Choice quantification in process algebra
Luttik, S.P.

Citation for published version (APA):
When I started work as an onderzoeker in opleiding, affiliated with the CWI and the University of Amsterdam, my first project was to specify a part of the IEEE 1394 (see Luttik, 1997). It was my first encounter with the process specification language $\mu$CRL, and with the choice quantifier (also referred to as 'sum operator'). Syntactically, $\mu$CRL is an extension of the algebraic process theory ACP, which I had come across before, and, indeed, the choice quantifier reminded me of the notation sometimes used in ACP specifications to abbreviate large alternative compositions. However, in ACP, this notation is informal, and its use is explicitly restricted to cases in which the abbreviated alternative composition is finite. In contrast, the choice quantifier belongs to the official syntax of $\mu$CRL, and it may refer to an infinite alternative composition.

Since my first contact with $\mu$CRL, I have been interested in choice quantification, and especially in its mathematical theory. Jan Friso Groote and I tried to axiomatise choice quantification in the context of a finite fragment of $\mu$CRL in several semantic settings. First, we investigated the completeness of a set of equational axioms with strong bisimulation as a semantics (Groote and Luttik, 1998a). Our main conclusion was that a complete set of axioms could not be found in general, because for certain data, the associated notion of bisimulation was too complex to have an axiomatisation. We also formulated general restrictions on the data, under which our set of axioms was complete. Then, we extended our results to a setting with branching bisimulation as a semantics (Groote and Luttik, 1998b), and I extended this result further, to settings with weak-, delay- and $\eta$-bisimulation as semantics (Luttik, 1999a).

In my view, a drawback of the axiomatisations we had found was that they treat choice quantification as a binder, a construction that relies on the syntactic structure of its argument. As such, our axiomatisations could not be viewed at the same time as an abstract algebraic definition of the mathematical notion that choice quantification refers to. In other words, choice quantification had in our theory not the same semantic status as the operations of a purely algebraic theory such as ACP. Building on the techniques of algebraic logic, I therefore proposed an alternative treatment of choice quantification, which abstracts from the syntactic aspect of choice quantification (Luttik, 1999b).

Apart from conducting the above mentioned investigations, I participated in other research. Eelco Visser and I coauthored a paper on the specification of rewriting strategies (Luttik and Visser, 1997). Together with Piet Rodenburg and Rakesh Verma, I wrote a paper on correctness criteria for transformations of rewrite systems (Luttik et al., 1998). Wan Fokkink and I proved that a finite $\omega$-complete specification of interleaving is obtained by adding to the algebraic theory

When time had come to present a dissertation, I could have chosen to just put all my papers together. Clearly, given the diversity of subjects, this would have resulted in a very fragmented account. Instead, I preferred to try and write a coherent report about my study of choice quantification, the main theme of my research thus far. One of the things I had learned was that the mathematical definitions underlying $\mu$CRL are considerably more complex than those underlying, e.g., the algebraic theory ACP, which is firmly founded on the standard theory of universal algebra. In fact, I found that the mathematical basis for $\mu$CRL had not been defined in sufficiently precise detail.

This dissertation, then, is concerned with the mathematical theory of choice quantification, with a bias towards an algebraic approach. It is organised as follows. Chapter 1 explains the advantages of using choice quantification in a process specification, and briefly touches on the subjects of the later chapters. Chapter 2 explores the semantic connection between $\mu$CRL and ACP, providing an abstract algebraic definition of infinite sums in basic process algebras with deadlock. Chapter 3 defines the fragment of $\mu$CRL which is the main focus of the rest of the book. It establishes a connection with the structures discussed in Chapter 2; in particular, it explains how choice quantification relates to alternative composition.

Chapter 4 demonstrates a correspondence between choice quantification in $\mu$CRL and quantification in first-order logic. It considerably improves on the first part of (Groote and Luttik, 1998a), showing that, with respect to the data inside $\mu$CRL expressions, choice quantification can simulate both universal and existential quantification of first-order logic. We put this in perspective by showing that the input prefix mechanism of value-passing CCS can only simulate universal quantification.

The results of Chapter 4 motivate the restrictions imposed on the data domain in later chapters. Chapter 5 discusses a sound and complete deductive system, and is based on the second part of (Groote and Luttik, 1998a). Chapter 6 is based on (Luttik, 1999a), and presents an alternative to the deductive system discussed in Chapter 5; this alternative is more attractive from an algebraic point of view. It is shown that the systems of Chapters 5 and 6 are equivalent in expressive and deductive power. Chapter 7 presents the conclusions.

Acknowledgments

My supervisors, Jan Bergstra and Jan Friso Groote, always expressed their confidence in me, for which I am very grateful. Jan had time for me whenever I wanted to discuss my work or my personal situation. He always gave inspiring and valuable advice. Jan Friso made his ‘theme’ at the CWI into a stimulating research environment, in which I had the freedom to do the research that I wanted to do. His enthusiasm was of great support to me.

Piet Rodenburg played an invaluable part in my development as a researcher. I could always drop by his office to ask him a question, to test an idea for a proof, or
just to chat about one thing or another. He taught me a lot about logic, algebra, and science in general. His proof reading ability is superhuman.

My cooperation with Wan Fokkink was most pleasant, and when he succeeded Jan Friso as theme leader, he turned out to be an excellent boss as well, providing me, both literally and figuratively, with the room in which I could write this book. The circumstances in my last six months at the CWI, in my corner on the third floor, were perfect. I could close my door and concentrate on the writing. But whenever I felt like it, I could swing it open again, to find in Jaco van de Pol an enthusiastic neighbour, ready to exchange ideas. He read and commented on large parts of this dissertation.

I thank the members of the reading committee, Maarten Boasson, Jan van Eijck, Wan Fokkink, Paul Klint, Kees Middelburg, Piet Rodenburg and Davide Sangiorgi for reviewing the manuscript and for their comments. A last minute discussion with Jan van Eijck enabled me to improve the presentation.

I wish to express my gratitude to all the participants of PAM, and especially to top speakers Sjouke Mauw and Vincent van Oostrom, who were always willing and able to fill gaps in my program. Vincent was usually there when I spent the weekend at the CWI, and I have benefited from our long conversations. I wish to extend my gratitude to all the former colleagues of the CWI, and in particular to Doekos Bosscher, David Griffioen, Alban Ponse, Michel Reniers, Judi Romijn, Yaroslav Usenko and Mark van der Zwaag.

Jan Willem Klop and Roel de Vrijer were so kind as to enable me to complete this book at the VU. My new colleagues there, and especially my new office mate Mirna Bognar, made me quickly feel at home.

The cooperation with Eelco Visser, at the end of my first year, has meant a lot to me. He has become a good friend, and has been very encouraging ever since.

The most important source of relaxation in my life is jazz music; I played the piano in quite a few (big) bands, and I wish to thank my fellow musicians. I am most grateful to all my friends, and in particular to Anna, Eric, Floortje, Ingmar, Luuk, Ramin, Rob and Sanne. Simona's presence and support made a great difference to me and, indeed, to this book; multumesc frumos!

My final words of thanks are for my family: Mama & Jan, Papa & Karin, and my three sisters Léonie, Tirza and Janine. Jan carefully read the introduction and suggested many improvements with regard to English usage. I am happy to have Tir and Nien for my 'paranimfen'.

Bas Luttik

Amsterdam, January 2002