Quantum optics and multiple scattering in dielectrics

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Appendix D

Laplace operators and time-dependent coefficients

In section 6.2 the electric-field operator $\tilde{E}(\imath)$ was given in terms of the operators at $t = 0$. Here we give the analogous expressions for the other Laplace operators. Furthermore, we show how to evaluate the time-dependent coefficients $M_{mn}(t)$ like in Eq. (6.20) for the electric-field operator. Finally, we list the expressions for the coefficients of the other operators.

The expression for $\tilde{E}(\imath)$ in (6.15) has the following analogous expressions for the other Laplace operators:

\[
\tilde{A}(\imath) = \frac{1}{\tilde{D}(\imath)} \left\{ -E(0) + pA(0) - \frac{\alpha}{\varepsilon_0} \left[ \frac{p^2}{\omega_c^2} \bar{\chi}(\imath) - 1 \right] X(0) - \frac{1}{\alpha} p\bar{\chi}(\imath)[P(0) - \tilde{B}(\imath)] \right\},
\]

\[
\tilde{X}(\imath) = \frac{1}{\tilde{D}(\imath)} \left\{ -\frac{\varepsilon_0}{\alpha} p\bar{\chi}(\imath)E(0) + \frac{\varepsilon_0}{\alpha} p^2 \bar{\chi}(\imath)A(0)
+ \left( \frac{k^2 c^2}{\omega_c^2} + \frac{p^2}{\omega_c^2} + 1 \right) p\bar{\chi}(\imath)X(0) + \frac{\varepsilon_0}{\alpha^2} (p^2 + k^2 c^2)\bar{\chi}(\imath)[P(0) - \tilde{B}(\imath)] \right\},
\]

\[
\tilde{P}(\imath) = \frac{1}{\tilde{D}(\imath)} \left\{ -\alpha \left[ \frac{p^2}{\omega_c^2} \bar{\chi}(\imath) - 1 \right] E(0) + \alpha p \left[ \frac{p^2}{\omega_c^2} \bar{\chi}(\imath) - 1 \right] A(0)
+ \frac{\alpha^2}{\varepsilon_0} \left[ \frac{p^2}{\omega_c^2} \bar{\chi}(\imath) - 1 \right] \left( \frac{k^2 c^2}{\omega_c^2} + \frac{p^2}{\omega_c^2} + 1 \right) X(0)
+ p \left( \frac{k^2 c^2}{\omega_c^2} + \frac{p^2}{\omega_c^2} + 1 \right) \bar{\chi}(\imath)[P(0) - \tilde{B}(\imath)] \right\}. \tag{D.1}
\]

If we now apply the inverse Laplace transformation to these expressions, we find the full time dependence of the operators $A$, $X$, and $P$. The inverse Laplace transformation is a contour integration over the Bromwich contour that includes the whole imaginary $\imath$.
Laplace operators and time-dependent coefficients

axis. After transforming to frequency variables, the contour includes poles from $\tilde{D}^{-1}(p)$, which are in the lower half plane and moreover poles on the real frequency axis arising from $\tilde{B}(p)$. The latter are important in the calculation of the long-time solutions of the operators in section 6.4. However, in the calculation of the coefficients $M_{mn}(t)$, which we will discuss here, they play no role. For example, the coefficient $M_{AE}(t)$ becomes

\[
M_{AE}(t) = -\frac{1}{2\pi i} \int_{-i\infty + \eta}^{i\infty + \eta} dp \, e^{pt} \tilde{D}^{-1}(p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \frac{e^{-i\omega t}}{\varepsilon(\omega)\omega^2 - k^2 c^2} = \frac{1}{4\pi k^2 c} \int_{-\infty}^{\infty} d\omega \left( \frac{e^{-i\omega t}}{n(\omega)\omega - k} - \frac{e^{-i\omega t}}{n(\omega)\omega + k} \right) = \frac{1}{k^2 c} \sum_j \text{Im}(e^{-i\Omega_j t} \frac{v_{pj}}{c}).
\]

(D.2)

Note that $M_{AE}(t)$ is exponentially damped because all $\Omega_j$ in the exponentials have negative imaginary parts. The other coefficients can be calculated in a similar way. The results are

\[
M_{AA}(t) = M_{EE}(t),
M_{AX}(t) = -\frac{\alpha k}{\omega^2 c^4} \sum_j \text{Im} \left[ e^{-i\Omega_j t} \frac{v_{pj}}{c} \left( 1 - \frac{v_{pj}^2}{c^2} + \frac{\omega_0^2}{k^2 c^2} \right) \right],
M_{AP}(t) = \frac{1}{\alpha} \sum_j \text{Re} \left[ e^{-i\Omega_j t} \frac{v_{pj}}{c} \left( \frac{v_{pj}}{c} - \frac{c}{v_{pj}} \right) \right],
M_{XE}(t) = \varepsilon_0 M_{AP}(t),
M_{XA}(t) = \varepsilon_0 M_{EP}(t),
M_{XX}(t) = -\frac{k^2 c^2}{\omega^2 c} \sum_j \text{Re} \left[ e^{-i\Omega_j t} \frac{v_{pj}}{c} \left( \frac{v_{pj}}{c} - \frac{c}{v_{pj}} \right) \left( 1 - \frac{v_{pj}^2}{c^2} + \frac{\omega_0^2}{k^2 c^2} \right) \right],
M_{XP}(t) = -\frac{\varepsilon_0 k^2 c}{\alpha^2} \sum_j \text{Im} \left[ e^{-i\Omega_j t} \frac{v_{pj}}{c} \left( \frac{v_{pj}}{c} - \frac{c}{v_{pj}} \right)^2 \right],
M_{PE}(t) = \varepsilon_0 M_{AX}(t),
M_{PA}(t) = \varepsilon_0 M_{EX}(t),
M_{PX}(t) = \frac{\alpha^2 k^3 c^3}{\omega^2 c^4} \sum_j \text{Im} \left[ e^{-i\Omega_j t} \frac{v_{pj}}{c} \left( 1 - \frac{v_{pj}^2}{c^2} + \frac{\omega_0^2}{k^2 c^2} \right)^2 \right],
M_{PP}(t) = M_{XX}(t).
\]

(D.3)

With the sum rules discussed in section 6.3, one can see that the “diagonal” coefficients in this list equal 1 at time $t = 0$, whereas the other coefficients have the initial value 0.