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**DOI**

[10.1103/PhysRevA.55.3918](https://doi.org/10.1103/PhysRevA.55.3918)

**Publication date**

1997

**Published in**

Physical Review A

[Link to publication](#)

**Citation for published version (APA):**

de Jong, F. B., Spreeuw, R. J. C., & van Linden van den Heuvell, H. B. (1997). Quantum Zeno effect and V-scheme lasing without inversion. *Physical Review A*, 55, 3918. <https://doi.org/10.1103/PhysRevA.55.3918>

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# Quantum Zeno effect and V-scheme lasing without inversion

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(Received 12 December 1996)

We show that the quantum Zeno effect (QZE) plays an essential role in discussions of lasing without inversion (LWI) in a V-level scheme. We investigate this role using Monte Carlo wave-function simulations, which provide insight into the physical origin of inversionless lasing. The QZE appears as a result of spontaneous emission and the corresponding collapse of the wave function. Whereas the usual explanation of LWI in terms of quantum interference requires *coherence*, the QZE explains why *incoherent* processes are also needed. [S1050-2947(97)06605-5]

PACS number(s): 42.50.Gy, 42.50.Lc, 32.80.Qk

## I. INTRODUCTION

The interest in lasing without inversion (LWI) derives from the potential for short-wavelength lasing and the interesting interplay between coherence in atoms and in light. There has been a large number of theoretical contributions (see overviews in [1–4]) and amplification without inversion was demonstrated experimentally [5–9]. Recently actual *lasing* without inversion in atomic Rb [10] and Na [11] has been reported. In the latter experiment lasing occurred at a frequency slightly higher than that of the driving field. These experiments show that a transition between two incoherently coupled levels can be made to lase by the addition of a coherent driving field that couples one of the levels to a third level.

The physical mechanism of LWI in the V scheme is usually discussed in terms of quantum interference. In this paper we show that, in addition, the quantum Zeno effect is also essential. We find that neither quantum interference nor the quantum Zeno effect is sufficient by itself; both are needed. Whereas for quantum interference one needs *coherence*, for the quantum Zeno effect one needs *incoherent* processes.

In this paper we investigate the role of the quantum Zeno effect by means of Monte Carlo wave-function (MCWF) simulations of the three-level V scheme. The quantum Zeno effect is the suppression of coherent transitions between quantum states due to frequent measurements [12,13] and has been observed experimentally in a V scheme [13]. Other MCWF simulations of the V scheme showing the quantum Zeno effect have been performed in the context of quantum measurement theory [14,15].

## II. V-SCHEME LASING WITHOUT INVERSION

We consider the V scheme as shown in Fig. 1. The transition  $b \leftrightarrow c$  is driven on resonance by a saturating coherent driving field with Rabi frequency  $\Omega_c$ . The laser transition  $a \leftrightarrow b$  is probed by a coherent field with Rabi frequency  $\Omega_a$ . Both excited-state levels are incoherently pumped, with rates  $\lambda_a, \lambda_c$ , from the common ground state and it is assumed that level  $|c\rangle$  decays faster than level  $|a\rangle$  ( $\gamma_c > \gamma_a$ ). Analysis of the Bloch equations shows that there exists a parameter regime for which there is amplification for the probe field without inversion between any combination of

two levels, including the levels of a dressed state basis [16].

An analytical expression for the gain can be obtained but even for resonant coupling this expression is very complex and cannot easily be interpreted physically. The Bloch equation for the population of the excited state  $\rho_{aa}$ , in the rotating frame and using the rotating wave approximation, is

$$\dot{\rho}_{aa} = \lambda_a(\rho_{bb} - \rho_{aa}) - \gamma_a \rho_{aa} - \frac{i}{2} \Omega_a (\rho_{ba} - \rho_{ab}). \quad (1)$$

The probe gain is equal to the decrease of population caused by the coherent coupling with the probe, as represented by the last term in Eq. (1). In the steady state the probe gain per unit time is

$$\Omega_a \text{Im}(\rho_{ab}) = -\gamma_a \rho_{aa} + \lambda_a(\rho_{bb} - \rho_{aa}). \quad (2)$$

This shows that the energy required for the probe gain comes from absorption of the incoherent pump  $\lambda_a$ . Note that in order for the probe to gain energy from the incoherent pump the steady state populations  $\rho_{aa}$  and  $\rho_{bb}$  *cannot* be inverted.

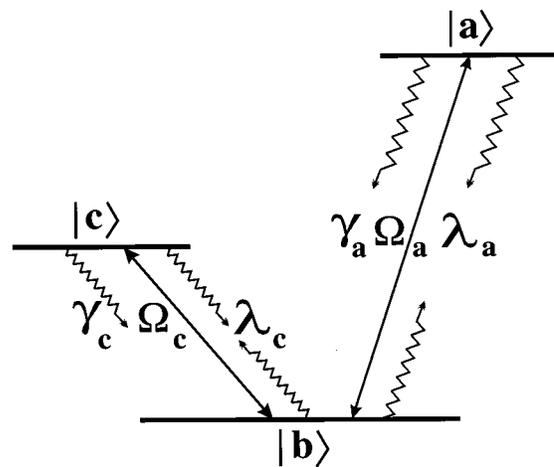


FIG. 1. Driven V-scheme. The zigzag lines indicate the incoherent processes, the straight lines the coherent processes.  $\Omega_a$  and  $\Omega_c$  are the Rabi frequencies of the laser field and strong driving field, respectively,  $\gamma_a$  and  $\gamma_c$  represent the spontaneous decay rates. The incoherent pumping rates of the excited-state levels are given by  $\lambda_a$  and  $\lambda_c$ .

The gain in the V scheme is usually explained in terms of destructive quantum interference between different excitation paths ( $|b\rangle \rightarrow |a\rangle$  and  $|b\rangle \rightarrow |c\rangle \rightarrow |b\rangle \rightarrow |a\rangle$ ) yielding a cancellation of absorption [2]. The strong driving field on the transition  $b \leftrightarrow c$  causes the wave function of the coupled states to change sign with the Rabi frequency  $\Omega_c$ . If the Rabi frequency of the laser field  $\Omega_a$  is small compared to the Rabi frequency of the driving field  $\Omega_c$ , the amplitude of state  $|b\rangle$  changes sign many times during one Rabi cycle of the laser field, thus causing the contributions to the amplitude of state  $|a\rangle$  to cancel each other. This reduces both absorption and emission processes.

Besides quantum interference also the quantum Zeno effect plays an important role. Due to the strong driving field and the fast decay of state  $|c\rangle$ , spontaneous emission from level  $|c\rangle$  is the dominant incoherent process. Each detection of a spontaneously emitted photon collapses the wave function into the ground state  $|b\rangle$ . Frequent such measurements can reduce the probability for slow coherent absorption on the laser transition  $a \leftrightarrow b$ . On the other hand, the inverse, emission process is hardly affected. The quantum Zeno effect thus accounts for the asymmetry between absorption and emission processes which is needed for lasing without inversion. Indeed, the situation considered in this paper is very similar to the experiment by Itano *et al.* [13].

Frequent measurements can also be caused by the incoherent pump between  $|b\rangle$  and  $|c\rangle$ . In this context we can characterize the quantum Zeno regime by the condition that the incoherent processes on the transition  $b \leftrightarrow c$  are stronger than the coherent and incoherent processes on the transition  $a \leftrightarrow b$ , i.e.,  $\gamma_c + \lambda_c \gg \lambda_a, \gamma_a, \Omega_a$ .

### III. MONTE-CARLO WAVEFUNCTION SIMULATION

The Monte Carlo wave-function technique [17] is ideally suited as a tool to study the quantum Zeno effect. The latter is the result of frequent wave-function collapses due to measurements, which in the MCWF technique take the form of quantum jumps [14,15]. Whereas the Bloch equations describe an ensemble of atoms, the MCWF method calculates the time evolution of the wave-function of a single atom, a so-called quantum trajectory. We use here the method as introduced by Dalibard *et al.* [17]. The trajectory is calculated by integrating the time-dependent Schrödinger equation using a non-Hermitian effective Hamiltonian. Incoherent processes such as spontaneous emission and incoherent pumping are incorporated as random quantum jumps that cause a collapse of the wave function to a single state. Averaging over many realizations of the trajectories reproduces the ensemble results of the Bloch equations.

Using the rotating-wave approximation and transforming to the rotating frame, the effective Hamiltonian is

$$H_{\text{eff}} = \hbar \begin{pmatrix} -\frac{i}{2}(\lambda_a + \gamma_a) & \frac{1}{2}\Omega_a & 0 \\ \frac{1}{2}\Omega_a & -\frac{i}{2}(\lambda_a + \lambda_c) & \frac{1}{2}\Omega_c \\ 0 & \frac{1}{2}\Omega_c & -\frac{i}{2}(\lambda_c + \gamma_c) \end{pmatrix}, \quad (3)$$

where the Rabi frequencies  $\Omega_c$  and  $\Omega_a$  describe the coherent couplings by the strong driving field and laser field, respectively. The incoherent processes are the level decays  $\gamma_c$  and  $\gamma_a$  and the incoherent pumping mechanisms  $\lambda_c$  and  $\lambda_a$ . This Hamiltonian is used to propagate the wave function  $|\phi(t)\rangle = \alpha(t)|\tilde{a}\rangle + \beta(t)|\tilde{b}\rangle + \gamma(t)|\tilde{c}\rangle$ , where the tilde refers to the rotating frame. This coherent evolution is interrupted by the incoherent processes, which generate four types of quantum jumps with rates,

$$R_{a \rightarrow b}(t) = (\gamma_a + \lambda_a)|\alpha(t)|^2, \quad R_{b \rightarrow a}(t) = \lambda_a|\beta(t)|^2, \quad (4)$$

$$R_{c \rightarrow b}(t) = (\gamma_c + \lambda_c)|\gamma(t)|^2, \quad R_{b \rightarrow c}(t) = \lambda_c|\beta(t)|^2.$$

During a time  $dt$  the probability for a quantum jump is  $R_{\text{tot}}(t)dt$ , with  $R_{\text{tot}}(t)$  the sum of all rates. We propagate the wave function  $|\phi(t)\rangle$  together with the probability  $P_0(t)$  that no quantum jump has taken place (since the last quantum jump),

$$\frac{d}{dt}|\phi(t)\rangle = -\frac{i}{\hbar}H_{\text{eff}}|\phi(t)\rangle \quad (5)$$

$$\frac{dP_0(t)}{dt} = -R_{\text{tot}}(t)P_0(t), \quad P_0(0) = 1,$$

where the last jump occurred at  $t=0$ . After each time step the wave function is renormalized. If  $P_0$  decreases below a random number between 0 and 1 we implement a random quantum jump. The coherent evolution continues with a new initial state dictated by the wave-function collapse.

The average over many realizations of  $\langle i|\phi(t)\rangle\langle\phi(t)|j\rangle$  starting from  $|\phi(0)\rangle$  reproduces the solution of the Bloch equations,  $\rho_{ij}(t)$ , with initial values  $\rho_{ij}(0) = \langle i|\phi(0)\rangle\langle\phi(0)|j\rangle$ . The gain is equal to the average number of photons that the probe gains per unit time. This quantity can be calculated directly from the quantum-jump counts in a MCWF simulation or from the time average of  $\langle a|\phi(t)\rangle\langle\phi(t)|b\rangle$  as we shall see in the next section.

### IV. QUANTUM ZENO EFFECT

In Fig. 2(a) we show part of a quantum trajectory for the excited state probability  $|\alpha(t)|^2$ , using typical parameters for which LWI occurs. In order to simplify the analysis we have set  $\lambda_c=0$ . In this case only three types of quantum jumps are possible:  $b \rightarrow a$ ,  $a \rightarrow b$ , and  $c \rightarrow b$ , indicated by +, -, and \*, respectively. The dominant incoherent process (\*) is the decay on the transition  $b \leftrightarrow c$ , indicating that we are in the quantum Zeno regime. We see that when  $|\alpha|^2 \approx 0$  the coherent buildup of excited state probability  $|\alpha|^2$  is frequently interrupted by the quantum jumps from  $c \rightarrow b$  (\*). On the other hand, when  $|\alpha|^2 \approx 1$ , the coherent evolution can proceed for longer time without being interrupted. Thus the frequent measurements on the transition  $b \leftrightarrow c$  (by detecting the spontaneously emitted photons) suppress the coherent absorption on the transition  $a \leftrightarrow b$ . This is the quantum Zeno effect.

According to Eq. (2) the probe gain is equal to the jump rate for  $b \rightarrow a$  (+) minus the jump rate for  $a \rightarrow b$  (-). The difference in jump counts is equal to the number of photons

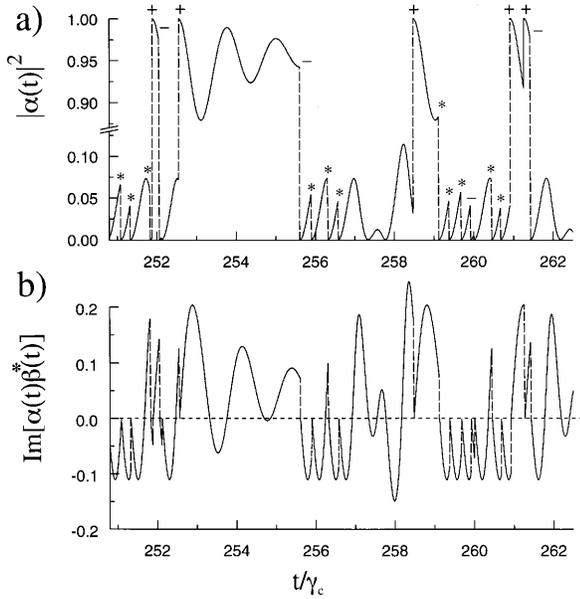


FIG. 2. Quantum trajectories: (a) The excited-state population  $|\alpha(t)|^2$  as a function of time as simulated by the Monte Carlo technique. Note that the vertical scale has been interrupted. The quantum jumps into  $|a\rangle$  ( $|\alpha|^2 \rightarrow 1$ ) are caused by the incoherent pump on the transition  $a \leftrightarrow b$  and are marked by a “+.” The jumps into  $|b\rangle$  ( $|\alpha|^2 \rightarrow 0$ ) are caused either by decay from  $|a\rangle$ , marked with a “-” or by decay from  $|c\rangle$ , marked with a “\*.” The differential count of + and - jumps gives the number of photons gained by the probe. (b) The imaginary part of the instantaneous coherence on the transition  $a \leftrightarrow b$ . The parameters for this simulation are  $\Omega_a = 1$ ,  $\gamma_a = 0.1$ ,  $\lambda_a = 0.5$ ,  $\Omega_c = 5$ ,  $\gamma_c = 2$ , and  $\lambda_c = 0$ .

that the laser field gains. Thus, in this particular part of the quantum trajectory one photon is added to the laser field. Averaging the trajectory over a long time reproduces the gain predicted by the Bloch equations.

For comparison we also show the imaginary part of the instantaneous coherence  $\alpha(t)\beta(t)^*$  for the same trajectory, Fig. 2(b). According to Eq. (2) the gain is proportional to the time average, which is positive because the loss processes (below the dotted line) are more frequently interrupted than the gain processes (above the dotted line).

Calculations for the steady state populations show that there is no population inversion for these parameters. We have thus found the quantum Zeno effect at work under the typical gain condition of Fig. 2. In the next section we show that the quantum Zeno effect is a necessary condition for gain.

## V. GAIN IMPLIES QUANTUM ZENO EFFECT

The gain on the laser field is associated with a coherent evolution from  $|a\rangle$  to either  $|b\rangle$  or  $|c\rangle$ . Such coherent processes are recognized in the jump record as a jump  $b \rightarrow a$ , followed by a coherent evolution interrupted by either another jump  $b \rightarrow a$  or a jump  $c \rightarrow b$ . Following [18] two different gain processes can be identified from these jump sequences as is illustrated in Fig. 3. A coherent evolution starting in  $|a\rangle$  and ending in  $|b\rangle$  represents the stimulated emission process. If a coherent evolution that started in  $|a\rangle$  ends in  $|c\rangle$  one driving field photon is absorbed and one laser

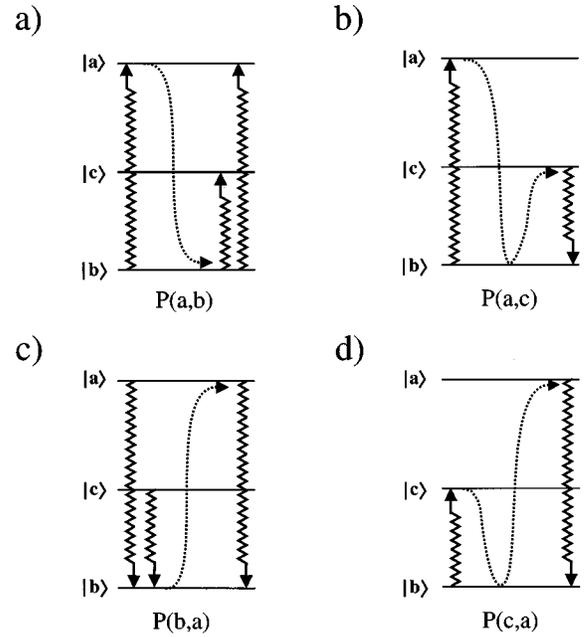


FIG. 3. Gain and loss processes: (a) Stimulated emission process. A jump  $b \rightarrow a$  is followed by another jump  $b \rightarrow a$  or a jump  $b \rightarrow c$ . The coherent evolution between these jumps started in  $|a\rangle$  and ended in  $|b\rangle$ , thereby gaining one laser photon. (b) Two-photon Raman process: a jump  $b \rightarrow a$  is followed by a jump  $c \rightarrow b$ . Thus the coherent evolution took the system from  $|a\rangle$  to  $|c\rangle$ , thereby emitting one laser photon and absorbing one photon from the driving field. (c), (d) are the inverse, and loss processes of (a) and (b).

photon is gained. This is a two-photon Raman process. The reverse processes are also possible and result in a decrease in the number of laser photons. For the probability that a coherent evolution randomly selected from a quantum trajectory starts in  $|i\rangle$  and ends in  $|j\rangle$  we will use the notation  $P(i,j)$  from Ref. [18]. The gain exceeds absorption if

$$P(a,b) + P(a,c) > P(b,a) + P(c,a). \quad (6)$$

In order to show that gain implies the quantum Zeno effect, we will now give a formulation of the quantum Zeno effect in terms of conditional probabilities. The probability for a particular process  $P(i,j)$  can be written as the probability  $P(i)$  to start a coherent evolution in  $|i\rangle$  times the conditional probability  $P(j|i)$  that, given that a coherent evolution started in  $|i\rangle$ , it will end in  $|j\rangle$ :

$$P(i,j) = P(i)P(j|i). \quad (7)$$

The quantum Zeno effect states that if coherent evolutions that start in  $|b\rangle$  or  $|c\rangle$  [which contributes to absorption, Fig. 2(b)] are interrupted frequently, the absorption ( $b \rightarrow a$ ) is reduced. This means that the conditional probability that, given that a coherent evolution started in  $|b\rangle$  or  $|c\rangle$ , a laser photon will be absorbed is small. Coherent evolutions starting in  $|a\rangle$  do not suffer these frequent interruptions, so the conditional probability that, given that a coherent evolution started in  $|a\rangle$ , a laser photon will be gained is larger. We can now express the quantum Zeno regime in terms of conditional probabilities:

$$P(b|a) + P(c|a) > \frac{P(b)}{P(b) + P(c)} P(a|b) + \frac{P(c)}{P(b) + P(c)} P(a|c). \quad (8)$$

On the left-hand side we recognize the conditional probability that, given that a coherent evolution started in  $|a\rangle$ , it will end in  $|b\rangle$  or  $|c\rangle$  (thus gaining a laser photon). The right-hand side represents the conditional probability that, given that a coherent evolution started in  $|b\rangle$  or  $|c\rangle$ , it will end in  $|a\rangle$  (thus absorbing a laser photon). Note that on the right of the inequality the conditional probabilities  $P(a|b)$  and  $P(a|c)$  are weighted by the relative probabilities to start in  $|b\rangle$  or  $|c\rangle$ .

To show that the gain condition implies that the system is in the quantum Zeno regime we rewrite the gain condition Eq. (6) in terms of conditional probabilities using Eq. (7) and divide by  $P(a)$ :

$$P(b|a) + P(c|a) > \frac{P(b)}{P(a)} P(a|b) + \frac{P(c)}{P(a)} P(a|c). \quad (9)$$

The comparison of Eqs. (9) and (8) shows that if  $P(a) < P(b) + P(c)$  the gain condition implies the quantum Zeno regime. The probabilities  $P(a)$  and  $P(c)$  obey the simple relation

$$\frac{P(a)}{P(c)} = \frac{\lambda_a}{\lambda_c}, \quad (10)$$

which results from the fact that in order to start in  $|a\rangle$  or  $|c\rangle$  the system must have jumped from  $|b\rangle$ . So for  $\lambda_c < \lambda_a$  the gain condition implies that the system is in the quantum Zeno regime; for  $\lambda_c < \lambda_a$ , Eq. (6)  $\Rightarrow$  Eq. (8).

The Bloch equations show that increasing the incoherent pump  $\lambda_c$  will reduce the gain. Moreover, increasing  $\lambda_c$  will lead the system further into the quantum Zeno regime. This can be recognized from the ratio of reciprocal conditional probabilities  $P(j|i)/P(i|j)$ , which is equal to the ratio of incoherent processes responsible for leaving a coherent evolution from level  $|i\rangle$  and  $|j\rangle$  (see Appendix):

$$\frac{P(b|a)}{P(a|b)} = \frac{\lambda_a + \lambda_c}{\lambda_a + \gamma_a}, \quad \frac{P(c|a)}{P(a|c)} = \frac{\lambda_c + \gamma_c}{\lambda_a + \gamma_a}. \quad (11)$$

Therefore, by increasing  $\lambda_c$  we will eventually violate the inequality (6) for the gain, but not the inequality (8) for the quantum Zeno effect. The presence of the quantum Zeno effect is therefore a necessary but not sufficient condition for gain, for all values of  $\lambda_c$ . The reason that it is not sufficient is that also quantum interference is essential for gain.

By ignoring all incoherent processes on the transition  $b \leftrightarrow c$  ( $\lambda_c = \gamma_c = 0$ ), we have also studied the effect of quantum interference without the presence of the quantum Zeno

effect. In this case only the coherent evolutions  $P(a,b)$  and  $P(b,a)$  contribute to gain or absorption. The conditional probability for the emission process is smaller than for the absorption process [see Eq. (11)] and also the probability to start in  $|a\rangle$  is smaller than the probability to start in  $|b\rangle$ . Therefore the gain condition is not satisfied. Apparently, both the quantum Zeno effect and quantum interference are essential for inversionless lasing in the V scheme.

## VI. CONCLUSION

We have analyzed the three-level V scheme for lasing without inversion, using the Monte Carlo wave-function simulation method, which has allowed us to identify the quantum Zeno effect. This gives a new intuitive perspective on lasing without inversion, in particular on the need for a sufficiently high spontaneous emission rate. We have shown that in the quantum Zeno regime an asymmetry exists between conditional probabilities for emission and absorption processes, and that inversionless lasing can only occur in the quantum Zeno regime. We have performed simulations for parameter regimes in which either quantum interference or the quantum Zeno effect were absent and conclude that only the combination of the two can result in inversionless lasing.

## ACKNOWLEDGMENTS

The work in this paper is part of the research program of the ‘‘Stichting voor Fundamenteel Onderzoek van de Materie’’ (Foundation for Fundamental Research on Matter) and was made possible by financial support from the ‘‘Nederlandse Organisatie voor Wetenschappelijk Onderzoek’’ (Netherlands Organization for the Advancement of Research). The research of R.S. has been made possible by financial support from the Royal Netherlands Academy of Arts and Sciences.

## APPENDIX

Following Ref. [18] the conditional probabilities are given by

$$P(j|i) = \int_0^\infty G_j |c_{ij}(\tau)|^2 d\tau, \quad (12)$$

with  $G_j$  the total rate of incoherent processes out of  $|j\rangle$  and

$$c_{ij}(\tau) = \langle j | e^{-iH_{\text{eff}}\tau} | i \rangle. \quad (13)$$

For an effective Hamiltonian such that  $(H_{\text{eff}})^* = (H_{\text{eff}})^\dagger$  it can be shown that  $c_{ij}(t) = c_{ji}(t)$ , giving

$$\frac{P(j|i)}{P(i|j)} = \frac{G_j}{G_i}. \quad (14)$$

Substituting  $G_a = \lambda_a + \gamma_a$ ,  $G_b = \lambda_a + \lambda_c$ , and  $G_c = \lambda_c + \gamma_c$  gives Eq. (11).

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