Bose-Einstein condensation in low-dimensional trapped gases
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Chapter 1 Introduction

1.1 Background

Nature provides us with three spatial coordinates to describe various phenomena which happen in our daily life. We often do not use all the coordinates as it is sufficient to build a description or a model in 2-dimensional or even in 1-dimensional terms using either symmetry or necessity considerations. Low-dimensional systems were always used as textbook examples aimed to make the description of phenomena technically easier and the phenomena themselves physically transparent. This does not mean, however, that all low-dimensional problems are only poor dependent relatives of the rich family of colorful and challenging 3D physics. Dilute low-dimensional gases present an example of many-body systems that stand by themselves and for which the dimensionality is essential.

Bose-Einstein condensation (BEC) is a quantum statistics phenomenon which occurs in a 3D system of bosons (particles with an integer spin) when the characteristic thermal de Broglie wavelength of particles exceeds the mean interparticle separation. Under this condition it is favorable for particles to occupy a single ground state, and below a critical temperature the population of this state becomes macroscopic. This phenomenon has been predicted as a result of the work of Bose [1] and Einstein [2] in the mid twenties. Since that time, a number of phenomena have been considered as manifestations of BEC: superfluidity in liquid helium, high-$T_c$ superconductivity in some materials, condensation of hypothetical Higgs particles, BEC of pions and so on. Bose-Einstein condensation in dilute gases has been observed in 1995 in pioneering experiments with clouds of magnetically trapped alkali atoms at JILA [3], MIT [4] and RICE [5].

Since then the field of ultracold quantum gases has developed from the point of proof of principle to a mature field, and hundreds of BEC experiments with different atoms, atom numbers, temperatures, interatomic interactions and trapping geometries have been performed. Many experiments focus attention on creating quasi2D and quasi1D trapped gases by tightly confining the particle motion to zero point oscillations in one or two directions. Then, kinematically the gas is 2D or 1D, and the difference from purely 2D or 1D gases is only related to the value of the interparticle interaction which now depends on the tight confinement. The presence of a shallow confinement in the other direction(s) allows one to speak of a trapped 1D(2D) gas. Recently, several groups have realized quasi2D and quasi1D regimes for trapped condensates [6; 7; 8; 9]. One-dimensional systems of bosons become especially interesting in view of ongoing experiments on atom lasers and atom chip interferometry. These experiments deal with very elongated atomic clouds where the problem of phase coherence is of fundamental importance.

Uniform purely 1D and 2D many-body systems are relatively well understood and described. However, the presence of the trapping potential adds new features to the well-known problems and attracts our attention to the low-dimensional systems again. The trapping geometry introduces a finite size of the system and significantly modifies the
structure of energy levels. The effective interparticle interaction plays an important role in low dimensions and in trapped gases it strongly depends on the tight confinement. A number of questions arise naturally: How do these features modify the well-known conclusions about the BEC phase transition, superfluidity and phase coherence properties of the low-dimensional quantum gases? What are the regimes of quantum degeneracy in these novel low-dimensional systems?

Trying to find answers to these questions we rely on a long prehistory of the subject. In uniform low-dimensional Bose systems the true BEC and long-range order are absent at finite temperatures [10; 11]. At sufficiently low temperatures the density fluctuations are suppressed as in the 3D case. However, long-wave fluctuations of the phase of the boson field provide a power law decay of the density matrix in 2D and an exponential decay in 1D (see [12] for review). The characteristic radius of phase fluctuations exceeds the healing length, and locally the system is similar to a true condensate. This state of the system is called quasicondensate or condensate with fluctuating phase (see [13]).

The presence of the quasicondensate is coupled to the phenomenon of superfluidity. In uniform 2D systems one has the Kosterlitz-Thouless superfluid phase transition associated with the formation of bound pairs of vortices below a critical temperature. The Kosterlitz-Thouless transition has been observed in monolayers of liquid helium [14; 15]. Recently, the observation of KTT in the 2D gas of spin-polarized atomic hydrogen on liquid-helium surface has been reported [16]. BEC of excitons – electron-hole bound pairs in semiconductors – has been discussed for a long time (see [17; 18; 19; 20] for review). There are now promising prospects for reaching quantum degeneracy in quasi2D trapped exciton gases [21; 22; 23; 24]. Bose-Einstein condensation of composite bosons formed out of tightly bound pairs of electrons (bipolaron mechanism) is one of the explanations of the high critical temperature in superconducting copper-oxide layers and other HTSC materials [25]. Non-BCS mechanisms of superconductivity have been reported in quasi1D wires [26].

From a theoretical point of view, the situation in dilute gases is unique compared with liquids or HTSC materials. In a gas the characteristic radius of interaction between particles, $R_e$, is much smaller than the mean interparticle separation, giving rise to a small parameter (gaseous parameter). In the ultra-cold limit, collisions are dominated by the $s$-wave scattering and the scattering amplitude is characterized by a single parameter of the dimension of length. In low dimensions this quantity is an analog of the 3D scattering length and replaces the full knowledge of the interaction potential. The existence of the small gaseous parameter allows an \textit{ab initio} theoretical description of gases and makes the physical picture more transparent than in phenomenological theories of liquids.

1.2 This Thesis

In this Thesis we develop a theory for describing regimes of quantum degeneracy and BEC in ultra-cold low-dimensional trapped gases. The main emphasis is put on the phase coherence and on the role of interparticle interaction in trapped degenerate gases. The results of the Thesis are linked to the ongoing and future experimental studies.

In Chapter 2 we introduce fundamental concepts related to BEC in low dimensions and give a brief overview of literature on low-dimensional quantum gases.
In Chapter 3 we investigate the interparticle interaction in the quasi2D regime and discuss the influence of the tight confinement on the properties of degenerate quasi2D Bose and Fermi gases. The first section is dedicated to the mean-field interaction in quasi2D Bose gases. We show that it is sensitive to the strength of the tight confinement, and one can even switch the sign of the interaction by changing the confinement frequency. We find that well below the BEC transition temperature $T_c$ the equilibrium state of a quasi2D Bose gas is a true condensate, whereas at intermediate temperatures $T < T_c$ one can have a quasicondensate.

After the recent progress in cooling 3D Fermi gases to well below the temperature of quantum degeneracy, achieving superfluidity is a challenging goal and it may require much lower temperatures. In the second section we show that quasi2D atomic Fermi gases are promising for achieving superfluid regimes: the regime of BCS pairing for weak attraction between atoms, and the regime of strong coupling resulting in the formation of weakly bound quasi2D bosonic dimers. For the BCS limit, we calculate the transition temperature $T_c$ and discuss how to increase the ratio of $T_c$ to the Fermi energy. In the other extreme, we analyze the stability of a Bose condensate of the quasi2D dimers.

Active studies on achieving the quasi2D regime [27; 28; 29; 30; 31; 6; 7] for cold gases raise a subtle question of how 3D collisions and interactions transform into 2D. In a nondegenerate gas, the crossover from 3D to 2D should take place when the level spacing for the tightly confining potential becomes comparable with the mean kinetic energy of particles. In Chapter 4 we develop a theory of pair interatomic collisions at an arbitrary energy in the presence of tight confinement in one direction. We identify regimes in which collisions can be no longer regarded as three-dimensional and the 2D nature of the particle motion is important. We describe the collision-induced energy exchange between the axial and radial degrees of freedom and analyze the recent experiments on kinetic properties, thermalization and spin relaxation in a tightly confined gas of Cs atoms [27; 28; 29; 30].

Chapter 5 is dedicated to quasi1D trapped Bose gases. We discuss the regimes of quantum degeneracy and obtain the diagram of states for this system. We identify three regimes: the BEC regimes of a true condensate and quasicondensate, and the regime of a trapped Tonks gas (gas of impenetrable bosons). We show that the presence of a sharp cross-over to the BEC regime requires extremely small interaction between particles.

The phase coherence in a degenerate 3D Bose gas depends on the geometry. If the sample is sufficiently elongated, long-wave thermal fluctuations of the phase in the axial direction acquire 1D character and can destroy BEC. In Chapter 6 we show that in very elongated 3D trapped Bose gases, even at temperatures far below the BEC transition temperature $T_c$, the equilibrium state is a quasicondensate. At very low temperatures the phase fluctuations are suppressed and the quasicondensate turns into a true condensate.

A simple and efficient method of observing phase fluctuations in an elongated trapped Bose-condensed gas has been recently demonstrated in Hannover. The method relies on the measurement of the density distribution after releasing the gas from the trap. In the second section of Chapter 6 we analyze this experiment and show how the phase fluctuations in an elongated BEC transform into the density modulations in the course of free expansion.