (3.47)), but in terms of \( \kappa \) it reads

\[
C_p = \cosh(\kappa a) + \frac{\kappa a^2 F}{2\omega^2} \sinh(\kappa a).
\]

(3.53)

Equation (3.52) should lead to the dispersion relations \( \omega(\kappa) \) for the guided modes, if they exist. When increasing the frequency, new guided modes appear that at first are only just captured by the structure so that \( \kappa = 0^+ \). As we did for \( s \)-polarized guided modes, we look for the guided modes in this small-\( \kappa \) limit. Note that for \( \kappa \) very small and positive, the constant \( C_p \) can be written as \( 1 + (\chi \kappa)^2 /2 \), up to second order in \( \kappa \), with \( \chi \) defined as \( a\sqrt{1 + FC^2/(2a\omega^2)} \). We can estimate the Bloch wave vector for small \( \kappa \) as follows:

\[
C_p = 1 + (\chi \kappa)^2 /2 + o(\kappa^3) = \cosh(\chi \kappa) + o(\kappa^3) \Rightarrow K_p = i\chi \kappa /a
\]

(3.54)

It follows that for small and positive \( \kappa \), solutions of equation (3.52) will only exist when \( \sinh[(N + 1)\chi \kappa] \) equals \( \sinh(N\chi \kappa) \), or equivalently \( \chi = 0 \). If for the optical potential we take \( F = D_{\text{eff}}(\omega /c)^2 \), then we find that there are guided modes for \( p \)-polarized light.
whenever $1 + D_{\text{off}}/(2a) = 0$. So in a crystal of $N$ planes with a positive effective thickness, there are no guided modes corresponding to $p$-polarized light. This is in agreement with the result in [65] for infinite crystals.

In conclusion, all guided modes in the finite crystal of plane scatterers correspond to $s$-polarized light and their properties have already been studied in the previous chapter. For dielectric slabs of finite thickness it is known that $s$-waves can be guided more easily than $p$-waves [60]. The non-existence of guided $p$-waves for plane scatterers is a manifestation of this difference between the two polarization directions.

3.4 Spontaneous emission

3.4.1 General theory applied to planes

The spontaneous emission rate $\Gamma$ of an atom embedded in an inhomogeneous dielectric is given by [111]

$$\Gamma(\mu, \mathbf{R}, \Omega) = \pi \sum_i \frac{\omega_i}{2\hbar} |\mu \cdot \mathbf{E}_i(\mathbf{R})|^2 \delta(\omega_i - \Omega). \quad (3.55)$$

This can be found by applying Fermi's golden rule. The spontaneous-emission rate (3.55) depends on the atomic transition frequency $\Omega$, on the $z$-coordinate of its position $\mathbf{R} = (x, y, z)$ and on the magnitude and orientation of its dipole moment $\mu$. The $\mathbf{E}_i$ are the normal mode solutions of the wave equation (3.5) for the electric field. The spontaneous-emission rate can alternatively be expressed in terms of the Green function of the medium. A derivation will be presented in section 5.3.1, but the rate to be found in Eq. (5.36) will already be used now:

$$\Gamma(\mu, \mathbf{R}, \Omega) = -\frac{2\Omega^2}{\hbar\varepsilon_0 c^2} \text{Im} [\mu \cdot \mathbf{G}(\mathbf{R}, \mathbf{R}, \Omega) \cdot \mu]. \quad (3.56)$$

Here, $\mathbf{G}$ is the dyadic Green function of the electric-field wave equation (3.5). For systems consisting of dielectric layers (not necessarily plane scatterers), it is easiest to first calculate the Green function in the plane representation $\mathbf{G}(\mathbf{k}_\parallel, z, z, \Omega)$. We only need to Fourier transform this Green function to real space as in Eq. (3.50) in order to find the Green function of Eq. (3.56) that determines the spontaneous-emission rates. A slight complication is that until now we chose a representation for $\mathbf{G}(\mathbf{k}_\parallel, z, z, \Omega)$ which is co-rotating with the incoming wave vector $\mathbf{k}_\parallel$ that we now want to integrate over. We need a fixed basis $\{\hat{x}, \hat{y}, \hat{z}\}$ and we can choose it such that the atomic dipole becomes $(\mu_x, 0, \mu_z)$ in the new representation. We perform the two-dimensional integral $\int d^2k_\parallel$ by writing it in polar coordinates $\int_0^{2\pi} d\phi_\parallel \int_0^\infty dk_\parallel$. By doing the angular integral first, only diagonal elements of the dyadic Green function survive. The total spontaneous emission-rate is the sum of two contributions, the perpendicular and the parallel decay rate, which are given
by, respectively,
\[
\Gamma_z (z, \Omega) = - \frac{3c\Gamma_0 \mu_z^2}{\Omega \mu^2} \text{Im} \int_0^\infty dk || k || G^{zz}(k || z, z, \Omega) \tag{3.57}
\]
\[
\Gamma_x (z, \Omega) = - \frac{3c\Gamma_0 \mu_x^2}{\Omega \mu^2} \text{Im} \int_0^\infty dk || k || \left( G^{ss}(k || z, z, \Omega) + G^{uv}(k || z, z, \Omega) \right) \tag{3.58}
\]

The parallel decay-rate has a contribution both from \( s \)- and \( p \)-polarized light (\( G^{ss} \) and \( G^{uv} \), respectively) whereas the perpendicular decay-rate only has a \( p \)-polarized decay channel (through \( G^{zz} \)). When the dipole is oriented parallel to the plane, the perpendicular decay-rate is zero, and vice versa. The spontaneous-emission rates in Eq. (3.57) and (3.58) are integrals over all possible lengths of the in-plane wave vector. Both rates can be subdivided into a propagating (or radiative) rate corresponding to the integration of \( k || \) from 0 to \( \omega/c \), and a guided rate which is the integral from \( \omega/c \) to infinity.

More precisely, we can subdivide the parallel decay-rate into three parts: an \( s \)-polarized radiative rate (sr), a \( p \)-polarized radiative rate (pr) and an \( s \)-polarized guided rate (sg). There are no guided modes for \( p \)-polarized light and as a consequence the Green tensor \( G^{zz} \) has no imaginary part (zero density of states) for \( k || \gg \omega/c \). The perpendicular decay-rate \( \Gamma_z \) is therefore purely radiative:

\[
\Gamma_{sx} (z, \omega) = - \frac{3c\Gamma_0}{2\Omega} \left( \frac{\mu_x}{\mu} \right)^2 \text{Im} \int_0^{\omega/c} dk || k || G^{ss}(k || z, z, \Omega), \tag{3.59a}
\]
\[
\Gamma_{px} (z, \omega) = - \frac{3c\Gamma_0}{2\Omega} \left( \frac{\mu_x}{\mu} \right)^2 \text{Im} \int_0^{\omega/c} dk || k || G^{uv}(k || z, z, \Omega), \tag{3.59b}
\]
\[
\Gamma_{sz} (z, \omega) = - \frac{3c\Gamma_0}{\Omega} \left( \frac{\mu_z}{\mu} \right)^2 \text{Im} \int_0^{\omega/c} dk || k || G^{zz}(k || z, z, \Omega). \tag{3.59c}
\]
\[
\Gamma_z (z, \omega) = - \frac{3c\Gamma_0}{\Omega} \left( \frac{\mu_z}{\mu} \right)^2 \text{Im} \int_0^{\omega/c} dk || k || G^{zz}(k || z, z, \Omega). \tag{3.59d}
\]

With all the spontaneous-emission rates spelled out now, let us first study spontaneous-emission rates near a single plane.

### 3.4.2 Spontaneous emission near one plane scatterer

The Green function in the plane representation can be calculated directly using the single-plane T-matrix \( T \) of section 3.2.6 in the Dyson-Schwinger equation

\[
G(k || z, z', \omega) = G_0(k || z, z', \omega) + G_0(k || z, z', \omega) \cdot T(k || z, \omega) \cdot G_0(k || z, z', \omega). \tag{3.60}
\]

This Green function can be used in the expressions of the previous section 3.4.1 to obtain spontaneous-emission rates near a single plane.

What happens to the spontaneous-emission rates very close to the plane? (By "close" it is meant: at distances small compared to the wavelength of light, but still large compared to atomic distances where local-field effects come into play that we do not consider
Spontaneous emission rates of dipoles with zero distance to the single plane scatterer, as a function of the dimensionless parameter \( \xi = \pi D_{\text{eff}} / \lambda \). Solid line: the rate \( \Gamma_z \), which is the sum of a rate into radiative (dashed line) and into guided modes (dotted line). The rate \( \Gamma_z \) is the dash-dotted line.

In the limit that the atomic position \( z \) becomes equal to the plane position \( z_\alpha \), the spontaneous-emission rates for the two orientations can be calculated analytically. Let \( \Gamma_0 \) be the spontaneous-emission rate in vacuum, equal to \( \mu^2 \Omega^3 / (3\pi \hbar c^3) \). For a dipole perpendicular to the planes we find

\[
\Gamma_z(z_\alpha, \Omega) = \Gamma_0 \left( 2 + \frac{3}{2\xi^2} \right) - \frac{3}{2} \Gamma_0 \left( 1 + \frac{1}{\xi^2} \right) \frac{\arctan(\xi)}{\xi}, \quad (3.61)
\]

where the dimensionless parameter \( \xi \) is defined as \( \pi D_{\text{eff}} / \lambda \). The three contributions to the parallel decay-rate at the position of the plane can also be expressed in terms of the parameter \( \xi \) alone:

\[
\begin{align*}
\Gamma_{z\parallel}(z_\alpha, \Omega) &= \frac{3}{4} \Gamma_0 \left[ 1 - \xi \arctan(1/\xi) \right], \quad (3.62a) \\
\Gamma_{z\parallel}(z_\alpha, \Omega) &= \frac{3}{4\xi^2} \Gamma_0 \left[ 1 - \frac{\arctan(\xi)}{\xi} \right], \quad (3.62b) \\
\Gamma_{z\parallel}(z_\alpha, \Omega) &= \frac{3\pi\xi}{4} \Gamma_0. \quad (3.62c)
\end{align*}
\]

In figure 3.4 the relative rates are plotted as a function of \( \xi \). The results can be checked in two limiting cases: the weakest test is the limit of invisible planes where \( D_{\text{eff}} = 0 \) and therefore \( \xi = 0 \). Then both \( \Gamma_z \) and \( \Gamma_x \) approach the free-space value, as they should. The other test is the limit of a perfect mirror, when \( \xi = D_{\text{eff}} = \infty \). This limit is not visible in the figure, but we find \( \Gamma_z/\Gamma_0 = 2 \) and \( \Gamma_x/\Gamma_0 = 0 \). To be precise, we first have taken the limit of the perfect mirror and then come closer with the atom to the mirror. In doing so, the contribution to the spontaneous-emission rate of the guided waves goes to zero. Taking the limits in the reverse order would give the unphysical result that the contribution of the guided waves diverges when the positions of the atom and mirror coincide. The spontaneous-emission rates for atoms near perfect mirrors agree with the values in the literature [16, 78]. These results can be explained in terms of constructive or destructive interference of the light emitted by the atom and its perfect mirror.
3.4 Spontaneous emission

Figure 3.5:
Spontaneous-emission rates of dipoles near a single partially transmitting plane scatterer, relative to \( \Gamma_0 \). Solid lines correspond to total spontaneous-emission rates \( \Gamma_x \) for dipoles parallel to the plane, dotted lines are radiative contributions to \( \Gamma_x \), and the dashed lines denote \( \Gamma_z \). In (a), the effective thickness \( D_{\text{eff}} \) of the plane equals 0.46\(a\) and in (b) \( D_{\text{eff}} = 10a \).

In our model, effects on spontaneous emission become more pronounced when the parameter \( \xi \) is larger and this can be accomplished by choosing a higher frequency. However, this only makes sense physically in the frequency regime where we can neglect dispersion in the dielectric function \( \varepsilon(\omega) \) of the dielectric slabs that are being modelled. Otherwise, the dispersion shows up in the effective thickness \( D_{\text{eff}} = d|\varepsilon(\omega) - 1| \), with the effect that there is a frequency for which the parameter \( \xi \) is maximal. Dielectrics do not make perfect mirrors for high frequencies. Theoretically, this point is important for a consistent description of quantum optics in dielectrics [79].

In figure 3.5(a), spontaneous-emission rates as a function of position are plotted for \( D_{\text{eff}} = 0.46a \). The values for \( z = 0 \) are finite in both cases (although not shown in figure 3.5) and indeed correspond to Eqs. (3.61) and (3.62a). For both dipole orientations, far away from the plane the rate approaches the free-space value. Close to the plane, \( \Gamma_z \) is increased; the \( \Gamma_x \) is also larger than \( \Gamma_0 \), but this decay-rate consists of a rate into propagating modes that is less than \( \Gamma_0 \) and a guided-mode-rate. The contributions of radiative and guided s-waves for an atom with \( \mu = \mu_x \) are the same as for scalar waves with “scalar dipole moment” \( \mu \), but since the total decay-rate \( \Gamma_0 \) for vector waves is larger than for scalar waves, the relative contributions of s-waves to \( \Gamma/\Gamma_0 \) are smaller for vector waves (by a factor 3/4). In figure 3.5(b), the same rates are plotted, but now for a much better reflecting plane with an effective thickness of \( D_{\text{eff}} = 10a \). There the limiting values of spontaneous-emission rates near the plane are approached: \( \Gamma_z \) climbs up but does not reach the value of 2\(\Gamma_0 \) yet, whereas the propagating part of \( \Gamma_x \) is practically zero when \( D_{\text{eff}} \) equals 10\(a\). The partial emission-rate into the guided mode has a much larger amplitude near the plane but falls off much more rapidly away from the plane.
3.4.3 Spontaneous emission near a ten-plane scatterer

In our formalism we can study spontaneous-emission rates in the vicinity of an arbitrary number of planes. Here we choose to present results for \( N = 10 \), as we did for scalar waves. The spontaneous-emission rates calculated in this section are based on Eqs. (3.59a-3.59d). The following Dyson-Schwinger equation gives the relevant Green functions in terms of the T-matrix of section 3.2.6:

\[
G(k||, z, z', \omega) = G_0(k||, z, \omega) + \sum_{\alpha, \beta=1}^{N} G_0(k||, z_\alpha, \omega) \cdot T_{\alpha\beta}^{(N)}(k||, \omega) \cdot G_0(k||, z_\beta, z', \omega).
\]  

(3.63)

The first thing that one should like to do in a formalism for vector waves is to compare the total emission rates for dipoles parallel (\( \Gamma_x \), solid lines) with dipoles perpendicular (\( \Gamma_z \), dashed lines) to the planes. In figure 3.6(a-d) orientation-dependent spontaneous-emission rates are plotted for several frequencies. For clarity in the pictures, the positions of the planes at 1, 2, ..., 10 are not shown as vertical lines this time. The most striking difference is that \( \Gamma_x \) becomes very spiky near the planes because of the guided modes to which only parallel dipoles can couple in the vicinity of the planes. We can also see that \( \Gamma_z \), which is purely radiative, on average increases due to the presence of the planes whereas the radiative part of \( \Gamma_x \) on average decreases near the planes. In the previous section we saw the same trend near a single plane where in the limit of a perfect mirror \( \Gamma_x/\Gamma_0 \to 0 \) and \( \Gamma_z/\Gamma_0 \to 2 \). Figure 3.1 showed that the optical modes of \( p \)-polarized light have discontinuities at the plane positions. There are also discontinuities in the spontaneous-emission rates, but these discontinuities are too small to be visible in figure 3.6. They are small because the discontinuities per mode are averaged in the emission rate.

The dotted lines in figure 3.6 are the radiative parts of \( \Gamma_x \). These are similar to the radiative rates for scalar waves, but not identical since in \( \Gamma_x \) not only \( s \)-polarized light but also \( p \)-polarized light contributes. In particular, far away from the planes, the emission rate of dipoles parallel to the planes consists of 75 percent \( s \)-polarized and 25 percent \( p \)-polarized light.

For scalar waves the ten-plane structure can act an omnidirectional mirror, whereas in section 3.3.1 it was found that it is not an omnidirectional mirror for vector waves. Correspondingly, the radiative LDOS for scalar waves at \( a/\lambda = 0.5 \) dropped down to (almost) zero inside the ten-plane omni-directional mirror, whereas the emission rates in figure 3.6(c) show that the radiative LDOS for vector waves stays nonzero inside the crystal. In the inner unit cells, dipoles parallel to the planes emit predominantly guided light, but the small amount of light that leaves the structure is strongly \( p \)-polarized. This is the case around \( a/\lambda = 0.5 \) only, where \( s \)-polarized light is omnidirectionally reflected; at higher and lower frequencies the emitted light can have both polarization directions.

3.5 Radiative line shifts

Not only spontaneous-decay rates change inside a dielectric medium, but there are also radiative shifts of the atomic transition frequencies. Both effects will be derived in detail
3.5 Radiative line shifts

Figure 3.6:
Spontaneous-emission rates $\Gamma_x$ (solid line) and $\Gamma_z$ (dashed) near a ten-plane crystal. The dotted line is the radiative part of $\Gamma_x$. The figures (a)-(d) correspond to four frequencies: (a) $a/\lambda = 0.2$; (b) $a/\lambda = 0.4$; (c) $a/\lambda = 0.5$; (d) $a/\lambda = 0.6$.

In section 5.3.1. As given in Eq. (5.38), the atomic transition frequency in a medium will be shifted with respect to the value $\Omega$ in free space by an amount

$$\Delta(R, \Omega) = \left( \frac{\mu^2 \Omega^2}{\hbar c^2} \right) \text{Re} \langle \mathbf{\hat{\mu}} \cdot [\mathbf{G}(R, R, \Omega) - \mathbf{G}_0(R, R, \Omega)] \cdot \mathbf{\hat{\mu}} \rangle,$$

which depends both on position and frequency. In a T-matrix formalism, the Green function is the sum of the free-space Green function and the term $\mathbf{G}_0 \cdot \mathbf{T} \cdot \mathbf{G}_0$, see Eq. (3.63). Line shifts are therefore determined by the latter term only. Proceeding much the same way as for the decay rates in section 3.4.1, the line shift at position $R = (x, y, z)$ in a crystal of plane scatterers is found to be

$$\Delta(R, \Omega) = \left( \frac{3e\Gamma_0}{2\Omega} \right) \text{Re} \left[ \frac{\mu_x^2}{2\mu^2} (I_x + I_e) + \frac{\mu_z^2}{\mu^2} I_z \right],$$
Spontaneous emission of vector waves in crystals of plane scatterers

Figure 3.7:
Line shifts of the optical transition frequency relative to the free-space value $\Omega$. Shifts are given as a function of the distance of the atom to a single plane, both for atoms with dipole moments pointing parallel (solid lines) and perpendicular (dashed lines) to the plane. Shifts are scaled to the free-space decay rate $\Gamma_0$. In (a), $D_{\text{eff}} = 0.46a$ and in (b) $D_{\text{eff}} = 10a$. In both figures, $a/\lambda = 0.5$.

where the integrals $I_s, I_v$ and $I_z$ are given by

$$I_s = \sum_{\alpha,\beta} \int_0^{\infty} dk_\parallel k_\parallel G_0^{ss}(k_\parallel, z, z_\parallel, \Omega) T^{ss}_{\alpha\beta}(k_\parallel, \Omega) G_0^{ss}(k_\parallel, z_\parallel, z, \Omega), \quad (3.66)$$

$$I_v = \sum_{\alpha,\beta} \int_0^{\infty} dk_\parallel k_\parallel G_0^{vv}(k_\parallel, z, z_\parallel, \Omega) T^{vv}_{\alpha\beta}(k_\parallel, \Omega) G_0^{vv}(k_\parallel, z_\parallel, z, \Omega), \quad (3.67)$$

$$I_z = \sum_{\alpha,\beta} \int_0^{\infty} dk_\parallel k_\parallel G_0^{uv}(k_\parallel, z, z_\parallel, \Omega) T^{uv}_{\alpha\beta}(k_\parallel, \Omega) G_0^{uv}(k_\parallel, z_\parallel, z, \Omega). \quad (3.68)$$

In these integrals, the magnitude $k_\parallel$ of the wave vector parallel to the planes goes from zero to infinity. The matrix elements $T^{ss}_{\alpha\beta}(k_\parallel, \omega)$ in the integral $I_s$ have poles on the real-$k_\parallel$ axis corresponding to the guided modes. The real part of $I_s$ can be evaluated by taking the Cauchy principal-value integral at these poles. Although there are no guided modes for $p$-polarized light, there is a nonzero contribution to the integral $I_z$ from in-plane wave vectors larger than $\Omega/c$. There was no such contribution to the spontaneous-emission rate $\Gamma_{\text{z}}$ in Eq. (3.59d). The difference stems from the fact that the decay rate is an on-shell quantity, whereas line shifts $\Delta(\Omega)$ have off-shell contributions. In other words, unlike the rate $\Gamma(\Omega)$, the shift $\Delta(\Omega)$ also depends on optical modes with eigenfrequencies different from $\Omega$. Actually, it is a weighted sum over all optical modes.

In figure 3.7(a), line shifts are presented as a function of distance to a single plane, for dipoles pointing parallel and perpendicular to the plane. Away from the plane, the shifts show damped oscillatory behavior. The magnitudes are very small, typically ten percent or less of the free-space decay rate. Close to the planes, the off-shell contributions of large wave vectors become important. For both dipole orientations, the line shifts even
diverge when approaching the plane. At distances on the order of 0.1\(\lambda\) and smaller, the perpendicular dipoles feel larger frequency shifts than the parallel ones. On the other hand, far away from the plane the shifts are larger for parallel dipoles. Figure 3.7(b) shows shifts near a plane with much larger effective thickness. The amplitudes of the shifts are increased as compared to figure 3.7(a), but the overall picture has not changed.

In order to avoid the infinite line shifts at the position of the plane, the cutoff parameter \(\Lambda\) could have been kept finite. Then the discontinuities in modes and in Green functions would have become smooth. By keeping \(\Lambda \gg \Omega/c\) finite, only the line shifts at distances of typically \(\Lambda^{-1}\) or less are modified and become finite. The emission rates presented in this chapter already were finite and they will not be affected either at distances greater than \(\Lambda^{-1}\) when a finite cutoff is chosen to keep line shifts finite.

### 3.6 Conclusions and outlook

In this chapter a multiple-scattering theory was set up for the scattering of vector waves by parallel planes. The Green function had to be regularized and this was done by introducing a high-momentum cutoff. In the end, the regularization parameter was sent to infinity. An effective scattering theory for plane scatterers emerged with a nonzero \(T\)-matrix that no longer depends on the cutoff. This is in contrast with point scatterers where two cutoffs must be kept finite in order to define a nonzero \(T\) matrix [69]. The only reason to keep a finite cutoff for plane scatterers would be to avoid diverging line shifts at the positions of the planes.

The planar symmetry of the crystal of parallel planes enabled us to separate \(s\)- and \(p\)-polarized modes. The scattering by a nonabsorbing plane scatterer satisfies a separate optical theorem for each polarization. The radiative and guided modes of \(s\)-polarized light could be mapped onto modes for scalar waves. The \(s\)-polarized light has continuous mode functions, whereas the mode function of the \(p\)-polarized light is discontinuous at the positions of the planes. As a consequence, the \(p\)-polarization contributions to the spontaneous emission rate show discontinuities as well. Spontaneous emission rates were calculated, both for dipoles parallel and perpendicular to the planes.

One could wish to create a crystal such that 100\% of the spontaneously emitted light is guided light. In our model of identical equidistant plane scatterers this will not happen. If there was only \(s\)-polarized light, then it would be possible, as it was for scalar waves [figure 2.7(a)]. Deep inside an omnidirectional mirror all spontaneously emitted light is guided light. For vector waves, however, equidistant plane scatterers can not be an omnidirectional mirror.

Omnidirectional mirrors consisting of dielectric layers do exist [59, 60, 80]. The occurrence of omnidirectional reflection is promoted by the fact that a single dielectric layer and a multilayer stack both reflect \(p\)-polarized light at the Brewster angle [55]. This is to be contrasted with our plane scatterer model where the Brewster angle is shifted away to 90°. Therefore, the Brewster effect does help slabs but does not help planes to create the omnidirectional reflector.

What use can plane scatterers be in the future? One obvious idea is to study crystals built up of non-parallel planes. This turns out to be problematic, however, not so much
because the planes have an overlap of measure zero, but rather because the scattering of each plane as a whole must be described in its own plane representation. For that reason one can not treat the individual planes as wholes. Instead, one is forced to think of the planes as being built up of infinitely many line scatterers or point scatterers.

Albeit for parallel planes, the plane-scattering formalism can be used to study much more than was presented here. With the knowledge of the complete Green function, one could calculate near-field and far-field spectra. One could study the effect of more than one plane in a unit cell. Furthermore, the planes need not be identical and in fact the result (3.44) for the T-matrix also holds for planes chosen at arbitrary positions with different effective thicknesses. The model can therefore be used to study the effect of disorder, both in the positions and in the effective thicknesses of individual planes. Disorder can be increased further to study random dielectrics rather than crystals of plane scatterers.

Crystals of plane scatterers can also be used as a model system to study the modification of several quantum optical processes of embedded atoms. Transient effects in the spontaneous-emission rates are a first example, thereby generalizing work done on a one-dimensional cavity formed by two planes [81]. It is also interesting to study (two-atom) superradiance modified by the crystal (see chapter 5). Or one could generalize the formalism by allowing light absorption or gain in the planes, by choosing the effective thickness complex and frequency-dependent. In that case, a quantum optical description would require the identification of quantum noise operators (see chapter 6) for the planes. Also, Casimir forces can be calculated between an arbitrary number of imperfect mirrors. For two passive mirrors in one dimension, these forces are always attractive [82]. For planes with optical gain, the Casimir force might change sign.