Bose-Einstein condensation into non-equilibrium states
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Chapter 6
Condensation into non-equilibrium states and focusing of a condensate

6.1 Introduction

Equilibrium condensates [4, 28] are produced by quasi-static growth, where heat extraction limits the formation rate. The condensate nucleates as a small feature in the center of the trap and grows as long as heat is extracted from the sample. To observe the formation kinetics, the gas has to be brought out of equilibrium, in practice by shock cooling. Since the first experiment on condensate growth, by Miesner et al. [78], this is done by fast RF removal of the most energetic atoms from the trap. Starting from a thermal gas just above the phase transition temperature ($T_C$), the condensate appears as the result of thermalization. Miesner et al. [78] observed the growth under adiabatic conditions. Köhl et al. [65] continued the extraction of heat and atoms, also during growth. In both experiments, the condensate was observed to grow from the center of the trap, like in the quasi-static limit.

Kagan et al. [57] pointed out that qualitatively different stages have to be distinguished in the formation of equilibrium condensates with a large number of atoms. The early stage (kinetic stage) is governed by Boltzmann kinetic processes and leads to a preferential occupation of the lowest energy levels. Once a substantial fraction of the atoms gathers within an energy band of the order of the chemical potential of the emerging condensate during formation, their density fluctuations are suppressed in a fast interaction-dominated regime governed by a non-linear equation for the boson field. The appearing phase-fluctuating condensate then grows and the condensed fraction approaches its equilibrium value. However, the phase fluctuations still persist, giving rise to dynamically evolving flow patterns in search for the true equilibrium state. In elongated 3D trapped gases the phase fluctuations can be pronounced even under equilibrium conditions as was predicted by Petrov et al. [82] and observed experimentally by Dettmer et al. [31].

In this chapter we describe the formation of condensates into non-equilibrium states and a new path towards equilibrium in elongated traps. In contrast to the previous experiments our results were obtained starting from thermal clouds deep in the cross-over regime to hydrodynamic behaviour. The condensates are much longer than equilibrium
condensates with the same number of atoms. Moreover, they display strong phase fluctuations and a dynamical evolution similar to that of a quadrupole shape oscillation decaying towards equilibrium. We identify $1/\omega_z$ as a characteristic time that should be addressed explicitly for elongated cylindrical harmonic traps, i.e. for traps with $\omega_p \gg \omega_z$ where $\omega_p$ and $\omega_z$ are the radial and axial angular frequencies, respectively. We show that these exotic condensates emerge as the result of local thermalization when the nucleation time is short as compared to $1/\omega_z$. The dynamical evolution of the condensate in the trap has to be dealt with explicitly to properly interpret time-of-flight absorption images. In this context we introduce condensate focusing as an alternative to Bragg scattering [95] for measuring the phase-coherence length of phase-fluctuating Bose-Einstein condensates.

In the previous experiments on condensate formation the phase fluctuations were not studied. The results of Miesner et al. [78] were compared to an analytical expression for adiabatic growth of a condensate from a thermal cloud, derived by Gardiner et al. [38]. Although a qualitative agreement between theory and experiment was readily obtained it turned out to be impossible to obtain detailed agreement at the quantitative level [11, 29, 39, 70]. In the experiment of Köhl et al. [65] quantitative agreement with the quantum kinetic approach (see refs. [11, 29, 38, 39, 70] was obtained for strong truncation, whereas for weak truncation the observed behavior differed distinctly from theory.

### 6.2 Preparation of degenerate samples out of equilibrium

Evaporative cooling and preparation of cold near-$T_c$ samples was previously described in detail in Section 5.2. The starting conditions of experiments on condensate formation are represented by a static, purely thermal cloud of $N_t \approx 5 \times 10^6$ atoms at a temperature $T_0 = 1.3(1) \ \mu$K. This cloud is placed in a magnetic trap characterised by the frequencies $\omega_p = 2\pi \times 477(2) \ \text{Hz}$ and $\omega_z = 2\pi \times 20.8(1) \ \text{Hz}$ (given in the absence of RF dressing).

As discussed in Chapter 5 these samples are prepared far deeper in the cross-over to the hydrodynamic regime, as compared to the previous experiments [78] and [65].

Once the thermal cloud is prepared we distinguish three distinct stages (see Figure 6.1). First, in the truncation stage, the radio frequency is set to the value $\nu_{r, b} = 660 \ \text{kHz}$. This stage has a duration $t_r = 1 \ \text{ms}$, which is chosen to be long enough ($t_r > 1/\omega_p$) to allow atoms with radial energy $\varepsilon_r$ larger than the RF truncation energy $\varepsilon_r$ to escape from the trap, yet is short enough to disallow evaporative cooling. We found that in this stage 50% of the atoms are removed. Notice that due to the finite radial escape horizon $(\lambda_0/l_p \approx 0.4)$ the ejection is not expected to be complete. Furthermore, the escape efficiency is anisotropic as
6.3 Kinetics of the condensate growth

![Graph showing the preparation of the thermal sample, truncation, thermalization, and TOF expansion stages.](image)

**Figure 6.1.** Schematic representation of the evaporation sequence used for obtaining non-equilibrium degenerate atomic clouds.

A result of gravitational sag. The truncation energy \( \varepsilon_{tr} \) covers the range 3 \( \mu \)K – 5 \( \mu \)K depending on the position of the truncation edge in the gravity field and is lowered by an additional 1 \( \mu \)K due to RF-dressing (Rabi frequency \( \Omega_{rf} \approx 2\pi \times 14 \) kHz). At the start of the second stage, the thermalization stage, the radio frequency is stepped back up for a time \( t_{th} \) to the frequency \( \nu_{r,a} \) to allow the gas to thermalize under formation of a condensate. The value \( \nu_{r,a} \) is chosen to eliminate any appreciable evaporative cooling. The third stage, the expansion stage, starts by switching off the trap and covers the time of flight \( \tau \) after which the sample is absorption imaged on the \( |5S_{1/2}, F = 2 \rangle \rightarrow |5P_{3/2}, F = 1, 2 \) or 3) transition.

To follow the evolution of the trapped gas after the truncation we took time-of-flight absorption images for a range of evolution times \( t \equiv t_{tr} + t_{th} \) and a fixed expansion time \( \tau \). The images show a bimodal distribution, indicating that the truncation procedure results in BEC (see Figure 6.6).

### 6.3 Kinetics of the condensate growth

Without making an attempt to study in detail the actual kinetics of the condensate growth we present the growth data for the non-equilibrium condensates. We use a simple relaxational model to describe the growth curve

\[
N_0(t_{th}) = N_{0,fn} \left(1 - e^{-t_{th}/\tau} \right),
\]

(6.1)
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Figure 6.2. Growth kinetics of the non-equilibrium condensates following the RF truncation. The two data sets present in the figure correspond to the different expansion times $\tau = 5.3$ ms and 8.3 ms. The solid line is a fit of a simple relaxational model described by Equation (6.1).

where $t_{gr}$ is the characteristic growth time, $N_{0,\text{fin}}$ is the final number of particles in the condensate. In some datasets it was possible to detect the appearance of the condensate already during the 1-ms truncation stage. The condensate fraction at the end of this stage typically would not exceed 1%.

In Figure 6.2 we show the growth of the condensate fraction $N_0/N$ as a function of evolution time $t$. Truncation time $t_{th} = 1$ ms is visible as an offset of the first data point on the horizontal axis. The condensate fraction grows to a final value of 6% with a characteristic time of 6 ms. This corresponds to 30 times the elastic collision time, which is in accordance with previous experiments [65, 78]. It is appropriate to note here that the characteristic growth time is much shorter than the axial oscillation period in the trap of 49 ms. This already gives a hint to a non-equilibrium nature of the forming condensate which is described in the following sections.

### 6.4 Formation of a non-equilibrium BEC

Rather than discussing the details of the growth kinetics we emphasize that our condensates nucleate into non-equilibrium states. Let us try to follow carefully the effect of a truncation on a thermal near-$T_C$ cloud.
6.4.1 *Local critical temperature*

At first it is important to understand the conditions leading to formation of non-degeneracy in the samples after the truncation. To do this we compare the critical temperature for various parts of the cloud with the developing temperature profile. It is instrumental to realize that in such experiments the truncation occurs in a real space, unlike the commonly accepted picture of a truncation in an energy space. In our samples with highly anisotropic geometry the extraction of the heat is mostly done in the radial direction. Since the mean free path is much smaller than the axial size of the cloud, one can separate the cloud into the regions of the size of the order of the mean free path, and consider for each of these regions the problem of Bose-Einstein condensation in an infinitely long cylinder (see Figure 6.3). An expression for local $T_C$ in such a system has already been derived in Section 2.5.2:

$$T_C(z) \approx \frac{1.28 \hbar \omega}{k_B} \left[ n_{1D}(z) r_p \right]^{2/5},$$  \hspace{1cm} (6.2)

where $r_p = [\hbar/m\omega_\rho]^{1/2}$ is the radial oscillator length and $n_{1D}(z)$ is the atom number per unit length at position $z$.

6.4.2 *Truncation and local temperature*

Following the arguments similar to the one in previous section we can introduce the concept of a local temperature. In fact, in Section 2.5.3 we have already derived an expression for the temperature profile developed in the sample after an abrupt truncation:

$$T_{loc} = \frac{P(7/2, \eta_t) P(5/2, \infty)}{P(5/2, \eta_t) P(7/2, \infty)} T_0$$  \hspace{1cm} (6.3)

where $T_0$ is the temperature before the truncation,

$$\eta_t = \frac{\varepsilon - m\omega^2 z^2/2}{k_B T_0}$$  \hspace{1cm} (6.4)

is the local truncation parameter, and $P(a, \eta)$ is the incomplete gamma function.

Following the concept of local formation underlying ref. [57], a simple description of formation of a non-equilibrium Bose-Einstein condensate under the truncation of the evaporation barrier is demonstrated in Figure 6.4. Starting with a thermal cloud of $N_i = 5 \times 10^6$ atoms in an 11.5 mK deep harmonic trap at a temperature $T_0 = 1.3 \mu$K, we calculate that 55% of the atoms remain trapped after all atoms with energy $\varepsilon > \varepsilon_{tr} = 3.4 \mu$K are removed. Following with the initial density distribution (a), we can produce the profiles of the local
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Figure 6.3. Introduction of a local critical temperature as a result of high density and elongated geometry of the cloud. For the regions of the condensate with the size of the order of the mean free path the condensation problem can be considered as that of an infinitely long cylinder.

$T_C$ and the local temperature $T_{loc}$ after the truncation (b). It is evident from Figure 6.4(b) that after the truncation the temperature is rapidly lowered to the values below critical over a large region of the sample. This is in contrast with the quasi-static case, where the equilibrium temperature (which would look like a horizontal line in graph (b)) is lowered relative to the density-dependent profile of $T_C$. In the latter case the condensate formation would start from a region with the highest density and propagate to the outer regions of the cloud. In the non-equilibrium case the condensate is formed on a time scale short compared to the $1/\omega_z$ (as discussed in Section 6.3). Comparing Equations (6.2) and (6.3) we can estimate the size of such condensate as well as the condensate fraction (Figure 6.4(c)). We find that the local temperature $T_{loc}(z)$ is lower than the local $T_C(z)$ over a length of the order of $l_z$, the axial size of the cloud, defined as $l_z=[2k_BT_0/m\omega_z^2]^{1/2}$. This clearly shows that the condensate formed after the truncation is oversized in the axial direction, compared with the equilibrium condensate with the same number of particles and in the same potential. This is shown in the plots of the one-dimensional density distributions in Figure 6.4(d). Evidently, this difference in sizes must lead to oscillations of the non-equilibrium condensate in the trap, which is confirmed experimentally. In view of the simplicity of the model we consider this as good qualitative agreement with experiment.

In Figure 6.5 we plot the Thomas-Fermi half-length $L_z(t,\tau)$ obtained with the standard fitting procedure of a bimodal distribution to our data (see Section 3.7.3 and ref. [64]). For the shortest expansion time, $\tau = 2.8$ ms, the axial size of the condensate image equals to good approximation ($\tau \ll 1/\omega_z$) the axial size of the condensate in the trap. We
Figure 6.4. Simulation of formation of an oversized Bose-Einstein condensate as a result of truncation experiment. All graphs are plotted vs the axial coordinate z. (a) Initial density profile. (b) Local temperature (solid line) and $T_c$ (dashed line). (c) Condensate fraction. (d) Density profiles of the thermal cloud and the condensate after thermalization.
Figure 6.5. (b) Condensate length $L_z(t, \tau)$ versus evolution time $t$ for three different expansion times $\tau$. The dark grey line is a guide to the eye. The light grey line represents a fit to a damped quadrupole shape oscillation in the limit of a linear response model. Data set corresponding to 25.3 ms expansion time is also shown for the radial direction. (a) The oscillations of the radial size do not exceed the noise level.

see that $L_z(t)$ is initially oversized by a factor $L_z(0)/L_z(\infty) = 2.2(3)$ and rapidly decreases to reach its equilibrium size after roughly one strongly damped shape oscillation (see open triangles Figure 6.5). Thus, the condensate is clearly not in equilibrium.

6.5 Oscillation modes

In Figure 6.5 we also show the oscillation in the axial size of the condensate as observed for 15.3 ms (open squares) and 25.3 ms (crosses) of expansion. Longer expansion times enhance the contrast of the data and the amplitude of the oscillation is increased as compared to the 2.8 ms results. For $\tau = 25.3$ ms the shape oscillation is seen to exceed the noise for at least 100 ms. For evolution times $t \geq 20$ ms this oscillation can be described by a linear response model which is discussed in detail in Section 6.7. We measure a damping time of $\tau_Q = 50(9)$ ms and a frequency ratio $\omega_Q/\omega_z = 1.54(4)$. The latter is slightly lower than the frequency expected for a quadrupole shape oscillation of a pure condensate in very elongated traps, $\omega_Q/\omega_z \approx (5/2)^{1/2} \approx 1.58$ [21, 60]. A 5\% negative frequency shift was observed for the quadrupole mode in Na condensates just below $T_C$ [94] and is consistent
with theory [27]. In radial direction the oscillation amplitude did not exceed the noise for the conditions of Figure 6.5.

The shock-cooling by truncation in the presence of gravitational sag also gives rise to a vertical center-of-mass oscillation of the sample in the trap with an amplitude of 0.5 \( \mu \text{m} \) and a damping time of 40 ms. Such short damping time is related to the presence of a large thermal cloud. See e.g. refs. [36, 107, 110, 111] for discussion on the damping of low-energy excitation at finite temperatures.

### 6.6 Non-equilibrium phase fluctuations

Phase coherence properties are among the most interesting aspects of Bose-condensed gases. Since the discovery of BEC [4, 19, 28], various experiments have proved the presence of phase coherence in trapped condensates. The MIT group [6] has found the interference of two independently prepared condensates, once they expand and overlap after switching off the traps. The MIT [95], NIST [48] and Munich [15] experiments provided evidence for the phase coherence of trapped condensates through the measurement of the phase coherence length and/or single particle correlations. In equilibrium 3D condensates, the fluctuations of density and phase are important only in a
narrow temperature range near $T_C$. Outside this region, the fluctuations are suppressed and the condensate is phase coherent.

A case of special interest exist in very elongated 3D condensates, where the axial phase fluctuations are found to manifest themselves even at temperatures far below $T_C$. Then, as the density fluctuations are suppressed, the equilibrium state is a condensate with fluctuating phase (quasicondensate) similar to that in 1D trapped gases [81]. With decreasing the temperature below a sufficiently low value, the 3D quasicondensate gradually turns into a true condensate. The equilibrium phase fluctuations in elongated 3D traps were predicted by Petrov et al. [82] and first observed by Dettmer et al. [31].

The process of condensation induced by shock cooling used in our experiments presents a picture of the condensate forming simultaneously across a large region of the atomic cloud (see Section 6.4). This evidently leads to the absence of phase correlation between different parts of the condensate and formation of large phase fluctuations of non-equilibrium origin. When the trap is switched off, the gradients in phase lead to local variations in expansion velocity. After some expansion they give rise to irregular stripes in the density profile (see Figure 6.6) similar to the ones observed in Hannover [31]. The damping of shape oscillations described in Section 6.4.2 and the phase fluctuations provide a fundamentally different path to equilibrium, as compared to the quasi-static formation of a condensate. Investigation of the statistical characteristics of the stripes in the density profile gives access to a wealth of information on the properties of the phase fluctuations. However, experiments described in this thesis were performed in the setup where the sample was observed at an angle of 73° to the trap axis. This limits the observation of the stripes at long expansion times. The detection setup has recently been modified and the physics of non-equilibrium phase fluctuations is currently under investigation in our group. The information on the phase coherence length can still be extracted by the use of a novel method described in the following section.

### 6.7 Focusing of a condensate

Since the first observation of Bose Einstein condensation (BEC) coherent atom optics has developed into an important field [88], providing tools for the investigation of macroscopic quantum phenomena in dilute atomic gases below their critical temperature. Many properties of quantum gases can be extracted by studying the interference between overlapping Bose-Einstein condensates after expansion from magnetic or optical traps [64]. With atom interferometry and quantum information processing as long term goals, atom waveguides as well as atom chips are being developed [9, 12, 52]. Mirrors, beam splitters
and beam shaping optics of various types have been demonstrated [17, 68, 86]. Bloch et al. [16] demonstrated the focusing of an atom laser beam by a harmonic potential.

To further investigate the formation process in our experiments we introduce the concept of condensate focusing. In our case one-dimensional focusing results from axial contraction of the expanding cloud when the gas is released from the trap during the inward phase of a shape oscillation.

### 6.7.1 Focusing principle

To describe the principle of focusing of a condensate in free flight (see Figure 6.1a), we consider a cloud of atoms confined in an axially symmetric harmonic trapping potential with angular frequencies \( \omega_z \) (axial) and \( \omega_\rho \) (radial) and small aspect ratio \( \beta = \omega_z / \omega_\rho \ll 1 \). We presume the cloud to dilate periodically in shape with angular frequency \( \omega_\rho \) in such a way that a linear velocity field \( v_z (z) = -a_z (t) z \) is present along the z-axis. At time \( t = 0 \) the gas is released from the trap by the sudden removal of the trapping potential. For \( t \leq 0 \) the axial size, normalized to its value at release, is given in linear response by

\[
  b_z (t) = 1 - a_z \sin \omega_\rho t, \tag{6.5}
\]

where \( a_z \) is the rescaled axial amplitude of the oscillation. For the oscillation shown in Figure 6.1b, the axial size at \( t = 0 \) is contracting and we look for a focus at some later time \( t > 0 \).

Let us first consider a pure Bose-Einstein condensate drive on the low-frequency mode of a quadrupole shape oscillation for which \( \omega_\rho \approx 1.58 \omega_z \) [59, 60]. At \( t = 0 \) the axial size is given by the equilibrium Thomas-Fermi radius, \( L_z = [2 \mu / m \omega_z^2]^{1/2} \), where \( \mu \) is the chemical potential of the gas and \( m \) is the atomic mass. Within the Thomas-Fermi approximation the evolution of the axial and radial sizes of the cloud is given by the scaling equations (2.40) and (2.41),

\[
  \ddot{b}_i = \frac{\omega_i^2}{b_i b_z b_\rho^2}, \quad \text{with } i \in \{z, \rho\}, \tag{6.6}
\]

subject to the initial conditions

\[
  b_z (0) = b_\rho (0) = 1,
  \quad \dot{b}_z (0) = -a_z \omega_\rho, \quad \dot{b}_\rho (0) = 0. \tag{6.7}
\]

As \( \beta \ll 1 \) we find to a good approximation for the radial expansion \( b_\rho = [1 + \omega_\rho^2 t^2]^{1/2} \). The axial expansion at \( t \gg 1 / \omega_\rho \) is given by
Figure 6.7. a) BEC-focusing observed as a contraction of the Thomas-Fermi size as a function of time; b) Evolution of the axial size before and after trap release at \( t = 0 \) for an oscillation amplitude \( a_z = 0.4 \). Solid line: condensate evolution for \( \zeta = 0.41 \); c) Black squares: experimental data points; Grey curve: fit of Equation (6.10) to the black squares, corresponding to \( \zeta = 0.37 \) and \( \kappa = 0.20 \); Dashed line: plot of Equation (6.8) for \( \zeta = 0.37 \). Dotted line: as dashed line by showing the optical resolution limit.

\[
b_2 = \left| 1 - \zeta \omega_z t \right|
\]  

(6.8)

where the contraction parameter \( \zeta \) is defined as \( \zeta = (a_z \omega_Q / \omega_z - \beta \pi / 2) \). The result is shown as the solid line in Figure 6.1b. Hence for \( \zeta > 0 \) the axial size decreases to produce a (one-dimensional) focus at time \( t_{\text{focus}} = 1/(\zeta \omega_z) \). This is the case if the axial contraction velocity at release, \(-a_z \omega_Q\), dominates over the axial expansion velocity \( \beta \pi \omega_z / 2 \) induced by the kick during the initial stages \( (t \ll \omega \rho) \) of the expansion, i.e. \( a_z > \beta \). As the radial size remains finite and \( L_z \) decreases, around \( t_{\text{focus}} \) the chemical potential is restored and the focus reaches a minimum size \( b_z(t_{\text{focus}}) \approx 2 \beta^2 \), independent of the value of \( a_z \). This result is obtained by using the approximation \( b_\rho(t_{\text{focus}}) \approx b_\rho(t_{\text{focus}}) = \omega_\rho t_{\text{focus}} \) and integrating Equation (6.6) for \( i = z \). Matching the resulting slope
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\[ b_z \approx \left[ \frac{2\omega^2}{b^2(t_{\text{focus}})b_z(t_{\text{focus}})} \right]^{1/2} \]  

(6.9)

with the contraction velocity \( b_z(1/\omega_p \ll t \ll t_{\text{focus}}) \) we get the mentioned result. The compression can be very light, e.g. \( 2\beta^2 \approx 4 \times 10^{-3} \) for the conditions in our apparatus. In such cases the optical resolution of the imaging system used for detection is likely to limit the minimum observable focal size as was reported in ref. [16].

6.7.2 Focal broadening

Beyond a certain expansion time the kinetic energy of the original condensate can no longer be neglected as it gives rise to spreading of the condensate wavefunction. This effect may be accounted for by writing

\[ b_z(t) \approx \left[ (1 - \zeta \omega_z t)^2 + \kappa \omega_z^2 t^2 \right]^{1/2}, \]

(6.10)

where \( \kappa \) is a parameter determining the size of the focal waist. Note that for \( \kappa = 1 \), Equation (6.10) represents the spreading of a minimum uncertainty wavepacket released under conditions of axial contraction. Notice further that in this case no appreciable focusing is observed (see dotted line in Figure 6.1b) except for shape oscillations driven far outside \( (\zeta > 1) \) the linear regime. In general, Equation (6.10) gives rise to substantial focusing only if \( \zeta > \kappa \) (at \( \zeta = \kappa \) the condensate is compressed by 30%). The condition \( \zeta > \kappa \) is satisfied for elongated Thomas-Fermi condensates at \( T = 0 \), because the momentum spread is strongly reduced compared to that of the oscillator ground state. This situation is described by approximating the waist parameter with the value \( \kappa = \hbar \omega_z / 2 \mu \). Then, for \( t < t_0 = 2\beta^2 m L_z^2 / \hbar \), the spreading can be neglected even with respect to the compression minimum \( 2 \beta^2 L_z \). For our conditions we calculate \( t_0 \approx 7 \) ms.

For similar reasons it is virtually impossible to focus a collisionless thermal cloud. To illustrate this we consider a simple Boltzmann gas at temperature \( T \) with an oscillation described for \( t < 0 \) by Equation (6.5) and released from the trap at \( t = 0 \). If the collisional mean free path is much larger than the radial size of the cloud, the expansion proceeds ballistically and the momentum of the individual atoms is conserved (free expansion). The scaled axial size evolves according to

\[ b_z(t) = \left[ (1 - a_z \omega_z t)^2 + \omega_z^2 t^2 \right]^{1/2}, \]

(6.11)

which represents the convolution of two gaussians: the density profile of equilibrium width \( l_z = [2k_BT/\hbar \omega_z^2]^{1/2} \) and the velocity field \( v_z(z) = -a_z \omega_z z \). In the absence of a shape
oscillation ($a_z = 0$) this expression reduces to the well-known result used in time-of-flight analysis of collisionless thermal clouds [64, 109]. Presuming the same value of $\zeta = a_z \omega_0 / \omega_z$ as for the condensate (i.e. the solid line in Figure 6.1b), the thermal cloud is represented by the dotted line in Figure 6.1b.

Returning to elongated condensates we point out that at temperatures above the phase fluctuation temperature, $T > T^* = 15(\hbar \omega_z)^2 N / 32 \mu$, equilibrium phase fluctuations will dominate the focal broadening [31, 49, 82]. In this case the waist parameter may be approximated by $\kappa \approx (L_z / L_\phi)^2 \hbar \omega_z / \mu \approx (l_h / L_\phi)^2$, where $L_\phi$ is the phase coherence length and $l_h = [\hbar / \mu \omega_z]^{1/2}$ the axial harmonic oscillator length.

### 6.7.3 Applications of BEC focusing

The first application concerns the phase fluctuations discussed in Section 6.6 and deals with the focus shown in Figure 6.1c. The grey line represents a fit of Equation (6.10) to the data and yields $\zeta = 0.37$ and $\kappa = 0.20$. In this experiment the focus is strongly broadened, $b_z (t_{\text{focus}}) \gg 2 \beta^2$ exceeding the optical resolution limit of 3.3 μm – compare to the minimum size of 15.2 μm in Figure 6.1c. Hence, for the conditions of this experiment we may write $\kappa \approx (l_h / L_\phi)^2$ and find a phase coherence length of $L_\phi \approx 0.45 l_h \approx 1 \mu m$.

Condensate focusing offers improved detection of small condensate fraction. Near the focus the axial condensate size is compressed by a factor $1 / b_z (t_{\text{focus}})$. In time-of-flight absorption imaging the signal-to-noise can be improved accordingly by choosing the time of detection equal to $t_{\text{focus}}$. This is advantageous, particularly close to $T_C$ where the condensate fraction is small and has to be detected against the background of a large thermal cloud.

Condensate focusing also provides some advantage in detecting small thermal clouds as the separation time of the two components is reduced. Therefore, in time-of-flight absorption imaging detection can be shifted to shorter expansion times when the drop in optical density of the thermal cloud ($D_h \propto [\omega_p \omega_z t_{\text{sep}}]^{-1}$) is less and an improvement in the signal-to-noise ratio of a factor of two can be obtained.

### 6.8 High condensate fractions in non-equilibrium states

So far we dealt with the data where the condensate fraction was of the order of a few percent. We used another method, different from the one described in Section 6.2, to produce out-of-equilibrium samples with high condensate fractions. In the first 10 seconds of evaporative cooling of these clouds the RF knife is linearly lowered from 50 MHz to 4 MHz, similar to the way it was described in Section 5.4. From 4 MHz onwards the speed of
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Figure 6.8. Giant quadrupole oscillations of sample with 50 – 90 % condensate fraction. A characteristic damping time is 141 ms, although a simple exponentially decaying fit describes the data rather poorly. The number of particles drop explains the reduction of the equilibrium size of the condensate.

the linear RF ramp is increased by an order of magnitude to 5.5 MHz/s. At this point it satisfies the condition

$$\left| \frac{d\nu_{tr}}{dt} \right| > \frac{\nu_{tr,a} - \nu_0}{\omega_z}.$$  \hspace{1cm} (6.12)

The density is sufficiently high to keep evaporative cooling efficient and to reach progressively colder temperatures. However, the local nature of the cooling results in formation of non-equilibrium condensates similar to those described earlier in the chapter. Due to the continuous extraction of the heat from the system one can reach higher condensate fractions to the point where no thermal cloud can any longer be observed.

The number of particles in the condensate depends on the frequency at which the RF ramp is stopped. Once the final frequency is reached, RF knife remains at this frequency for the duration of what we refer to as plain evaporation time $t_{pe}$. The densities at this point reach $10^{15}$ cm$^{-3}$ and the heat load due to three-body recombination products is the main limiting factor on the lifetime of a condensate. Evaporation knife remaining at the final frequency acts as a heat shield and a heat-removal mechanism, which significantly increases the lifetime.
Figure 6.9 (a) Stripes in the density profile of an almost pure condensate caused by the non-equilibrium phase fluctuations. (b) Absorption image of the condensate produced quasi-statically is shown for comparison. Small thermal fraction is visible in the wings of the cloud. (c) & (d) Axial (horizontal) cross-sections through the optical density profile of (a) & (b) respectively. Solid line in (d) represents a fit to the condensed fraction, while the dashed line is a fit to the thermal cloud. The images are taken after 10.3 ms expansion time.

Quadrupole shape oscillations observed for these condensates (Figure 6.8) have a much higher amplitude, with the oscillations also visible in the radial direction. The difference in damping time between oscillations shown in Figure 6.8 and Figure 6.5 is spectacular. While the oscillations for small condensate fractions are almost critically damped, the damping time of the giant oscillations of the pure condensates is as long as 140 ms, with the small-amplitude oscillations exhibiting even longer damping times.
One can also observe strong non-equilibrium phase fluctuations in these condensates. Exhibited in the form of interference stripes in the beginning (see Figure 6.9 (a) & (c)), they are being damped out over the course of a few hundred milliseconds. These stripes show a higher contrast as compared to those displayed in Figure 6.6 due to the near absence of the thermal cloud in the trap. For comparison in Figure 6.9 (b) & (d) we also give an absorption picture and an optical density cross-section for a normal condensate produced with quasi-static ramping of the evaporation barrier. The analytical description of the condensates produced with such fast RF ramps is much more difficult as compared to the truncation method discussed above, and their use is limited to specific cases where a detailed analytical picture is not required.