Bose-Einstein condensation into non-equilibrium states

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Chapter 3
Experimental setup

3.1 Introduction

Experiments on Bose-Einstein condensates involve a wide range of techniques and methods. They include laser cooling, magnetic trapping, radio-frequency evaporative cooling, optical detection, vacuum technology and many others. Making no attempt to be a complete reference on the subject, this chapter introduces those aspects of the experimental setup, which are relevant to this thesis. A more detailed description of various aspects of the setup can be found in [33].

3.2 Overview

From the very first steps the design of our experimental setup was optimised for creation of samples with large atom numbers and experiments with high-density clouds. In Figure 3.1 we show a schematic view of the experimental setup. Two main parts can be distinguished here: the two-dimensional magneto-optical trap acting as an atomic beam source [32] in the bottom of the apparatus and the upper part, which combines the recapture MOT, the magnetic trap and the imaging system.

The path towards Bose-Einstein condensation begins with loading a magneto-optical trap (MOT) from an intense cold atom source [32]. The atom source operates at flux numbers of $5 \times 10^9$ atoms/s with an average velocity of atoms of 8 m/s. The number of particles stored in a density-limited MOT is typically $1.2 \times 10^{10}$. To maximise the number of atoms captured in the MOT, a dedicated high-power laser was designed and built (see Chapter 4). Atoms in the MOT are further cooled during an optical molasses stage to a temperature of 40 $\mu$K. After optical pumping into $|F = 2, m_F = 2 \rangle$ state (with approximately 60% loss) the atoms are transferred into a weak roughly isotropic magnetic trap with the frequencies $\omega = 2\pi \times 7.5$ Hz. This frequency allows to match a 3 mm $1/e$ radius of the MOT cloud to the size of the cloud in the magnetic trap. To achieve a higher elastic collision rate and meet the condition for the runaway regime of evaporative cooling the trap is adiabatically compressed within 6.615 s. At the end of compression stage the temperature rises to 760 $\mu$K and the density increases to $7 \times 10^{11}$ cm$^{-3}$. After this the evaporation barrier is ramped down within 10.6 s to reach the condensation point. The critical temperature,
Figure 3.1. Schematic outline of the experimental setup. 2D MOT in the lower part of the setup acting as a cold atomic beam source. The upper part of the setup is taken by the recapture MOT, magnetic trap and the imaging optics.
$T_C = 1.5 \mu K$ and the number of particles at the transition point is $\sim 10^7$.

Both the atomic beam source and the recapture MOT are placed inside a cube created by six square coils (0.94 m side) which serve to compensate the magnetic field of the Earth. An additional function for one pair of these coils aligned with the axis of magnetic trap is the active control of the trap bottom.

In the course of experiments with BECs one has to control more than 300 events over the course of one minute. The precision and relative timing of some of these events can be as short as 1 $\mu$s, which places high demands on the control system. The control of the experiment is performed with a real-time automation system developed in-house and based on LabVIEW programming environment and the hardware from National Instruments and Viewpoint. Processing of the obtained data can be done in parallel with the experimental runs.

### 3.3 Vacuum system

Both trapping atoms in a MOT and their storage in a magnetic trap place stringent requirements on a vacuum system, as background gas collisions is a dominant factor in the life time of a magnetic trap. The schematic layout of the vacuum setup is presented in Figure 3.2. The setup consists of two ultra-high vacuum (UHV) chambers connected through a differential pumping hole. The atomic beam source is produced in the lower chamber which contains saturated rubidium vapour at room temperature, approximately $4 \times 10^{-7}$ mbar [89]. The beam of cold atoms is guided through the hole in the aluminium mirror to the upper chamber evacuated to a pressure below $3 \times 10^{-11}$ mbar.

The chambers are essentially rectangular quartz cells fused to thick circular quartz disks. The wall thickness of the cells is 4 mm. In the cross-section the cells present a square of $30 \times 30$ mm. They are coupled to the opposite sides of a stainless-steel vacuum manifold with pairs of concentric viton O-rings. The space between the rings is pumped out to a few times $10^{-3}$ mbar to avoid limitations due to the permeability of viton. The outside surface of the cells is covered with an anti-reflection coating of better than 0.2% at normal incidence at 780 nm. This minimises multiple reflections of the laser beams propagating through the cells. The pumping of the upper chamber is done by a 40 l/s ion pump and a titanium sublimation pump. The vapour cell of the atomic beam source is connected to a 2 l/s ion pump. The ion pumps are placed sufficiently far from the trap to avoid the influence of their magnetic fields on the captured atoms. The pressure is controlled by the ionisation gauge (Varian, model UHV-24 nude). Further, the life time (typically 65 s) of the sample in the magnetic trap gives the ultimate indicator of the vacuum quality.
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**Figure 3.2.** Vacuum system. The lower part of the differentially pumped chamber accommodates the atomic source, while around the upper UHV cell the magneto-optical and magnetic traps are built.
Two mirrors with protected gold coating are installed inside the UHV manifold. They permit introduction of the laser beams at an angle close to the vertical axis of the trap, e.g. MOT beams, optical pumping etc.

3.4 Laser system

Diode lasers are a common source of light used in spectroscopic and laser cooling applications at 780 nm. The essential requirements presented to a laser in such experiments are narrow linewidth (<1 MHz), ability to tune the frequency, long- and short-term frequency stabilisation, and sufficient optical power. Diode lasers can have all these properties in addition to a low price and the ease of operation.

Figure 3.3 presents a block diagram of the laser system. The basis of the system is formed by a grating-stabilised diode laser (GSL1) (Toptica, DL100) [85, 106]. This laser is based on a single-mode laser diode (Hitachi, model HL7851G, 50 mW). Frequency stabilisation of the laser is realised with Doppler-free saturation spectroscopy together with the Dichroic-Atomic-Vapour Laser Lock [26]. The frequency is locked to a crossover signal between two hyperfine transition of D2 line: $|5S_{1/2}, F = 2\rangle \rightarrow |5P_{3/2}, F = 3\rangle$ and $|5S_{1/2}, F = 2\rangle \rightarrow |5P_{3/2}, F = 1\rangle$. Light produced by GSL1 is split into four beams, each being frequency-shifted by an acousto-optic modulator (AOM). The first beam is used as a seed for a broad-area diode laser (BAL) system (see Chapter 4), which provides optical power for the MOT as well as for the near-resonance absorption detection of the atomic cloud. The second beam is used for injection locking of a single-mode 80-mW diode laser (Sanyo, model DL-7140-001), which serving the atomic beam source. Another beam is applied for optical pumping of the atoms into a $|5S_{1/2}, F = 2, m_F = 2\rangle$ state. The last beam is used for the fluorescence detection of the atomic beam source and as an optical plug for the source.

A dedicated repumping laser (GSL2, identical to GSL1) is used in the setup to prevent atoms from accumulating in the $|5S_{1/2}, F = 1\rangle$ state. This laser is tuned to the $|5S_{1/2}, F = 2\rangle \rightarrow |5P_{3/2}, F = 1\rangle$ line and is used in the MOT, atomic beam source, probing of the atomic beam and optical pumping of the atoms. For frequency stabilisation the repumping laser employs Doppler-free saturation spectroscopy with a frequency modulation technique [13, 34]. The summary of the frequencies and the powers of all laser beams is presented in Table 3.1.

An additional diode laser not shown in Figure 3.3 was built for the use in detection setup. The main distinction of this laser was tunability and frequency stabilisation over a range of $-3.5 \text{ GHz}$ to $+3.5 \text{ GHz}$ with respect to $F = 2 \rightarrow F = 3$ transition. This was achieved by locking to the mixed-down beat signal between this laser and the master
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**Figure 3.3.** Block diagram of the laser system. Precise control of the frequencies of various laser beams is done with AOM's. Optical transport and spatial filtering of the beams is done in most cases with single-mode optical fibres.

(GSL1) No data presented in this work were obtained with this laser, and no further description is given. For information on a similar system one can refer to ref. [91].

### 3.5 Magnetic trap

A schematic view of the Ioffe-quadrupole magnetic trap used in our experiments is presented in Figure 2.2 (see also Figure 3.1). The functions of all coils were already briefly described in Section 2.2. Four parallel current bars made of four elongated "racetrack" coils produce quadrupole magnetic field in x-y plane. Small circular dipole coils in the centre ("pinch" coils) with currents running in the same direction create confining potential along z-axis. The same coils are used for the MOT, when the currents are sent in the opposite directions. Large outer compensation coils are used for reduction of the large offset field.
3.5 Magnetic trap

<table>
<thead>
<tr>
<th>Name of the beam</th>
<th>Transition</th>
<th>Detuning (MHz)</th>
<th>Power (mW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MOT</td>
<td>$F = 2 \rightarrow F = 3$</td>
<td>-33</td>
<td>120</td>
</tr>
<tr>
<td>Absorption imaging</td>
<td>$F = 2 \rightarrow F = 3$</td>
<td>-70 to +15</td>
<td>0 to 120</td>
</tr>
<tr>
<td>Atomic beam source</td>
<td>$F = 2 \rightarrow F = 3$</td>
<td>-12</td>
<td>47</td>
</tr>
<tr>
<td>Optical pumping</td>
<td>$F = 2 \rightarrow F = 2$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Source probe</td>
<td>$F = 2 \rightarrow F = 3$</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>Source plug</td>
<td>$F = 2 \rightarrow F = 3$</td>
<td>0</td>
<td>0.9</td>
</tr>
<tr>
<td>Source repumper</td>
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<td>0</td>
<td>0.4</td>
</tr>
<tr>
<td>MOT repumper</td>
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<td>0.1</td>
</tr>
<tr>
<td>OP repumper</td>
<td>$F = 1 \rightarrow F = 2$</td>
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</tr>
<tr>
<td>Source probe repumper</td>
<td>$F = 1 \rightarrow F = 2$</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1 Summary of the values for detuning and optical power in various laser beams used in the setup.

created by the pinch coils. This setup allows to avoid regions where magnetic field crosses zero value. The four coils of the Ioffe bars allow flexibility of the radial field control. In particular, it is important to compensate the magnetic field against the trap minimum shift due to gravitation in the transfer phase from the MOT to the magnetic trap. This is achieved by reducing the current in the upper coil of the quadrupole. For fine tuning and modulation of the trap potential all Ioffe coils and the compensation coils have additional double-winding coils placed next to them. This coils were for example used to realise a magnetic double-well potential by combining a static Ioffe-Pritchard trap with a time-orbiting potential (TOP). The atoms in such trap were successfully cooled and condensed to produce two spatially separated condensates [99].

In the quest for achieving high densities we produce the trap with the field gradient $\alpha = 353$ G/cm and the curvature $\beta = 286$ G/cm$^2$. This requires driving currents of approximately 400 A through the copper wires of square cross-section $(4 \times 4$ mm$^2$ for pinch and Ioffe coils, $5 \times 5$ mm$^2$ for compensation coils). The wires have a central hole of 2 and 2.5 mm respectively for active cooling of the trap, as the power dissipated by the coils reaches 5.4 kW. As a result, the coils are heated by only about 10 K, which results in high stability of the trapping field. In order to maximise thermal stability of the trap, a titanium holder for the coils is mounted on four quartz bars, with support elements designed to reduce the thermal drifts of the coil positions. The most sensitive indicator of the thermal effects in the trap is the stability of the offset field $B_0 = 886(1)$ mG in the centre of the trap. Translated into the units of the evaporation barrier ($\nu_0 = 620$ kHz), at full current the short-
term thermal effects are less than 1 kHz/s. In the long term the trap shows drifts of approximately 5 kHz/hr.

For time-of-flight measurements on the cold atomic clouds the switch-off time of trapping field is an important parameter. The current flow through the coils is controlled by the programmed values of the power supplies as well as by the switching circuitry based on IGBT switches (IXYS, model IXGN200N60A). Fast switching-off behaviour is achieved by damping the energy of the coils into large electrolyte capacitors preloaded to 200 V. The full current of 400 A is measured to vanish on a time scale of $\sim 60 \ \mu$s, with the inductance of the coils being in the range of 30 $\mu$H. For high stability of the currents flowing through the trap coils all control elements are also water-cooled.

To minimise the noise on the magnetic field the pinch and compensation coils are driven in series. The compensation coils are bypassed by a passive bypass used for fine adjustment of the $B_0$ value. The trap is compensated in such a way that the bottom of the trap is touching the zero-field value. At this point no modulation of the current through the axial coils would affect the bottom of the trap. The actual offset field $B_0$ is then created by an additional pair of coils, also used in the Earth magnetic field compensation. This allows independent control of the frequencies of the trap.

Trap frequencies can be calculated using Equations (2.6), (2.7) and measured by exciting the centre-of-mass oscillations of the cloud. All experiments described in this thesis were performed in a trap with frequencies measured to be $\omega_p = 2\pi \times 477(2)$ Hz and $\omega_z = 2\pi \times 20.8(1)$ Hz. These frequencies are given in the absence of evaporation knife which leads to trap deformation (see Section 3.6)

### 3.6 RF evaporative cooling

The principle of evaporative cooling outline in Section 2.5.1 is realised in practice by transferring the atoms to the untrapped Zeeman states with an oscillating magnetic field. For rubidium the frequency of the evaporation field lies in the radio-frequency (RF) range of approximately 50 MHz to 500 kHz. The transition occurs in the spatial region where the resonance condition is satisfied:

$$g_F \mu_B |B(r)| = \hbar \omega_{rf},$$  

(3.1)

where $\omega_{rf}$ is the angular frequency of the oscillating field. It is related to the truncation energy $\varepsilon_t$ discussed in Section 2.5.1 through the following relation:

$$\varepsilon_t = m_F \hbar (\omega_{rf} - \omega_0),$$  

(3.2)
where $\omega_0 = \mu B_0 \gamma_0 |B(0)| \hbar$ is the resonance frequency corresponding to the centre of the trap (i.e. the bottom of the potential). The probability of the transition into an untrapped state is defined by the amplitude of the evaporation field and the speed at which the atom is passing through the resonance region. This problem was solved for a two level atom in [112] and is discussed in [90]. Additional studies of the transition probability for the $F = 2$ state of $^{87}$Rb were done by Valkering [102]. The probability is a function of the Landau-Zener parameter which is proportional to the square of Rabi frequency $\Omega_R$. At a certain amplitude of the evaporation field the transition probability approaches unity and this defines the minimum amplitude required for evaporation at a given temperature.

In the course of evaporative cooling the evaporation knife is ramped down in frequency from 50 MHz to a few hundred kHz. The total duration of the ramp is 10.6 s. For experiments described further in this thesis the precise timing and high stability of the RF signal are of great importance. No commercial device available on the market at the time could satisfy the set requirements. This motivated design and construction of the RF generator employing direct digital synthesis of the signal (DDS) and based on AD9852 chip from Analog Devices. The generator could be programmed from the LabVIEW interface as a part of the whole event sequence of the experimental cycle. The waveform produced by the synthesiser was typically made of a number of linear frequency ramps combined with several intervals of constant frequency generation.

After passing through a variable 60 dB attenuator the RF signal from the synthesiser is amplified by a power amplifier (Amplifier Research, model 25A250A). The level of the signal is controlled by a 12-bit analogue output of the computer connected to the variable attenuator. The output of the amplifier is coupled to a two-winding circular antenna 31 mm in diameter, which is positioned 16 mm from the trap centre. The final amplitude of the RF field in the trap varies from $15 \times 10^{-6}$ T in the beginning of the ramp to $4 \times 10^{-6}$ T in the end.

It is important to consider the effects of the oscillating magnetic field leading to an energy shift of magnetic sublevels in the “dressed” states picture. These shifts distort the effective shape of the potential and should also be taken into account in measurement of the trap bottom. By diagonalising the time-dependent Hamiltonian of the atom in an oscillating magnetic field, one can calculate the following expression for the dressed potential (in the presence of gravity):

$$U(\rho) = m_F \hbar \left( \omega_\rho - \left[ \Omega_R^2 + \left( g_F \mu B(\rho) / \hbar - \omega_\rho \right)^2 \right]^{1/2} \right) + mg_\rho \rho + \frac{m g_0^2}{2 \omega_\rho^2}, \quad (3.3)$$

where $B(\rho)$ is defined by Equation (2.5) at $z = 0$, and $\Omega_R$ is the Rabi frequency.
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\[ \Omega_R = \frac{g_F \mu_B B_{rf}}{2h}. \]  

(3.4)

Here \( B_{rf} \) is the amplitude of rf-field. One can solve Equation (3.3) to produce an expression for the effective trap depth and an approximate expression for the new trap frequency \( \omega_{\rho}' \), which will be lower than the "non-dressed" frequency \( \omega_{\rho} \) for values of \( \omega_{\rho} \) near the bottom of the trap. A similar analysis can be made for the axial trap frequency. Such systematic effects on the frequency have to be taken into account if the RF barrier is switched on and is close to the bottom of the trap, e.g. in experiments with cold samples where the RF knife is kept at a constant frequency to act as a heat shield.

Another procedure where this effect plays a role is the measurement of the trap bottom \( B_0 \), which is performed as a matter of daily routine. The evaporation knife is lowered until no particles can be detected in the trap. Due to the power broadening described above this occurs before the RF-frequency reaches the resonance value corresponding to \( B_0 \). This offset in frequency is measured by comparing the values of the evaporation knife with those of an continuous atom laser [14] and is found to be 10(1) kHz. A calculation based on Equation (3.3) confirms this result.

### 3.7 Imaging of cold atomic clouds

In this section we consider different aspects of the detection of cold atomic clouds. Description and characterisation of the optical system is followed by a discussion of absorption detection. Analysis of the data and various limitations of imaging are discussed in the next sections. Finally, we present a short discussion of lensing — an important effect frequently occurring in imaging dense small clouds.

#### 3.7.1 Optics

The detection setup in a BEC experiment must be versatile enough to enable the detection of clouds ranging in size from several millimetres to several microns. Features like the optical resolution, noise characteristics and dynamic range all contribute to the quality of the produced data. The schematic outline of the detection optics is presented in Figure 3.4. In the absorption imaging method used throughout this thesis, the shadow in the near-resonant laser beam directed at the sample is transformed by the lenses and imaged on the CCD array. Detection of cold clouds in the trap as well as of fine structures in expanded clouds requires high numerical aperture of the detection optics. However, the position of the sample inside the vacuum cell makes the use of standard microscope objectives impossible. The relay telescope made out of two confocal achromatic lenses produces an
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![Schematic outline of the detection system. Solid lines represent the laser beam shining on the cloud. The diffraction orders are indicated by the dashed lines.](image)

intermediate image of the cloud in a plane far enough from the magnetic trap to allow convenient placement of the microscope objective and the CCD camera. The other purpose of the relay telescope is to enable phase-contrast imaging of the samples [5]. The system is turned into a phase-contrast microscope by insertion of a positive or negative $\pi/2$ phase-shifting plate in the confocal plane of the telescope. The phase-contrast method – a standard tool in microscopy – can be successfully used for non-destructive in situ imaging of cold clouds, as it is sensitive to the phase, rather than intensity, variations in the sample.

The task of the relay telescope is to produce the relayed image with minimal distortion and loss in resolution. In fact, it is the performance of this telescope that is limiting factor in the optical resolution of the setup. The telescope is made out of two high-quality achromats of 100 mm and 200 mm focal length (Melles Griot 06LA1011 and 01LA0189). At 780 nm these lenses provide a near-diffraction limited performance. Numerical aperture of the first lens ($F = 100$ mm) is $NA = 0.15$. The ratio of the focal length sets the primary magnification of the telescope and was calibrated with a Ronchi ruling to be $M_T = 2.01(1)$. Depending on the imaging requirements one can change DIN microscope objectives with minimal re-focusing of the system. The main secondary microscope magnifications are $M_\mu = 3.98$, $M_\mu = 2.39$. For imaging large clouds (e.g. a MOT cloud) we would replace the microscope with a single achromat lens which would give magnification of $M_\mu = 0.25$. The data presented in this thesis were measured with calibration factor of 1.88(1) $\mu$m/pixel in the object space, with $\Delta_p = 15$ $\mu$m being the size of square pixels of the CCD camera. Calibration of the optical system assembled on a single rail was done with a Ronchi ruling away from the setup. After the calibration the system was placed at a known distance from the magnetic trap. Imperfections in longitudinal
placement of the rail would result in the magnification error of well below that quoted for the primary magnification of the relay telescope.

The imaging device is a cooled CCD camera (Princeton Instruments, model TE/CCD-512EFT). The controller of the camera gives a choice between 12- and 16-bit analogue-to-digital conversion. A feature specific to this particular camera is the ability to operate in the so-called “kinetic transfer” mode. In this mode only part of the chip is exposed to light, while the rest of the chip is used as a storage area. One can thus take a burst of images at high speed (limited only by the array shift time) and read them out later at a slow speed. This feature can be particularly useful in non-destructive imaging of the cloud.

Resolution of the optical system is one of the crucial parameters. In the literature on Bose-Einstein condensation it is sometimes quoted in confusing terms. The diffraction performance of our imaging system was analysed both by measurement of a point-spread function (by looking at the end of a single-mode optical fibre) and by a standard tool in microscopy: a positive 1951 USAF resolution target. In line with the definition of a Raleigh criterion, a stripe pattern was defined as resolved if the transmitted intensity was modulated at least by 20%. The smallest resolved pattern had a repetition period of 6.9 micron, which corresponds to a resolution of $R = 3.3 \, \mu \text{m}$ (1/e half-width). This measurement includes the effect of the aberrations added by the 4-mm thick wall of the quartz cuvette in the optical path. This value matches the one measured by imaging the output of a single-mode fibre: $R = 3.1 \, \mu \text{m}$. Calculation of the resolution for a diffraction-limited lens with a Raleigh criterion gives $r = 0.61\lambda/NA = 3.2\,\mu\text{m}$. To compare with often quoted FWHM or $1/e^2$ resolution figures one should multiply the given numbers by an appropriate pre-factor. Resolution effects should be taken into account in the imaging of small objects such as cold clouds in situ, stripes due to phase fluctuations etc. While convolution of the instrumentation function with a gaussian density profile is a trivial task, additional care should be taken in the analysis of clouds with other (e.g. parabolic) density profiles.

### 3.7.2 Absorption detection

The main method of observation of the atomic clouds in our experiments is imaging the absorption profile produced by the cloud in a near-resonant laser beam. While most of the detection was done on the $|5S_{1/2}, F = 2\rangle \rightarrow |5P_{3/2}, F = 3\rangle$ transition, in some measurements we could benefit from using the weaker transitions $|5S_{1/2}, F = 2\rangle \rightarrow |5P_{3/2}, F = 1, 2\rangle$. Such measurements would usually involve dense small clouds where refraction effects (described in Section 3.7.4) were especially pronounced. In this case, using a small
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(or zero) detuning from a weak transition would result in a higher image quality. For driving these transitions we used a separate laser briefly described in Section 3.4.

The intensity distribution in the detection beam after passing through the absorbing cloud follows directly from Lambert-Beer's law:

\[ I(y,z) = I_0(y,z) e^{-D(y,z)}, \tag{3.5} \]

where \( I_0(y,z) \) is the initial density profile before the absorption and \( D(y,z) \) is the optical density:

\[ D(y,z) = \sigma_x \int n(x,y,z) dx = \sigma_x \eta(y,z). \tag{3.6} \]

Here \( \sigma_x \) is the photon absorption cross-section. The detection light is linearly polarised and in the zero-approximation the expression for the cross-section can be obtained by averaging over all \( \pi \)-transitions (see e.g. ref. [76] for discussion on the transition strengths):

\[ \sigma_x = \frac{7}{15} \frac{3 \lambda^2}{2 \pi} \frac{1}{1+(2\delta/\Gamma)^2}, \tag{3.7} \]

where \( \delta \) is the detuning from the optical transition, \( \Gamma \) is the full natural linewidth, and \( \lambda \) is the wavelength of the laser. However, since the detection is not done on a closed transition, in the duration of the detection pulse optical pumping results in re-distribution of the atoms between different Zeeman sublevels. This changes the 7/15 factor in the expression for the cross-section and can lead to systematic errors in determination of the number of atoms.

The choice of the detection pulse duration is dictated by limitations of the ballistic blur caused by scattered photons to the sample [64]. In our experiments it was typically 40 \( \mu s \). At the same time short detection times force one to go to higher optical powers of the detection beam to maximise the use of the dynamic range of the camera and increase the signal-to-noise ratio of the image. This is especially true for the use of high-power microscope objectives, when the light collection efficiency goes down. This increase in powers presents no problem if the detuning of the detection beam is large enough to stay away from saturation of the transition. However, once the sample expands one is forced to reduce the detuning to keep the optical density well above the noise floor. In such cases varying across the sample saturation effects should be taken into account by solving the following differential equation:

\[ \frac{\partial I(x,y,z)}{\partial x} = -n(x)\sigma_0 \left[ 1 + \left( \frac{2\delta}{\Gamma} \right)^2 + \frac{I(x,y,z)}{I_s} \right]^{-1} I(x,y,z). \tag{3.8} \]
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where \( I_s \) is the saturation intensity. For the microscope objective \( M_\mu = 3.98 \) the power of the detection beam was 2.5 mW, which corresponded to 0.85\( I_s \) on resonance.

A single data shot of the optical density distribution contains in fact three images and is normalised according to the following rule:

\[
D(y,z) = -\ln \frac{I(y,z)}{I_0(y,z)} = -\ln \frac{I_{abs}(y,z) - I_{bg}(y,z)}{I_{ff}(y,z) - I_{bg}(y,z)}. \tag{3.9}
\]

Here \( I_{abs}(y,z) \) is the beam profile with the shadow of the cloud, \( I_{ff}(y,z) \) is the flat-field profile taken in the absence of the cloud, and \( I_{bg}(y,z) \) is the background light illumination taken in the absence of the detection beam.

Ideally, the lower limit on the detuning of the detection laser is set by the noise performance of the whole imaging system and by the dynamic range of the analogue-to-digital converter of the CCD camera. For a 12-bit camera the minimum optical density would be \( D_0 = 8 \). However, in practice the observed maximum optical density is limited to \( D_0 = 5 \). This difference is due to the broad spectral background typical for diode lasers. This aspect of the spectral purity of the detection is discussed in Chapter 4. To avoid the systematic errors due to this effect the detuning of the detection beam is usually made large enough to keep the maximum optical density below 2.5.

### 3.7.3 Fitting parameters

All information about the condensates and cold clouds is extracted from analysis of the optical density profile defined by Equation (3.9). The total number of particles can be determined directly from the pixel sum of the image:

\[
N = \frac{A_p^2}{\sigma_x} \sum_{i,j} D(y_i,z_j). \tag{3.10}
\]

where \( A_p \) is the size of the square pixel.

More complete information is obtained by fitting a two-dimensional surface to the array of data described by Equation (3.9). The thermal cloud profile is fitted by the following function:

\[
D_{th}(y,z) = \sigma_x \eta_{th}(y,z) = \sigma_x \eta_{th}(0,0) g_2 \left( \hat{z} \exp \left( \frac{y^2}{l_y^2} - \frac{z^2}{l_z^2} \right) \right) / g_2(\hat{z}). \tag{3.11}
\]

In the limit of collisionless gas the temperature can be obtained from the radial \( l_y \) or axial \( l_z \) 1/e size parameters of the profile [64].
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\[ T = \frac{m}{2k_B} \left[ \frac{\omega_i^2}{1 + \omega_i^2 \tau^2} \right] l_i, \; i \in \{y, z\}. \]  

(3.12)

In practice for cigar-shaped clouds the temperature can also be determined directly from the axial size at short expansion times \( \tau \ll 1/\omega_z \). If the cloud can no longer be described by a collisionless gas model, the temperature determination becomes less trivial. This is discussed in detail in Chapter 5.

The condensed fraction of the cloud has a parabolic density profile, which, after integration along the line of detection, yields the following distribution for the optical density:

\[ D_c(y, z) = \sigma_c \eta_c(0, 0) \left( \max \left\{ 1 - \frac{y^2}{L_y^2(\tau)} - \frac{z^2}{L_z^2(\tau)} , 0 \right\} \right)^{3/2}. \]  

(3.13)

Here the Thomas-Fermi size parameters \( L_y \) (or \( L_\rho \)) and \( L_z \) are given by [21]:

\[ L_\rho(\tau) = L_\rho(0) \sqrt{1 + \omega_\rho^2 \tau^2}. \]  

(3.14)

\[ L_z(\tau) = L_z(0) \left[ 1 + \left( \frac{\omega_z}{\omega_\rho} \right)^2 \left( \frac{\omega_\rho \tau \arctan(\omega_\rho \tau)}{2} - \ln \sqrt{1 + \omega_\rho^2 \tau^2} \right) \right]. \]  

(3.15)

The chemical potential is then given as

\[ \mu = \frac{m \omega_\rho^2 I_\rho^2(0)}{2} = \frac{m}{2} \left( \frac{\omega_\rho^2}{1 + \omega_\rho^2 \tau^2} \right) I_\rho^2(\tau). \]  

(3.16)

The number of particles in the condensate is expressed through the chemical potential as (see Equation (2.22))

\[ N_0 = \left( \frac{2\mu}{\hbar \omega} \right)^{3/2} \frac{r_h}{15a}, \]  

(3.17)

and the central density of the condensate is given by \( n_0(0) = \mu/\bar{\mu} \).

3.7.4 Lensing

In this section we briefly discuss the refractive effects of cold atomic clouds. In the description of absorption detection in Section 3.7.2 we only considered the effect of the imaginary part of the complex dielectric susceptibility. However, the real part of the susceptibility responsible for refraction cannot be neglected in sufficiently dense atomic
clouds. Depending on the detuning such clouds can behave as a combination of the GRIN lens and a usual spherical lens. Lensing can lead to systematic errors in the determination of the cloud size and the number of particles. Thus, it is important to understand regimes in which lensing can appear.

The key limitations become obvious from the analysis of the expression for a refractive index of a two-level atom, classical derivation of which follows from e.g. [66]:

\[
N = \sqrt{\mu (1 + 4\pi \chi n)} \approx 1 + \frac{3\lambda^3 \Gamma n}{16\pi^2} \left[ -\frac{\delta}{\delta^2 + \Gamma^2/4} + i\frac{\Gamma/2}{\delta^2 + \Gamma^2/4} \right].
\] (3.18)

Here \( n \) is the density of the atoms. For more detailed discussion one can refer to [30], while quantum mechanical derivation of susceptibility of rubidium vapour is discussed in [3].

**Figure 3.5. Simulation of lensing effect on the condensate density profile (radial direction) for blue (a) and red (b) detuning. (Courtesy of Imogen Colton).**
The optical density can thus be written down as

\[ D = D_0 \frac{1}{1 + 4 \frac{\delta^2}{\Gamma^2}}, \]  

(3.19)

where \(D_0\) is the maximum resonant optical density of the sample. All changes in the phase of the propagating light are described by the real part of Equation (3.18):

\[ \text{Re}(N) = 1 - \frac{3 \lambda^2 n}{16 \pi^2} \frac{4 \delta^2}{\Gamma^2}. \]  

(3.20)

It is clear that for blue detuning the real part of the refractive index is smaller than unity and the cloud acts as a diverging lens, while for red detuning it acts as a converging lens. In Figure 3.5 we show an example of simulation of the lensing effect on a pure condensate with a parabolic density profile [24].

If the phase shift induced by a cloud is sufficiently large to make the cloud act as a lens with the focal length comparable to its own size, lensing severely affects the image and this region should be avoided. This effect can be especially pronounced in imaging the clouds in the magnetic trap or at short expansion times. In such cases one can benefit from using weaker transitions, as described in Section 3.7.2, or go to the extreme of large detunings, where absorption is negligible, and use phase-contrast detection. The intensity distribution of the phase-contrast image is proportional to the phase shift induced by the cloud and is therefore proportional to density.

It is possible to extract information from images affected by lensing by doing sophisticated data processing. Moreover, there is current work on the use of non-interferometric methods of imaging the dense atomic samples using phase information. [25, 101].