Chapter 3

Social Interaction in a Spatial Voting Model

3.1 Introduction

The best known model of electoral competition is the spatial competition model, where the preferred policy position of each voter is modelled as a point in some kind of policy or issue space. In elections each voter casts his vote for that party whose policy platform is closest (e.g. in terms of weighted Euclidean distance) to his own ideal position. The implicit assumption is that each voter is able to evaluate the consequences of all policy positions and has a (complete and transitive) preference ordering over all these positions. Given the distribution of preferences of the population of voters, political parties select platforms that generate the maximum expected number of votes. In the basic spatial competition model it is assumed that: i) voters have stable preferences on the issue space and ii) political parties have complete information about the distribution of the ideal positions of voters. These assumptions are very demanding. We can hardly expect a political candidate to know the exact distribution of preferences. Furthermore, it seems that the ideal points of most voters are determined partly by interaction with other voters. In particular, voters may be unaware of the exact consequences of different policy positions and moreover, there may be some social pressure to adopt certain positions.

\footnote{This chapter is based on Sadiraj, Tuinstra and van Winden (2000).}
CHAPTER 3. SOCIAL INTERACTION IN A SPATIAL VOTING MODEL

on some political issues. In fact, voting behavior is typically subject to hypes and herd behavior. An issue related to this social interaction, but largely ignored in spatial voting theory, is the organization of voters into interest groups. These interest groups often play an important role in the competition between political candidates.

In this chapter we propose spatial models of electoral competition that relax the assumptions regarding the stability of voters' preferences and the complete information and full rationality of political agents. Furthermore, we allow for the endogenous emergence of interest groups. These interest groups coordinate the voting behavior of boundedly rational voters and influence the information of political parties about voter preferences. Thus, our model integrates both types of models that exist in the literature of interest groups that concentrate on campaign contributions, exchange (affecting policy choices) and support (affecting election outcomes) models.¹ Our work is related to Kollman, Miller and Page (1992, 1998) where the assumption that political candidates exactly know the distribution of voters' preferences is relaxed. Instead, political candidates are assumed to experiment with different policy positions in order to find the position where the probability of winning is the highest. An important motivation for their research was to investigate the relevance of the so-called "chaotic" results, i.e. the fact that for multi-dimensional issue spaces, in general, the challenger can always find a policy position that defeats the incumbent (see e.g. McKelvey (1976, 1979), Schofield (1978)). In Kollman, Miller and Page (1992) the platforms of parties converge to moderate platforms, therefore mitigating these "chaos" results. This chapter addresses a similar research question. It presents agent-based models that aim to capture the nature of individual voting behavior. First we introduce interest groups, in which voters are organized that feel strongly about certain positions on certain issues. These interest groups interact with the election procedure by, on the one hand, helping political parties in exploring certain parts of the electoral landscape by financing polls and, on the other hand, influencing the interest group members in how to cast their vote in the upcoming election. Secondly, we present a dynamic model of political opinions. Voters preferences are not independent of each other. Voters interact and influence each other in their preferred policy positions. The social interactions may

¹See van Winden (1999) for a survey of formal models of interest groups.
3.2. ADAPTIVE POLITICAL PARTIES

lead to clusters of political opinions. In the model, which integrates both clustering and interest group phenomena, the size and importance of the interest groups is determined endogenously and depends upon the interaction structure within the population of voters and on the election outcomes. These election outcomes are again influenced by the sizes and positions of the interest groups.

We will try to provide an answer to some of the following questions. Will policy platforms converge to some point in the policy space? Will there be convergence of the policy platforms of the two political parties? Will the probability that the challenger wins the election be close to 1 (as the “chaos” result predicts) or close to \( \frac{1}{2} \) (as the median voter model predicts)? And in particular, what influence does the social interaction process and the emergence of interest groups have upon these convergence properties? The models presented here are a first step in trying to resolve these issues in a framework with social interactions and interest groups. In this chapter, we focus on simulation methods to explore what kind of behavior emerges. For the model with interest groups that will be introduced in Section 3.3, Chapters 4 and 5 will offer a more rigorous approach.

This chapter is organized as follows. In Section 3.2 we discuss the adaptive political party models following Kollman, Miller and Page (1992). In Section 3.3 we present a model with endogenous interest groups and investigate their influence on the election outcome. Section 3.4 constructs a dynamic model of political opinions. In Section 3.5 the dynamics of the model that integrates both interest groups and social interactions is investigated. Section 3.6 concludes.

3.2 Adaptive political parties

Let there be a population of \( N \) voters, indexed \( j = 1, \ldots, N \) and let there be \( I \) issues, indexed \( i = 1, \ldots, I \). For each issue there are \( K_i \) different positions, indexed \( k = 1, \ldots, K_i \). The policy or issue space is \( \mathcal{X} = \{1, \ldots, K_1\} \times \ldots \times \{1, \ldots, K_I\} \). Voters are associated with an ideal point \( x \in \mathcal{X} \) and their utility with respect to a certain policy outcome \( y \in \mathcal{X} \) is given by the negative of the (weighted) Euclidean distance between this policy outcome.
CHAPTER 3. SOCIAL INTERACTION IN A SPATIAL VOTING MODEL

and their ideal point, i.e., utility for voter $j$ of policy outcome $y$ is given by

$$u_j(y) = - \sum_{i=1}^{I} s_{ji} (x_{ji} - y_i)^2,$$

where $s_{ji} \in S = \{s^1, s^2, \ldots, s^L\}$ is nonnegative and corresponds to the weight or strength that voter $j$ attaches to issue $i$. Each voter $j$ is therefore identified by a $2I$-tuple $(x_{j1}, s_j) = (x_{j1}, x_{j2}, s_{j1}, s_{j2}) \in X \times S^I$. If a voter can choose between two different political candidates he votes for the candidate which has a platform that generates the highest utility.

A configuration of voters is generated as follows. First, for each voter $j$, an ideal point $(x_{ji})_{i \in I}$ is generated by independently drawing $x_{ji}, i = 1, \ldots, I$, from the discrete uniform distribution on $1, 2, \ldots, K_i$. Subsequently, the strengths for agent $j$ on issue $i$ ($s_{ji}$) are independently drawn from a discrete distribution on $S$.

Given the initial configuration of voters an electoral landscape is constructed as follows. There are two political parties entering the election, the incumbent and the challenger. We assume that the incumbent does not change its policy from the previous period. Each voter votes for the political candidate yielding him the highest utility as given by (3.1). In case this utility is the same for both political candidates the voter votes with probability $\frac{1}{2}$ for the challenger and with probability $\frac{1}{2}$ for the incumbent. For every position $z$ in the issue space the height of the electoral landscape can now be determined as the expected fraction of voters voting for the challenger, if it selects position $z$. This implies that at every position where the height of the landscape is above $\frac{1}{2}$ the challenger is expected to win the election.

We assume political parties are only interested in getting elected and not interested in the policy outcome, i.e., they view policy as a means to winning. As Downs (1957, p.28) puts it, “Parties formulate policies in order to win elections, rather than win elections in order to formulate policies.” Therefore the objective of the challenger is to find the

2 Alternatively, one could consider the case where strengths on issues where a voter takes a more extreme position will on average be higher than the strengths on issues where the voter takes a more moderate position (see Kollman, Miller and Page (1998)).

3 It is, however, possible to extend the model to partisan political parties that are also interested in the policy outcome.
positions of the high points of the landscape. It is also consistent with a situation where candidates are interested in maximizing the number of seats in the legislature. The problem of the challenger therefore reduces to a search problem: it has to find the optimum of some complicated nonlinear function (i.e. the electoral landscape). We assume that the challenger applies random search, that is, the challenger randomly selects a number of positions in the issue space and tests them against the incumbent's policy using a poll. Such a poll consists of a randomly drawn subsample of the population of voters. The challenger observes the fraction of the poll that favors its policy over the incumbent's policy and uses this as an estimate for the true height of the landscape at that position. If there are platforms with a polling result of at least $\frac{1}{2}$ then the challenger chooses the position with the highest polling result; otherwise, it chooses the incumbent position since in that case half of voters are expected to vote for the challenger. Subsequently, the election takes place and the candidate that wins will be the incumbent for the next period. This procedure is repeated for each election that follows.

Let us now discuss some simulations with this basic model. For all simulations in this chapter we use the following specification of the model. We have two issues ($I = 2$), with 5 positions per issue ($K_1 = K_2 = 5$) and three different strengths per issue, characterizing indifference, moderate importance and high importance attached to an issue ($S = \{0, \frac{1}{2}, 1\}$). Furthermore, we consider 20 consecutive elections and work with a population of $N = 301$ voters. For each poll 10% of the population is sampled. The choice to consider only two issues is less restrictive than it might seem. There is some

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4Kollman, Miller and Page (1992) also consider more sophisticated search procedures such as hill-climbing and genetic algorithms. Each of these search procedures has certain characteristics that correspond to particular ways in which parties might select platforms or candidates. However, the results are qualitatively the same for the different procedures.

5The fraction of voters in the poll who favor the challenger at a certain position seems to be a sensible estimator for the altitude of the electoral landscape at that position. Let the altitude of the electoral landscape at a certain position be $p$. Then randomly drawing (without replacement) $n$ voters out of the population of $N$ voters, the fraction of voters in the poll who favor the challenger has mean $p$ and variance $\frac{N-n}{N-1} \frac{p(1-p)}{n}$ (see e.g. Ross (1993, p.52)).

6For an argument on parties' incentives in adopting vote-maximizing versus marginal-winning positions see Kramer (1977).
empirical evidence that politics takes place in a low-dimensional space (see e.g. Poole and Rosenthal (1991), who show that at most two dimensions are necessary to explain the roll call voting behavior of members of Congress in the U.S.).

Figure 3.1 shows the value of three different measures describing the outcomes of the model averaged over 100 trials. For each run a new configuration of voters is drawn. There are 20 elections with either 2 or 8 polls per election. The first measure (the solid lines in Figure 3.1(a)) corresponds to the Euclidean distance between the incumbent (that is, the policy position of the candidate that wins the election) and the center of the distribution of voter preferences. In this chapter we refer to the center as the policy position that is constructed in the following way. For each issue we consider the distribution of ideal points in that issue and determine the (uni-dimensional) median voter in that issue. We then define the center of the whole issue space as that (multi-dimensional) policy position for which each element corresponds to the uni-dimensional median in the corresponding issue. From Figure 3.1(a) it follows that the distance between the election outcome and this center position decreases over time, implying that some kind of median-voter result still applies in our model. Furthermore, more information about the electoral landscape (i.e., an increase in the number of polls), leads to faster convergence. The second measure (broken curves) gives the Euclidean distance between the policy positions of the incumbent and the challenger at each election. The simulations show that the separation between parties' platforms decreases as elections go by. However, running more polls generates a higher separation. The reason is that an increase in the number of polls increases the probability of selecting positions that are close to the incumbent position. For such positions the polling result may be $\frac{1}{2}$ (or higher) due to the stochastic nature of the model. Therefore the challenger might select this neighboring position instead of the incumbent position, which leads to a higher average separation. This also suggests that the challenger's probability of winning will be reduced if he runs more polls. Figure 3.1(b) confirms this. This panel contains observations on the empirical frequency of election victories of the challenger. Indeed, an increase in the number of polls leads to a decrease in the probability of winning the election for the challenger. From the theory on spatial competition we know that if a Condorcet winner exists then under complete
3.2. ADAPTIVE POLITICAL PARTIES

information the probability the challenger wins an election is always equal to $\frac{1}{2}$. However, the simulation shows that in our specification of the model, an increase in the number of polls reduces that probability. For 8 polls, its value is somewhere between 0.2 and 0.3. Summarizing Figure 3.1, the simulation results for our basic model show that policy platforms seem to converge to the center of the distribution of voter preferences.

In Sections 3.3 and 3.4 we will extend the model set out above in two directions. First, in Section 3.3 we will introduce interest groups. Rather than making the standard assumption that these interest groups simply exist and are of fixed size, we model interest groups as endogenously emerging institutions. Interest groups have two functions in our model. First, they are a means to coordinating voting behavior. Secondly, they provide some information about the electoral landscape to the political candidates in order to influence the outcome of the election process. Our approach therefore differs from that theory on interest groups which focuses on lobbying and campaign contributions and uses game theoretic models to describe the interaction between political parties and interest groups. Instead, we focus on the existence of interest groups as a way to transmit information from voters to political candidates (for surveys on the theory of interest groups we refer to Austen-Smith (1997) and van Winden (1999)).

The second direction in which we extend the model is that we model incompletely informed adaptive voters whose behavior might be influenced by the behavior of other voters. The theory on spatial competition assumes voters as well as political parties to be fully rational. This assumption is very restrictive. The model set out above introduces adaptive political parties. In section 3.4 we will extend this model by incorporating incomplete informed and adaptive voters. The full rationality assumption implies that voters’ decisions are determined by their preferences, which are perfectly known by voters. Casual empiricism, however, suggests that voting behavior of most individuals is susceptible to the voting behavior of other agents in the economy. In particular, herd behavior and the occurrence of hypes seem to be the rule rather than the exception. It is therefore evident that social interaction between voters should play a key role in explaining election results (cf. Schram and van Winden (1991)). In this chapter we take a first step in constructing a model that takes these phenomena into account. In particular, we assume...
Figure 3.1: Time series of: (a) different measures; (b) frequencies the challenger wins an election, for the basic model.
3.3. INTEREST GROUPS

that agents' perceptions of the issue space change through social interaction. This leads to endogenous clustering of voters in the policy space. This clustering again can explain the emergence of successful interest groups.

3.3 Interest groups

Voters with identical ideal positions on certain issues may decide to get organized in interest groups in order to play a, hopefully pivotal, role in determining the election outcome. For convenience we will assume that interest groups are only interested in one issue, an extension to multi-issue interest groups is however straightforward.\(^7\) Interest group \(m\) is denoted by \((i^m, k^m)\), where \(i^m \in \{1, 2, \ldots, I\}\) is the issue on which interest group \(m\) focuses and \(k^m \in \{1, 2, \ldots, K_{i^m}\}\), is the (unique) position that this interest group takes on that issue. For example, the interest group characterized by \((i^m, k^m) = (2, 5)\) is an interest group that takes position 5 on issue 2. Notice that there are potentially \(\sum_{i=1}^{I} K_i\) different interest groups. An interest group emerges as follows. The potential members of interest group \(m\) are those voters that have position \(k^m\) on issue \(i^m\). These potential members decide whether to join or not. After this process of interest group formation is over, it is endogenously determined which interest groups become effective. In particular, we assume that this depends upon whether the interest group has collected enough funds to financially support a candidate.

Now let us consider in some more detail the way we model the decision to join an interest group. The main incentive for a potential member to join is that an interest group provides a means to exert some political influence. Intuitively, the larger the interest group, the higher the probability of having some influence, so this incentive is increasing in the number of interest group members. There may also be some positive feedback from the interest group size because one can identify oneself with this interest group. Another benefit of interest group membership is that it reduces the decision-making cost of voting by focusing the attention on one issue. On the other hand, there are some costs of joining

\(^7\)An ideological political party, that is, a party that is more concerned with policy outcomes than with winning the election per se, can in fact be interpreted as an interest group with multi-issue concerns.
an interest group. In particular, joining implies following the interest group's stance on how to cast the vote in the upcoming election, and this might come at a cost in terms of the election outcome for the issues not relevant to that interest group. Furthermore, upon joining a contribution has to be paid. We model the interest group formation process as follows. Voters are drawn in a random order to determine sequentially whether to join some interest groups or not. This procedure is repeated once, so each voter has to decide whether to join or not at most two times. Consider voter \( j \), characterized by \((x_j, s_j) = (x_{j1}, \ldots, x_{jI}, s_{j1}, \ldots, s_{jI})\). There are \( I \) potential interest groups the voter can join, one for each issue. For each relevant interest group \( m \) the number \( v_{jm} \) is determined as

\[
(3.2) \quad v_{jm} = V\left(\frac{n_{jm}^g}{N}, s_{jm}, |k^m - y_{im}|\right)
\]

where \( n_{jm}^g \) is the current number of members of interest group \( m \) (which of course depends upon the decisions made by voters that decided prior to voter \( j \)) and where \( y_{im} \) is the incumbent’s position on the interest group’s issue. The number \( v_{jm} \) corresponds to the (nonpecuniary) benefits of joining the interest group and is positively related to the relative size of the interest group, positively related to the current distance between the voter (and therefore the interest group) and the incumbent on the issue relevant to this interest group and positively related to the weight this voter attaches to this issue.\(^8\) Now for each interest group he joins the voter has to pay a contribution \( c \). Therefore he ranks the different interest groups according to the values of the associated benefits \( v_{jm} \). If the highest value of these benefits exceeds the contribution \( c \), he joins the corresponding interest group, and if it does not, he joins no interest groups at all. If the first interest

\(^8\)For our simulations the function \( V \) is specified as

\[
v_{jm} = s_{jm} (y_{im} - k^m)^2 \exp \left(1 + \frac{n_{jm}^g}{N}\right).
\]

Notice that voters with \( s_{jm} = 0 \) never join. Also observe that in order to get an interest group started at a distance \( d \) from the incumbent the contribution \( c \) should not exceed \( d^2 \exp(1) \) (recall that the interest group formation process starts out with \( n_{jm}^g = 0 \), and that \( s_{jm} \leq 1 \)). In our simulations we take \( c = 1.4 < \exp(1) \), hence expecting interest groups to emerge anywhere. Furthermore, the cost of running a poll (50) is taken such that an interest group could finance at most 1 poll.
3.3. INTEREST GROUPS

group is joined the voter considers the next interest group in the ranking. If its benefits exceed the contribution of the voter also joins this interest group, and so on. Notice that the decision to join or not depends upon the incumbent's position as well. This implies that the farther away the incumbent is from the interest group's position, the bigger the interest group becomes. The idea is that people are more inclined to join an interest group the higher their dissatisfaction with the policy of the current government. Notice that this mechanism is also incorporated in the model from the previous chapter.

In our model interest groups influence the election process in three ways: they coordinate the voting behavior of their members, they provide information about the electoral landscape to the political candidates, and they try to influence policy outcome by imposing conditions on polling. Let us first discuss the information they provide. Each interest group possesses certain funds raised by the contributions of its members. These funds are offered to the challenger conditional on: (i) running a certain number of polls (determined by the total contributions and the cost of running a poll) in policy positions coinciding with the interest group's position on the relevant issue; (ii) commitment of the challenger to select the platform with the highest poll result, provided this result is higher than or equal to $\frac{1}{2}$. The second condition is meant to avoid a sophisticated exploitation of information from the challenger. A nice interpretation of the above procedure is the following. Within the challenging party there is a primary election in order to determine who will challenge the current incumbent. The party itself comes up with some representatives but the interest groups can also support some representatives from the challenging party. Then a primary is held (this corresponds to the polls) and the representative winning this primary becomes the final challenger. The number of polls to be run is determined by the size of the funds of the interest groups and the cost of running a poll (which we take to be equal to 50 in our simulations). This endogenously selects which interest groups are recognized by parties and become effective, all other interest groups remain passive and play no further role in the election procedure.

The voting behavior of the effective interest groups' members is coordinated as follows. Once the two political candidates are known each effective interest group decides which party to support and then all members of the interest group vote for that party (if a voter
is a member of more than one effective interest group that support different candidates, it follows the advice of the interest group that gave him the highest benefit $v_{jm}$). An interest group decides which party to support as follows. If exactly one of the candidates takes the interest group's position on the relevant issue, the interest group supports that party. If one candidate is closer to the interest group's position than the other candidate, the former is supported. If both candidates have the same position as the interest group, or the distance from the interest groups positions on the relevant issue is the same, the interest group members votes according to their own utility, as given by (3.1).

During an electoral campaign, apart from the conditioned platforms where polls are financed by interest groups, the challenger is assumed to run some polls on platforms selected randomly in the issue space (in our simulations, the challenger runs 2 independent polls, each again consisting of a randomly drawn sample of 10% of the population). It then selects the policy position with the best polling result among the positions with an altitude of at least $\frac{1}{2}$. If there are no such polled positions then the incumbent position is selected. All voters organized in interest groups vote for the party supported by the interest group, voters that belong to more than one interest group follow the interest group with the highest value of (3.2), all other voters vote according to the weighted Euclidean distance from the different policy positions to their ideal points, as given by (3.1). The party with the majority of votes wins the election.

Before we study some simulations let us try to develop an intuition for what might happen. We have a population of voters with separable and symmetric preferences, uniformly distributed over the issue space and therefore we expect the generalized median voter to exist. The Generalized Median Voter Theorem (see Hinich and Munger (1997)) says that, once the median is located, no other platform can defeat it. It has already been argued in Section 3.2 that because the probability of choosing any point in the space (the median in particular) is strictly positive, in the limit the median is located with probability 1. Due to the finiteness of our issue space this happens in finite time. Hence, the model predicts that in the long-run the incumbent converges to the median and the challenger has a probability of $\frac{1}{2}$ to win the election. Now consider what happens when we introduce interest groups. Notice that for any voter, membership of interest group $m$ is partly equivalent
with changing the structure of preferences. More explicitly, instead of corresponding to weighted Euclidean distance, preferences become, in some sense, lexicographic: weighted Euclidean distance only becomes important when the interest group's issue is indecisive. The consequence of this is that, given the incumbent's position, the set of policy platforms defeating $y$ is expanded\(^9\) (this is confirmed in Figure 3.7(d)). This is due to the fact that interest groups are more likely to form far away from the incumbent and hence tilt the electoral landscape at the expense of the incumbent. Obviously, this leads to a higher probability for the challenger to win an election. Furthermore, the probability that the median is located at a given election, is higher for the model with interest groups as compared to the basic model. If an interest group indeed emerges at a (uni-dimensional) median position, polls financed by such an interest group have a much higher probability of locating the (multi-dimensional) median ($1/K_1$ versus $1/(K_1 \times \ldots \times K_I)$ for a regular poll). Note that, given the cost of joining and the cost of running a poll, if the location of the incumbent favors the organization of the median voters, the median is located much faster than in the basic model from Section 3.2. On the other hand, if the distribution of voters allows for the formation of interest groups everywhere except at the incumbent position (per issue) then the electoral system can move away from the median and cycles in winning platforms may appear. To illustrate this point consider the case where, once the incumbent is at the median, two groups located on different issues and different from the median organize in interest groups. Then the policy position corresponding to the intersection may, for certain stochastic realizations of voter preferences, defeat the center, only due to the fact that interest groups coordinate voting behavior. The increase in the winning set, combined with the increase in the number of polls, results in a higher probability that the challenger wins the election.

Figure 3.2 shows some distance measures for the simulations we have run. As in the basic model both the distance between parties and the distance between the policy outcome and the median decrease over time. The most notable differences between the basic model and the model with interest groups is that the separation between party platforms for the latter is larger (see Figure 3.7(a)) and that there is no convergence

\(^9\)Chapter 5 provides a proof for this result in a slightly modified version of the present model.
CHAPTER 3. SOCIAL INTERACTION IN A SPATIAL VOTING MODEL

Figure 3.2: Time series of different measures for the model with interest groups (2 random polls and 6 conditioned polls financed by interest groups).

to the center. Also, the frequency of election victories for the challenger shows that the existence of interest groups increases the probability of winning the election for the challenger. Furthermore, in each election the average number of polls financed by interest groups is 6. Summarizing, there are two main consequences of the presence of interest groups: i) an increase in the number of polls; ii) an increase in the size of the winning set. The extra number of polls financed by the interest groups that the challenger can run help him explore the electoral landscape better. This, combined with a larger set of the platforms defeating the incumbent, increases the probability the challenger wins an election. The first effect by itself can not increase the frequency with which the challenger wins significantly, as Figure 3.1(b) shows. It is the combination of both effects that increases the probability of winning.

More results on the influence of interest groups on policy outcomes are shown in Figure 3.3. This figure shows for the basic model and for the model with interest groups, the distance between the policy outcome for the first issue and position 1, 2 and 3 for the first issue. Clearly, the existence of interest groups leads to much more volatile policy
3.4 Social interaction and clusters of political opinions

In the previous section we have seen how voters can interact with political parties via interest groups. In this section we go one step further and investigate interaction between individual voters. This interaction may lead to clustering of voters’ opinions and there-
fore influence the interest group formation process. Suppose we have created an initial configuration of the population of voters as described in Section 3.2. We now want to model the observation that voters can be influenced by other voters.

Our model assumes that voters regularly meet each other and in light of these meetings might reconsider their ideal positions. In particular, if a voter with a certain position on a particular issue learns that there are many people having a different, but close-by position on this issue he may be persuaded to take the latter position. We can identify two reasons for this. The first is that there may be a group externality in the sense that people like to join groups, for example because this gives them the possibility to identify themselves with such a group. This group externality might also arise from peer group pressure or social pressure. The environment voters are living and/or working in may pressure voters into conforming to this environments’ political ideas (Schram and van Winden (1996)). The second reason why voters might change to another ideal position stems from an incomplete information argument. In particular, if voters are not completely certain about how to evaluate the consequences of different policy platforms, they might use the size of the group at positions in their neighborhood to help them in evaluating the implications of these positions. This incomplete information argument seems to be in particular relevant for the competition between different political parties, where it may be difficult for individual voters to judge the merits of (the political statements of) these different parties. Our approach is related to the recent literature on herding behavior (see e.g. Banerjee (1992), Bikchandani, Hirshleifer and Welch (1992) and Kirman (1993)).

In order to model the social interaction we assume that each voter $j$ attaches a certain value to the consequences of changing from a current position to some other position $z$. The decision function for changing has the following general form

$$U_j(z, n_z/N),$$

that is, it depends upon the position $z$ to which the voter considers moving and it depends upon the relative size of the social group residing at that new position. However, it seems reasonable to assume that voters do not know the exact distribution of preferences over the issue space. Instead, we propose that voters meet other voters and are informed about their ideal positions and in this way obtain an informed guess of the distribution
of preferences. We model this as follows. Each voter \( j \) decides whether to reconsider his perception of the issue space or not. This happens with some probability \( \varepsilon \in [0, 1] \), which is the same for all voters. If he reconsiders, he aspires to meet \( \lambda_j \) voters in order to get a “plausible” idea about the existing distribution. Because some voters are more active in meeting other voters than others, we assume that the variable \( \lambda_j \) is drawn from the discrete uniform distribution on \( \{0, 1, 2, \ldots , \Lambda \} \).\(^{10}\) Now voters are drawn in random order to meet these other people. Let \( i_1, i_2, \ldots , i_N \) be the order of these voters, where \( i_1 \) corresponds to the first voter that is drawn. This first voter meets \( \lambda_{i_1} \) voters randomly drawn from the population \( N \setminus \{i_1\} \) and learns their ideal position. Furthermore, it is assumed that these other voters learn his position.\(^{11}\) Now the \( s \)th voter meets \( \lambda_{s}^* \) other voters drawn from the population \( N^*_s \), where \( \lambda_{s}^* = \max \{\lambda_s - \#(\text{voters already met by voter } s), 0\} \) and \( N^*_s = N \setminus \{i_s \text{ and voters already met by voter } s\} \). Notice that people can meet more voters than they originally intended to and therefore \( \lambda_j \) actually corresponds to a lower bound of the number of voters voter \( j \) meets. Each voter now has some private information about the distribution of ideal points. In particular, if we let \( N^j \) be the number of people voter \( j \) meets and \( n^j_z \) the number of people voter \( j \) meets that have ideal position \( z \), then \( \frac{n^j_z}{N^j} \) serves as an estimate for \( \frac{n_z}{N} \). Each voter uses these estimates to evaluate the positions in the issue space.

It seems reasonable to construct the above decision function by adjusting the utility function (3.1) discussed in Section 3.2 by a term that models the group externality, i.e. for voter \( j \) with ideal position \( x \) the net benefit from moving to a position \( y \) is described as

\[
U_j \left( z, \frac{n^j_z}{N^j} \right) = u_j(z) + W_j \left( \frac{n^j_z}{N^j} \right),
\]

where \( W_j(.) > 0 \) is the group externality function which is assumed to be non-decreasing in its argument. From (3.3) it is clear that there are two, partially offsetting, effects from

\(^{10}\)In our simulations we take \( \Lambda = \left[ \frac{N}{10} \right] \), where \( \left[ \frac{N}{10} \right] \) is the largest integer smaller than or equal to \( \frac{N}{10} \).

\(^{11}\)An alternative way to model this would be to draw, with replacement, from the population of \( N \) voters, pairs of voters who learn each others ideal point. On the other hand, we could also assume that the information transmission is unilateral, i.e. only one voter in the pair learns the ideal position of the other voter. This would resemble information transmission by opinion leaders generated in, for example, newspaper articles or talkshows on TV.
moving to another position. The first is the negative effect of giving up one’s own ideal position, the second is the possibly positive effect of moving to a position where there are more voters.\footnote{In our simulations we use \( W_j \left(n_j, \frac{n_j}{N} \right) = \alpha \ln \left( \frac{n_j}{N} \right) \), where \( \alpha \geq 0 \) measures sensitivity to group size. Modeling the group externality function this way is consistent with social impact theory (Latané (1981)). This theory is experimentally and empirically supported and confirmed in a number of sociological studies, such as Latané & Bourgeois (1996). The estimated value in Latané (1981) is \( \alpha = 0.46 \). In our simulations we take \( \alpha = 0.5 \).} This updating of ideal positions is assumed to take place once in between elections, and prior to the formation of interest groups and the search procedure of the challenger. Figure 3.4 gives an example of: (a) the initial distribution of ideal points, (b) the distribution after 30 updates, and (c) the distribution after 40 updates. The height of the bars corresponds to the number of voters at each position. Remember that the process of social interaction starts with voters’ ideal positions that are uniformly distributed over the issue space, as Figure 3.4(a) illustrates.

Clearly, the social interaction gives rise to some kind of clustering of voters’ ideal positions. Whether the dynamics generate one, two or more big clusters, depends on the stochastic elements of the model. However, opinions in minorities do survive, as Figure 3.4 visualizes. Note that from update 30 to 40 there is no visible change in the distribution. For the robustness of the last observation, and in order to analyze the clustering feature we use the following measure, known as the \textit{Herfindhal} index of concentration,

\[
C = \frac{1}{N^2} \sum_z n_z^2.
\]

This measure increases as the distribution of the population of voters over different positions becomes more uneven. To get some intuition for this measure notice that, for the extreme case where all voters cluster into only one position, \( C \) equals 1. On the other hand, if there is no clustering at all and the population of voters is evenly divided over all possible positions then the number of voters at every single position would be \( \frac{N}{K_1 \times \ldots \times K_i} \) and \( C \) reduces to \( \frac{1}{K_1 \times \ldots \times K_i} \) (which is \( \frac{1}{25} = 0.04 \) in our simulations, corresponding to the social configuration shown in Figure 3.4(a)), which therefore constitutes a lower bound for
3.4. SOCIAL INTERACTION AND CLUSTERS OF POLITICAL OPINIONS

Figure 3.4: The evolution of the distribution of voters' ideal positions over the policy space. The height of the bars corresponds to the number of voters at each position in the:
(a) initial configuration; (b) 30-th configuration; (c) 40-th configuration.
the level of clustering. Finally, to be able to compare the values of $C$ from our simulations to the level of clustering, suppose that we find a value of this measure equal to $C = C_0$. This is equivalent to a situation where all voters are clustered *evenly* into $\frac{1}{C_0}$ positions. $C$ increases as the number of updates increases. For the social configuration presented in Figure 3.4((b) and (c)), we have $C = 0.21$, which is approximately equivalent to all voters being organized evenly into 5 positions. More information about the nature of the clustering process can be extracted from Figure 3.5. This figure presents the evolution of the Herfindhal index over 40 updates for different values of $\alpha$ and $\Lambda$. From this figure we conclude that i) an increase in the parameter representing individual sensitivity to group size, $\alpha$, leads to higher clustering for a given sample size $\Lambda$. Notice, however, that heterogeneity in opinions survives, since the clustering measure has a maximum value of 0.45; ii) the lower the sample size parameter $\Lambda$, the longer it takes for the system to cluster; and iii) for a given value of the parameter $\alpha$, there seems to be no monotonic relation between the clustering measure $C$ and the sample size parameter $\Lambda$. One effect of a high value of $\Lambda$ is that it increases the predictability of the social dynamics. Given the uniform distribution of ideal points and strengths, if information about the distribution of voters is almost perfect ($\Lambda = 90\%$) the system will cluster at those positions that are most crowded according to the realization of the initial distribution of voters in the issue space. Hence, clustering depends very much upon the initial size of positions. For low values of $\Lambda$, voters are not well informed and their estimates of the distribution of voters might very well be biased. Chance is then an important determinant of the emergence of new clusters. Furthermore, Figure 3.5 shows that for all values of parameters $\Lambda$ and $\alpha$, the Herfindhal index is lower than 0.5 which is equivalent to a situation where all voters are clustered *evenly* into more than 2 positions.

Now that we understand the clustering procedure, let us consider what happens in our model when we allow for the social dynamics. Let us prohibit the formation of interest groups for the moment. Figure 3.6(a) shows the average Euclidean distance between the policy outcome and the median (solid lines) and the average Euclidean distance between incumbent and challenger (broken lines), for 2 and 8 polls respectively. Figure 3.6(b) shows the empirical frequencies of election victories of the challenger, given that he runs
3.4. SOCIAL INTERACTION AND CLUSTERS OF POLITICAL OPINIONS

Figure 3.5: The evolution of the Herfindhal index over 40 updates for different values of the parameter of the individual sensitivity to group size, $\alpha$, and the parameter of information sampling size, $\Lambda$.

2 and 8 polls, respectively.

From these graphs we see that, for the first couple of elections, when the process of clustering has not fully matured yet, the policy outcome converges to the median and the separation between party platforms decreases. These results are similar to the ones found in the basic model (compare Section 3.2). In the early stages of the interactions, the social clusters that have appeared are only moderate in size and hence do not break the symmetry of the distribution of voter preferences. However, as time goes by clustering increases and the symmetry of the distribution of voter preferences may break. As can be seen from Figure 3.6(a) eventually there seems to be a slight increase in the different distance measures. The most notable effect of the social dynamics therefore seems to be
Figure 3.6: Time series of: (a) different measures; and (b) frequencies the challenger wins an election, for the model with social interaction.
3.5. THE FULL MODEL

that it prohibits (also in the long run) party platforms to converge to each other or to the center of the distribution. The implicit reason for non-convergence is that the center of the distribution may lose the property of being a dominant position (as Figure 3.4(b) – (c) illustrate).

3.5 The full model

Now that we have separately introduced social dynamics and interest groups in our spatial competition framework, we are ready to study the full model. First observe that whereas the social dynamics is a long run process that depends only upon its own past, the interest group formation process also depends upon the social dynamics and election outcomes. Prior to an election, after social interaction has taken place, voters start to get organized in interest groups that try to influence the outcome of that election. The sequence of events of the full model is therefore as follows:

1. Voters ideal positions and strengths are randomly drawn from a uniform distribution.

2. Interest groups develop and are activated as in Section 3.3.

3. The challenger runs polls and selects its policy platform for the upcoming election as described in Section 3.3.

4. The election is run and the party that wins the election becomes the new incumbent. Interest groups dissolve.

5. Social interaction takes place as described in Section 3.4, leading to a new configuration of voter preferences..

6. The sequence of events restarts at Step 2.

The results of the simulations for the full model are shown, together with the simulation results for the models from Sections 3.2, 3.3 and 3.4, in Figures 3.7(a) – (d). Figure 3.7(a) shows the average distance (over 100 trial runs again) between the incumbent and the
CHAPTER 3. SOCIAL INTERACTION IN A SPATIAL VOTING MODEL

center of the distribution. Clearly this distance is smallest for the basic model and highest for the full model. Furthermore, in the early elections (say the first 5) this difference can be attributed mainly to the effect of the interest groups. Later on, however, the social dynamics as well as the existence of the interest groups seems to be responsible for the difference. In fact, over time the effect of the interest groups decreases whereas the effect of the social dynamics increases. The latter effect ensures that there is no apparent convergence in the full model. The reason should be clear by now: in the long run the clustering really becomes significant and the appearance of large clusters drives the policy outcome away from the center of the distribution, which is moving as well. Figure 3.7(b) shows the average separation between the policy platforms of the two parties. Clearly, this separation is much larger for the full model and the model with interest groups than for the basic model and the model with social dynamics. The existence of interest groups therefore seems to play an important role in explaining the separation between political parties. Similarly, the probability for the challenger of winning an election is significantly larger when interest groups exist, as can be seen from Figure 3.7(c). There is also a positive effect of the social dynamics on the winning probability for the challenger, but this effect is clearly smaller. The probability of winning is influenced by the existence of interest groups via two channels. First, since the interest groups transmit information about the electoral landscape to the challenger, it is easier for the challenger to find an element of the winning set. This is a transient effect, however, since it will also lead the political parties to the center of the distribution faster. Second, the coordination of voting behavior, induced by the interest groups, changes the electoral landscape and increases the size of the winning set, given the incumbent’s position. This is illustrated by Figure 3.7(d) where the average number of winning points for the challenger over 100 trial runs are given. Clearly, interest groups lead to a sizable increase in the number of winning points.
Figure 3.7: Time series of: (a) distance between the incumbent and the challenger; (b) distance between the incumbent and the center; (c) frequencies the challenger wins an election; (d) number of positions that defeat the incumbent position, for different models.
3.6 Concluding remarks

Although it is widely accepted as the basic approach for studying issues related to voting, the spatial competition model has some serious drawbacks as a realistic description of election outcomes in democratic societies. In this chapter we have attempted to extend the spatial competition model into several directions that might give it a more plausible appeal. These directions are centered around two important features. First, we recognize the adaptive nature of political parties as well as the adaptive nature of voters' opinions. While the former was already investigated in an influential paper by Kollman, Miller and Page (1992), we incorporate the latter by introducing social interaction between voters. That is, we assume that voters (explicitly or implicitly) learn about other voters' beliefs and adapt their own beliefs (or preferences) accordingly. Second, we introduce interest groups into the spatial competition framework. These interest groups coordinate voting behavior and transmit information about voter preferences to the political parties. We further allow for the endogenous emergence of interest groups, a feature which, until now, has been neglected in the vast literature on interest groups.

Our simulations show that in the presence of interest groups the winning platforms, provided they exist, are selected faster than in their absence. Similarly, the challenger's probability of winning the election increases if there are interest groups, due to the fact that their presence tends to increase the winning set. Only in the basic model the policy outcome eventually converges to the center of the distribution of voter preferences.

The framework set out in this chapter has been a first attempt to model the complex relations between adaptive voters, adaptive political parties and interest groups. This framework should be taken as a starting point for the development of more elaborate and rigorous models to analyze these issues. We hope that we have succeeded in arguing that such models can contribute significantly to our understanding of the complex political-economic world we are living in.