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AGE DEPENDENCE OF THE VEGA PHENOMENON: THEORY

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ABSTRACT

In a separate paper, we have reexamined the observations of IR excess obtained with the *Infrared Space Observatory* satellite and discussed the ages of stars with excess. The amount of dust (measured by the luminosity fraction $f_d = L_{\text{IR}}/L_*$) seen around main-sequence stars of different ages shows several interesting trends. To discuss these results in the context of a physical model, we develop in this paper an analytical model for the dust production in Vega-type systems. Previously it has been claimed that a power-law slope of about -2 in the diagram plotting amount of dust versus time could be explained by a simple collisional cascade. We show that such a cascade in fact results in a power law $f_d \propto t^{-1}$ if the dust removal processes are dominated by collisions. A power law $f_d \propto t^{-2}$ only results when the dust removal processes become dominated by Poynting-Robertson drag. This might be the case in the Kuiper Belt of our own solar system, but it is certainly not the case in any of the observed disks. A steeper slope can, however, be created by including continuous stirring into the models. We show that the existence of both young and old Vega-like systems with large amounts of dust ($f_d \simeq 10^{-3}$) can be explained qualitatively by Kuiper Belt-like structures with *delayed stirring*. Finally, the absence of young stars with intermediate amounts of dust might be due to the fact that stirring due to planet formation may not be active in young low-mass disks. The considerations in this paper support the picture of simultaneous stirring and dust production proposed by Kenyon and Bromley.

Subject headings: circumstellar matter — infrared: stars

1. INTRODUCTION

Debris disks are thought to form after the depletion of gas in young circumstellar disks. During the first ~ 10 Myr of a star’s life, small dust grains grow by coagulation and finally produce planetesimals (Lissauer 1993). When the average velocity of the collisions increases because of stirring by planets or large planetesimals forming in the disk, the encounters between the planetesimals become destructive, initiating a collisional cascade. Large quantities of small dust grains are produced in such a cascade. These grains emit infrared and submillimeter radiation, making the debris disk visible (e.g., Aumann et al. 1984; Zuckerman & Becklin 1993).

Since most Vega-like excess stars are spatially unresolved by infrared telescopes, the quantity used to measure the amount of dust is the “fractional luminosity,” f_d : the ratio of the dust emission L_{IR} and the stellar luminosity L_*

$$f_d = \frac{L_{\text{IR}}}{L_*}. \quad (1)$$

f_d can be interpreted as a “covering fraction.” It measures the fraction of the sky seen from the star that is covered by dust, and therefore the fraction of the stellar radiation that will be absorbed and reprocessed to the infrared.

Studying the age dependence of the Vega phenomenon has been at the focus of several studies using the *Infrared Space Observatory* (ISO). The Vega phenomenon appears to be much more widespread around younger stars than around older stars. Studying a volume-limited sample of

stars near the Sun, Habing et al. (1999, 2001) found that more than 60% of the observed stars below an age of 400 Myr show a Vega-like excess, while only 9% of older stars can be classified as Vega-like. This result is also confirmed by submillimeter observations (e.g., Zuckerman & Becklin 1993; Wyatt, Dent, & Greaves 2003). Looking at a few selected Vega-like stars, Kalas (2000) noted a $\sim t^{-1}$ decline. Studying members of young clusters with different ages, Spangler et al. (2001) found a trend in the average amount of dust seen around stars of different ages that was described by a power-law dependence of f_d versus time. Spangler et al. tentatively interpreted this result as a global trend due to a collisional cascade.

A thorough study of the required parameters, age and fractional luminosity, was done in Decin et al. (2003, hereafter Paper I) to reexamine the available ISO data. It led to the following implications for models. (1) Debris disks are more common around young stars than around old ones, but (2) a general power law for the dust mass versus age with slope of approximately -2 , as found by, e.g., Holland et al. (1998) and Spangler et al. (2001), could not be confirmed. Two main issues are causing the difference in results. There clearly is a strong decrease in circumstellar material from weak-line T Tauri stars to the youngest main-sequence stars, but we reject weak-line T Tauri stars as debris disks (Lagrange, Backman, & Artymowicz 2000) because these disks are likely to still be gas-rich and governed by different physics (Artymowicz 1996). Also, we find a significant number of old stars with large f_d -values, contradicting a global power law decrease of f_d . There might still be a

decline of dust mass for the youngest stars, with slope of approximately -1.3 , but surely not for the older ones. (3) We found that the maximum excess does not depend on age, also demonstrating the absence of a global declining trend applying to all stars. (4) A final interesting result from Paper I is that there is an apparent scarcity of young stars with intermediate or low infrared excesses.

Little modeling on the general time evolution of Vega-like disks has been done so far. The collisional evolution of the solar system asteroid belt has been modeled extensively, mainly to understand the observed present day size and rotation distribution of large asteroids. Similar models for Kuiper Belt objects have been described by Davis & Farinella (1997), *exemplar gratis*. Production of dust in the asteroid belt has been studied by Durda & Dermott (1997), focusing on the detailed size distribution produced by the collisional cascade at small sizes, which is influenced by both collisions and interaction with radiation. Studies of collisional cascades in the Kuiper Belt have also considered the formation of dust, but only in very basic ways (Stern & Colwell 1997b). The most detailed numerical models of combined growth and destruction of bodies in disks around stars are recent calculations by Kenyon & Luu (1999a, 1999b) and Kenyon & Bromley (2001, 2002a, 2002b), who study the evolution of the outer protoplanetary disk. We will get back to these papers in the discussion.

The goal of the present paper is to better understand the age dependence of the dust mass in Vega-type stars in terms of a simple model. Therefore, an analytical model is developed in § 2, which describes the evolution of debris dust. This model is then confronted with the observations and discussed in § 3.

2. COLLISIONAL MODEL

We use a very simple collisional model to derive the amount of dust produced by a collisional cascade of large bodies, and to compute the decay of the dust present in the system with time. A similar model has been used to estimate the lifetime of Vega-like disks by Kenyon & Bromley (2001), but we go further and compute the amount of dust as a function of time under the assumption that the evolution of the disk is purely due to a collisional cascade. In deriving the model, we make use as much as possible of general laws governing all kinds of collisions, and refer as little as possible to detailed material properties. The underlying model will be that of the Kuiper or asteroid belt of our own system: a number of large bodies, colliding to produce and replenish the dust we see.

2.1. Collisional Removal of Comets

Let us assume that all the dust in the system is produced as the end result of a collisional cascade involving large bodies. We therefore start with a number N_c of comets. These comets have a radius a_c , a geometrical cross section for collisions between comets of $\sigma_{\text{coll}} = 4\pi a_c^2$, and a mass $m_c = (4\pi/3)\rho_c a_c^3$, where ρ_c is the density of the cometary material.

For the collisions between the comets, we use a particle-in-a-box model: all comets are moving through a given volume V , which could be an entire planetary disk or just a limited region like the asteroid or Kuiper belt. We assume that the occupied volume is a section of a wedge-shaped disk

between distances r_1 and r_2 with a local half-height $H(r) = h \times r$, where h is the normalized height H/r of the disk. The volume of such a wedge segment is

$$V = \frac{4\pi}{3} h (r_2^3 - r_1^3) \equiv \frac{4\pi}{3} h \hat{r}^3, \quad (2)$$

where \hat{r} is a number close to r_2 if r_1 is not too close to r_2 . If we use numbers typical for the Kuiper Belt ($r_1 \simeq 35$ AU, $r_2 \simeq 50$ AU, $h \simeq 0.2$) we find $\hat{r} = 43.5$ AU and $V = 6.25 \times 10^{44}$ cm³.

The particle-in-a-box model assumes that there is a fixed collision velocity between different comets, which we denote with v_{coll} . For simplicity, we write this collision velocity as a fraction ν of the Kepler velocity:

$$v_{\text{coll}} = \nu v_K(\hat{r}) = \nu \sqrt{\frac{GM}{\hat{r}}}, \quad (3)$$

where G is the gravitational constant and M is the mass of the star. One can easily see that ν is related to the relative height h of the disk. Collisions between bodies in a disk are due to differences in the inclinations and eccentricities of orbits. A particle on a circular orbit with inclination i will cross the midplane of the system with a vertical speed $v_K \tan i$. Therefore, a disk with a normalized height of $h = \tan i$ will show typical collision velocities $h v_K$. We can therefore assume $\nu = h$ whenever we numerically evaluate expressions.

With the collision velocity v_{coll} and the volume given, we can compute the ‘‘sweeping time’’ t_s , which is the time needed by a single comet to sweep the entire volume accessible to the cometary cloud

$$t_s = \frac{V}{v_{\text{coll}} \sigma_{\text{coll}}}. \quad (4)$$

Note that the sweeping time is actually independent of the collision velocity, since the volume occupied by the cometary cloud also grows with v_{coll} . The collision time for comets is then given by

$$\tau_{\text{comet}} = \frac{t_s}{N_c}. \quad (5)$$

For dust production to occur, the collisions between comets must be destructive. Assuming that any such collision removes two comets from the cloud, we get for the time evolution of the number of comets

$$\frac{dN_c}{dt} = -2 \frac{N_c^2}{t_s}, \quad (6)$$

with the solution

$$N_c(t) = \frac{N_c(0)}{1 + 2N_c(0)t/t_s}. \quad (7)$$

For small times, the number of comets will be constant, while for long times we find

$$N_c(t) \simeq \frac{t_s}{2t} \quad \text{for } t \gg \frac{t_s}{2N_c(0)}. \quad (8)$$

The number of comets after a given time is therefore initially constant and later turns into a power law with slope -1 .

2.2. Dust Production

Little is known about the dust production in collisions between comets or asteroids, as the dust production rate will depend to some extent on the internal structure of the body. If the body has already been fractured many times in previous collisions and now basically is a “bag of sand,” a massive collision will release large amounts of dust. If, on the other hand, the body still has some internal strength, it can be expected to produce a distribution of fragment sizes. While the experimentally determined distributions probably give good results for the larger fragments, the production rate of dust grains is very uncertain. Therefore, we follow a different road and assume that dust production is the result of a collisional cascade in which increasingly small particles collide with each other to produce smaller and smaller fragments. It has been known for many years that the size distribution produced by such a process assumes the shape of a power law with the slope $f(m) \propto m^{-q}$ or (for spherical particles) $f(a) \propto a^{2-3q}$, where q can be shown to be equal to 11/6 for a self-similar collisional cascade, largely independent of material properties (Dohnanyi 1968; Williams & Wetherill 1994; Tanaka, Inaba, & Nakazawa 1996). Significant deviations from this law appear only when the gravity component of the impact strength is considered—but for bodies smaller than 1 km, the binding energy is dominating. The steady-state power law is based on the competition between production and removal processes. At the smallest sizes, this equilibrium will be disturbed by the noncollisional grain removal processes like radiation pressure (blowout) and Poynting-Robertson drag (Durda & Dermott 1997). Also, collisions with β meteorites may play a role (Artymowicz 1996; Krivova, Krivov, & Mann 2000). The disturbance in the power law can actually progress like a wave to larger particle sizes (Durda & Dermott 1997). For the simple estimates in this paper, we ignore these additional processes. We assume that a power-law distribution is produced by the collisions, and that the distribution continues with the same slope down to the particles that are removed from the system by another process. We write the size distribution as

$$f(a) = f_a a^\gamma, \quad (9)$$

where $\gamma = 2 - 3q$ is equal to -3.5 .

It is easy to show that the collisional timescales are slowest at the large-particle end of the distribution. Then, the collisions between large comets determine the injection of material in the cascade and, therefore, directly the dust production rate at the small particle end of the cascade. The steady-state dust production rate $\mathcal{R}_{\text{gain}}$ in such a model is proportional to the number of comet collisions per time, i.e.,

$$\mathcal{R}_{\text{gain}} = k_1 N_c^2, \quad (10)$$

where k_1 is a constant.

Since the dust production rate is directly fixed by the collisional cascade, the dust removal processes govern the amount of dust visible in a given source.

2.3. Dust Removal

Before we look into the details of dust removal, we can already provide the general time dependence of the dust content in a debris disk driven by a purely collisional

evolution. From equation (10) it is clear that the dust production rate is proportional to rate of collisions between comets, thus $\propto N_c^2$. Since the number of comets decreases as t^{-1} , the dust production rate is proportional to t^{-2} . However, we cannot conclude from this that the amount of dust present in steady state shows the same time dependence—the mechanism that is responsible for grain losses has to be considered as well. If grain losses are due to collisions among the visible grains themselves, the loss rates will generally be proportional to the number of visible grains squared. However, if some other process removes the visible grains, the loss rates will depend only linearly on the number of the grains. Thus, we have

$$\mathcal{R}_{\text{loss}} = k_2 n_{\text{vis}}^\alpha, \quad (11)$$

where α is either 1 or 2. In steady state we have

$$\mathcal{R}_{\text{gain}} = \mathcal{R}_{\text{loss}}, \quad (12)$$

and therefore

$$n_{\text{vis}} = \left(\frac{k_1}{k_2}\right)^{1/\alpha} N_c^{2/\alpha}. \quad (13)$$

Thus, for internal collisional processes dominating the grain loss, we can expect the number of visible grains to be proportional to the number of comets left in the system, corresponding to a t^{-1} dependence of the dust amount. If other processes dominate grain removal, a N_c^2 (corresponding to t^{-2}) dependence should be expected.

In the following we show under what circumstances which process dominates, and therefore which power law we should expect from the observations.

2.3.1. Poynting-Robertson Lifetime

Poynting-Robertson (PR) drag removes angular momentum from particles in orbit around a star and causes them to spiral inward. The PR drag reduces the size of a circular orbit according to (Burns, Lamy, & Soter 1979)

$$\dot{r} = -\frac{GM_*}{r} \frac{2\beta}{c} = -2v_K^2 \frac{\beta}{c}, \quad (14)$$

where c is the speed of light and β is the ratio of radiative and gravitational acceleration of a particle. The radiative force acting on the particle depends on the absorption and scattering cross sections (Burns et al. 1979). For the simple estimates in our study, we do not wish to discuss the detailed grain properties. Instead, we limit ourselves to large grains (compared to the wavelength of the radiation emitted by the star) and assume the grains to be perfect absorbers. In this limit, the absorption cross section of the particle is πa^2 , and scattering does not contribute to the radiation pressure (van der Hulst 1981). The ratio β is then inversely proportional to the particle size, so we can write (Backman & Paresce 1993)

$$\beta(a) = \frac{3L_*}{16\pi c GM_* a \rho_c} = \frac{1}{2} \frac{a_b}{a}, \quad (15)$$

where a_b is the blowout size, i.e., the size of particles that will be blown out of the system by radiation pressure. We define these to be the particles with $\beta = 1/2$. Particles with $\beta = 1/2$ will leave the system when ejected in a collision

between particles unaffected by radiation pressure (Burns et al. 1979).

The PR timescale of particles of radius a moving in orbits with a semimajor axis r is then given by

$$\tau_{\text{PR}}(a) = \frac{r}{\dot{r}} = \frac{r^2 c}{2GM_*} \frac{a}{a_b} \quad (16)$$

$$= 2 \times 10^6 \text{ yr} \left(\frac{r}{50 \text{ AU}} \right)^2 \frac{M_\odot}{M_*} \frac{a}{a_b}. \quad (17)$$

PR drag is frequently used in the literature to compute the dust loss in debris disks (e.g., Jura et al. 1998; Jourdain de Muizon et al. 1999; Li & Lunine 2003). It certainly provides an upper limit on removal timescales. However, as we show below, collisions will always dominate the removal process in the currently observable debris disks.

2.3.2. Collisional Lifetime

The collisional cross section between two particles with radii a_1 and a_2 is given by

$$\sigma(a_1, a_2) = \pi(a_1 + a_2)^2. \quad (18)$$

Of course, not all collisions will be destructive—only collisions with particle sizes above a certain threshold can destroy the bigger particle. We denote by ϵa_1 the size of the smallest particle that can (at a given collision velocity) still destroy a target particle with radius a_1 . The total destructive collision cross section provided by the particle size distribution in order to destroy particles with radius a_1 is then given by

$$\sigma_{\text{tot}}(a_1) = \int_{\epsilon a_1}^{a_c} f(a) \pi(a_1 + a)^2 da \quad (19)$$

$$= \int_{\epsilon a_1}^{a_c} f_a a^\gamma \pi(a_1 + a)^2 da. \quad (20)$$

In a power-law distribution with $\gamma < -3$, the cross section will be dominated by collisions with grain sizes ϵa_1 :

$$\sigma_{\text{tot}}(a) \simeq -f_a \frac{\pi \epsilon^{\gamma+1}}{\gamma+1} a^{\gamma+3} \equiv \epsilon_0 f_a a^{\gamma+3}, \quad (21)$$

where we have defined

$$\epsilon_0 = -\frac{\pi \epsilon^{\gamma+1}}{\gamma+1}. \quad (22)$$

The collisional lifetime due to collisions with other particles in the cascade is therefore given by

$$\tau_{\text{coll}}(a) = \frac{V}{v_{\text{coll}} \sigma_{\text{tot}}(a)} = \frac{V}{\nu v_K} \frac{1}{\epsilon_0 f_a a^{\gamma+3}}, \quad (23)$$

which for $\gamma = -3.5$ is proportional to $(a)^{1/2}$. However, the absolute value of the timescale depends on the normalization constant f_a of the equilibrium size distribution.

We still need to compute ϵ . Collisions are destructive if the kinetic energy of the collision is approximately equal to the binding energy of the two bodies. In general, the binding energy is composed of a component due to the material properties, and a gravitational component. However, for bodies of 1 km or less in diameter, the gravitational component can be ignored. For a collision to be destructive we

TABLE 1
THE ϵ_0 PARAMETER AS A FUNCTION OF COLLISION VELOCITY v_{coll}
AND BINDING ENERGY S

S (ergs g ⁻¹)	ϵ_0 FOR v_{coll} (km s ⁻¹)			
	0.01	0.1	1.0	10
10 ⁴	3.16 × 10 ¹	1.52 × 10 ³	7.05 × 10 ⁴	3.47 × 10 ⁶
10 ⁵	2.80 × 10 ⁰	2.22 × 10 ²	1.04 × 10 ⁴	4.81 × 10 ⁵
10 ⁶	3.16 × 10 ¹	1.52 × 10 ³	7.05 × 10 ⁴
10 ⁷	2.80 × 10 ⁰	2.22 × 10 ²	1.04 × 10 ⁴

therefore require

$$\frac{1}{2} \mu v_{\text{coll}}^2 = S(m_1 + m_2), \quad (24)$$

where μ is the reduced mass and S is the binding energy per mass. We then ask what the minimum mass m_2 is that can still destroy a body with mass m_1 in a collision with a given velocity. Using $m_2 = \epsilon^3 m_1$, we find

$$\epsilon^3 = \frac{v_{\text{coll}}^2}{4S} - \sqrt{\frac{v_{\text{coll}}^4}{16S^2} - \frac{v_{\text{coll}}^2}{2S}} - 1. \quad (25)$$

Note that ϵ contains the only important velocity dependence of our estimates. The higher the collision velocity, the larger the size interval of destructive impactors, and the larger the destruction rates of a given particle size. We will use a binding energy of $S = 2 \times 10^6$ ergs s⁻¹, a value typical for the icy bodies in the outer solar system (Kenyon & Luu 1999a). Table 1 lists the value of ϵ_0 as a function of S and v_{coll} .

2.3.3. Sublimation

Sublimation of ice can be a removal process for small grains in the inner regions of the disk. Ice sublimation is an extremely strong function of the grain temperature. In a range between 90 and 120 K, the sublimation time τ_{subl} can be approximated by the following expression (Backman & Paresce 1993)

$$\tau_{\text{subl}} \approx 10^6 \frac{a}{1 \mu\text{m}} \left(\frac{T_g}{100 \text{ K}} \right)^{-55} \text{ yr}, \quad (26)$$

where T_g is the grain temperature. Vega-like disks show emission that is usually dominated by 60 μm excess; only very few stars show detectable access at 25 μm (e.g., Laureijs et al. 2002). The disk emission usually peaks around 60 μm or at even longer wavelengths (e.g., Zuckerman & Becklin 1993). Using Wien's law, this corresponds to dust temperatures below 100 K. The sublimation times for such temperatures are generally comparable to or longer than PR drag timescales, even for the A stars among Vega-like stars (e.g., β Pic, α Lyr, and α PsA, see Backman & Paresce 1993). In stars of later spectral type, grains at similar distance from the star will be colder. While grain sublimation may play a role in setting the inner boundaries of debris disks, it does not dominate the removal processes. We therefore ignore grain sublimation in the following.

2.4. Steady-State Size Distribution

In steady state in a collisional cascade, f_a can be determined by equating the mass flux through the collisional cascade with influx of new material at the top end. The mass

flux through the chain will, in general, depend on the details of the collision physics, but for the purpose of this study, a simple estimate is sufficient. The collisional mass loss of particles with sizes in a scale-free size interval between a and $2a$ is given by

$$\dot{M}(a) = \frac{f(a)a(4\pi/3)\rho_c a^3}{\tau_{\text{coll}}(a)} = f_a^2 a^{2\gamma+7} \frac{(4\pi/3)v_{\text{coll}}\rho_c \epsilon_0}{V}. \quad (27)$$

For the equilibrium slope $\gamma = -3.5$, this value is independent of grain size. This mass loss is an excellent estimate for the mass flux through the cascade if in a typical collision the largest fragment has about half the diameter of the impactors. Equating $\dot{M}(a)$ with the inflow of mass into the collisional cascade (eq. [6] multiplied by the m_c), we find

$$f_a = N_c \sqrt{\frac{2m_c \sigma_{\text{coll}}}{(4\pi/3)\rho_c \epsilon_0}} = N_c a_c^{5/2} \sqrt{\frac{8\pi}{\epsilon_0}}. \quad (28)$$

2.5. Dust Visible in Steady State

The dust visible in the steady-state solution of a collisional cascade consists of two components. One component is small dust grains that are being blown away by stellar radiation (the “wind grains”). Each of these grains contributes to the radiation (scattering and IR/submillimeter emission) only for the duration of about 1 Kepler time, typically a few hundred years. The second component consists of grains that are too large to be blown away, but are already small enough to provide significant surface for interaction with photons (the “orbiting grains”). Such grains contribute to the radiation as long as they live in the given orbit, i.e., for a collision time or a Poynting-Robertson drag time, whatever is shorter. The relative importance of both components can be easily estimated. The critical size for blowout of grains is a_b . Of the grains smaller than this size, there will be a size a_{rad} , which will dominate the radiation interaction of the grains below the blowout limit. We assume that the destruction of larger particles produces the two different sizes according to the steady-state power law; i.e., the small grains are produced more frequently by factor $(a_b/a_{\text{rad}})^{-\gamma}$. The production of grain surface (which is most important for the visibility of dust) proceeds then with a ratio $(a_b/a_{\text{rad}})^{-\gamma-2}$. Therefore, the orbiting grains will dominate if their lifetime τ meets the condition

$$\tau \geq (a_b/a_{\text{rad}})^{-\gamma-2} t_K. \quad (29)$$

For a debris disk around a main-sequence star, the blowout size is typically 1–10 μm (depending upon spectral type; Artymowicz 1988), while a_{rad} should be of the order of 1 μm . Therefore, as long as the lifetime of the smallest orbiting grains is at least 10–100 orbits, orbiting grains will always dominate the visibility of debris disks. In the following discussion, we therefore focus on the steady-state abundances of grains with sizes just above the blowout size. We call this size of the visible dust grains a_{vis} .

2.6. Steady-State Abundance of Grains

The number of grains visible in the disk at a given time will be determined by the dominating grain-loss mechanism.

2.6.1. Collisionally Dominated Grains

If the visible dust grains are still being removed by collisions and have their steady-state abundance, we find

$$n_{\text{coll}} = f(a_{\text{vis}}) da = f_a a^\gamma da \approx N_c a_c^{5/2} \sqrt{\frac{8\pi}{\epsilon_0}} a_{\text{vis}}^{\gamma+1}, \quad (30)$$

where we have again assumed $da \approx a$.

2.6.2. PR Drag-dominated Grains

If the visible grains are removed by PR drag, this means that the collisional cascade is effectively terminated at the size a_{vis} . Then, the number of grains can be estimated by equating the mass flux through the collisional cascade with PR driven losses. Therefore, we have

$$n_{\text{PR}} = \frac{m_c}{m_{\text{vis}}} \dot{N}_c \tau_{\text{PR}}(a_{\text{vis}}) = N_c^2 \frac{3a_c^5 c}{a_{\text{vis}}^3 r^2 v_K}. \quad (31)$$

2.6.3. General Case

The number of grains visible in the disk is then given by

$$n_{\text{vis}} = \min(n_{\text{coll}}, n_{\text{PR}}), \quad (32)$$

and the covering fraction f_d of the disk is

$$f_d = n_{\text{vis}} Q_{\text{abs}}(a_{\text{vis}}) \pi a_{\text{vis}}^2, \quad (33)$$

where Q_{abs} is the usual absorption efficiency of the dust particles. At high disk masses, the collisional destruction always dominates the life time of grains, while at low disk masses, PR drag can become important. The disk mass at which the switch in processes takes place is given by the equation $n_{\text{coll}} = n_{\text{PR}}$. We find

$$M_{\text{coll} \rightarrow \text{PR}} = \frac{4\pi}{9} \sqrt{\frac{8\pi}{\epsilon_0}} \frac{\sqrt{a_c a_{\text{vis}} G M_* r^3 \rho_c}}{c}. \quad (34)$$

2.7. Dependencies and Typical Numbers

The equations derived above show many properties of a collisional cascade and the amount of radiation reprocessing by dust that can be expected from such a cascade. If we insert typical numbers into the above equations, we find the following results:

$$\begin{aligned} \tau_{\text{comet}} &= 7.4 \times 10^7 \text{ yr} \left(\frac{\hat{r}}{50 \text{ AU}} \right)^{3.5} \frac{a_c}{1 \text{ km}} \frac{\rho_c}{1 \text{ g cm}^{-3}} \\ &\times \left(\frac{M_\odot}{M_*} \right)^{0.5} \frac{10 M_\oplus}{M_{\text{disk}}}, \end{aligned} \quad (35)$$

$$\begin{aligned} &= 4.9 \times 10^7 \text{ yr} \left(\frac{\hat{r}}{50 \text{ AU}} \right)^{1.5} \\ &\times \left(\frac{1 \mu\text{m}}{a_{\text{vis}}} \frac{a_c}{1 \text{ km}} \frac{226 M_\odot}{\epsilon_0 M_*} \right)^{0.5} \frac{Q}{1} \frac{10^{-3}}{f_d}, \end{aligned} \quad (36)$$

$$\begin{aligned} \tau_{\text{coll}} &= 3.9 \times 10^2 \text{ yr} \left(\frac{\hat{r}}{50 \text{ AU}} \right)^{3.5} \\ &\times \left(\frac{a_c}{1 \text{ km}} \frac{1 M_\odot}{M_*} \frac{a_{\text{vis}}}{1 \mu\text{m}} \frac{226}{\epsilon_0} \right)^{0.5} \frac{10 M_\oplus}{M_{\text{disk}}}, \end{aligned} \quad (37)$$

$$\tau_{\text{PR}} = 2.0 \times 10^6 \text{ yr} \left(\frac{\hat{r}}{50 \text{ AU}} \right)^2 \frac{M_{\odot} a}{M_* a_b}, \quad (38)$$

$$M_{\text{coll} \rightarrow \text{PR}} = 1.7 \times 10^{-3} M_{\oplus} \left(\frac{\hat{r}}{50 \text{ AU}} \right)^{1.5} \times \left(\frac{226}{\epsilon_0} \frac{a_c}{1 \text{ km}} \frac{a_{\text{vis}}}{1 \mu\text{m}} \frac{M_*}{M_{\odot}} \right)^{0.5} \left(\frac{\rho_c}{1 \text{ g cm}^{-3}} \right), \quad (39)$$

$$f_{d, \text{coll}} = 2.3 \times 10^{-3} \left(\frac{50 \text{ AU}}{\hat{r}} \right)^2 \left(\frac{1 \mu\text{m}}{a_{\text{vis}}} \frac{1 \text{ km}}{a_c} \frac{226}{\epsilon_0} \right)^{0.5} \times \frac{M_{\text{disk}}}{10 M_{\oplus}} \frac{1 \text{ g cm}^{-3}}{\rho_c} \frac{Q_{\text{abs}}}{1}, \quad (40)$$

where we have used a value of 226 for the ϵ_0 parameter, which is appropriate for our standard model ($S = 2 \times 10^6$ ergs g^{-1} , $v_{\text{coll}} = 0.1 v_K \simeq 0.45 \text{ km s}^{-1}$).

First of all, we see that the dust removal timescales through collisions are much shorter than the removal timescales through PR drag. This means that typical debris disks will be collisionally dominated down to the smallest dust sizes, which will then be blown out by radiation pressure. Equation (39) shows that the disk mass (i.e., the mass of all comets in the collisional system) must be as low as $1.7 \times 10^{-3} M_{\oplus}$ for the transition to occur. From equation (40) we can see that the f_d -value at that stage will typically be 4×10^{-7} —well below the detection limits of *ISO* and *IRAS*.

Figure 1 shows the dependence of the dust visible in the system on the different parameters of the model. Figure 1a shows the data taken from Paper I. The solid line in the other five panels of the figure represents the time dependence of a standard model with $M_{\text{disk}} = 10 M_{\oplus}$, $\hat{r} = 43 \text{ AU}$, $\nu = 0.1$, $a_c = 1 \text{ km}$ around an A0 star. The other lines indicate the changes due to the variation of a single parameter. From Figure 1b, it can be seen that only the more massive disks ($>10 M_{\oplus}$) reach collisional steady state within $\tau_{\text{comet}} \simeq 10^8 \text{ yr}$. This is the time when most comets have seen at least one collision, and when the slope in the f_d -time relation turns from zero to -1 . Models with disk masses below $10 M_{\oplus}$ result in a constant dust production for 10^8 yr or more, after which the curve turns into the common power law with slope -1 .

The speed at which collisional equilibrium is achieved is also influenced by the size a_c of the parent bodies (Fig. 1e). If, for a given disk mass we reduce the size of the parent bodies, the number of such particles will increase and the collision time will become smaller. Collisional equilibrium is established after one or a few collision times. We can see this effect clearly in the curves. For the standard size of 1 km, collisional equilibrium is reached after about 10^8 yr . For 10 km bodies this takes 10^9 yr , and for 100 m bodies, collisional equilibrium is already fully established after 10^7 yr . Another very important parameter is the collision velocity $\nu = v_{\text{coll}}/v_K$ (Fig. 1c). Changing the collision velocity changes the amount of dust seen in the system significantly, in a counterintuitive way: increasing the collision

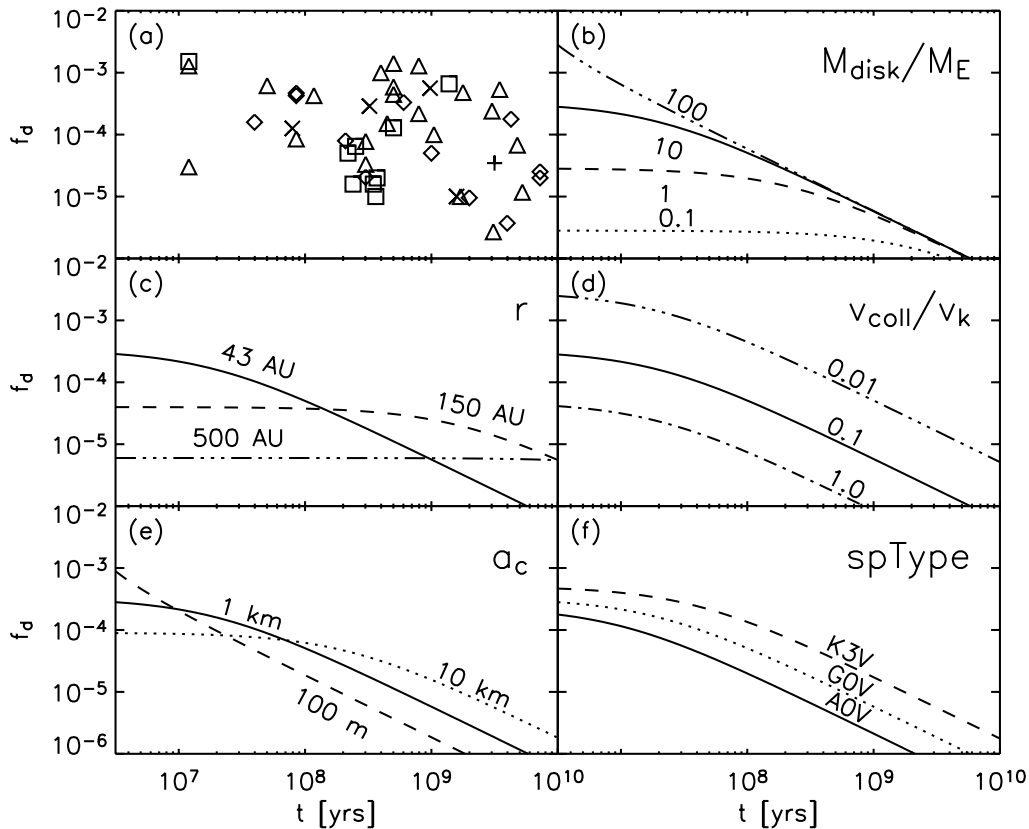


FIG. 1.—Dependence of the fractional luminosity of dust produced by a cloud of comets as a function of time. (a) The observations (Paper I). Explanation of the symbols: *squares*, A main-sequence stars; *triangles*, F main-sequence stars; *diamonds*, G dwarfs; *crosses*, K dwarfs; and *plus sign*, K giant. (b)–(f) The dependence on different parameters, starting from a standard model (solid line in all panels). See text.

velocity *decreases* the amount of dust seen from a steady-state cascade. This behavior results from the fact that the *grain removal* processes also strongly depend on the collisional velocities. Normally one would also expect that increasing the velocity would lead to faster depletion of the cometary cloud. However, this is not the case, because the stirring of the orbits also increases the volume in which the particles move (see eq. [2])—the collision times of comets are therefore not influenced by the relative velocity. The distance of the cometary cloud (Fig. 1c) to the star also influences the observed f_d -values, in two ways. First, at a larger distance, the same amount of dust covers a smaller fraction of the solid angle seen from the star. The effect decreases the f_d -value proportionally to r^{-2} . Also, moving to larger distances increases all timescales. We can see in the figure that at larger distances, it takes much longer for collisional equilibrium to be established. The time of almost constant f_d is extended to 10^9 yr if we move the dust production site from 43 to 150 AU. Finally, we can look at the dependence on the spectral type of the star (Fig. 1f). Moving from an A0 star to later types slightly increases the amount of dust seen. The most important effect is that the size that dominates the visible dust will be smaller for low-luminosity stars, since the blowout limit moves to smaller grain sizes. An additional small effect is the lower mass of late-type stars, which will reduce collision timescales. An additional parameter that influences the outcome of the calculations is the binding energy S , which is dependent on the internal composition and structure of the comets. The binding energy determines the value of ϵ_0 through equations (22) and (25). This parameter influences the speed at which material is processed through the collisional cascade, and therefore the amount of visible dust. The calculations shown in this paper all use $S = 2 \times 10^6$ ergs g^{-1} , a value appropriate for icy comet-like bodies (Kenyon & Luu 1999a, and references therein), and we assume this value to hold for the entire size range in the collisional cascade. Using $S = 10^7$ ergs g^{-1} , a value more appropriate for asteroid-type bodies, Table 1 shows that the value of ϵ_0 would decrease by approximately a factor of 10. Equations (30), (36), (37), (39), and (40) all show a $\epsilon_0^{-1/2}$ dependence, so the corresponding timescales and masses as well as the amount of visible dust would increase by a factor of about 3. Similarly, if the material the comets are made of were exceptionally weak (Kenyon & Luu 1999a), the numerical values would decrease accordingly.

An important result is the *nowhere* in the parameter space covered by Figure 1 is a slope of t^{-2} observed. This means that in all models shown here, the collisional removal of dust grains dominates over the PR drag. This was to be expected from equation (39), where we showed that the transition from collisionally dominated dust removal to PR drag-dominated removal processes happens only at rather low disk masses, which are unobservable with current instrumentation.

2.8. Summary of the Collisional Model

1. Timescales do not depend critically on the collision velocities, if only the velocities are energetic enough to be destructive.

2. A power-law dependence of the amount of observed dust as a function of time can only be expected after about one collisional time for the bodies starting the cascade.

Before that time, an undisturbed cascade produces an approximately constant amount of dust.

3. For 1 km-sized comets, the collision times are of order of 10^9 yr for disk masses (comets) of $1 M_{\oplus}$. To reach power-law behavior within 10^8 yr and below, high disk masses ($10 M_{\oplus}$) are required. Alternatively, the collisional cascade could be started by 100 m bodies, provided that sufficient stirring can be achieved for these bodies.

4. An undisturbed collisional cascade predicts $f_d = \text{const}$ for $t < \tau_{\text{comet}}$ and $f_d \propto t^{-1}$ at later times.

5. Disks in which PR drag dominates dust removal would show $f_d \propto t^{-2}$ behavior. However, the transition from collisionally dominated to PR drag dominated disks happens at disk masses of typically $10^{-3} M_{\oplus}$, much less than required to support the observed debris disks.

6. Adding more mass to the disk does *not* extend the lifetime of the debris disk. For given collisional velocities, disks of different masses all converge (after one collision time τ_{comet}) to the same curve. For more massive disks, this happens very quickly. The reason for this behavior is that in massive disks, the removal timescales are much shorter. In fact, in a more complete model that also treats planet formation, massive disks would form planets quickly and remove comets by gravitational scattering, further shortening the lifetime of the debris disk.

7. The lower blowout sizes of late-type stars help somewhat to increase the amount of dust seen around old stars, but not enough to explain the old Vega-like stars with significant debris disks.

3. DISCUSSION

In this section we relate the findings of the simple model to the results of Paper I, namely, the main features of the distribution of stars in the $\log t$ - $\log f_d$ diagram.

3.1. The Initial Decrease for Young Stars

Looking at only the youngest stars in the sample of Paper I, and mainly at the A stars, there may indeed be an initial decrease in the amount of dust present. Because of the absence of very young stars with low f_d -values, it seems clear that the stars at the edge of the empty region in the lower left corner of the diagram must have evolved from larger f_d -values at younger ages. In this case, the lower boundary of the data points would be marking out the fastest path to decreasing the amount of dust in the system. Spangler et al. (2001) fitted a power law with a slope -1.7 to their sample, but did include T Tauri stars that are probably not gas-free debris disks. For the reduced sample, the slope of the lower boundary is about -1.3 , much closer to the slope of -1 that we have derived for collisional cascades. The data clearly are currently not good enough to make a strong statement about the correctness of this slope. Hopefully, *SIRTF* will provide a much more solid database for this study. We would like to make a remark here. Suppose the slope really is steeper than -1 : what would that mean? A pure collisional cascade will not produce this. However, we can see from Figure 1 that an increase in collision velocity is connected with a decrease in the f_d -value. Therefore, a slope steeper than -1 can be produced by a collisional cascade that is *continuously stirred*. We have simulated this in a very simple way, by increasing the collision velocity linearly in time starting from an initial $\nu = 0.01$ and ending at $\nu = 1.0$

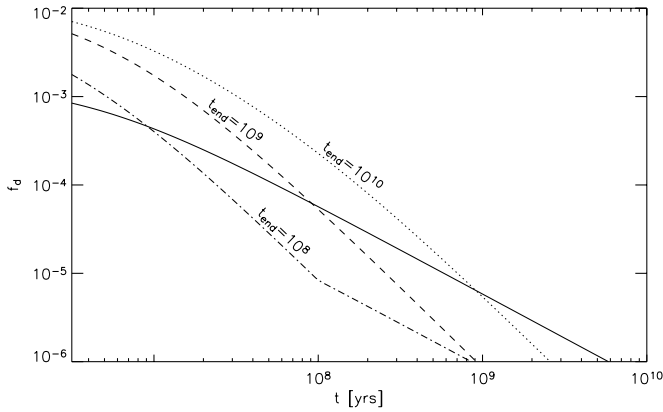


FIG. 2.—Fractional luminosity f_d –time t relation with continuous stirring. *Solid line*: The standard model without stirring. The other lines include stirring, with ν starting at 0.01 and increasing linearly with time to a maximum value of 1.0 after 10^8 , 10^9 , and 10^{10} yr.

after a given stirring time. The result of this experiment can be seen in Figure 2. Compared to the curve with a fixed $\nu = 0.1$, the stirring curves all show a steeper slope.

3.2. The Upper Limit

Vega-like stars never seem to have f_d -values larger than 10^{-3} . This upper limit is independent of age; i.e., at all ages do we find stars with $f_d \sim 10^{-3}$, but not higher. This upper limit can be understood from the cascade dynamics. Equation (36) expresses the collision timescale for comets, just like equation (35), but with the dependence on disk mass replaced by f_d . From this equation we can see that the timescale for survival of the cometary cloud drops below 10^8 yr when f_d exceeds 10^{-3} . An even stronger limit comes from work by Artymowicz (1996). He showed that in a gas-free debris disks, the amount of dust is limited by the creation of “dust avalanches.” When f_d reaches values of 10^{-2} , the breakup of particles in the inner regions of the disk creates β meteorites, which are small particles driven out by radiation pressure. For low f_d -values, such particles will not suffer additional collisions on the way out, because the disk is radially optically thin. However, at f_d -values between 10^{-3} and 10^{-2} , the probability that such β meteorites will cause additional collisions on their way out of the system becomes so high that a self-accelerating avalanche effect is created. On basically a Kepler time, the disk will be cleaned of small visible dust grains. Thus, even if the collisional equilibrium discussed in the present paper does allow larger f_d -values for a limited period of time, the avalanche mechanism effectively limits f_d to about 10^{-3} . Stars with higher f_d -values must contain significant amounts of gas. The physics in such disks is different from “normal” Vega-like systems, and studying the age dependence must therefore make a clear distinction between the two types of disks.

3.3. Independence of the Upper Limit of the Age

Models of collisional cascades indicate that the dust content of a system should decrease with time. This seems to be inconsistent with the fact that the observed debris disks can have $f_d \approx 10^{-3}$ at all ages. Assuming that the debris state starts at the same time for all stars, a systematic decrease of dust abundance should result in an absence of large f_d -values in old stars. This is not what is observed. We have

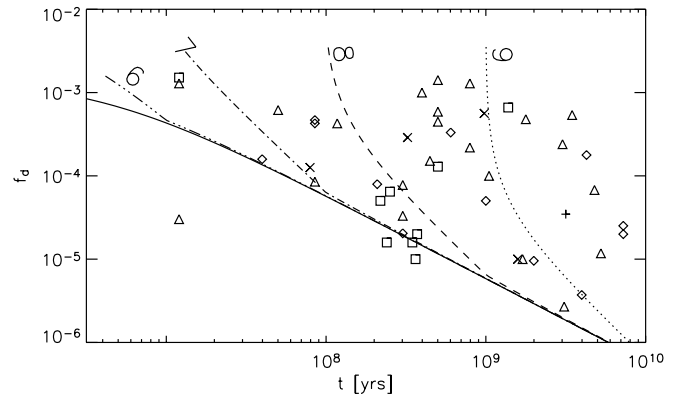


FIG. 3.—Fractional luminosity f_d –time t relation for different starting times of stirring. *Solid line*: The standard model without stirring. The other lines show models with different starting times for stirring. The starting time ($\log t_0$) is noted at the curve. The stirring increases the velocity linearly between t_0 and $10t_0$, where t_0 is 10^7 , 10^8 , and 10^9 yr.

seen above that a more or less constant amount of dust can be supported before the disk goes into collisional equilibrium, i.e., if the parent bodies are large, if the distance from the star is large, or if the initial disk mass present in large planetesimals is small. However, Figure 1 also shows that the types of solutions with constant dust for a long time always are at levels significantly lower than $f_d = 10^{-3}$. Therefore, a long duration of the debris state cannot be the explanation for old Vega-like stars. A more likely scenario is that different planetesimal disks are starting to become active debris disks at different times. We can study this in a toy model, in which we turn on the stirring of the planetesimal disk only after a given waiting period. Figure 3 shows the result of this calculation. In this way, indeed, solutions can be reached that cover the observed points. To find the reasons for the stirring, one needs to run much more complete models that self-consistently include the formation of planets. Such models have recently been introduced by Kenyon & Bromley (2001, 2002b). They run a code that computes the time evolution of the size distribution in an annulus of a disk around a star. Processes included are growth through coagulation, collisional destruction, dynamical stirring, and damping. It is found that stirring by bodies of 500 km or larger that form in the disk can lead to the onset of a collisional cascade, which will produce debris dust (Kenyon & Bromley 2001). Furthermore, since the timescales for the growth of larger bodies are larger in the outer disk regions, planet formation, and therefore debris production, proceeds from the inner to the outer disk in rings and can lead to multiple collisional cascades at different times around the same star (Kenyon & Bromley 2002b). At least qualitatively, the *ISO* observations and the estimates put forward in the present paper are consistent with this picture. The question remains whether this mechanism can also explain very old Vega-like stars at ages of several Gyr.

3.4. The Absence of Young Stars with Low f_d -Values

The observations currently show no stars in this region. We have discussed in Paper I that the samples are currently too small to securely exclude the presence of stars in this region. However, let us for the moment take the relative absence of such stars as significant. Then this indicates than

normal debris disk do not populate this region. Looking at Figure 1, there are three main ways to populate the area of young stars with low f_d -values.

1. *Low initial disk masses below $10 M_{\oplus}$.*— Our collisional model produces low f_d -values for low initial disk masses. However, the collisional velocity in the model is fixed. Considering a self-stirring scenario as proposed by Kenyon & Bromley (2002b), it is clear that enough mass must be present initially to allow for the formation of at least Pluto-like bodies. Kenyon & Bromley (2002b) use a model with $100 M_{\oplus}$ of solid material to reach the required velocities within 100 Myr. Stern & Colwell (1997a) find that at least $30 M_{\oplus}$ are needed in the early Kuiper Belt in order to grow Pluto. Both calculations indicate that the timescale for planet formation and therefore for stirring is approximately inversely proportional to the disk mass. It is therefore likely that disks with less than about $10 M_{\oplus}$ of solid material in the Kuiper Belt region will not be able to start a collisional cascade in the first 100 Myr because no large bodies will be formed in the belt.

2. *High collision velocities very early on.*— Low f_d -values can also be produced by very high collision velocities in the disk, already very early, after about 10^7 yr. However, such high velocities cannot be due to slow stirring by Pluto-sized objects forming in the disk. Large velocities are possible by embedding a giant planet in the disk, but in this case all planetesimals may have been removed by close encounters with the planet within the first 10 Myr.

3. *Collisional cascade at large distance from star.*— A collisional cascade at perhaps 150 AU or farther out seems to be capable of producing low f_d -values at young ages. However, in this case the question must be raised: if there is enough mass in the disk at such large radii to produce stirring and a collisional cascade, why is there not also material closer to the star, which would form planets faster, start the collisional cascade earlier, and produce dust more efficiently? If that additional material were present, the star would become a “normal” debris disk with $f_d \approx 10^{-3}$. The collisional cascade at large distances will then only dominate the IR output of the system after the initial debris ring closer to the star loses its capacity to produce dust because most planetesimals in this regions have been removed.

Therefore, we speculate that the absence of debris disks with intermediate or low f_d -values around *young* stars is *due to the fact that stirring is only possible in higher mass disks*. Low-mass disks, if they exist, will remain quiet for the entire life of the star.

4. CONCLUSIONS

We have studied a simple model for collisional cascades in debris disks. While such a model does not lead to a self-consistent description of the stirring and dust production in a debris disks, it allows us to study the time behavior of such systems. Comparing with observations of debris disks, we come to the following conclusions.

1. A collisional cascade with constant collision velocities leads to a power-law decrease of the amount of dust seen in a debris disk. The slope of that power law is -1 for the parameter space valid for all observed debris disks. Only at much lower masses (typically $10^{-3} M_{\oplus}$) will this slope turn over to -2 .

2. A collisional cascade that is continuously stirred (i.e., where the collision velocities are increasing with time) can produce slopes steeper than -1 .

3. If the initial decrease of f_d for young stars is confirmed by further observations, it may be due to a combination of stirring and collisional cascade, as described by Kenyon & Bromley (2002b).

4. The observed upper limit of $f_d \approx 10^{-3}$ for debris disks has to do with the dynamics of dust production in a collisional cascade and is due to the avalanche effect described by Artymowicz (1996). According to that paper, larger f_d -values require gas to dominate the dust dynamics, but, following the definition by Lagrange et al. (2000), such disks are excluded from the class of debris disks.

5. The most likely explanation for the presence of debris disk with f_d -values up to 10^{-3} at ages well above a Gyr is the *delayed onset of collisional cascades* by late planet formation further away from the star. A prediction from this result is that the debris disks around older stars should be (on average) farther away from the star than young debris disks.

6. The tentatively observed absence of young debris disks with f_d significantly lower than 10^{-3} may be real and caused by the effect of stirring. Low-mass disks that could produce lower f_d -values cannot produce the planets needed to stir the disk quickly enough (Kenyon & Bromley 2002b).

7. The trends observed in debris disks so far need confirmation by a much larger sample, which we hope will be available after the launch of *SIRTF*.

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