Chapter 9

Summary and Conclusions

In this final chapter, we summarise the main result and conclusions presented in this thesis, and future work to enhance the numerical model.

In the present study, the lattice Boltzmann method is presented as a robust and accurate haemodynamics numerical solver at mesoscale. The capabilities and shortcomings of the method are discussed. It is demonstrated that the lattice Boltzmann is of second order in space and time at low Mach numbers. The stress tensor is obtained from the non-equilibrium parts of the distribution functions without any need to approximate the shear-rate.

Various steady and unsteady numerical simulations are performed, all yielding excellent agreement with analytical solutions, other numerical methods, and/or available experimental data. Machine accuracy was obtained for some simple flow problems such as the channel and the Couette flows, even with the bounce-back rule, which is known to be of first order behaviour.

For 2D unsteady flows driven by a body force, a shift in time has been observed and analysed. The lattice Boltzmann BGK model is found to be more accurate when a half time step correction is added to the time coordinates. The effects of the Womersley, the Reynolds and the Strouhal numbers have also been studied in a number of simulations which showed that the shift in time is reduced at high Reynolds numbers. The obtained accuracy in time for time-dependent flows is of first order.

Using a quasi-incompressible D3Q19 model for the 3D simulations of oscillatory tube flow we have recovered the analytical Womersley solution with an average error of about 15% with bounce-back on the link at relatively high Mach number which reduces to less than a percent at low Mach numbers, at a cost to computational time. A body fitted curved boundary condition, recently proposed by Bouzidi et al. (2001) is found to produce better results and is of second order accuracy.

As the purpose of building this numerical solver is to use it as an interactive flow solver in the promising Cross Grid environment now under development, performance of the method was enhanced by introducing the Mach number annealing as an acceleration technique for unsteady flows. With Mach number annealing, it is possible to perform simulations that are of the order of the annealing factor times faster than non-annealed simulations. Composite annealing procedures in which both
Reynolds and Mach numbers may be annealed would be adopted for further acceleration in the future. The influence of walls, inlet and outlet boundary conditions on accuracy and performance is studied in detail as a function of Mach and Knudsen numbers. It is found that the bounce-back on the links could be more efficient if used at low Mach numbers when the Mach number annealing technique is used. With the Mach number annealing, we recommend the bounce-back on the links as a better alternative than other sophisticated boundary conditions which are difficult to use in the field of haemodynamics.

Another application of interest in simulation environments is changing the geometry on-the-fly and investigating the robustness of the numerical method in producing accurate results without a need to restart the simulation. The lattice Boltzmann method is found to be fully adaptive, as demonstrated by simple test cases. More investigation is necessary to demonstrate the real benefit of this feature. This is recently under study in our group.

Simulation results of steady and unsteady flow in a model of the human aortic bifurcation reconstructed from Magnetic Resonance Angiography are presented as a typical haemodynamic application. The computational model under study involves only the bifurcation region, directly after the IMA, and includes parts of the left and right iliac arteries. We have conducted a number of steady and unsteady flow simulations for the aorta model. Results on velocity fields and stress are successfully obtained and are qualitatively compared to literature.

As the shear stress plays a crucial role in cardiovascular diseases and since it is directly and independently computed in the lattice Boltzmann solver, we strongly encourage researchers from haemodynamics to consider this method as an alternative blood flow solver. More benefits are seen from easy grid generation and straightforward parallelism, easy and feasible adaptation to changing geometry. Further investigation of the complete aorta model and experimental validations are under development in our group.

A main conclusion in the thesis is that, even with the most simple lattice Boltzmann methods, comparable results to the most sophisticated traditional solvers are possible. However, the thesis has left open many questions to be answered. Some of them are

- The method is not yet matured in solving models involving fluid-structure interactions. Some recent developments based on coupling the fluid lattice Boltzmann with solid models (Chopard et al., 2002, personal communication) to investigate thrombosis are promising.
- The lattice Boltzmann method with simplified BGK approximation has many troubles at high Mach and Reynolds numbers, but not at haemodynamic Womersley numbers. However, this issue is quite sensitive to the boundary conditions.

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1It is worth noting that Chopard suggested to the author in 2000 that this coupling would be useful if applied in Haemodynamics. At that time we were not sure if even the lattice BGK yields acceptable results. When that has been cleared out, there is no time left for this interesting direction.
and the lattice BGK boundary conditions are known to be viscosity dependent, except for a few of them. This has direct influence on the stability of the method. A solution is to consider using generalised lattice Boltzmann equations (GLBE). However, the computational cost raised by using GLBE needs to be investigated versus the accuracy and compared to the engineering accuracy.

- It is well known that the non-Newtonian models produce significantly different results than Newtonian ones. Although the global structure remains similar, the profile of the stress tensor is significantly different near the walls. It is relatively easy to perform non-Newtonian flows in lattice Boltzmann models, but GLBE may be a better option than the lattice BGK.

- As early turbulence may build up in the circulation, turbulent models need to be approached in an ideal haemodynamic solver. A number of turbulent models are available in previous literature, most of which use GLBE.

- With the development in imaging techniques and the increase in microscopic understanding of atherosclerosis, traditional CFD solvers may not be useful and mesoscopic models may join wider acceptance.

In conclusion, the lattice Boltzmann methods are accurate and robust computational fluid dynamics solvers in the mesoscale, with elegant computational characteristics. This makes them feasible alternates to traditional macroscopic solvers in a wide range of applications.