Essays on Political and Experimental Economics
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Chapter 8

A Model of Other-Regarding Preferences

8.1 Introduction

Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) demonstrate that their models of inequality aversion can rationalize seemingly inconsistent sets of data from several kinds of experiments. Charness and Rabin (2000) find that, in reciprocity free environments, about 70% of subjects are motivated by quasi-maximin preferences and 20% by inequality aversion. In Chapter 7, we have argued that both inequality aversion and quasi-maximin preferences are inconsistent with the results from our dictator experiments. Instead of inequality aversion or quasi-maximin preferences, the new data support a model of “other-regarding” preferences with the property that utility is globally increasing in both one’s own and others’ material payoffs. In this chapter we address the question of whether the other-regarding preferences model can rationalize data from several other types of experiments, as can the inequality-aversion models. In Section 8.2, we specify the many-agent extension of the model of other-regarding preferences. In Sections 8.3-8.6, we demonstrate that the model of other-regarding preferences is consistent with most data from experiments that can be rationalized by models of inequality aversion. We also discuss the limitations of both models of other-regarding preferences and models of inequality.

\[\text{This chapter is taken from Cox, Sadiraj and Sadiraj (2001b).}\]
8.2 The many-agent model of egocentric other-regarding preferences

The generalization of the egocentric other-regarding utility function (7.5) to the case of \( n \geq 2 \) agents is written as

\[
\begin{align*}
(8.1) \quad u^i(y_1, \ldots, y_{i-1}, m, y_{i+1}, \ldots, y_n) &= \left[ m^\alpha + \sum_{j \neq i} \theta_j^i (y_j^\alpha - m^\alpha) \right]^{1/\alpha} \\
&= \left[ (1 - \sum_{j \neq i} \theta_j^i) m^\alpha + \sum_{j \neq i} \theta_j^i y_j^\alpha \right]^{1/\alpha},
\end{align*}
\]

where

\[
\theta_j^i = \begin{cases} 
\theta_j^i/(n-1), & \text{if } m < y_j \\
\theta_j^i/(n-1), & \text{if } m \geq y_j.
\end{cases}
\]

The egocentric property generalizes to the \( n \)-agent case as follows. First, define

\[
y_{-k} = (y_2, \ldots, y_{k-1}, y_{k+1}, \ldots, y_n),
\]

and write the utility function of agent \( i \) as \( u^i(m, y_k, y_{-k}) \). Then the egocentric property is

\[
u^i(b, a, y_{-k}) > u^i(a, b, y_{-k})
\]

for all \( a, b, \) and \( y_{-k} \) such that \( b > a \geq 0 \), for \( k = 2, 3, \ldots, n \). This has to hold for all \( i = 1, 2, \ldots, n \).

Monotonicity, egocentricity\(^1\), and convexity imply the parameter restrictions,

\(^1\)Note that \( u^i(0, b, 0, \ldots, 0) = \left[ \frac{\theta_+}{\theta_-} b^\alpha \right]^{1/\alpha} \) and \( u^i(0, 0, 0, \ldots, 0) = [(1 - \theta_+) b^\alpha]^{1/\alpha} \). The ego-centricity property implies that \( u^i(b, 0, 0, \ldots, 0) > u^i(0, b, 0, \ldots, 0) \) which is equivalent with \((1 - \theta_+)(n-1) > \theta_-\).
8.3. GAMES WITH PROPOSER COMPETITION

(8.2) \[ 0 < \alpha < 1; \, 0 \leq \theta^i_+ < 1; \, 0 \leq \theta^-_i \leq \theta^i_+; \text{ and } \theta^-_i < (n-1)(1-\theta^i_+), \]

for all \( i \in \{1,2,\ldots,n\} \).

8.3 Games with proposer competition

Roth, Prasnikar, Okuno-Fujiwara, and Zamir (1991) report results from experiments in four countries with the following game of proposer competition. There are \( n-1 \) proposers who can simultaneously propose shares \( s_j \in [0,1] \), \( j = 1,2,\ldots,n-1 \). The responder can accept or reject the highest share offered, \( \bar{s} = \max\{s_1,s_2,\ldots,s_{n-1}\} \). If the responder accepts the highest offer then the proposer that made the offer gets \( 1 - \bar{s} \), the other proposers get 0, and the responder gets \( \bar{s} \). If more than one proposer made the accepted offer then one of the proposers is randomly selected, with equal probability for all tied proposers, to get \( 1 - \bar{s} \). If the responder rejects the highest offer then all players get 0. Results from experiments in all four countries were that the accepted (highest) proposal converged to 1 in five to six periods. The model of other-regarding preferences predicts this outcome, as can be seen from the following.

First, we offer an informal demonstration that conveys intuition about the implications of the other-regarding preferences model for the subgame perfect equilibrium in the game with proposer competition. Then we state a proposition and provide a formal proof.

Because for the responder, \( i = R \), both \( \theta^R_- \) and \( \theta^R_+ \) are nonnegative and \( \sum \theta^R_j(= \frac{k \theta^R_j + (n-1-k) \theta^R}{n-1} \text{ for some } k \in \{0,\ldots,n-1\}) \) is less than 1, utility function (8.1) implies that the responder prefers the money payoffs implied by any \( \bar{s} > 0 \) to the outcome in which everyone gets 0. Thus the responder will accept all offers. Because for any proposer, \( j = P \), both \( \theta^P_- \) and \( \theta^P_+ \) are nonnegative and \( \sum \theta^P_j \) is less than 1, utility function (8.1) implies that a proposer prefers the payoffs from the set of offers in which he is one of \( k \) proposers, \( 1 < k \leq n-1 \), who submits \( \bar{s} > 0 \), and has probability \( 1/k \) of receiving \( 1 - \bar{s} \), to the payoffs from \( s^P < \bar{s} \) where he gets 0 for sure. Furthermore, from the egocentricity property, for a sufficiently small value of \( \varepsilon > 0 \) a proposer prefers the payoffs from the set
CHAPTER 8. A MODEL OF OTHER-REGARDING PREFERENCES

of offers in which he submits \( s^P = s + \varepsilon \) to the payoffs from the set of offers in which \( k \) proposers, \( 1 < k \leq n - 1 \), submit offers of \( \bar{s} = \bar{s} < 1 \). Thus one expects the following.

**Proposition 8.1** Let \( \bar{s} \) be the highest offer submitted by proposers in the game of proposer competition. One has:

1. The responder will accept any offer \( \bar{s} \geq 0 \);

2. There can be no subgame perfect equilibrium in which proposers offer \( \bar{s} < 1 \); and

3. There is a unique subgame perfect equilibrium in which at least two proposers offer \( \bar{s} = 1 \) and the responder accepts it.

**Proof.** Let the maximum offer be \( \bar{s} \geq 0 \).

1. Assume that the responder rejects it. So, all players get money payoffs 0 and hence utility 0. From the monotonicity assumption on money payoffs, the responder derives a positive utility by deviating and accepting \( \bar{s} \). Hence, it is a dominant strategy for the responder to accept any positive offer \( \bar{s} > 0 \). First, note that if the maximum offer is rejected then the utility of a proposer \( i \) who offers \( s \) is 0, otherwise it is given by:

- if \( s = \bar{s} \) and \( k - 1 (\geq 0) \) other proposers submit \( \bar{s} \), then expected utility becomes

\[
u^i(s) = \begin{cases} 
\left( \frac{1}{k} \left[ (1 - (n - 1) \frac{\theta^i}{n-1}) (1 - \bar{s})^\alpha + \frac{\theta^i}{n-1} (\bar{s})^\alpha \right] \right)^{\frac{1}{\alpha}} & \text{if } 0 \leq \bar{s} \leq 0.5 \\
+ (1 - \frac{1}{k}) \left[ \frac{\theta^i}{n-1} (1 - \bar{s})^\alpha + \frac{\theta^i}{n-1} (\bar{s})^\alpha \right]^{\frac{1}{\alpha}} & \text{otherwise;}
\end{cases}
\]

- if \( s < \bar{s} \) then

\[
u^i(s) = \left[ \frac{\theta^i}{n-1} (1 - \bar{s})^\alpha + \frac{\theta^i}{n-1} (\bar{s})^\alpha \right]^{\frac{1}{\alpha}}.
\]

Second, we show that there are no subgame perfect equilibria with a maximum offer \( \bar{s} \in (0.5, 1) \). We distinguish two cases:
8.3. GAMES WITH PROPOSER COMPETITION

Case 1: More than one proposer offers the maximum offer, \( \bar{s} \in (0.5, 1) \). Let 
\( k, 1 < k \leq n - 1 \) denote the number of proposers who offers \( \bar{s} \). Let \( i \) be 
one of the proposers who offer \( \bar{s} \). If proposer \( i \) offers \( \bar{s} + \varepsilon \) instead of \( \bar{s} \) then 
\[
u^i(\bar{s} + \varepsilon) = [(1 - (n - 2) \frac{\theta^i_+}{n-1} - \frac{\theta^i_-}{n-1})(1 - \bar{s} - \varepsilon)^\alpha + \frac{\theta^i_-}{n-1}(\bar{s} + \varepsilon)^\alpha]^\frac{1}{\beta}.
\]
Since \( \theta^i_+ > 0 \), one has:

- if \( \theta^i_+ = 0 \), then \( \theta^i_- = 0 \) and \( \lim_{\varepsilon \to 0} \nu^i(\bar{s} + \varepsilon) = (1 - \bar{s}) > \frac{1}{k}(1 - \bar{s}) = \nu^i(\bar{s}) \);
- if \( \theta^i_+ > 0 \), then first denote \( a_j \) the utility of proposer \( i \) when proposer \( j \) gets 
\( 1 - \bar{s} \), other proposers get 0 and the responder gets \( \bar{s} \). Note that from the 
egocentricity property \( a_j < a_i \) for all \( j \neq i \) and hence \( \nu^i(\bar{s}) = \frac{1}{k} a_i + \frac{k-1}{k} a_j < a_i \). Since \( \lim_{\varepsilon \to 0} \nu^i(\bar{s} + \varepsilon) = a_i \), there exists an \( \varepsilon^* > 0 \) such that \( \nu^i(\bar{s} + \varepsilon) < \nu^i(\bar{s} + \varepsilon) \), for all \( \varepsilon \in (0, \varepsilon^*) \). Therefore, proposer \( i \) is better off by offering 
\( \bar{s} + \varepsilon, \varepsilon \in (0, \varepsilon^*) \), instead of \( \bar{s} \), and thus, \( \bar{s} \) cannot be played in a subgame 
perfect equilibrium.

Case 2: There is only one proposer who offers the maximum offer \( \bar{s} \). Let that 
proposer be \( i \). It can be shown in the same way as in Case 1 that for some 
\( \varepsilon > 0 \), such that \( \{\max_j (s_j, 0.5) < \bar{s} - \varepsilon < \bar{s}, \nu^i(\bar{s}) < \nu^i(\bar{s} - \varepsilon) \). Indeed, the last inequality is equivalent with 
\[
(1 - \frac{n - 2}{n-1} \theta^i_+ - \frac{1}{n-1} \theta^i_-)(1 - \bar{s} + \varepsilon)^\alpha + \frac{\theta^i_-}{n-1}(\bar{s} - \varepsilon)^\alpha
\geq
(1 - \frac{n - 2}{n-1} \theta^i_+ - \frac{1}{n-1} \theta^i_-)(1 - \bar{s})^\alpha + \frac{\theta^i_-}{n-1}(\bar{s})^\alpha,
\]
which can be rewritten as 
\[
\frac{\theta^i_-}{n-1}(\bar{s}^\alpha - (\bar{s} - \varepsilon)^\alpha) <
(1 - \frac{n - 2}{n-1} \theta^i_+ - \frac{1}{n-1} \theta^i_-)((1 - \bar{s} + \varepsilon)^\alpha - (1 - \bar{s})^\alpha).
\]
Since \( \theta^i_- < \min\{(n-1)(1 - \theta^i_+), \theta^i_+\} \) (see 8.2), a sufficient condition for this to 
hold is 
\[
(\bar{s} - \varepsilon)^\alpha + (1 - \bar{s} + \varepsilon)^\alpha > \bar{s}^\alpha + (1 - \bar{s})^\alpha.
\]

Denoting 
\[
f : x \to x^\alpha + (1 - x)^\alpha,
\]
and noting that
\[ f'(x) = a \left( x^{\alpha - 1} - (1 - x)^{\alpha - 1} \right) < 0, \text{ for all } x \in (0.5, 1), \]

one finds that (8.3) is true.

2. In the same way it can be shown that \( \bar{s} \leq 0.5 \) cannot be played in a subgame perfect equilibrium either.

3. Let \( \bar{s} = 1 \) and \( i \) be one of the proposers who offers it. Suppose that proposer \( i \), deviates and makes an offer \( s \) slightly less than 1. Since the highest offer is 1 and the responder accepts it, one has
\[ u^i(s) - u^i(1) = \left( \frac{\theta^i_1}{n-1} \right)^{\frac{1}{\alpha}} - \left( \frac{\theta^i_0}{n-1} \right)^{\frac{1}{\alpha}} = 0. \]

Therefore, the set of strategies, "at least two proposers offer \( \bar{s} = 1 \), and the responder accepts it", is a (weak) subgame perfect equilibrium. What is left to be shown is that \( \bar{s} = 1 \) being offered by only one proposer \( i \) cannot be played in a subgame perfect equilibrium. Indeed, in such a case proposer \( i \) is better off by deviating and offering an amount \( 1 - \varepsilon \), since there exists \( \varepsilon \in (0, \min_{j \neq i} \{1 - s_j\}) \cap (0, \varepsilon^*) \) such that\(^2\)
\[ u^i(1 - \varepsilon) > u^i(1). \]

This and (2) prove the uniqueness. ■

8.4 Games with responder competition

Güth, Marchand, and Rulliere (1997) report an experiment with a game in which a proposer proposes a share \( s \in [0, 1] \) to \( n - 1 \) responders. A responder can accept or reject

\[^2\]First, note that \( u^i(1 - \varepsilon) > u^i(1) \) if and only if \( \left( \frac{n-1}{\theta^i_1} - \frac{(n-2)\theta^i_1}{\theta^i_0} - 1 \right) \varepsilon^\alpha > 1 - (1 - \varepsilon)^\alpha \). Let \( F(\varepsilon) = \eta \varepsilon^\alpha + (1 - \varepsilon)^\alpha \), where \( \eta = \frac{n-1}{\theta^i_1} - \frac{(n-2)\theta^i_1}{\theta^i_0} - 1 \) (which is positive since \( \theta^i_1 < \min\{(n-1)(1 - \theta^i_1), \theta^i_2\} \) (see 8.2)). Second, note that \( F(0) = 1 \) and that \( F'(\varepsilon) = \alpha \eta \varepsilon^{\alpha - 1} - \alpha(1 - \varepsilon)^{\alpha - 1} > 0 \) for all \( \varepsilon \in (0, \varepsilon^*) \) where \( \varepsilon^* = 1/(1 + (1/\eta)^{1/(1-\alpha)}) \).
the proposal. If only one of the responders accepts the offer then she gets \( s \), the other responders get 0, and the proposer gets \( 1 - s \). If more than one responder accepts the proposal then one of the responders is randomly selected to get \( s \). If all of the responders reject the proposal then all players get 0. The experiments were run with a design in which responders were asked to pre-commit to acceptance thresholds for a period before observing the proposal for that period. Results from a limited number of experiments were that the average responder threshold had declined to less than 0.05 by the fifth period, 71% of responders chose a threshold of zero, and 9% of the responders chose a threshold of 0.02. Also by period 5, on average the proposals had decreased to 0.15. The predictions of the other-regarding preferences model are as follows.

First, we present an informal demonstration that conveys intuition about the properties of the equilibrium. Consider any proposal \( \theta^R > 0 \). If all responders reject the proposal, everyone gets 0. Since both \( \theta^R \) and \( \theta^R \) are nonnegative and \( \sum \theta^R \) is less than 1, utility function (8.1) implies that a responder \( R \) prefers to accept any \( \theta > 0 \) rather than accept the outcome where everyone gets 0. Similarly, if \( k \) other responders, \( 1 \leq k \leq n-2 \), accept \( \theta > 0 \), utility function (8.1) implies that a responder also prefers to accept \( \theta \) because the payoffs determined by probability \( 1/(k+1) \) of receiving \( \theta \) are preferable to those where the responder receives 0 for sure, given parameter restrictions (8.2). Since responders will accept all proposals, the proposer will propose that offer which maximizes his utility. One responder will be randomly selected to receive \( s^P \) and the proposer will receive \( 1 - s^P \), hence the proposer’s utility implied by (8.1) is

\[
(8.4) \quad u^P = \begin{cases} 
(1 - (n - 1) \frac{\theta^P}{n-1}) (1 - s)^{\alpha} + \frac{\theta^P}{n-1} s^{\alpha} \right)^{1/\alpha}, & \text{if } 0 \leq s \leq 0.5 \\
\left( 1 - (n - 2) \frac{\theta^P}{n-1} - \frac{\theta^P}{n-1} \right) (1 - s)^{\alpha} + \frac{\theta^P}{n-1} s^{\alpha} \right)^{1/\alpha}, & \text{if } 0.5 \leq s \leq 1.
\end{cases}
\]

Differentiation of (8.4) reveals that the proposer’s optimal proposal, \( s^P \) will be positive or 0, depending on the relative values of \( \theta^P \) and \( n \).

**Proposition 8.2** Let \( s^P \) be the proposer’s subgame perfect equilibrium proposal. One has:

1. At least one responder will accept any proposal;
2. The proposer’s offer is

\[
    s^P = \begin{cases} 
        0, & \text{if } \theta_+^P = 0, \\
        \left(1 + ((n-1)\left(1 - \theta_+^P\right)/\theta_+^P\right)^{1/(1-\alpha)}\right)^{-1}, & \text{if } \theta_+^P \in (0, 1 - 1/n) \\
        0.5, & \text{otherwise.}
    \end{cases}
\]

**Proof.**

1. Let the proposer offer \( s \geq 0 \). Assume that all responders reject it. So, all players get money payoffs \( 0 \) and hence utility \( 0 \). From the monotonicity assumption on money payoffs, a responder \( i \) derives a positive utility by deviating and accepting \( s \). Hence, the strategy choices in which all responders reject an offer \( s \) cannot be played in a subgame perfect equilibrium. Thus, an offer \( s \) will be accepted in equilibrium by at least one responder.

2. Since any offer will be accepted in equilibrium, only \( s^P = \arg\max\{u^P(s) \mid s \in [0, 1]\} \) will be played in a subgame perfect equilibrium. Straighforwardly, one finds that the utility of the proposer takes its maximum at \( s^P \), given as follows.

- \( 0 \leq s \leq 0.5 : s^o = \arg\max u^P(s) \) is given by

\[
    s^o = \begin{cases} 
        0, & \text{if } \theta_+^P = 0 \\
        s^a, & \text{if } \theta_+^P < 1 - 1/n \\
        0.5, & \text{otherwise},
    \end{cases}
\]

where \( s^a = \left(1 + ((n-1)(1 - \theta_+^P)/\theta_+^P\right)^{1/(1-\alpha)}\right)^{-1} \).

- \( 0.5 < s \leq 1 \) : it can be easily shown that the maximum is reached at \( s^b = (1 + \eta)^{-1} \) where \( \eta = (n-1) - (n-2)\theta_+^P \). Furthermore, \( \theta_+^P < \min\{(n - 1)(1 - \theta_+^1), \theta_+^1\} \) (see 8.2) implies \( s^b < 0.5 \) and therefore \( s^o = 0.5 \).

Summarizing, the optimal amount for the proposer is

\[
    s^P = \begin{cases} 
        0, & \text{if } \theta_+^P = 0 \\
        s^a, & \text{if } \theta_+^P < 1 - 1/n \\
        0.5, & \text{otherwise},
    \end{cases}
\]

where \( s^a < 0.5 \).


8.4. GAMES WITH RESPONDER COMPETITION

A discrete-variable case. An immediate extension of Proposition 8.2 can be written for a discrete-variable case. Suppose that the proposer can choose an offer \( s \in S \) where \( S \) is a discrete set \( S \subseteq [0,1] \). Denote \( g = \min S \setminus \{0\} \) and assume that \( g < 0.5 \).

**Corollary 8.1** The proposer's offer, \( s^p \) for the discrete-variable case of the experiment is 0 if

\[
\theta^p_+ < 1/(g + 1),
\]

where \( g = \frac{s^\alpha}{(n-1)(1-(1-s)\alpha)} \).

**Proof.** Proposition 8.2 and the monotonicity of function (8.4) in \((0, s^\alpha)\) and \((s^\alpha, 0.5)\) for \( s^\alpha \in (0,g) \) imply that if \( u(0) > u(g) \) then the proposer's offer would be 0. It can be easily shown that \( s^\alpha < g \) is equivalent to \( \theta^p_+ < (n - 1)/\left( (g^{-1} - 1)^{1-\alpha} + n - 1 \right) \) and \( u(0) > u(g) \) is equivalent to \( \theta^p_+ < 1/\left( \frac{s^\alpha}{(n-1)(1-(1-s)\alpha)} + 1 \right) \). The statement follows by noting that

\[
\left( \frac{g^\alpha}{(n-1)(1-(1-g)^\alpha) + 1} \right)^{-1} \leq (n-1) \left( (g^{-1} - 1)^{1-\alpha} + n - 1 \right)^{-1}.
\]

Suppose \( g \geq 0.01 \) and \( \alpha \geq 0.76 \). Table 8.1 shows the values of the rhs of inequality (8.5) for some given number, \( n \), of responders, \( n \geq 5 \).

Referring to the distribution of preference parameters shown in Figure 7.6, at least 69% of subjects have parameters \( \theta^p_+ \leq 0.5 \). Noting that the minimum value in the third column of Table 8.1 is 0.502 and using Corollary 8.1, we can conclude that if the number of responders is at least 5, then at least 69% of proposers would offer 0. Hence, the proposal average will be at most 0.155(= 0 \times 0.69 + 0.5 \times 0.31). Remember that the empirical proposal average had decreased to 0.15 by period 5.
CHAPTER 8. A MODEL OF OTHER-REGARDING PREFERENCES

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</table>

Table 8.1: The upper bounds of the parameter $\theta^P_i$ that induce an offer of zero in equilibrium.

8.5 Voluntary contributions public good games

In a voluntary contributions public good game, $n \geq 2$ players simultaneously choose the amounts they will contribute to a public good. Typically, each subject is given an endowment, $y$ and asked to choose an amount $g_i \in [0, y]$ to invest in the public good. Investments in the public good yield the constant rate of return $a \in [1/n, 1)$ whereas the rate of return in the private good equals 1. Thus, the monetary payoff to subject $i$ from participating in one period of a voluntary contributions public good experiment is

$$(8.6) \quad \pi(g_i, g_{-i}) = y - g_i + a \sum_{j=1}^{n} g_j.$$ 

The traditional model of self-regarding preferences predicts that for a finite number of periods and under common information all subjects will contribute zero in all periods of a public good experiment. Typically, a large proportion of subjects contribute positive amounts in early rounds of a multi-period experiment but average contribution rates move towards zero as the experiment continues. In the large number of experiments reported in Fehr and Schmidt’s Table II (Fehr and Schmidt (1999)), the percentage of complete free riders ($g_i = 0$) in the last period varied between 54% and 89%.

The egocentric other-regarding preferences model has the following implications for the final period of a voluntary contributions public good game. Let $G = g_1 + g_2 + \ldots + g_n$ denote the total contribution in the public good. Then the utility of contribution profile
(8.7) \[ u^i(g_i, g_{-i}) = \left[ \left( 1 - \sum_{j \neq i} \theta_j^i \right) (y - g_i + aG)^{\alpha} + \sum_{j \neq i} \theta_j^i (y - g_j + aG)^{\alpha} \right]^{\frac{1}{\alpha}}. \]

**Proposition 8.3** Predicted outcomes for the public good game depend on the rate of return, \( a \) and the egocentricity parameters, \( \theta_+ \) and \( \theta_- \), as follows:

1. if \( \theta_+ < (1 - a) \), for all \( r = 1, 2, \ldots, n \), then the unique Nash equilibrium is for all contributions to equal 0;
2. if \( \theta_- > (1 - a) \), for all \( r = 1, 2, \ldots, n \), then the unique Nash equilibrium is for all contributions to equal \( y \);
3. if \( \theta_+ \geq (1 - a) \), for all \( i \in I \neq \emptyset \), and \( \theta_- \leq (1 - a) \), for all \( j \in J \neq \emptyset \) then

   (a) if \( |J| < n \) and \( |I| < n \) then there is no symmetric Nash equilibrium. Furthermore, in any asymmetric Nash equilibrium, \( g \);
   - \( g_j = 0 \), for all \( j \in J \), with \( \theta_j^+ \leq (1 - a)/2 \);
   - \( g_i > 0 \), for all \( i \in I \), with \( \theta_i^- > (1 - a) \);

   (b) if \( |J| < n \) and \( |I| = n \) then the only symmetric Nash equilibrium is for all contributions to equal \( y \); Furthermore, in any asymmetric Nash equilibrium, \( g \), \( g_r > 0 \), for all \( r = 1, 2, \ldots, n \), with \( \theta_r^- > (1 - a) \);

   (c) if \( |J| = n \) and \( |I| < n \) then the only symmetric Nash equilibrium is for all contributions to equal 0; Furthermore, in any asymmetric Nash equilibrium, \( g \), \( g_r = 0 \), for all \( r = 1, 2, \ldots, n \), with \( \theta_r^+ \leq (1 - a)/2 \);

   (d) if \( |J| = n \) and \( |I| = n \) then any contribution, \( g_r = \delta \in [0, y], r = 1, 2, \ldots, n \), is a symmetric Nash equilibrium.

**Proof.** Let \( g = (g_1, \ldots, g_n) \) be a vector of contributions to the public good game. Consider some player \( r, r = 1, 2, \ldots, n \). Let \( k_r^- \) and \( k_r^+ \) denote the numbers of contributions that are, respectively, less and not less than the contribution \( g_r \), and

\[ L^- = \{ l \in \{1, \ldots, n\} \mid l \neq r \text{ and } g_l < g_r \}, \]
and

\[ L^+ = \{ l \in \{1, \ldots, n\} \mid l \neq r \text{ and } g_l \geq g_r \}. \]

Differentiating the utility function (8.7) of player \( r \), with respect to its argument, \( g_r \), one finds that the sign of the partial derivative is determined by the sign of

\[
(a - 1)(1 - \frac{k_r \theta_r}{n - 1} - \frac{k_+ \theta_+}{n - 1})(y - g_r + aG)^{\alpha - 1}
+ a \left( \sum_{l \in L_1} \frac{\theta_r}{n - 1}(y - g_l + aG)^{\alpha - 1} + \sum_{l \in L^+} \frac{\theta_+}{n - 1}(y - g_l + aG)^{\alpha - 1} \right)
\]

the sign of which is determined by

\[
F'(g_r, g_{-r}) = \left( \frac{aS_r^r}{1 - a} + k_r^r \right) \frac{\theta_r^r}{n - 1} + \left( \frac{aS_+^r}{1 - a} + k_+^r \right) \frac{\theta_+^r}{n - 1} - 1
\]

where \( S_r^r = \sum_{l \in L_1} \left( \frac{y - g_l + aG}{y - g_l + aG} \right)^{1 - a} < k_r^r \) and \( S_+^r = \sum_{l \in L^+} \left( \frac{y - g_l + aG}{y - g_l + aG} \right)^{1 - a} \geq k_+^r \).

Denote \( x = \max_{l \neq r} \{ g_l \} \) and note that for player \( r \),

- for all \( g_r > x \),

\[
F'(g_r, g_{-r}) = \left( \frac{aS_r^r}{1 - a} + n - 1 \right) \frac{\theta_r^r}{n - 1} - 1,
\]

and that \( \lim_{g_r \to x} F'(g_r, g_{-r}) = \frac{\theta_r^r}{1 - a} - 1; \)

- for all \( g_r \leq x \),

\[
F'(g_r, g_{-r}) = \left( \frac{aS_+^r}{1 - a} + n - 1 \right) \frac{\theta_+^r}{n - 1} - 1,
\]

and that \( \lim_{g_r \downarrow x} F'(g_r, g_{-r}) = \frac{\theta_+^r}{1 - a} - 1. \)

Let \( \bar{\theta} = \min_r \{ \theta_r^r \} \) and \( \overline{\theta} = \max_r \{ \theta_+^r \} \). Only the following cases are possible:

- \( \bar{\theta} < (1 - a) \), which corresponds to part (1) of Proposition 8.3,

- \( \bar{\theta} > (1 - a) \), which corresponds to part (2) of Proposition 8.3,
8.5. VOLUNTARY CONTRIBUTIONS PUBLIC GOOD GAMES

- \( \theta \leq (1 - a) \leq \bar{\theta} \), which corresponds to part (3) of Proposition 8.3.

1. Let \( \bar{\theta} < (1 - a) \). We show that \( g = (0, 0, \ldots, 0) \) is the unique Nash equilibrium.

Let \( g \) be a vector of contributions.

- \( G > 0 \). Let player \( r \) contribute the maximum amount, \( g_r \).

If player \( r \) is the only one that has invested the maximum amount then at any contribution \( z < g_r \), such that \( z > g_l, \forall l \neq r \), one has \( k_+ = n - 1, k_- = 0 \). Using (8.9), one finds that

\[
F^r(z, g_{-r}) = \left( \frac{aS_r^-}{1-a} + n - 1 \right) \frac{\theta_r^-}{n-1} - 1 < \frac{\theta_r^-}{(1-a)} - 1,
\]

which is negative given \( \theta_r^- \leq \theta_r^+ \leq \bar{\theta} < (1 - a) \). Therefore \( g \) cannot be an equilibrium.

If player \( r \) is not the only one with the maximum contribution then one can derive the following.

If \( g \) is a vector with unequal contributions then since \( g_r \) is the maximum contribution, there exists at least one player \( l \) with contribution \( g_l < g_r \). Let \( x = \max_{i \in L \setminus \{r\}} \{g_i\} \). Clearly \( x < g_r \). Consider contribution \( z = g_r - \varepsilon > x \). Note that \( z > x \) implies that \( k_+^r > 0 \). Furthermore, let \( k_+^r(> 0) \) equals the number of players that have invested \( g_r \) and

\[
S^r_+ = \sum_{l: g_l = g_r} \left( \frac{y - z + aG}{y - g_r + aG} \right)^{1-a}
= k^r_+ \left( 1 + \frac{g_r - z}{y - g_r + aG} \right)^{1-a}
< k^r_+ \left( 1 + \frac{\varepsilon}{y - g_r} \right).
\]

Denoting \( q^* = \frac{g_r}{1-a} (< 1) \) and using (8.8), one has
\( \forall \varepsilon \in (0, \min\{ \frac{1-q^r}{v-a}, g_r - x \}) \),

\[
F^r(z, g_{-r}) = \left( \frac{aS_r^r}{1-a} + k^r \right) \frac{\theta^r}{n-1} + \left( \frac{aS_r^r}{1-a} + k^r \right) \frac{\theta^r}{n-1} - 1
\]

\[
< \frac{k \theta^r + k \theta^r}{(1-a)(n-1)} + \frac{\varepsilon \theta^r k^r}{y-g_r n-1} - 1
\]

\[
\leq q^r + \frac{\varepsilon a}{y-g_r} q^r - 1,
\]

which is negative. If \( g \) is a vector of equal contributions then consider a contribution \( z = g_r - \varepsilon > 0 \). Referring to that \( z, k^r = 0, k^r = n-1 \) and in the same way as at the asymmetric case it can be shown that \( S^r_r < (n-1)(1 + \frac{\varepsilon}{y-g_r}) \).

Thus, from (8.10) one has

\[
\forall \varepsilon \in (0, \min\{ \frac{1-q^r}{v-a}, g_r \}),
\]

\[
F^r(z, g_{-r}) = \left( \frac{aS_r^r}{1-a} + n-1 \right) \frac{\theta^r}{n-1} - 1
\]

\[
< \frac{\theta^r}{1-a} + \frac{\varepsilon}{y-g_r} \theta^r 1-a - 1
\]

\[
\leq q^r + \frac{\varepsilon a}{y-g_r} q^r - 1,
\]

which is negative. Hence, in either case player \( r \) is better off by deviating and offering less than \( g_r \) and \( g \) can therefore not be an equilibrium.

- \( G = 0 \). For any player \( r \), at any \( z > 0 \), using (8.9) one has

\[
F^r(z, 0) = \left( \frac{aS_r^r}{1-a} + n-1 \right) \frac{\theta^r}{n-1} - 1
\]

\[
< \frac{\theta^r}{1-a} - 1
\]

\[
\leq q^r - 1,
\]

which is negative, and thus \( g = (0,0,\ldots,0) \) is an equilibrium.

2.-3. see Appendix E.

Note that the distribution of preference parameters revealed in the dictator experiments implies the following. First, remember that the Isaac and Walker (1988) experiment
reported in Fehr and Schmidt’s Table II, was run with a marginal rate of return on investment in the public good of $a = 0.75$. Note that the sufficient conditions of either Proposition 8.3.3 (a) or Proposition 8.3.3 (b) are satisfied since $|J| < n$ (see Figure 7.6). In either case, the other-regarding preferences model predicts that in any asymmetric Nash equilibrium, $g_i > 0$, for all $i \in I$, with $\theta^i > (1 - a)$. Therefore the percentage of free-riders is not larger than 37%. Second, for all other experiments reported in Fehr and Schmidt’s Table II, the sufficient conditions of Proposition 8.3.3(c) are satisfied since $\theta^j \leq (1 - a)(\leq 0.5)$ for all $j = 1, 2, \ldots, n$ (see Figure 7.6). Thus, the only symmetric equilibrium is for all contributions to equal 0. Referring to the proof of that part (see Appendix E), one derives that, in any asymmetric Nash equilibrium $g_i$, $g_r = 0$, for all $r = 1, 2, \ldots, n$, such that

\[(8.11) \quad \theta^r_+ < (1 - a)/(1 + k/(n - 1))\]

where $k$ is the number of subjects with positive contributions in $g$. Being consistent with the distribution of preference parameters shown in Figure 7.6, we may assume that 69% of the subjects have parameters $\theta^r_+ \leq 0.35$. It should be clear that in any asymmetric equilibrium 69% of subjects, those who have parameters $\theta^r_+ \leq 0.35$, would invest 0 in the public good. Noting that in such a case $k \leq 0.31$, one can derive the predicted percentage of free-riders for all experiments (see Table 8.2).

Thus the egocentric other-regarding preferences model predicts that, on average, the percentage of subjects who would free ride completely is not less than 69%, whereas the empirical average percentage is 73%.

### 8.6 Ultimatum game

In an ultimatum game, the first mover proposes a division of a fixed amount of money between himself and the second mover. The second mover then either accepts the proposed division or rejects it, in which case both players receive zero. Of course, the model of self-regarding preferences predicts that a second mover strictly prefers any positive amount of money to $0 and that she is indifferent between accepting a proposal that gives her $0
# Chapter 8. A Model of Other-Regarding Preferences

<table>
<thead>
<tr>
<th>Study</th>
<th>Group Size</th>
<th>Marg. Ret.</th>
<th>RHS of Eq. (8.11)</th>
<th>Predicted Free-Riders (%)</th>
<th>Empirical Free-Riders (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isaac and Walker (1988)</td>
<td>4 (10)</td>
<td>0.3</td>
<td>0.495283, (0.520661)</td>
<td>≥69</td>
<td>83</td>
</tr>
<tr>
<td>Isaac and Walker (1988)</td>
<td>4 (10)</td>
<td>0.75</td>
<td>...</td>
<td>≤37</td>
<td>57</td>
</tr>
<tr>
<td>Andreoni (1988)</td>
<td>5</td>
<td>0.5</td>
<td>0.36036</td>
<td>≥69</td>
<td>54</td>
</tr>
<tr>
<td>Andreoni (1995a)</td>
<td>5</td>
<td>0.5</td>
<td>0.36036</td>
<td>≥69</td>
<td>55</td>
</tr>
<tr>
<td>Andreoni (1995b)</td>
<td>5</td>
<td>0.5</td>
<td>0.36036</td>
<td>≥69</td>
<td>66</td>
</tr>
<tr>
<td>Croson (1995)</td>
<td>4</td>
<td>0.5</td>
<td>0.353774</td>
<td>≥69</td>
<td>71</td>
</tr>
<tr>
<td>Croson (1996)</td>
<td>4</td>
<td>0.5</td>
<td>0.353774</td>
<td>≥69</td>
<td>65</td>
</tr>
<tr>
<td>Keser and van Winden (1996)</td>
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<td>0.5</td>
<td>0.353774</td>
<td>≥69</td>
<td>84</td>
</tr>
<tr>
<td>Ockenfels and Weiman (1996)</td>
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<td>0.33</td>
<td>0.482883</td>
<td>≥69</td>
<td>89</td>
</tr>
<tr>
<td>Burlando and Hey (1997)</td>
<td>6</td>
<td>0.33</td>
<td>0.488338</td>
<td>≥69</td>
<td>66</td>
</tr>
<tr>
<td>Falkinger, et al. (forthcoming)</td>
<td>8</td>
<td>0.2</td>
<td>0.590717</td>
<td>≥69</td>
<td>75</td>
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<td>Falkinger, et al. (forthcoming)</td>
<td>16</td>
<td>0.1</td>
<td>0.676353</td>
<td>≥69</td>
<td>84</td>
</tr>
</tbody>
</table>

Table 8.2: Predicted and empirical percentage of subjects who free ride completely.
8.6. **ULTIMATUM GAME**

and rejecting it. Thus, in an experiment with a discrete unit of divisibility, a first mover with self-regarding preferences is predicted to propose giving the second mover $0$ or $\varepsilon$, and a second mover with self-regarding preferences is predicted to accept either proposal. With a continuous payoff variable, the unique subgame perfect equilibrium is a proposal of $0$, which is accepted by the second mover.

Numerous ultimatum game experiments have been run in several countries, all of which produced data inconsistent with the predictions of the self-regarding preferences model. The robust results are that almost all proposals assign between 20% and 50% of the total payoff to the second mover and that some second movers reject low offers when they (infrequently) occur (Güth, Schmittberger, and Tietz (1990), Roth (1995)).

The ego-centric other-regarding preferences model has the following implications for the ultimatum game. A second mover’s utility function is globally increasing in both players’ money payoffs. Therefore a second mover will accept any proposal. Knowing this, a first mover will choose the proposal that maximizes her utility, which depends on the parameters $\theta_P^-$ and $\theta_P^+$ of her utility function. The limiting value of $\theta_+ = 0$ gives us the self-regarding preferences model, for which the first mover proposes 0 to the second mover. Let the amount to be divided between the two players be $\omega$. Statements (7.5) and (7.6) imply that the slope of the first mover’s indifference curves as they approach the 45-degree line from below is $-(1 - \theta_+)/\theta_+$. Therefore, if $\theta_+ \in (0, 1/2)$ then the first mover will propose to the second mover the amount $s \in (0, \omega/2)$ given by

$$s = \omega \left(1 + ((1 - \theta_+)/\theta_+)^{1/(1-\alpha)}\right)^{-1}.$$ 

Finally, if $\theta_+ \in [1/2, 1)$ then statements (7.5), (7.6), and (7.7) imply that the first mover will propose to the second mover the amount $s = \omega/2$.

**Proposition 8.4** A responder will accept all offers in an ultimatum game. The predicted outcomes for the ultimatum game depend on the proposer's parameter, $\theta_+$:

1. if $\theta_+ = 0$ then $s = 0$;

2. if $\theta_+ \in (0, 1/2)$ then $s = \omega \left(1 + ((1 - \theta_+)/\theta_+)^{1/(1-\alpha)}\right)^{-1}$.
3. if $\theta_+ \in [1/2, 1)$ then $s = \omega/2$.

**Proof.** It should be clear that it is a dominant strategy for the second mover to accept any offer $s \geq 0$ since $u^R(s) > u^R(0), \forall s > 0$.

Furthermore, note that the proposer's utility as a function of $s$ is:

\begin{equation}
    u^P(s) = [(1 - \theta)(\omega - s)^{\alpha} + \theta s^{\alpha}]^{1/\alpha},
\end{equation}

where

\[ \theta = \begin{cases} 
    \theta_+, & s < \omega/2 \\
    \theta_-, & s \geq \omega/2.
\end{cases} \]

Differentiating (8.12) with respect to $s$, one has

\begin{equation}
    \frac{du^P(s)}{ds} = u(s)^{1-\alpha}[\theta s^{\alpha-1} - (1 - \theta)(\omega - s)^{\alpha-1}]
\end{equation}

Let $s_- = \omega \left(1 + ((1 - \theta_+)/\theta_+)\right)^{1/(1-\alpha)}$, and $s_+ = \omega \left(1 + ((1 - \theta_+)/\theta_+)\right)^{1/(1-\alpha)}$ denote the solutions of the equation $\frac{ds}{ds} = 0$ for parameters $\{\theta_-, \alpha\}$ and $\{\theta_+, \alpha\}$ respectively. Note that $s_- < \omega/2$ since $\theta_- < 1/2$ from the parameter restrictions (7.7). So, by asking not less money payoff for himself, the first mover gets a utility of at most $\omega/2$, which corresponds to the division $s = \omega/2$. Since one of the properties of the utility function is continuity, the optimal division does depend only on the proposer's egocentricity parameter, $\theta_+$. Thus, in equilibrium the proposer will offer

\[ s = \begin{cases} 
    0, & \text{if } \theta_+ = 0 \\
    s_+, & \text{if } \theta_+ \in (0,1/2) \\
    \omega/2, & \text{if } \theta_+ \in [1/2,1),
\end{cases} \]

where

$\omega \left(1 + ((1 - \theta_+)/\theta_+)\right)^{1/(1-\alpha)}$.

It is clear from Proposition 8.4 that the egocentric other-regarding preferences model can explain all properties of the data from ultimatum game experiments with one exception, which is the low but significant incidence of rejections of proposals. In contrast, an inequality aversion model can explain these rejections (Fehr and Schmidt (1999, p.826)). Thus each type of model can explain data from an experimental environment that the
other type of model cannot explain: (i) other-regarding preferences can explain proposer behavior in our dictator game experiments but inequality aversion cannot; and (ii) inequality aversion can explain responder behavior in ultimatum game experiments but other-regarding preferences cannot. Of course, the other-regarding preferences model could be generalized to include inequality-averse preferences by changing the parameter restrictions in statement (7.7) to allow $\theta_-$ to take on negative values. This generalized model would include both other-regarding preferences and inequality-averse preferences and could explain all of the above-cited data, including that for proposer behavior in dictator games and responder behavior in ultimatum games. But explaining responder vetoes in ultimatum games solely by inequality-averse preferences is questionable because of the experiments reported by Blount (1995).

The experimental design in Blount (1995) reveals the incremental effects on responder behavior of attributions of the intentions of proposers in ultimatum games. The paper reports several treatments in three “studies.” Two of the treatments in study 1 are especially informative for the present discussion. The subjects were randomly assigned to two groups. Before learning which group he was in, each subject was asked to report a “minimum acceptable outcome” (MAO) that would be used in the event he was a responder. If a responder subsequently received a proposal greater than or equal to his stated MAO, the proposal would be implemented, otherwise it would be vetoed. In the “interested party” treatment, the proposals were made by the one-half of the subjects that had been randomly selected to be proposers. In the “random” treatment, the proposals were randomly selected from a uniform distribution of amounts varying from $0 to $10. The unit of divisibility in all treatments was $0.50. The results from these two treatments included the following. In the random treatment, about 80% of the subjects chose an MAO of $0 or $0.50. In contrast, less than 30% of the subjects chose an MAO of $0 or $0.50 in the interested party treatment. Of course, the difference between the complete distributions of responses for the two treatments is significant. These results show that models of preferences over outcomes cannot provide a complete explanation of responder behavior in the ultimatum game. The ultimatum game is beyond the boundary of empirical failure of outcome-preference models and within the domain of environments in which models
that incorporate perceived intentions of others, in addition to preferences over outcomes, are needed to provide a complete explanation of behavior.

### 8.7 Concluding remarks

The egocentric other-regarding preferences model can rationalize data from several other types of experiments, including experiments with proposer competition, responder competition, and voluntary public goods contributions.

The present version of our model cannot explain second-mover vetoes in the ultimatum game. Formally, models of inequality aversion can explain such vetoes (Fehr and Schmidt (1999)). An extended version of our model that permits $\theta_-$ to be negative for some agents could also explain the vetoes. But experiments reported by Blount (1995) make this a questionable approach. Her data lend support to the view that ultimatum game vetoes are a type of behavior that cannot be explained by models of preferences over outcomes, whether they be inequality-averse preferences or other-regarding preferences of the extended type; instead, explaining the vetoes requires introduction of perceptions of others’ intentions into theoretical models.