A term structure model of interest rates and forward premia: an alternative monetary approach
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Citation for published version (APA):

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Chapter 2

Theoretical Review

A fundamental assumption of many financial-economic models is that the price of domestic currency is expected to depreciate when domestic nominal riskless interest rates exceed foreign nominal riskless interest rates. This assumption is based on the well-known uncovered interest rate parity condition. However, empirical literature consistently rejects the null hypothesis that the forward foreign exchange premium is an unbiased forecast of the expected change in the currency spot price. In addition, rational expectation models of the forward foreign exchange premium have been unable to provide a plausible explanation for this anomaly. Other approaches, such as peso problems, risk premia, and irrational speculative bubbles, have made some progress in explaining the forward rate bias.

Our focus in this theoretical review is on rational expectation models that seek to explain this bias based on foreign exchange risk premium. Since our area of research is the term structure of interest rates we concentrate in particular on no-arbitrage and general equilibrium models of forward risk premium. Other studies of the risk premium based on rational behavior, such as the consumption based CAPM and the portfolio-balance models of forward foreign exchange risk premiums are not considered. In this review we do not cover those models based on irrational expectations, speculative bubbles, learning and peso problems.
The forward foreign exchange anomaly can best be illustrated by considering the
depreciation of the log-currency price on the forward foreign
exchange premium, expressed in log form:

\[ s_{t+1} - s_t = a + b(f_t - s_t) + u_{t+1}, \tag{2.1} \]

where \( s_t \) is the log of the spot domestic price of the foreign currency at time \( t \), \( f_t \) is the
log of the one-period forward exchange rate at time \( t \), and \( u_t \) is the regression error with
conditional mean equal to zero. Based on the assumption that the log-forward rate is
an unbiased predictor of the future log-spot currency price, the null hypothesis in this
regression is formulated as \( a = 0 \) and \( b = 1 \). Following Engel (1996) the consistent
estimator of the slope parameter \( \hat{b} \) in a large sample is defined as:

\[ p \lim (\hat{b}) = \frac{\text{Cov}(f_t - s_t, s_{t+1} - s_t)}{\text{Var}(f_t - s_t)}, \tag{2.2} \]

where \( \text{Cov}(\cdot) \) and \( \text{Var}(\cdot) \) denote the covariance and variance, respectively. Let the risk
premium on the forward foreign exchange contract be defined as

\[ \tau p_t = f_t - E^m_t (s_{t+1}), \tag{2.3} \]

where \( E^m_t (s_{t+1}) \) represent the market's expectation of the future exchange rate. As in
Engel (1996) assume that the market's subjective conditional probability distribution for
the exchange rate is not the same as the true distribution conditional on information
available to the market. If we allow for irrational expectations, i.e. \( E_t (s_{t+1}) \neq E^m_t (s_{t+1}) \),
then we obtain for the estimate of \( b \) in a finite sample the following expression:

\[ \hat{b} = 1 - \hat{b}_{rp} - \hat{b}_{ss} - \hat{b}_{ue}, \]
where

\[ \hat{b}_{rp} = \frac{\text{Cov} (F_t^m (s_{t+1}) - s_t, rp_t) + \text{Var} (rp_t)}{\text{Var} (f_t - s_t)}, \]

\[ \hat{b}_{ss} = \frac{\text{Cov} (f_t - s_t, E_t (s_{t+1}) - s_{t+1})}{\text{Var} (f_t - s_t)}, \]

\[ \hat{b}_{ie} = \frac{\text{Cov} (f_t - s_t, E_t^m (s_{t+1}) - E_t (s_{t+1}))}{\text{Var} (f_t - s_t)}, \]

and \( \hat{b}_{ie} \) captures the correlation of the forward premium with the expectation error, that is the deviation of market's expectation from rational (i.e. irrational expectation factor of the bias). The term \( \hat{b}_{ss} \) represents the deviation of the estimate of \( b \) in a particular sample from its probability limit due to, for instance, small-sample bias or sample variation. Note that \( \text{plim} \left( \hat{b}_{ss} \right) = 0 \). The term \( \hat{b}_{rp} \) captures the correlation with respect to the risk premium under rational expectation and correct sampling. If these terms (\( \hat{b}_{rp}, \hat{b}_{ss}, \) and \( \hat{b}_{ie} \)) are positive, it may provide an explanation for the forward bias. The irrational and speculative bubbles literature focuses on the \( \hat{b}_{ie} \)-term as an argument for the observed forward discount bias. The 'peso problem' approach provides arguments in favor \( \hat{b}_{ss} \) as the basis of this bias. Since our focus in this theoretical review is on rational expectation models that seek to explain the forward discount bias based on foreign exchange risk premium, we do not discuss these two terms as a possible explanation, i.e. we concentrate on \( \hat{b}_{rp} \neq 0 \) as a possible explanation of the bias.

We distinguish between two approaches, namely an approach that does not attempt to model the forward risk premium, but instead take it as exogenous, and another that derives this endogenously. The latter approach of foreign exchange risk premium models based on optimizing behavior can be divided into two broad categories: models based on no-arbitrage and general equilibrium conditions.
Exogenous forward risk premium approach

Some studies of the currency puzzle consider realistic economic condition that can possibly account for the negative Fama slope coefficient, with an exogenously given forward risk premium [e.g. Boyer and Adams (1988) and McCallum (1994)]. McCallum (1994) gives three explanations for the empirical rejection of the unbiased hypothesis. The first one is that the uncovered interest rate parity relation does not hold. The second argument refers to the possibility that \( b = 1 \) indeed, but that expectations are not rational. This argument implies that the economic agents are irrational, hence a negative estimate of \( b \) is obtained in empirical studies. The last one, which is analytically relevant, can be referred to as the policy response hypothesis, whereby account is taken of the fact that monetary policy is conducted in such a way to manage the interest rate differentials. This is modeled through the addition of an extra equation capturing the impact of monetary policy on \( R_t - R_t^* \), where \( R_t \) denotes the domestic nominal interest rates and \( R_t^* \) is the foreign nominal interest rate. Consequently McCallum models policy behavior as

\[
R_t - R_t^* = \lambda (s_t - s_{t-1}) + \psi (R_{t-1} - R_{t-1}^*) + \mu_t
\]  

(2.4)

with \( \mu_t \) representing random policy influences, \( \lambda \) the exchange rate response parameter, and \( \psi \) reflects the smoothing parameter. Following Fama (1984) the forward premium is decomposed in an expected rate of depreciation and a so-called risk premium. As a result he obtains

\[
s^{\text{f}}_{t+1} - s_t + r_{p,t+1} = R_t - R_t^*,
\]

(2.5)

where \( s^{\text{f}}_{t+1} \) is the expected spot exchange rate at time \( t + 1 \) and \( r_{p,t+1} \) is the risk premium. A so-called bubble-free linear solution of Equations (2.5) and (2.4) is presented by McCallum (1994) as follows
\[ \Delta s_t = \phi_1 (R_{t-1} - R^*_{t-1}) + \phi_2 \mu_t + \phi_3 \varepsilon_t \]  

(2.6)

with \( \phi_1 = -\psi/\lambda \), \( \phi_2 = -1/\lambda \), and \( \phi_3 = 1/(\lambda + \psi) \). He argues that the model is capable of explaining the empirical rejection of the unbiasedness hypothesis in a way that is consistent with the uncovered interest rate parity. When the forward risk premium follows a first-order serially-correlated process with correlation coefficient \( \nu \), he obtains

\[ E_t s_{t+1} - s_t = \frac{\nu - \psi}{\lambda} (R_{t-1} - R^*_{t-1}) . \]  

(2.7)

This specification is capable of generating a negative value for the Fama slope parameter when \( \nu \) is smaller than the smoothing parameter \( \psi \). The question now is whether this monetary feedback scheme is supported by a general equilibrium model.

**No-arbitrage models of foreign exchange risk premium.**

Under this category falls a broad class of models, which can in general be characterized by the following no-arbitrage condition:

\[ v_{i,t} = E_t (q_{t+1} d_{i,t+1}) , \]  

(2.8)

or

\[ 1 = E_t (q_{t+1} R_{i,t+1}) , \]  

(2.9)

where \( v_{i,t} \) denotes the currency value of a claim at time \( t \), \( d_{i,t+1} \) is the stochastic cash flow one period latter, \( R_{i,t+1} = d_{i,t+1}/v_{i,t} \) represents the gross one-period return on asset \( i \) and \( q_{i+1} \) is a positive state-price density process or pricing kernel. We can distinguish two approaches with respect to the pricing kernels. One approach uses a model of intertemporal utility maximization under uncertainty for the foreign exchange risk premium, where the pricing kernel might be the intertemporal marginal rate of substitution. Another ap-
proach, uses Equation (2.9) purely as an arbitrage condition and imposes restrictions on the state-price density process. This approach is reviewed below, under the sub-section "the term structure of interest rate models".

The first approach defines the state-price density process, for instance, when utility is additively time-separable as \( q_{t+1} = \rho u' (c_{t+1}) / u' (c_t) \), where \( \rho \) is the subjective discount factor in the individual's utility function and \( c_t \) denotes the individual's consumption flow at time \( t \). Note that Equation (2.9) holds for both domestic and foreign assets. Therefore, we obtain as in Engel (1996)

\[
0 = E_t \left( \frac{\rho u' (c_{t+1})}{u' (c_t)} \left( R^h_{t+1} - R^f_{t+1} \right) \right). \tag{2.10}
\]

Most of these studies of the forward discount bias specify a utility function which display constant relative risk aversion [e.g., Mark (1985), Hodrick (1989) and Modjtahedi (1991)],

\[
U(C_t) = \frac{1}{1-\gamma} C_t^{1-\gamma}, \tag{2.11}
\]

where \( \gamma \) is parameter of relative risk aversion. For analytical reason the null hypothesis [Equation (2.1)] in these models are formulated in real terms,

\[
0 = E_t \left( \frac{F_t - S_{t+1}}{P^h_{t+1}} \right), \tag{2.12}
\]

where \( F_t \) and \( S_{t+1} \) denotes, respectively, the level of the forward foreign exchange and the spot currency price and \( P^h_{t+1} \) is the home price level. It is assumed that the variables are log-normally distributed and therefore the rational expectations risk premium, \( r p_t = f_t - E_t (s_{t+1}) \), can be obtained from Equation (2.10) as:

\[
r p_t = 0.5 \text{Var}_t (s_{t+1}) - \text{Cov}_t (s_{t+1}, p^h_{t+1}) - \gamma \text{Cov}_t (s_{t+1}, c_{t+1}).
\]

It is generally accepted that the Jensen's inequality term, \( 0.5 \text{Var}_t (s_{t+1}) - \text{Cov}_t (s_{t+1}, p^h_{t+1}) \), is very small and unable to account for the forward bias of \( r p_t \neq 0 \) [see e.g., McCulloch
(1975), Engel (1984), Hodrick (1989), and Backus et al. (1993)]. As a result, the explanation of this bias lies with the last term, \( \gamma \text{Cov}_t (s_{t+1}, c_{t+1}) \). Most of these studies find that consumption data do not generate large variability, such that it can account for the large variance of ex-ante returns from foreign exchange. Therefore, it is inevitable that these studies obtain implausible large estimates for the parameter of relative risk aversion, \( \gamma \).

Since the time-additive preferences combined with modest degrees of risk aversion cannot account for the bias, Backus et al. (1993) uses a time non-separable utility function to derive the state prices, \( q_{t+1} \), in Equation (2.9). Habit persistence has been applied successfully in other frameworks. They argued that this kind of intertemporal non-separability, as applied by Constantinides (1991), greatly increases the equity-premium-puzzle theory's ability to generate mean excess returns on equity similar to those observed in the data.

Their expected utility function takes the following form

\[
U_t = E_t \sum_{k=0}^{\infty} \rho^k u (d_{t+k}), \quad u (d_t) = \frac{1}{1 - \gamma} \left( d_t^{1-\gamma} - 1 \right)
\]

(2.13)

\[
d_t = c_t - \lambda c_{t-1},
\]

(2.14)

where \( \lambda \) is the habit parameter that governs the intertemporal non-separability of preferences. For \( \lambda \) constraint to zero (i.e. the time-separable power utility function used in other studies), they also obtain implausible large estimates for the parameter of relative risk aversion, \( \gamma \). The unconstrained estimation of their models also do not provide plausible parameter values that accounts for the forward discount bias. Therefore, as concluded by Engel (1996), studies that rely on consumption data to explain the currency puzzle does not perform well.
General equilibrium models of foreign exchange risk premium.

In this context it is important to defined what is meant by general equilibrium models of exchange rate. In line with Dumas (1993) and Engel (1996), the models of foreign exchange risk premium we have considered so far can be characterized as partial equilibrium models. The explanation herefore is that the stochastic processes that govern asset returns in these models are exogenously given, while in a fully general equilibrium model these processes are determined endogenously by the underlying exogenous variables.

The Lucas (1982) two-country two-good two-currency "cash-in-advance" model forms the basis of most general equilibrium models of the risk premium [e.g. Backus and Chen (1997), Bekäert (1996), Bekäert (1994), Engel (1992), Macklem (1991), and Hodrick and Srivastava (1986)]. The Lucas model is a complete, dynamic, two-country, general equilibrium model. The world consists of two countries whose residents have identical preferences. In contrast they have different stochastic endowments consisting of the two goods and they get utility from both types of goods. In most of these studies the representative agent maximizes a time-separable additive utility function with constant subjective discount factor. Money is introduced in this economy through the cash-in-advance constraint and lump-sum money transfer from the government. In the Lucas model the investor learns the state of the economy at the beginning of the period, in particular the state is observed (arrival of new information) after the goods market is closed and before the asset market opens. Note that no new information arrives after the asset market is closed and before the goods market opens again. Then the individuals receive their share of cash from the sales of last period's endowments, receive money transfers, and trade cash and shares. After these transactions are completed, the consumer buys goods and pays with cash, and the period ends. This timing factor is a distinguishing feature of the Lucas model, as it entails only a transaction demand for money. In the Stockman's (1980) model, in contrast, consumers decide on their cash balances before they know the state of the economy. As argued by Svensson (1985) the demand for money is therefore
much more realistic than in Lucas, namely a combined transaction, precautionary, and store-of-value demand for money. However it may be impossible to obtain a solution for the stochastic equilibria in the Stockman’s (1980) model.

Engel (1996) obtains the risk premium in these Lucas-based models as follows,

\[ rp_t = -0.5 \left[ \text{Var}_t m_{t+1} - \text{Var}_t m^*_t \right] \]
\[ + \alpha (1 - \gamma) \text{Cov}_t (m_{t+1} - m^*_{t+1}, y_{t+1}) \]
\[ + (1 - \alpha)(1 - \gamma) \text{Cov}_t (m_{t+1} - m^*_{t+1}, y^*_{t+1}) \]  

(2.15)

and

\[ trp_t = f_t - f^*_t \]
\[ = -\alpha \gamma \text{Cov}_t (m_{t+1} - m^*_{t+1}, y_{t+1}) \]
\[ - (1 - \alpha) \gamma \text{Cov}_t (m_{t+1} - m^*_{t+1}, y^*_{t+1}), \]  

(2.16)

where \( m_t \) represents the log-money supply, \( y_t \) is the log of output, and asterisk denotes the foreign quantities. The parameters \( \gamma \) and \( \alpha \) capture coefficient of relative risk aversion and the expenditure share on domestic good, respectively. Engel (1996) shows that Equation (2.16) captures the equilibrium solution for the 'true' risk premium, where agents has risk neutral preferences. These equations imply that CIA-models, such as Hodrick and Srivastava (1986), that assume investors has risk-neutral preference and/or that output shocks are uncorrelated with money shocks can hardly account for the currency puzzle. As argued by Engel (1996), the \( trp_t \) is zero in this case and, since \( rp_t - trp_t = JIT \), the explanation of the puzzle is therefore based only on the Jensen inequality term, which is in general very small. In addition, it can be observed that the main source of the forward discount bias (\( rp_t \neq 0 \)) is the correlation between the rate of growth of money and output. The argument provided by Engel (1996) is that with constant expenditure shares, the exchange rate is a function only of domestic and foreign money. In general,
the risk premium on any asset is related to the covariance of the asset return with the marginal rate of substitution. Since, in this model the agents derive utility from the consumption of both goods, which is a function only of output, money supply must be correlated with output shocks for the exchange rate to be correlated with the marginal rate of substitution. Engel (1992) provides evidence why the Lucas framework cannot account for the forward discount bias. The main argument relates to the fact that the variability and the co-variability observed in the money and output data are not able to generate a large value for the $trp_t$.

Bekaert (1994) studies the forward bias in the context of a variant of Svensson’s two-country cash-in-advance model. The main difference between the Lucas (1982) model and the Svensson (1985) model is as mentioned above the timing of events. In the Svensson’s version of the Stockman’s (1980) model new information arrives after the asset market is closed and before the goods market opens. Given that uncertainty is resolved simultaneously with the money holdings decision and the (natural) restriction to equilibria with positive nominal interest rates results in binding liquidity constraints in the Lucas model. Whereas, in the Svensson’s model the cash-in-advance constraint is non-binding as the investors money holdings decision arrives before the state is revealed. As Svensson (1985) shows there is a wedge between the marginal utility of wealth and the marginal utility of consumption in his model due to the particular timing property, which is in contrast with the Lucas model. Therefore, real interest rates depend on monetary policy in the Svensson’s model, which is not the case in the Lucas model. In addition, this modeling approach of Svensson (1985) allows for velocity to enter directly in the exchange rate and risk premium expressions.

Bekaert (1994) solves for the exchange rate changes and its moments as a function of the exogenous processes of money and output. He allows general correlation between these two variables. In addition Bekaert uses a first-order Markov chain to approximate the law of motion of the forcing processes. The optimization problem of the home consumer is formulated as set of equations, representing respectively his preferences, the
CIA constraints, and the budget constraint:

$$E_0 \sum_{t=0}^{\infty} \rho^t U \left(x_t^d, y_t^d\right),$$

(2.17)

$$P_t^x x_t^d \leq M_t^d, \quad S_t P_t^y y_t^d \leq S_t N_t^d,$$

(2.18)

and

$$M_{t+1}^d + S_{t+1} N_t^d + \alpha_{t+1}' Q_t \leq \alpha_t' (Q_t + D_t) + (M_t^d - P_t^x x_t^d) + S_t (N_t^d - P_t^y y_t^d),$$

(2.19)

where $x_t^d$ and $y_t^d$ denotes the domestic demand for the home and foreign countries goods, $P_t^x$ and $P_t^y$ their respective prices, and $S_t$ the level of the spot exchange rate. The demand of home and foreign money is represented by $M_t^d$ and $N_t^d$, respectively. Asset prices and holdings are captured by $Q_t$ and $\alpha_t'$. The dividends, $D_t$, are nominal and expressed in the currency of the home country. The superscript $d$ denotes the quantity demanded ($s$ indicates the quantity supplied). By using the law of one price they obtain, as in Svensson (1985), the exchange rate as a forward-looking asset price

$$S_t = E_t \left[ U_2 (x_{t+1}^s, y_{t+1}^s) / P_{t+1}^y \right] / \left[ U_1 (x_{t+1}^s, y_{t+1}^s) / P_{t+1}^x \right],$$

(2.20)

where the subscripts on $U$ represent partial derivatives. Letting $i_t$ and $i_t^*$ denote home and foreign interest rates, Bekaert (1994) obtains:

$$1 + i_t = [E_t (n_{t+1})]^{-1}$$

(2.21)

and

$$1 + i_t^* = \left[ E_t \left( n_{t+1} \frac{S_{t+1}}{S_t} \right) \right]^{-1},$$

(2.22)
where

\[ n_{t+1} = \frac{\lambda_{t+1}}{\lambda_t} = \frac{E_{t+1} \left[U_1 \left( x_{t+1}^*, y_{t+1}^* \right) / P_{t+2}^* \right]}{E_t [U_1 \left( x_{t+1}^*, y_{t+1}^* \right) / P_{t+1}^*]} \]

The \( n_{t+1} \) represents the nominal (home currency) intertemporal marginal rate of substitution, defined as the ratio of the discounted value of one unit of the home currency tomorrow (\( \rho \lambda_{t+1} \)) and the value of the home currency unit today (\( \lambda_t \)). This set of equations characterizes his equilibrium solution for respectively the currency depreciation, the normalized forward bias, and the risk premium,

\[
\begin{align*}
DS_{t+1} &= \frac{(S_{t+1} - S_t)}{S_t}, \\
FB_{t+1} &= \frac{(S_{t+1} - F_t)}{S_t} = DS_{t+1} - FP_t, \\
RP_t &= E_t(DS_{t+1}) - FP_t, \\
\end{align*}
\tag{2.23}
\]

where the forward premium is defined as \( FP_t = (F_t - S_t)/S_t \). Bekärt (1994) chooses two widely used specification for the consumers preferences, namely, an addilog and homothetic utility function. He does not provide closed form solution for the covariance of the interest rate differential with the expected rate of depreciation. As a result it is not clear what the implications are of the Fama slope coefficient for the equilibrium interest rates in his model. He calibrates his model by choosing parameter values to minimize the distance between the model's mean, variance and first auto-covariance of exchange rate changes and the forward discount and the sample moments of those variables. His findings are that the volatility generated by the model for the \( rp_t \) is extremely small and cannot account for the observed volatility in the data. His model generates a value of one for the Fama slope coefficient in Equation (2.1).

Bekärt (1996) develops a Lucas-type two-country monetary general equilibrium model, where the cash-in-advance constraint is substituted by a transaction-costs function. That means that while agents incur transaction cost when buying goods in his model, money holding diminishes the transaction costs associated with purchasing consumption goods. The presence of transaction costs leads to money being valued in equilibrium. The trans-
action cost function is parametrized as a Cobb-Douglas function of the amount of the goods purchased and real money holdings. Agents' preferences are characterized by the so-called addilog utility function. This type of expected utility function exhibits durability in the short run and habit persistence in the long run. In this framework, agents are subjected to money supply and endowment shocks. He allows for time-varying heteroskedasticity in the exogenous forcing processes. Bekaert (1996) uses the law of one price to obtain the exchange rate level as the relative value of the marginal utility of pounds versus dollars,

$$S_t \lambda_t = \lambda_t^*, \quad (2.24)$$

where $\lambda_t$, the marginal utility of one dollar, is defined as the total expected marginal utility of consumption divided by the transaction cost adjusted price of one unit of the home consumption. Bekaert (1996) defines the home (foreign) interest rate as the net return on a nominal bond yielding one dollar (pound) at time $t$,

$$i_t = - \ln \left( E_t \left[ mrs_{t+4,4} \right] \right) \quad i_t^* = - \ln \left( E_t \left[ mrs_t^*_{t+4,4} \right] \right), \quad (2.25)$$

where $mrs_{t+4,4} = \rho^4 \lambda_{t+4}/\lambda_t$ denotes the dollar intertemporal marginal rate of substitution for four period returns. Solving for the optimization problem and the market clearing conditions, this equations yield explicit expressions for the exchange rate, the domestic and foreign interest rates, the forward premium and risk premium.

Bekaert (1996) calibrates the model parameters and produces simulated results for the variables in question, i.e. the forward discount, the exchange rate, and the forward risk premium. As argued by Engel (1996), the simulated moments for these variables are much closer to those observed in the data than the results of previous studies. Bekaert (1996) finds that the combination of time-varying uncertainty in the fundamentals and time aggregation is an important factor in explaining the risk premium puzzle. However, as concluded by Bekaert, the model still overpredicts the variability of the forward premium and severely underpredicts the variability of the risk premium.
Term structure models of interest rates

Most of the term structure models of interest rates that study the foreign exchange risk premium uses a no-arbitrage framework [e.g. Nielsen and Saá-Raquejo (1993), Ahn (1995), Saá-Raquejo (1994), and Backus et al. (2001)]. In this approach the state-price density process in Equation (2.9) is considered as a purely arbitrage condition. In these studies ad hoc restrictions are imposed on the pricing kernel, $q_{t+1}$. Backus et al. (2001) consider assets with returns denominated in both domestic and foreign currency, whereby Equation (2.9) hold for both. They convert pound returns in dollars and obtain $R_{t+1}^s = (S_{t+1}/S_t) R_{t+1}^f$. By using Equation (2.9) for both returns they derive

$$\frac{q_{t+1}^f}{q_{t+1}^s} = \frac{S_{t+1}}{S_t}. \quad (2.26)$$

Backus et al. (2001) relate the risk premium defined by Fama (1984) to the properties of the pricing kernel for the home country and the foreign country. As a result the pricing relation in Equation (2.9) becomes

$$0 = E_t (q_{t+1}^s (F_t - S_{t+1})). \quad (2.27)$$

Dividing Equation (2.27) by $S_t$ and rearranging terms they obtain the forward premium as

$$f_t - s_t = \log E_t q_{t+1}^f - \log E_t q_{t+1}^s. \quad (2.28)$$

Backus et al. (2001) consider the case of conditionally log-normal pricing kernels and asset returns, where $\log q_{t+1} \sim N(\mu_{1t}, \mu_{2t})$. Then, from Equations (2.26) and (2.28) they obtain the expected rate of depreciation and the forward risk premium, respectively, as

$$ed_t = E_t s_{t+1} - s_t = \mu_{1t}^f - \mu_{1t}^s \quad (2.29)$$
and

$$r_p = \frac{\mu^d_{2t} - \mu^s_{2t}}{2}. \quad (2.30)$$

Based on this closed expression for the expected depreciation and the risk premium, we can observe that the Fama (1984) conditions require in their framework, (1) negative correlation between differences in conditional means and conditional variances of the two pricing kernels and (2) greater variation in one-half the difference in the conditional variances. Since, Backus et al. (2001) do not require that $q_{t+1}$ be constrained by any particular equilibrium model, they search in the general class of affine term structure models for a random variable that has a stochastic process that satisfies these conditions. Backus et al. (2001) specify three types of stochastic process for $q_{t+1}$ that supports the affine structure. First, they use a discrete-time two-currency formulation of the Cox-Ingersoll-Ross (1985) model of term structure. For this particular specification they find that it generates the negative correlation between $ed_t$ and $rp_t$ of Fama’s condition (1), but that it cannot satisfy condition (2) and therefore cannot reproduce the Fama regression slope. The basic shortcoming of this formulation is the restriction that the foreign and domestic interest rates depend on independent state variables.

Second, Backus et al. (2001) consider the general class of affine models, with a common factor affecting both pricing kernels in the same way and currency-specific factors that affect the pricing kernel of only one currency. This is similar to the structure that has been applied by Ahn (1997, 1998). Since the common factor affects both pricing kernels in the same way, it has no effect on currency prices, interest rate differentials, or the Fama slope parameter. The currency specific factors account fully for the forward premium anomaly in their model. As argued by Backus et al. (2001) this result is at the cost of allowing for a positive probability of negative interest rates.

Third, they consider an interdependent factor model where the foreign and domestic interest rates depend in different ways on the same two state variables. Similar models are considered by Bakshi and Chen (1997), Nielsen and Saá-Raquejo (1993), and Saá-Raquejo (1994). This model accounts for the anomalous regression slope parameter for
an appropriate choice of parameter values.

Next, Backus et al. (2001), provide empirical evidence of the two models of affine term structure model that is consistent with the forward premium anomaly. They find that the anomaly imposes two conditions on affine models: either interest rates must be negative with positive probability or the effects of one or more factors on pricing kernels must differ across currencies. Even in the latter case of interdependent factor the estimates indicate some fundamental shortcomings. They argue that the "price of risk" coefficients needs to be large to account simultaneously for Fama's second condition and the unconditional variance of the depreciation rate. Another implication of fully interdependent, i.e. only common, factors is that it leads to implausible correlation structure between the domestic and foreign interest rates. In addition, as argued by Engel (1996) and Backus et al. (2001), the solution provided by their pricing kernel model is entirely arbitrary and it is the question whether (1) a general equilibrium model of term structure of interest rates can be constructed that is consistent with such a state price density process and (2) whether the estimated parameter values needed to account for the puzzle is supported by such an economy.

From this review we can conclude that the large variability observed in the expected change of currency prices and the extreme negative co-variability between the forward premium and the expected depreciation impose extreme restrictions on no-arbitrage and general equilibrium models of forward foreign exchange risk premium. As a result these models have been unable to account for these features of the data, under plausible parameter restrictions. We can observe that equilibrium models of the risk premium, in particular the term structure models, have not provided a fully integrated role for money and its dynamics. Furthermore, the multi-currency term structure models explain the puzzle through the factor-risk differentials. As a result they have to either allow for negative nominal interest rates or implausible correlation structure for the cross-country term structure in order to account for the anomaly. These results motivate us to develop a two-country general equilibrium model of term structure of interest rates that allows
money and real factors to play a valid role. We incorporate monetary endogeneity in a two-country CIR-type production economy, where we allow for the representative agents to hold money both for transaction purposes as for portfolio choice considerations.