A term structure model of interest rates and forward premia: an alternative monetary approach
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Chapter 5

An Equilibrium Model of Term Structure of Interest Rate and Foreign Exchange Forward Premia

In this chapter we determine the stochastic nominal interest rates and allow the stochastic spot foreign exchange rate to be state dependent in equilibrium. As a result our model is capable of explaining some of the empirical features obtained in literature, such as skewness and excess kurtosis in the nominal interest rates, the volatility smile in the exchange rate dynamics, and the forward exchange rate anomaly.

In order to allow for state dependent nominal interest rates and exchange rate in this economy, we assume that the growth rate of money and the real rate of return in production in country $i$ depends on the underlying state variables in that particular country, $Y(t)$ and $X_i(t)$. In line with CIR (1985b) we assume that the real rate of return of the productive process in the each countries is governed by

$$
\frac{d\eta_i(t)}{\eta_i(t)} = \alpha_{yi}Y dt + \alpha_{xi}X_i dt + \sigma_{\eta_{yi}} \sqrt{Y(t)}dw_y(t) + \sigma_{\eta_{xi}} \sqrt{X_i(t)}dw_x(t), \quad (5.1)
$$

where the parameters $\alpha_{yi}$, $\alpha_{xi}$, $\sigma_{\eta_{yi}}$, and $\sigma_{\eta_{xi}}$, are positive constants. The productive
process in each country is perfectly correlated with both sources of uncertainty in that particular country, as its diffusion term follows the same stochastic processes. In this economy the representative agents of both countries have full access to the equity markets in both countries. The real rate of return on foreign real investment \( j \) in terms of the local currency \( i \) is determined as

\[
\frac{d\tilde{\eta}_j(t)}{\tilde{\eta}_j(t)} = \bar{\alpha}_j(X_j, t) dt + S_{\tilde{\eta}_j}(X, Y, t) dw(t), \quad \text{for } i \neq j 
\]  

(5.2)

where

\[
\bar{\alpha}_j(X_j, t) = \alpha_{y_j} Y(t) + \alpha_{x_j} X_j(t) + \mu_{\epsilon_{ij}}(X, Y, t) + S_{\eta_{ij}}(X, Y, t) \\
S_{\tilde{\eta}_j}(X, Y, t) dw(t) = S_{\eta_j}(X_j, Y, t) dw(t) + S_{\epsilon_{ij}}(X, Y, t) dw(t).
\]

As argued by Clark, Goodhart, and Huang (1999) and Christiano and Eichenbaum (1992), the optimum monetary policy is state contingent and shock dependent. Following this argument, the monetary endogeneity assumption as formulated in equation (3.4) results in the following state dependent representation for the money supply process in country \( i \), for \( i \in \{ h, f \} \), by using equation (5.1):

\[
\frac{dM^*_i(t)}{M^*_i(t)} = \mu^*_m(X_i(t), Y(t), t) dt + S_{m^*_i}(X_i(t), Y(t), t) dw_{m^*_i}(t), 
\]

(5.3)

where

\[
\mu^*_m(X_i(t), Y(t), t) = (\mu_m + \gamma_i \alpha_m Y + \gamma_i \alpha_x X_i) \\
S_{m^*_i}(X_i(t), Y(t), t) dw_{m^*_i}(t) = \sigma_m dw_{m_i}(t) + \gamma_i \sigma_{\eta_x} \sqrt{Y(t)} dw_y(t) \\
+ \gamma_i \sigma_{\eta_x} \sqrt{X_i(t)dw_z(t)}
\]

and \( \mu_m \) is the autonomous component of the expected monetary growth rate in country \( i \). As in Bakshi and Chen (1996, 1997) such a representation leads to an empirically
plausible stochastic process for money supply in both countries and is consistent with
the properties of our economy. The conditional mean and variance of the rate of growth
of money supply in each country is linear in the underlying state variables that affect
economic development in that particular country. The ultimate effect depends on the
monetary response parameters, \( \gamma_i = [\gamma_h, \gamma_f] \).

5.1 The Equilibrium Stochastic Interest Rates

In the previous chapter we obtained the equilibrium nominal interest rates in an i.i.d.
economy, where several properties of the constant nominal interest rates were reviewed.
Some of the empirically important features of nominal interest rates could not be estab-
lished, such as the capture of higher moments and the covariability of the interest rates
in a two-country economy. In this section we consider these features for the stochastic
nominal interest rates in equilibrium. Given the process for the money supply in equation
(5.3), we endogenously obtain the equilibrium stochastic nominal interest rate in country
\( i \) as (see Appendix B.1 for the derivation):

\[
R_i(t) = \rho + \mu_m - \sigma^2_m + \gamma^*_i X_i + \lambda^*_i Y \quad i \in \{h, f\},
\]

where \( \gamma^*_i = \gamma_i \left( \alpha_x - \gamma_i \sigma^2_{\eta_m} \right) \) and \( \lambda^*_i = \gamma_i \left( \alpha_{\eta_m} - \gamma_i \sigma^2_{\eta_m} \right) \). The nominal interest rate in
country \( i \) is increasing in the time-preference parameter and the autonomous expected
growth rate of domestic money supply. This coincides with the results in the one-country
framework of Bakshi and Chen (1996). In addition the expected stochastic real rate of
return on productive investment has a positive impact on the equilibrium nominal in-
terest rates. Consistent with our results in the previous chapter, we find that monetary
endogeneity and portfolio selection considerations result in an equilibrium nominal in-
terest rate that is decreasing in the real and nominal uncertainties in this economy, as
measured by the variance of the money supply process.\footnote{Note that the velocity of money, determined in Chapter 4 as $\nu_i(t) = \varphi_{nu_i} R_i(t)$, becomes stochastic in this economy. It co-varies with the domestic nominal interest rates and follows the same dynamics.}

The nominal interest rate in each country in equation (5.4) is linearly dependent on two state variables, which is in line with the results obtained by Longstaff and Schwartz (1992) for the equilibrium real interest rate. In contrast to existing monetary equilibrium models in finance, the stochastic nominal interest rate depends on both the real and nominal sources of uncertainty as captured by the state variables [e.g., Foresi (1990), Bakshi \\& Chen (1996) and Basak \\& Gallmeyer (1999)]. The impact of both state variables on the level of the nominal interest rate in country $i$ depends on the value of the monetary response parameter, $\gamma^*_i$, in that particular country. In general we expect $\gamma^*_i$ and $\lambda^*_i$ to be positive constants, which implies that the expected money growth offsets the unanticipated movements in the money supply process. Consider a positive shock to any of the underlying state variables in this economy. Initially, the real rate of return on production and the money supply in that particular country increases, as can be observed from equations (5.1) and (5.3). Ceteris paribus, there will be an increased demand for productive investment and money balances. The increased demand for money balances is for both productive investment and consumptive purposes at the expense of nominal bond holdings. In this economy the agent's desired portfolio holdings of cash balances and nominal bonds depend on the expected nominal rate of return on these bonds, $R_i$, i.e. the opportunity cost of holding money. To restore the equilibrium holdings of bonds and money, requires an increase in the nominal interest rate to induce the agents to increase their demand for nominal bonds and to re-establish a new portfolio composition of money balances and bond holdings.

Applying Itô’s lemma on equation (5.4) results in the following dynamics for the nominal interest rate in country $i$, for $i \in \{h, f\}$:

$$dR_i(t) = \left[\kappa_{\nu_i}(\theta_{\nu_i} - Y(t)) + \kappa_{x_i}(\theta_{x_i} - X_i(t))\right] dt$$
\[ + \lambda^*_s \sigma_{\psi} \sqrt{Y(t)} dw_{\psi}(t) + \gamma^*_s \sigma_{X_1} \sqrt{X_1(t)} dw_{X_1}(t). \] 

(5.5)

As can be observed from equation (5.5), the domestic nominal interest rate displays mean reversion in the two state variables that affects the supply of the local currency in this economy. The long-term means with respect to the local and international state variables are \( \theta_{z_i} \) and \( \theta_{y_i} \), respectively. The parameters \( \kappa_{z_i} = \gamma_i \kappa_{z_i} \) and \( \kappa_{y_i} = \lambda_i \kappa_{y_i} \) determine the speed of adjustment toward the respective long-term means. Notice that the larger the domestic monetary response parameters, \( \gamma_i^* \) and \( \lambda_i^* \), the larger the speed of adjustment and the instantaneous interest rate volatility. The volatility of the nominal interest rates dynamics for both countries, as measured by the variance of the changes in the nominal interest rates, and the conditional covariance between these interest rates are defined respectively as

\[ V_{dR_h}(t) = \lambda_h^* \sigma_{Y_h}^2 (t) + \gamma_h^* \sigma_{X_h}^2 X_h(t) \]  

(5.6)

\[ V_{dR_f}(t) = \lambda_f^* \sigma_{Y_f}^2 (t) + \gamma_f^* \sigma_{X_f}^2 X_f(t) \]  

(5.7)

\[ \text{Cov} [dR_i, dR_j] = \lambda_* \lambda_*^j \sigma_{Y_i}^2 \sigma_{Y_j}^2 (t) i \neq j. \]  

(5.8)

Equations (5.6) and (5.7) show that our nominal interest rates model allows for stochastic volatilities, with nonzero instantaneous covariance between the changes in the nominal interest rates and the changes in the nominal interest rates volatility, given by:

\[ \text{Cov} [dR_i, dV_{dR_i}] = \lambda_i^* \sigma_{Y_i}^3 (t) + \gamma_i^* \sigma_{X_i}^3 X_i(t). \]  

(5.9)

The stochastic nominal interest rates have some economically plausible and empirically important properties. As Heston (1993) points out, stochastic volatility by itself cannot capture such an empirical phenomenon as “volatility smiles” in the option prices. Rather, it is the correlation between the volatility of the interest rate and the changes in the interest rate that can explain the presence of volatility smiles and skewness in interest-rate options. For example, Amin and Morton (1994) and Flesaker (1993) find evidence of
volatility smiles among Eurodollar futures options of all maturities. Another important property of this model is that it allows for the stochastic nominal interest rates in this two-country economy to be correlated in equilibrium. The covariance between the nominal interest rates in both countries, conditional on the information set, is a linear function of the international state variable. The effect of the international state on the covariability of these interest rates depends on the monetary response in both countries to shocks in this state.

Conditionally, the distribution function of the nominal interest rate is a bivariate non-central $\chi^2$ distribution times a constant. This is a direct result of the conditional distribution functions of the state variables, $X_i(t)$ and $Y(t)$, which are non-central $\chi^2$ distributions times a constant. As shown by Feller (1951) the conditions $2\kappa_x, \theta_z > \gamma_i^2 \sigma_i^2$, and $2\kappa_y, \theta_y > \lambda_i^2 \sigma_i^2$, for $i \in \{h,f\}$, imply that the stochastic process in equation (5.5) precludes negative nominal interest rates. The conditional probability density function of the nominal interest rates is a joint density of the two independent state variables. Unconditionally, the distribution of the nominal interest rate is a convolution of two independent gamma distributions. As $t \to \infty$, the steady state mean and variance are respectively:

$$E[R_i(t)] = \rho + \mu_{m_i} - \sigma_i^2 + \lambda_i^2 \sigma_i^2 + \gamma_i^2 \theta_z$$  \hspace{1cm} (5.10)

and

$$\text{var}[R_i(t)] = \frac{\theta_y \lambda_i^2 \sigma_i^2}{2\kappa_y} + \frac{\theta_z \gamma_i^2 \sigma_i^2}{2\kappa_x_i}. \hspace{1cm} (5.11)$$

We observe from equations (5.10) and (5.11) that the evolution of the nominal interest rate is stationary in the long-run. The long-run unconditional mean of the nominal interest rate in country $i$ depends, among others, on the autonomous rate of growth of domestic money supply and the long-run means of the state variables. The latter depends on the monetary response parameters in that particular country. In addition

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2See Feller (1951) and CIR (1985b) for a detailed discussion of the distributional properties of these type of square root processes.

3Note that this is also the long-run unconditional mean of the velocity of money, $\nu_i(t)$.
the co-variability between the nominal interest rates in this two-country economy as
$t \to \infty$ is given by:

$$\text{cov} \left( R_h(t), R_f(t) \right) = \frac{\theta_y \lambda_h \lambda_f \sigma_y^2}{2\kappa_y}. \quad (5.12)$$

Equation (5.12) shows that the domestic nominal interest rate co-moves with the foreign
nominal interest rate in this economy. The covariance between the two nominal interest
rates in the economy is stationary in the long-run. It is determined by the variability in
the international state variable and the monetary response in both countries on changes
in this state. Furthermore, it is also affected by the long term mean of the international
state variable.

An empirically appealing feature of the stochastic process in equation (5.5) is that
it can capture higher moments of the nominal interest rates, such as skewness, $\mu_3$, and
excess kurtosis, $Q$, respectively:

$$\mu_3 [R_i(t)] = \frac{\theta_y \lambda_i^3 \sigma_y^4}{2\kappa_y^2} + \frac{\theta_x \gamma_i^3 \sigma_z^4}{2\kappa_z^2}, \quad (5.13)$$

and

$$Q [R_i(t)] = \frac{3\sigma_y^2}{\theta_y \kappa_y} + \frac{3\sigma_z^2}{\theta_x \kappa_x}. \quad (5.14)$$

The presence of these higher order properties coincide with the empirical evidence pre-
presented by Babbs and Webber (1997), Das and Foresi (1996), El-Jahel, Lindberg and
Perraudin (1997), and Heston (1993b). Notice from equation (5.13) that the changes
in the domestic nominal interest rate are positively skewed. This implies greater prob-
abilities for upward movements as compared to downward movements in the nominal
interest rate. The probabilities for upwards movements depends, among others, on the
monetary response to shocks in the economy. The skewness is an increasing function
of the monetary response parameters in the economy. Equation (5.14) reveals that the
leptokurtosis in the changes in the nominal interest rate is independent of the monetary
response parameters, $\gamma_i^*$ and $\lambda_i^*$. The fourth central moment of the changes in the nom-
inal interest rates depends on the monetary response parameters, $\gamma_i^*$ and $\lambda_i^*$. However,
this dependency disappears when we divide this moment by the squared second central
moment.

5.2 The Term Structure of Nominal Interest Rates

Denote $N_t(R_i(t), t, \tau)$ as the nominal value denominated in currency $i$ of an interest rate
contingent claim with time-to-maturity $\tau$ and boundary conditions that do not depend
on wealth. Under a risk-neutral probability measure $Q$, Theorem 2(b) in Chapter 4 can
be applied to obtain the partial differential equation for the domestic nominal interest
rate contingent claim:

$$0 = \frac{1}{2} \lambda_i^2 \sigma_y^2 Y(t) N_{i,y} + \lambda_i^2 \kappa_y \left( \hat{\theta}_y - Y \right) N_{i,y} + \frac{1}{2} \gamma_i^2 \sigma_x^2 X_i(t) N_{i,x} + \frac{1}{2} \gamma_i^2 \sigma_x^2 X_i(t) N_{i,x} + \frac{1}{2} \gamma_i^2 \sigma_x^2 X_i(t) N_{i,x} - R_i(t) N_t,$$

where $\hat{\theta}_y$ and $\hat{\theta}_x$ are risk-adjusted parameters. As such, this partial differential equation
is the nominal counterpart of the fundamental valuation equation in equation (4.31).
Notice that the valuation equation has the same form when the contingent claim and the
interest rates are expressed in nominal terms as when both these variables are expressed
in real terms. Since most securities are denominated in nominal terms, it is convenient
for empirical and practical purposes to employ the valuation equation in which all values
are expressed in nominal terms.

Let $P_i(t, T)$ denote the nominal price denominated in currency $i$ at time $t$ of a pure
discount nominal bond that matures at $T$. This bond price must satisfy the PDE in
equation (5.15) subject to the following boundary conditions:

$$P_i(T, T) = 1 \quad i \in \{h, f\}.$$

\footnote{For notational convenience, in the remainder of the section, we suppress the "hat" for the risk-adjusted parameters.}
Theorem 3  The pricing formula for the nominal bond denominated in currency $i$, $P_i(t, T)$, for $i \in \{h, f\}$, is given by:

$$P_i(t, T) = A_i(t, T) e^{-(\rho + \mu_m - \sigma^2_m) t - B_{x_i}(t, T) \lambda_i(t) - B_{y_i}(t, T) Y(t)},$$

(5.16)

where

$$A_i(t, T) = \left( \frac{2 \varphi_{x_i} e^{\frac{\tau}{2} (\kappa_{x_i} + \varphi_{x_i}) (\tau - 1) + 2 \varphi_{x_i}}}{\kappa_{x_i} + \varphi_{x_i}} \right) \frac{2 \varphi_{x_i} e^{\frac{\tau}{2} (\kappa_{x_i} + \varphi_{x_i}) (\tau - 1) + 2 \varphi_{x_i}}}{\kappa_{x_i} + \varphi_{x_i}},$$

$$B_{x_i}(t, T) = \frac{2 \varphi_{x_i} e^{\frac{\tau}{2} (\kappa_{x_i} + \varphi_{x_i}) (\tau - 1) + 2 \varphi_{x_i}}}{\kappa_{x_i} + \varphi_{x_i}},$$

$$B_{y_i}(t, T) = \frac{2 \varphi_{y_i} e^{\frac{\tau}{2} (\kappa_{y_i} + \varphi_{y_i}) (\tau - 1) + 2 \varphi_{y_i}}}{\kappa_{y_i} + \varphi_{y_i}},$$

$$\varphi_{x_i} = \sqrt{\kappa_{x_i}^2 + 2 \gamma_i^* \sigma_{x_i}^2},$$

$$\varphi_{y_i} = \sqrt{\kappa_{y_i}^2 + 2 \lambda_i^* \sigma_{y_i}^2},$$

$$\tau = (T - t).$$

Proof: See Appendix B.2.

The nominal bond price in country $i$ in equation (5.16) is a decreasing convex function of both state variables that captures the underlying sources of uncertainty in the economy. Note that in this circumstance, the domestic nominal interest rate in equation (5.4) is increasing. Consistent with existing general equilibrium models of the nominal term structure, our model reveals that a decrease in the nominal interest rate in country $i$ is accompanied by an increase in the nominal bond price denominated in currency $i$. In conformity with the nominal interest rates, the bond prices in this two-country economy also co-vary with each other as both have the same sources of real risk.

In line with existing literature on bond pricing, the equilibrium price of the nominal bond is a decreasing function of maturity [e.g., Vasicek (1977), Bakshi and Chen (1996), and CIR (1985b)]. The nominal discount bond in country $i$ is decreasing in the time preference parameter, $\rho$. An increase in the autonomous expected growth rate of the supply of currency $i$, $\mu_m$, has a decreasing impact on the value of the bond denominated
in currency \( i \). An increase in the domestic autonomous source of monetary uncertainty, \( \sigma^2_{m_i} \), has an increasing impact on the bond price \( i \).

The yield-to-maturity for the nominal bond \( i \), \( R_i(X_i,Y,t,T) \), for \( i \in \{h,f\} \), is given by:

\[
R_i(X_i,Y,t,T) = \rho + \mu_{m_i} - \sigma_{m_i}^2 - \frac{1}{T} \left[ \log A_i(\tau) - B_{z_i}(\tau)X_i(t) - B_{y_i}(\tau)Y(t) \right],
\]

(5.17)

for \( \tau \equiv T - t \). The term structure of domestic nominal interest rates in equation (5.17) incorporates both local and international risk factors (real and monetary). As in the case of most two-factor models of the term structure of interest rates, the model in equation (5.17) is rich enough to give rise to yield curves of complex shapes. Another empirical appealing feature of the domestic nominal term structure of interest rates is that it permits rates of different maturities to be imperfectly correlated. In addition it can be observed that the international part of the yield curve moves together in the two countries, and that a divergence in the yield curves in this economy is due to the monetary policy stance and the developments in the local state variable in each country. As the time-to-maturity goes to zero, we find that the yield-to-maturity approaches the instantaneous domestic nominal interest rates. Further inspection of this model reveals that, as the time-to-maturity goes to infinity, the yield-to-maturity converges to a long-term nominal yield:

\[
R_i(X_i,Y,t,\infty) = \rho + \mu_{m_i}^* - \sigma_{m_i}^* - \frac{1}{\tau} \left[ \log A_i(\tau) - B_{z_i}(\tau)X_i(t) - B_{y_i}(\tau)Y(t) \right], \quad (5.18)
\]

The long-term nominal yield is a constant that is independent of the instantaneous domestic nominal interest rate and the current state of the monetary-production economy, which is represented by the state variables. Below we consider the relation between the term structure of nominal interest rate and the exchange rate in this two-country monetary-production economy.
5.3 The Equilibrium Exchange Rate Dynamics

In Chapter 4 we derived endogenously the spot foreign exchange rate level, \( \varepsilon_{ij}(t) \), from the equilibrium conditions [see equation (4.13)] and the dynamics of the spot exchange rate in an i.i.d. monetary-production economy. This was a convenient analytical tool to understand the dynamics of the exchange rate in this economy. In this section we examine the exchange rate process with empirically more plausible properties. Given the process for the production dynamics in equation (5.1), we endogenously determine the stochastic process for the instantaneous rate of depreciation of the domestic currency. By applying Itô's lemma on the level of the spot foreign exchange rate in equation (4.13) we obtain the rate of depreciation of the domestic currency at time \( t \) as

\[
\frac{d\varepsilon_{HF}(t)}{\varepsilon_{HF}(t)} = \mu_\varepsilon(X(t), Y(t)) dt + S_{\varepsilon_{HF}}(X(t), Y(t)) dw(t),
\]

where

\[
\mu_\varepsilon(X(t), Y(t)) = \alpha_{\varepsilon, h} X_h - \xi_{\varepsilon 1} X_f + \xi_{\varepsilon 2} Y \\
S_{\varepsilon_{HF}}(X(t), Y(t)) dw(t) = \sigma_{\varepsilon, h} \sqrt{X_h(t)} dw_{\varepsilon, h}(t) - \sigma_{\varepsilon, f} \sqrt{X_f(t)} dw_{\varepsilon, f}(t) \\
+ \left( \sigma_{\varepsilon, h} - \sigma_{\varepsilon, f} \right) \sqrt{Y(t)} dw_{\varepsilon, f}(t)
\]

\[
\xi_{\varepsilon 1} = \left( \alpha_{\varepsilon, f} - \sigma_{\varepsilon, f}^2 \right) \\
\xi_{\varepsilon 2} = \left( \alpha_{\varepsilon, h} - \alpha_{\varepsilon, f} + \sigma_{\varepsilon, f}^2 - \sigma_{\varepsilon, h} \sigma_{\varepsilon, f} \right)
\]

The properties of the stochastic spot exchange rate in equation (5.19) are consistent with empirical findings in literature. Empirical studies show that both the instantaneous mean and variance of the instantaneous rate of depreciation of the currency are not constant [e.g. Fama and Roll (1968), McFarland et al. (1982), and Nielsen and Saá-Raquejo (1993)]. The expected instantaneous rate of depreciation of the domestic currency is both time-varying and state-dependent. As indicated by equation (5.19) the expected
instantaneous rate of depreciation of the currency is decreasing in the foreign state variable and increasing in the local state variable. In line with the arguments put forward in Chapter 4, when the underlying sources of uncertainty in the economy increase, the spot foreign exchange rate adjusts instantaneously in order to restore the equilibrium rate of return on the two equities in this economy.

For example, the spot exchange rate depreciates when there is a positive shock in the domestic state variable. The rationale therefore is that a positive shock to the local state variable entails a positive technology shock, which ultimately leads to an increase in the real rate of return on investment in production of the domestic good, \( \alpha_i (X_i, Y, t) \). As the productive process of the foreign good remains unaffected, the agents in the economy, ceteris-paribus, prefer to invest in the local production technology. In order to restore the equilibrium real rate of return on the two equities in the economy and ensuring that the agents maintain their capital invested in both technologies, the spot exchange rate depreciates. The latter entails that the rate of return on foreign equity expressed in domestic currency, \( \tilde{\alpha}_j (X_j, Y, t) \), increases. Thus, rational investors expect the currency to depreciate in the face of a positive shock in the local state variables. Rational investors expect the contrary to occur when there is a positive shock to the foreign state variable.

The impact of the common state variable on the rate of depreciation depends on the differential between the relative effect of this state variables on each country. When the local impact is larger, i.e. \( \alpha_{y,h} > \alpha_{y,f} \) and \( \sigma_{\eta_{y,h}} > \sigma_{\eta_{y,f}} \), than a positive international shock causes the spot exchange rate to depreciate. In this case the same analysis applies as when there is a shock in the domestic state variable. Note that the real rate of return on the domestic productive investment increases more than that on the foreign productive investment, and therefore the spot exchange rate depreciates proportional to the difference \( (\alpha_{y,h} - \alpha_{y,f}) + (\sigma_{\eta_{y,f}} - \sigma_{\eta_{y,h}}) \sigma_{\eta_{y,f}} \) to restore the equilibrium between the two rate of returns. When the international state variable has a larger impact on the foreign country, i.e. \( \alpha_{y,h} < \alpha_{y,f} \) and \( \sigma_{\eta_{y,h}} < \sigma_{\eta_{y,f}} \), the effects on the rate of depreciation of the currency will be the same as that of the foreign state variable. It appreciates when
a positive shock occurs in the international state variable to restore the equilibrium on the goods/equity market.

The exchange rate volatility, as measured by the variance of the rate of depreciation of the domestic currency in equation (5.19), is a linear function of the three sources of uncertainty in this two-country world economy,

\[ V_e(X(t), Y(t)) = \sigma_{\eta_{x,h}}^2 X_h(t) + \sigma_{\eta_{x,f}}^2 X_f(t) + \left(\sigma_{\eta_{y,h}} - \sigma_{\eta_{y,f}}\right)^2 Y(t). \] (5.20)

This feature of our model coincides with empirical evidence that the rate of depreciation does not have a constant instantaneous variance [see Fama and Roll (1968), McFarland et al. (1982), and Nielsen and Saa-Raquejo (1993)]. A negative (positive) shock to any of the state variables in this economy will reduce (increase) the volatility of the spot exchange rate dynamics proportionally. Note that when the foreign impact of the common state is larger than the local impact, that is \( \sigma_{\eta_{y,f}} > \sigma_{\eta_{y,h}} \), it reduces the positive effect of an international shock on the exchange rate volatility. The instantaneous volatility of the spot exchange rate is not affected by the common state variable, when the common sources of uncertainty have the same impact on both countries, that is \( \sigma_{\eta_{y,h}} = \sigma_{\eta_{y,f}} \).

Equation (5.20) shows that our exchange rate model allows for stochastic volatility, with nonzero instantaneous covariance between the rate of depreciation of the spot exchange rate and the changes in the spot exchange rate volatility, given by:

\[
\text{Cov}[d\epsilon_{hf}/\epsilon_{hf}, dV_e] = \sigma_{x_h}^3 \sigma_{\eta_{x,h}} X_h(t) - \sigma_{x_f}^3 \sigma_{\eta_{x,f}} X_f(t) + \sigma_y \left(\sigma_{\eta_{y,h}} - \sigma_{\eta_{y,f}}\right)^3 Y(t). \] (5.21)

Note that the endogenously determined covariance can take both positive and negative values. This property of the spot exchange rate dynamics is important from an empirical point of view. As indicated by Bates (1996) it is the nonzero correlation between the volatility and the depreciation rate that can potentially explain the presence of volatility smile in the currency options. Next, we determine the relation between the spot foreign
exchange rate dynamics and the nominal interest rates in both countries. From the equations below we can observe that the covariances between the rate of depreciation of the spot exchange rate and the nominal interest rates dynamics and the covariability between their volatility are non-zero:

\[
\text{Cov}[d\epsilon_{hf}/\epsilon_{hf}, dR_h(t)] = \lambda_h^* \left( \sigma_{\eta_{h,h}} - \sigma_{\eta_{h,f}} \right) \sigma_Y Y(t) + \gamma_h^* \sigma_{z_h} \sigma_{z_h} X_h(t), \tag{5.22}
\]

\[
\text{Cov}[d\epsilon_{hf}/\epsilon_{hf}, dR_f(t)] = \lambda_f^* \left( \sigma_{\eta_{h,h}} - \sigma_{\eta_{h,f}} \right) \sigma_Y Y(t) - \gamma_f^* \sigma_{z_f} \sigma_{z_f} X_f(t), \tag{5.23}
\]

\[
\text{Cov}[dV_{Rh}(t), dV_{Rf}(t)] = \lambda_h^* \left( \sigma_{\eta_{h,h}} - \sigma_{\eta_{h,f}} \right)^2 \sigma_Y^4 Y(t) + \gamma_h^* \sigma_{z_h}^2 \sigma_{z_h}^2 X_h(t), \tag{5.24}
\]

\[
\text{Cov}[dV_{Rh}(t), dV_{Rf}(t)] = \lambda_f^* \left( \sigma_{\eta_{h,h}} - \sigma_{\eta_{h,f}} \right)^2 \sigma_Y^4 Y(t) + \gamma_f^* \sigma_{z_f}^2 \sigma_{z_f}^2 X_f(t). \tag{5.25}
\]

**Theorem 4** In equilibrium the expected rate of depreciation of the domestic currency is given by:

\[
\mu_{\epsilon_{hf}} (X(t), Y(t)) = \psi_{\epsilon_1} (R_h - R_f) + \mathcal{H} (V_{dR_h}, V_{dR_f}) - \psi_{\epsilon_1} \left( \mu_{m_h} - \mu_{m_f} - (\sigma_{m_h}^2 - \sigma_{m_f}^2) \right) \tag{5.26}
\]

and

\[
\mathcal{H} (V_{dR_h}, V_{dR_f}) = \psi_{\epsilon_2} V_{dR_h} - \psi_{\epsilon_3} V_{dR_f},
\]

where

\[
\psi_{\epsilon_1} = \left( [\alpha_h^* - \alpha_f^*] + (\sigma_{\eta_{h,h}} - \sigma_{\eta_{h,f}}) \sigma_{\eta_{h,f}} - \sigma_{\eta_{h,f}} \frac{\lambda_f^* \sigma_{z_f}^2}{\gamma_f^* \sigma_{z_f}^2} \right) \psi_{\epsilon_4}
\]

\[
\psi_{\epsilon_2} = \left( \alpha_z (\lambda_h^* - \lambda_f^*) - \gamma_h^* \xi_2 + (\gamma_f^* \alpha_z - \gamma_h^* \xi_2) \frac{\lambda_f^* \sigma_{z_f}^2}{\gamma_f^* \sigma_{z_f}^2} \right) / \gamma_h^* \sigma_{z_h}^2 \psi_{\epsilon_4}
\]

\[
\psi_{\epsilon_3} = \left( (\lambda_h^* - \lambda_f^*) \xi_1^2 + (\gamma_f^* \alpha_z - \gamma_h^* \xi_2) \frac{\lambda_f^* \sigma_{z_f}^2}{\gamma_f^* \sigma_{z_f}^2} \right) / \gamma_f^* \sigma_{z_f}^2 \psi_{\epsilon_4}
\]

\[
\psi_{\epsilon_4} = \lambda_h^* \left( 1 - \frac{\lambda_f^* \sigma_{z_f}^2}{\gamma_f^* \sigma_{z_f}^2} \right) - \lambda_f^* \left( 1 - \frac{\lambda_f^* \sigma_{z_f}^2}{\gamma_f^* \sigma_{z_f}^2} \right)
\]

\[
\alpha_h^* = \left( \alpha_{\eta_{h,h}} - \alpha_{z_h} \frac{\lambda_f^* \sigma_{z_f}^2}{\gamma_f^* \sigma_{z_f}^2} \right)
\]

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\[ \alpha^*_j = \left( \alpha_{y_j} - \alpha_{x_j} \frac{\sigma^2}{\gamma^2 \sigma^2_{e_j}} \right) \]

Proof: See Appendix B.3.

We observe from equation (5.26) that the uncovered interest rate parity hypothesis does not hold in this economy under the true probability measure \( P \). The instantaneous expected rate of depreciation of the domestic currency, equation (5.26), is a function of the nominal interest rates differential in the two countries and a so-called risk factor, \( \mathcal{H}(V_{dR_h}, V_{dR_f}) \). This risk factor depends, among others, on the volatility differential of the nominal interest rates in the two countries as defined in the equations (5.6) and (5.7). The effect of the volatility differential on the expected instantaneous rate of depreciation of the currency depends on the relative parameter values, which capture the impact of state variables on the production and money supply process in these countries. In general an increase in the volatility of the domestic nominal interest rate relative to the foreign nominal interest rates increases the expected instantaneous rate of depreciation of the currency, when the domestic parameters are larger than the corresponding parameters in the foreign country. An increase in the volatility of the foreign nominal interest rate under these circumstances leads to an appreciation of the spot exchange rate. The reverse is the case when the parameters that capture the impact of the state variables in foreign country are relatively larger.

Equation (5.26) explains the currency price puzzle observed in empirical literature, that is the tendency for high interest rate currencies to appreciate [see Engel (1995) for a survey]. Close examination of equation (5.26) shows that the parameter for the nominal interest rate differential, \( \psi_{e1} \), can take both negative and positive values. As can be observed from equation (5.26) there are several scenarios that can explain the currency price puzzle (\( \psi_{e1} < 0 \)). The most plausible explanation of the puzzle is when for example the common state has an asymmetric impact in the two countries. That is

\footnote{Note, however, that under the risk-neutral probability measure \( Q \) the uncovered interest rate parity condition does hold.}
when the common state variable has a stronger impact on output in one country than in the other, but the monetary response on this shock in that country is smaller than in the other country. Stated more formally, the currency puzzle is explained when one of the following (asymmetric) conditions are satisfied: $\alpha^*_h > \alpha^*_f > 0$ and $\lambda^*_h < \lambda^*_f$ or $0 < \alpha^*_h < \alpha^*_f$ and $\lambda^*_h > \lambda^*_f$. Note that in this region the differential parameter, $\psi_{t1}$, in equation (5.26) is in general negative. A less plausible alternative is when the parameter that captures the impact of the local state on the real rate of return of production is much larger than the parameter that captures the impact of the foreign state variable. Formally this means that the currency puzzle is explained when $0 > \alpha^*_h > \alpha^*_f$ and $\lambda^*_h > \lambda^*_f$ or $0 > \alpha^*_h > \alpha^*_f$ and $\lambda^*_h < \lambda^*_f$.

To illustrate the dynamics of the relation between the expected instantaneous rate of depreciation of the currency and the nominal interest rate differential we consider the more plausible case of the asymmetrical monetary response, namely $(\alpha^*_h > \alpha^*_f > 0$ and $\lambda^*_h < \lambda^*_f)$. Note that the same argument holds for $0 < \alpha^*_h < \alpha^*_f$ and $\lambda^*_h > \lambda^*_f$. Let there be a negative shock in the economy that decreases the foreign nominal interest rate relative to the nominal interest rate $[\lambda^*_h < \lambda^*_f$ and $\gamma^*_h < \gamma^*_f$ in equation (5.4)], therefore the nominal interest rate differential increases. As mentioned above the nominal interest rate differential parameter, $\psi_{t1}$, in equation (5.26) is negative under these conditions ($\psi_{t1} < 0$). The expected instantaneous rate of depreciation is then decreasing in the nominal interest rate differential, which explains the currency puzzle that relative high interest rate currencies appreciate. This relation between the nominal interest rate differential

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6 In order to determine the sign of the differential parameter, $\psi_{t1}$, in equation (5.26) we must notice that the diffusion parameters play a minor role compared with the drift parameters. In general it can be expected that $\alpha_{\ast}$, $> \alpha_{\ast}$, and that $\gamma^2 \sigma^2_{\ast} \sigma^2_{\ast} > \lambda^2 \sigma^2_{\ast} \sigma^2_{\ast}$, for $i \in \{h, f\}$, since this would otherwise imply that in general the foreign state variable has a larger impact than the local state variable on the real rate of return of production and on the money supply process, respectively. Although we expect that $\alpha_{\ast}$, $> \alpha_{\ast}$, it is plausible to assume that $\alpha^*_i = (\alpha_{\ast}, - \alpha_{\ast}, \lambda^2 \sigma^2_{\ast} / \gamma^2 \sigma^2_{\ast}) > 0$, for $i \in \{h, f\}$, since $\lambda^2 \sigma^2_{\ast} / \gamma^2 \sigma^2_{\ast} \in [0, 1]$, for $i \in \{h, f\}$. Then the currency puzzle is explained when one of the following conditions are satisfied: $\alpha^*_h > \alpha^*_f$ and $\lambda^*_h < \lambda^*_f$ or $\alpha^*_h < \alpha^*_f$ and $\lambda^*_h > \lambda^*_f$.

7 It is less plausible because we do not expect $\alpha^*_i = (\alpha_{\ast}, - \alpha_{\ast}, \lambda^2 \sigma^2_{\ast} / \gamma^2 \sigma^2_{\ast}) < 0$, for $i \in \{h, f\}$, since this implies that $\lambda^2 \sigma^2_{\ast} / \gamma^2 \sigma^2_{\ast} > 1$, for $i \in \{h, f\}$. Given the definition of $\lambda^*_i$ and $\gamma^*_i$ this only holds when $\alpha_{\ast} < \alpha_{\ast}$.
and the expected rate of depreciation of the domestic currency is consistent with the equilibrium conditions on all the markets in this two-country world economy.

Note that by using equations (5.1), (0.82), (0.83), and (0.84), we can write the expected instantaneous real rate of return on domestic and foreign production as function of the nominal interest rate differential,

$$\alpha_h (R_h, R_f, V_h, V_f) = \tilde{\alpha}_h (R_h - R_f) - \mathcal{H}_\alpha (V_h, V_f)$$
$$= \tilde{\alpha}_h \left[ (\mu_{m_h} - \mu_{m_f}) - (\sigma_{m_h} - \sigma_{m_f}) \right], \quad (5.27)$$

$$\alpha_f (R_h, R_f, V_h, V_f) = \tilde{\alpha}_f (R_h - R_f) - \mathcal{H}_\alpha (V_h, V_f)$$
$$= -\tilde{\alpha}_f \left[ (\mu_{m_h} - \mu_{m_f}) - (\sigma_{m_h} - \sigma_{m_f}) \right], \quad (5.28)$$

and

$$\mathcal{H}_\alpha (V_h, V_f) = v_1 V_h - v_2 V_f,$$
$$\mathcal{H}_\alpha (V_h, V_f) = v_3 V_h - v_4 V_f,$$

where

$$\tilde{\alpha}_h = \alpha_h^*/\psi_{4},$$
$$\tilde{\alpha}_f = \alpha_f^*/\psi_{4},$$

$$v_1 = \left[ \gamma_h^* \alpha_{y_h} - \alpha_{x_h} \left( \lambda_h^* - \lambda_f^* + \frac{\lambda_h^2 \sigma_f^2}{\gamma_f^2 \sigma_{x_f}^2} \right) \right] / \gamma_h^* \sigma_{x_h}^2 \psi_{4},$$

$$v_2 = \left( \alpha_{y_h} - \alpha_{x_h} \frac{\lambda_h^2 \sigma_f^2}{\gamma_f^2 \sigma_{x_f}^2} \right) / \gamma_f^2 \sigma_{x_f}^2 \psi_{4},$$

$$v_3 = \left( \alpha_{y_f} - \alpha_{x_f} \frac{\lambda_f^2 \sigma_h^2}{\gamma_h^2 \sigma_{x_h}^2} \right) / \gamma_h^2 \sigma_{x_h}^2 \psi_{4},$$

$$v_4 = \left( \gamma_f^* \alpha_{y_f} - \alpha_{x_f} \left( \lambda_h^* - \lambda_f^* + \frac{\lambda_h^2 \sigma_f^2}{\gamma_f^2 \sigma_{x_f}^2} \right) \right) / \gamma_f^* \sigma_{x_f}^2 \psi_{4}.$$

Equations (5.27) and (5.28) show that the expected real rate of return on production, $\alpha_i (R_h, R_f, V_h, V_f)$ for $i \in \{h, f\}$, is a decreasing function of the nominal interest rate
differential, when \( \alpha_i^* > 0 \) and \( \lambda_h^* < \lambda_f^* \) (i.e. \( \psi_{ct} < 0 \)). Given the assumption in this particular case that \( \alpha_h^* > \alpha_f^* \), the real rate of return of domestic production decreases relatively more than that of foreign production. Ceteris paribus, rational investors will shift their portfolio investments towards the foreign productive assets. Under these circumstances the exchange rate appreciates to restore the equilibrium between the real rate of return on domestic equity and that on foreign equity expressed in domestic currency [see equation (4.13)]. The decrease in the value of the foreign currency vis-à-vis the domestic currency decreases the real rate of return on the foreign equity denominated in the domestic currency such that it equates the real rate of return on the domestic physical capital. As a result the representative investors are indifferent between purchasing any of the two equities in this two-country world economy. This result is consistent with that obtained from the direct relation between the expected instantaneous rate of depreciation and the nominal interest rate differential in equation (5.26). It is also consistent with the equilibrium conditions on the other markets in equilibrium.

Under these conditions (that is the monetary response parameters are smaller in the domestic country) the decrease in the foreign nominal interest rate relative to the domestic nominal interest rate increases the relative opportunity cost of holding domestic money, i.e. \( R_h/R_f \). Ceteris paribus, the increased relative opportunity cost induces rational investors to shift away from domestic nominal money balances towards foreign nominal money balances [see Theorem 1]. Note, however, that in terms of domestic currency the relative opportunity cost of holding domestic money increases more, as the appreciation of the spot exchange rate decreases the opportunity cost of holding foreign currency expressed in domestic currency, \( \epsilon_{hf}(t)R_f(t) \). As can be observed from equation (4.16), a decrease in the rate of return on domestic production relative that of foreign production is consistent with a reduction of the price of domestic money relative to the price of foreign money. In terms of the price level, the price of the domestic good increases more than the price of the foreign good. Thus, the equilibrium on the money market, equation (4.20), is restored through the adjustment of the exchange rate and the price of
money. An increase of the nominal interest rate differential in this economy is consistent with an appreciation of the spot exchange rate, a relative decrease in the price of money, and a relative decline in real demand for domestic money balances.

The same arguments can be applied if we maintain this condition but instead allow for a positive shock in the economy, which reduces the nominal interest rate differential and depreciates the value of domestic currency. Also, the same reasoning applies for \( \alpha_h^* < \alpha_f^* \) and \( \lambda_h^* > \lambda_f^* \), whereby a positive (negative) shock increases (decreases) the nominal interest rate differential and the currency appreciates (depreciates). To analyze the economics of this relation consider a negative shock to the domestic economy that reduces the nominal interest rate differential under these conditions (the domestic monetary response parameters are smaller than the foreign monetary response parameters). As can be observed from equation (5.26), the nominal interest rate differential parameter, \( \psi_{e1} \), is negative since \( \alpha_h^* - \alpha_f^* < 0 \) and \( \psi_{e4} \) is positive (\( \lambda_h^* > \lambda_f^* \)). Therefore, the expected instantaneous rate of depreciation of the currency increases. Given that \( \alpha_h^* < \alpha_f^* \) and \( \psi_{e4} > 0 \), the real rate of return on domestic production decreases less than the foreign return on production [see equations (5.27) and (5.28)]. The depreciation of the spot exchange rate is consistent with developments on the equity market, since it increases the value of foreign currency relative to domestic currency and, therefore, the real rate of return on foreign equity expressed in domestic currency to the extent that it equates the real return on domestic production. As above, the money market equilibrium is restored through the adjustments of the relative price of money and the spot exchange rate when the nominal interest rates differential decreases due to a negative shock.

In the analysis above we have only considered those circumstances that explain the currency puzzle. Let us briefly review a case when the differential parameter is positive, that is for example when \( \alpha_h^* > \alpha_f^* \) and \( \lambda_h^* > \lambda_f^* \). The interest rate differential is increasing in economic shocks if the monetary response parameters are larger in the home country than in the foreign country. From equation (5.26) we can observe that the expected instantaneous rate of depreciation increases with the nominal interest rate differential.
when there is a positive shock in the economy. The real rate of return on production in each country is increasing function of the interest rate differential, as $\psi_{t4}$ in equations (5.27) and (5.28) is positive under these conditions. Since the foreign output parameter, $\alpha^*_f$, is smaller than that of the domestic output ($\alpha^*_h$), the real rate of return of domestic production increases more than that of foreign production. The depreciation in the value of the domestic currency under these circumstances restores the equilibrium between the real rate of returns on these equities expressed in the domestic currency. Ceteris paribus, the money market is not in equilibrium, as the relative opportunity cost of holding domestic money ($R_h/R_f$) is higher and rational investors are, therefore, shifting away from domestic money balances. The imbalance between the increased production of the domestic good and the reduced demand for domestic money is resolved through the depreciation of the value domestic currency. As a result, the opportunity cost of holding real foreign balances expressed in domestic currency increases. In addition note that under these circumstances, the local price of domestic money increases relative to the foreign price of money [see equation (4.16)]. As before the money market equilibrium [equation (4.20)] is restored through the adjustment of the nominal interest rates, the exchange rate, and the price of money. The same analysis can be performed when the other conditions are not satisfied and, as shown above, the differential parameter is positive while the exchange rate adjusts to restore the equilibrium on the world markets.

5.4 The foreign exchange forward premium

In the previous section we have solved the equilibrium exchange rate as a result of currency transaction in the spot market to finance international trade. In this section we determine the equilibrium forward exchange rate, which can be used by the representative agent in its role as international trader on the forward exchange market for hedging purposes or for portfolio management purposes to cover interest rate arbitrage and earn a return that is free of currency risk.
The representative agent of the home country can in this two-country economy invest one unit of the domestic currency in the local nominal riskless bond, which yields the nominal rate of return. Alternatively, he can purchase foreign currency on the spot exchange market and invest this amount in the foreign nominal riskless bond. By using the forward exchange market, the expected proceeds of this foreign investment at maturity can be converted into the local currency at a contractual forward rate. The return from both investments should be equal to offset any arbitrage opportunity in this economy. This argument holds for all maturities. Based on this no-arbitrage argument we can formulate the covered interest rate parity as

\[ F_e(t, T) = \frac{\epsilon_{hf}(t) P_f(R_f, t, T)}{P_h(R_h, t, T)}, \]  

(5.29)

where \( F_e(t, T) \) denotes the forward exchange rate determined at time \( t \) for delivery at time \( T \) with \( T > t \). We can use the closed-form expression for the pure discount nominal bond, equation (5.16), to obtain closed-form expression for the term structure of forward exchange rates from equation (5.29),

\[
\begin{align*}
    f_e(t, T) &= \epsilon_{hf}(t) + \log \frac{A_f(t, T)}{A_h(t, T)} + (\mu_{m_h}^* - \mu_{m_f}^*) \tau \\
    &\quad + B_{z_h}(t, T) X_h(t) - B_{z_f}(t, T) X_f(t) \\
    &\quad + [B_{y_h}(t, T) - B_{y_f}(t, T)] Y(t),
\end{align*}
\]

(5.30)

where \( f_e(t, T) = \log F_e(t, T) \), \( \epsilon_{hf}(t) = \log \epsilon_{hf}(t) \), \( \mu_{m_i}^* = \mu_{m_i} - \sigma_{m_i}^2 \), and the parameters are defined as in equation (5.16). Equation (5.30) shows that the log-forward exchange rate at time \( t \) for delivery at time \( T \) is a linear function of the state variables and the log-exchange rate at time \( t \). The differential between the autonomous expected rate of growth of money in the two countries, i.e. \( \left( \mu_{m_h} - \mu_{m_f} \right) \), has a positive effect on the log-forward exchange rate. Note that the parameters that capture the impact of the state variables, \( B_{z_h}(t, T), B_{z_f}(t, T), B_{y_h}(t, T), \) and \( B_{y_f}(t, T) \), take positive values. For example, a domestic shock increases the forward exchange rate of the domestic currency.
The rationale for this depreciation of the forward exchange rate is that a positive domestic shock will in this circumstances lead to an increase in the domestic nominal interest rate and a decrease in the bond price. The lower domestic bond price entails an increase in the nominal rate of return at maturity. Thus the forward exchange rate at time $t$ for delivery at time $T$ must depreciate to restore the equilibrium between the nominal rate of return on the two nominal bonds in this two country economy. The same reasoning applies for a shock to the other state variables. The impact of the international state variable, $Y(t)$, depends on the parameter differential $B_{ys}(t, T) - B_{yf}(t, T)$. If international uncertainty has a larger effect on the foreign bond price, that is when $\lambda_f > \lambda_s$, the forward exchange rate is decreasing in the international state variable. In the previous section we have examined the relation between the nominal interest rate differential and the expected rate of depreciation of the currency value. The question is how this relates to the forward exchange rate premium, $f_e(t, T) - e_{hf}(t)$.

Note that when maturity shortens, that is when $\tau$ approaches zero, equation (5.30) converges to the instantaneous forward premium, $f_e(t, T') - e_{hf}(t) = R_h(t) - R_f(t)$, where $T'$ denotes a very short maturity. By substituting this expression in equation (5.26) we obtain

$$
\mu_e(X(t), Y(t)) = \psi_{t1}(f_e(t, T') - e_{hf}(t)) + \psi_{t1}(\mu^*_m - \mu^*_r) + \mathcal{H}(V_{AR_h}, V_{AR_f})
$$

Equation (5.31) shows that the expected instantaneous rate of depreciation of the spot exchange rate when expressed as a function of the forward exchange rate premium depends on the parameter value $\psi_{t1}$ to adjust to the changes in this premium. The same conditions as before apply to determine whether the expected rate of depreciation is negatively correlated with the forward premium.

Following Fama (1984) we can decompose the forward foreign exchange premium into a "risk premium" on the currency, $r_p(t, T)$, and an expected rate of depreciation of the currency, $q(T, t)$. Closed form solution for the forward risk premium, $r_p(t, T)$ can be obtained by taking the difference between the forward foreign exchange rate,
equation (5.30), and the conditional expectation at time $t$ of the future exchange rate at time $T$. Note that the conditional distribution function of the exchange rate is not known. As a result the conditional expectation of the exchange rate cannot directly be determined from equation (4.13). As in Nielsen and Saá-Raquejo (1993) we can use the homogeneous Kolmogorov backward equation of the spot exchange rate under the true probability measure $\mathbb{P}$, adapted to the natural filtration $\mathbb{F}(t) = \{\mathcal{F}(t)\}_{t \geq 0}$, to obtain the conditional expectation of the future spot exchange rate at time $T$. Let $F(\epsilon_{hf}(t), X(t), Y(t), t, T)$ be the value of a security whose payoff is the exchange rate. Assume that $F(\epsilon_{hf}, X_h, X_f, Y, t)$ is continuously differentiable in $t$ and twice continuously differentiable in its other arguments $\epsilon_{hf}(t), X(t)$, and $Y(t)$. Since risk-neutrality imposes zero condition on the drift term, the price process $F(\epsilon_{hf}, X_h, X_f, Y, t)$ must satisfy the following partial differential equation

$$
0 = F_t + \epsilon F_{t\mu_t}(X_h(t), X_f(t), Y(t)) + \frac{1}{2} \sigma^2_{\epsilon} F_{xx} \left( \sigma^2_{\eta_h} X_h(t) + \sigma^2_{\eta_f} X_f(t) + \left( \sigma_{\eta_h} - \sigma_{\eta_f} \right)^2 Y(t) \right)
+ \mu_{x_h}(X_h(t), t) F_{x_h} + \frac{1}{2} \sigma^2_{x_h} X_h(t) F_{x_h x_h}
+ \mu_{x_f}(X_f(t), t) F_{x_f} + \frac{1}{2} \sigma^2_{x_f} X_f(t) F_{x_f x_f}
+ \mu_{y}(Y, t) F_{y} + \frac{1}{2} \sigma^2_{y} Y(t) F_{yy}
+ \epsilon \sigma_{\epsilon x_h} X_h(t) F_{x_h} - \epsilon \sigma_{\epsilon x_f} X_f(t) F_{x_f} + \epsilon \sigma_{\epsilon y} Y(t) F_{y} \quad (5.32)
$$

subject to the boundary condition

$$
F(\epsilon_{hf}, X_h, X_f, Y, T) = \epsilon(T),
$$

where

$$
\sigma_{\epsilon x_h} = \sigma_{x_h} \sigma_{\eta_h},
\sigma_{\epsilon x_f} = \sigma_{x_f} \sigma_{\eta_f},
\sigma_{\epsilon y} = \sigma_y \left( \sigma_{\eta_h} - \sigma_{\eta_f} \right).
$$
Following CIR (1981, 1985b) and Longstaff and Schwartz (1992), we apply the standard separation of variable technique to solve this partial differential equation and obtain the logarithm of the expected value of the exchange rate at time $T$ conditional on the filtration $\mathcal{F}_t$,

$$
\log E [\epsilon_{hf}(T) | \mathcal{F}_t] = \epsilon_{hf}(t) + \log A^*_x(\tau) + B^*_{z_h}(\tau) X_h(t) + B^*_{z_f}(\tau) X_f(t) + B^*_y(\tau) Y(t),
$$

(5.33)

where

$$
A^*_x(\tau) = A_{z_h}^*(\tau) A_{z_f}^*(\tau) A^*_y(\tau)
$$

$$
B^*_{z_f}(\tau) = \frac{2 \left( \alpha_{z_f} - \sigma_{z_f}^2 \right) \left( e^{\varphi_{z_f}^* T} - 1 \right)}{\left( \varphi_{z_f}^* + \kappa_{z_f} + \sigma_{z_f} \right) \left( e^{\varphi_{z_f}^* T} - 1 \right) + 2 \varphi_{z_f}^*}
$$

$$
B^*_{z_h}(\tau) = \frac{2 \alpha_{z_h} \left( e^{\varphi_{z_h}^* T} - 1 \right)}{\left( \varphi_{z_h}^* + \kappa_{z_h} - \sigma_{z_h} \right) \left( e^{\varphi_{z_h}^* T} - 1 \right) + 2 \varphi_{z_h}^*}
$$

$$
B^*_y(\tau) = \frac{2 \xi \left( e^{\varphi_{y}^* T} - 1 \right)}{\left( \varphi_{y}^* + \kappa_{y} - \sigma_{y} \right) \left( e^{\varphi_{y}^* T} - 1 \right) + 2 \varphi_{y}^*}
$$

$$
A_{z_h}^*(\tau) = \left( \frac{2 \varphi_{z_h}^* e^{\left( \varphi_{z_h}^* + \kappa_{z_h} - \sigma_{z_h} \right) T/2}}{\left( \varphi_{z_h}^* + \kappa_{z_h} - \sigma_{z_h} \right) \left( e^{\varphi_{z_h}^* T} - 1 \right) + 2 \varphi_{z_h}^*} \right)^{\frac{\kappa_{z_h} \varphi_{z_h}^*}{\sigma_{z_h}^2}}
$$

$$
A_{z_f}^*(\tau) = \left( \frac{2 \varphi_{z_f}^* e^{\left( \varphi_{z_f}^* + \kappa_{z_f} + \sigma_{z_f} \right) T/2}}{\left( \varphi_{z_f}^* + \kappa_{z_f} + \sigma_{z_f} \right) \left( e^{\varphi_{z_f}^* T} - 1 \right) + 2 \varphi_{z_f}^*} \right)^{\frac{2 \kappa_{z_f} \varphi_{z_f}^*}{\sigma_{z_f}^2}}
$$

$$
A^*_y(\tau) = \log \left( \frac{2 \varphi_{y}^* e^{\left( \varphi_{y}^* + \kappa_{y} - \sigma_{y} \right) T/2}}{\left( \varphi_{y}^* + \kappa_{y} - \sigma_{y} \right) \left( e^{\varphi_{y}^* T} - 1 \right) + 2 \varphi_{y}^*} \right)^{\frac{\kappa_{y} \varphi_{y}^*}{\sigma_{y}^2}}
$$

with
Equation (5.33) shows that the log of the expected exchange rate at time $T$ is a positive function of the home state variable, as $B^*_{z_h}(\tau) \geq 0$ for $\forall \tau$. The foreign state variable has a decreasing effect on the log-expected value of the currency at time $T$ given that $B^*_{z_f}(\tau) \geq 0$ for $\forall \tau$. The impact of the international state variable depends on the parameter value, $\xi_{z,f} = \alpha_{y,h} - \alpha_{y,f} + (\sigma_{\eta_{y,f}} - \sigma_{\eta_{y,h}})\sigma_{\eta_{y,f}}$. In general the magnitude of this parameter is determined by the value of the drift parameters, since the diffusion parameters are relatively small. Therefore, when the international state variable has a larger impact on the foreign country, the log-expected exchange rate at time $T$ is decreasing in the international state variable. The log of the expected value of the domestic currency at time $T$ is increasing in the international state variable, when the latter has a larger impact on the home country compared with the foreign country. The present value of the exchange rate has a positive effect on the expected value at time $T$.

Now we can obtain close form solutions for the term structure of forward "risk" premium, defined as $rp(t,T) = \log F_t(t,T) - \log E_t[\epsilon_{hf}(T)|\mathcal{F}_t]$, and the term structure of expected depreciation rate at time $T$ condition on the information set at time $t$ as $q(t,T) = \log E_t[\epsilon(T)|\mathcal{F}_t] - \epsilon_{hf}(t)$, from equations (5.30) and (5.33),

$$rp(t,T) = \log \frac{A_f(t,T)}{A_h(t,T) A^*_f(T,t)} + \left(\mu^*_{m_h} - \mu^*_{m_f}\right) \tau - \left[B_{z_f}(t,T) - B^*_{z_f}(T,t)\right] X_f(t) + \left[B_{y_f}(t,T) - B^*_{y_f}(T,t)\right] Y(t) \tag{5.34}$$
and

\[ q(t,T) = \log A^* (T,t) + B^*_{x_t} (T,t) Y(t) + B^*_{x_h} (T,t) X_h(t) - B^*_{x_f} (T,t) X_f(t). \] (5.35)

The parameters are defined as above [see equations (5.16) and (5.33)]. Equation (5.34) shows that the so-called risk premium on the forward exchange rate, that is the expected excess return on the currency, can take both positive and negative values. The risk premium on the forward exchange rate is an increasing function of the differential between the long term mean rate of growth of money, \( \mu_{m_h} - \mu_{m_f} \). The impact of the state variables on the risk premium of the forward foreign exchange rate depends on the parameter values. It is increasing in the domestic state variable, \( X_h(t) \), when this state has a larger effect on the bond price relative to its impact on the expected value of the currency at time \( T \). This ultimately depends on the values of \( \alpha_{x_h} \) and \( \gamma_h \), which capture the impact of the domestic state variable on the expected value of the exchange rate at time \( T \) and the rate of return on the nominal domestic bond, respectively. A relatively large domestic output parameter, \( \alpha_{x_h} \), implies that a positive shock in the domestic state variable increases the expected exchange rate relative to the rate of return on the domestic nominal bond. As a result the expected excess return on the currency decreases. The risk premium is decreasing in the domestic state variable when \( \alpha_{x_h} \) is relatively small.

The risk premium on the forward exchange rate, depending on the differential \( B_{x_f} (t,T) - B_{x_f}^* (T,t) \), is negatively related to the foreign state variable. As above, when \( \alpha_{x_f} \) is relatively large the local uncertainty in the foreign country has a larger impact on the expected currency value at time \( T \) relative to the rate of return on the foreign nominal bond. Consequently, the expected excess return on the currency is increasing in the foreign state variable. The impact of the international sources of uncertainty on the expected excess return on the currency value depends on its effect on the two nominal bonds and the expected currency value at time \( T \). These effects can be examined as follows. From
the analysis above, we know that a large domestic monetary response $\lambda_h^*$ relative to the foreign monetary response $\lambda_f^*$ implies that international uncertainty has a larger impact on the nominal rate of return on domestic bond than on that of the foreign bond. This positive return differential can be either mitigated or reinforced by the expected currency value at maturity. Note that for $\lambda_h^* < \lambda_f^*$ the return differential, $B_{y_h} (t, T) - B_{y_f} (t, T)$, is negative. As mentioned above, the value of the expected exchange rate at time $T$ is decreasing in the international state variable when $\xi_{t2} = \alpha_{y.h} - \alpha_{y.f} + (\sigma_{\eta_{y.f}} - \sigma_{\eta_{y.h}}) \sigma_{\eta_{y.f}}$ is negative. That is when $\alpha_{y.h} < \alpha_{y.f}$. Thus in this case the expected excess return on the currency is increasing in the international state variable. All the other combinations of $\alpha_{y.h}$ and $\alpha_{y.f}$ versus $\lambda_h^*$ and $\lambda_f^*$ give either an increasing or decreasing relation between the international sources of risk and the forward risk premium.

The uncovered interest rate parity hypothesis assumes that the variance of the risk premium is zero, $\text{var} \{ r_p (t, T) \} = 0$. By using equation (5.34) we obtain the variance of the risk premium as

$$
\text{var} \{ r_p (t, T) \} = \left[ B_{x_f} (t, T) - B_x^{*} (T, t) \right]^2 \text{var} \{ X_f (t) \}
+ \left[ B_{x_h} (t, T) - B_x^{*} (T, t) \right]^2 \text{var} \{ X_h (t) \}
+ \left[ B_{y_h} (t, T) - B_{y_f} (t, T) - B_y^{*} (T, t) \right]^2 \text{var} \{ Y (t) \},
$$

which can become zero under certain circumstances, that is only when the effect of the state variables on nominal rate of return of the bonds is equal to that on the expected value of currency at time $T$. Thus in general, as argued above, the uncovered interest rate parity hypothesis does not hold in this economy.

By using the decomposition of Fama (1984) we can write the forward foreign exchange rate premium as

$$
f_c (t, T) - e_{h_{f}} (t) = (f_c (t, T) - \log [E_t e_{h_{f}} (T)])
+ (\log [E_t e_{h_{f}} (T)] - e_{h_{f}} (t)) \text{ for } T > t.
$$

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The uncovered interest rate parity assumes that the forward foreign exchange rate is equal to the expected spot exchange rate at time $T$, $f_e(t,T) = \log [E_t e_{hf}(T)]$. This assumption implies that the foreign exchange risk premium (or say the expected excess return on the currency) is zero and that the forward exchange rate premium is equal to the expected rate of depreciation of the currency. Equations (5.26) and (5.31) show that this assumption does not hold in this economy under the true probability measure $\mathbb{P}$.

Most empirical studies of the forward foreign exchange premium perform the following regression:

$$e_{hf}(T) - e_{hf}(t) = \beta_1 + \beta_2 (f_e(t,T) - e_{hf}(t)) + \text{residual},$$

where $E_t [e_{hf}(T)] = e_{hf}(T) + \text{residual}$.\footnote{Alternatively, as in Fama (1984), the following regression can be performed:}

$$f_e(t,T) - e_{hf}(T) = \beta_3 + \beta_4 (f_e(t,T) - e_{hf}(t)) + \text{residual}.$$

From a theoretical perspective one expects that $\beta_1 = 0$ and $\beta_2 = 1$, which implies that $f_e(t,T) = E_t e_{hf}(T)$. The main results obtained in most of these studies of the forward premium can be summarized as follows [see Fama (1984, 1994), MacDonald and Taylor (1991), and, particularly, Engel (1996) for a survey of recent evidence]:

- Both components of the forward exchange rate premium, $rp(t,T)$ and $q(T,t)$, are time varying.
- The risk premium, $rp(t,T)$, and the expected depreciation rate, $q(T,t)$, display negative co-variability, which gives the following results:

$$\beta_2 = \frac{\text{cov} [q(t,T), rp(t,T) + q(t,T)]}{\text{var} [rp(t,T) + q(t,T)]} < 0$$

- The variability of the risk premia accounts for most of the variability in the forward foreign exchange premium, i.e. $|\text{cov} (q(t,T), rp(t,T))| > \text{var} (q(t,T))$
Note that the last two results account for the anomaly of the forward exchange rate premium observed in empirical literature. Equations (5.34) and (5.35) shows that our model is consistent with the first empirical result mentioned above, as both the term structure of forward risk premia and the term structure of expected rate of depreciation are state dependent.

For illustrative purposes we consider the case of the spot nominal interest rate differential, \(\frac{\text{cov}[q(t,t+1),R_h(t)-R_f(t)]}{\text{var}(R_h(t)-R_f(t))}\). We obtain the covariance between the expected rate of depreciation and the nominal interest rate differential and the variance of the nominal interest rate differential, respectively, as

\[
\text{cov}[q, R_h - R_f] = \xi_2 [\lambda_h^* - \lambda_f^*] \text{var}[Y] + \gamma_h^* \alpha_{z_h} \text{var}[X_h] \\
+ \gamma_f^* (\alpha_{z_f} - \sigma_{z_f}^2) \text{var}[X_f]
\]  

(5.38)

and

\[
\text{var}(R_h - R_f) = \gamma_f^2 \text{var}(X_f) + \gamma_h^2 \text{var}(X_h) \\
+ [\lambda_h^* - \lambda_f^*]^2 \text{var}(Y).
\]  

(5.39)

Since each state variable in this two-country world economy follows a square root process, its unconditional distribution is a gamma distribution. From the unconditional second moment of the gamma distribution we obtain

\[
\text{var}(X_h) = \theta_{x_h} \sigma_{x_h}^2 / 2 \kappa_{x_h}, \quad \text{var}(X_f) = \theta_{x_f} \sigma_{x_f}^2 / 2 \kappa_{x_f}, \quad \text{and} \quad \text{var}(Y) = \theta_y \sigma_y^2 / 2 \kappa_y.
\]

Our model extends existing general equilibrium and existing multi-currency affine term structure models of the forward premium puzzle [e.g. Bekaert (1996, 1994), Bansal et al. (1995), Hodrick and Srivastava (1986), Nielsen and Saá-Raquejo (1993), Saá-Raquejo (1994), Ahn (1995), Bakshi and Chen (1997), Backus et al. (1993), and Backus et al. (2001)], since equations (5.38) and (5.38) provide a tractable explanation of the Fama (1984) conditions based on the general equilibrium conditions and realistic parameter values. Equation (5.38) explains the currency price puzzle observed in empirical
literature, when $\xi_{e2} > 0$ and $\lambda_h^* < \lambda_f^*$ or for $\xi_{e2} < 0$ and $\lambda_h^* > \lambda_f^*$. From the definition of $\xi_{e2}$ in equation (5.19), we observe that our model accounts for the puzzle mainly through the expected return differential $(\alpha_{y,h} - \alpha_{y,f})$. Under certain conditions the asset-risk differential $(\sigma_{\eta_y,f} - \sigma_{\eta_y,h})$ also accounts for this puzzle. This result is an extension of existing studies, which explain the puzzle based only on the risk differentials [Backus et al. (1993), Nielsen and Saá-Raquejo (1993), Saá-Raquejo (1994), Ahn (1995), and Backus et al. (2001)].

Alternatively, the puzzle can also be explained when $\alpha_{z,f}$ is extremely small, such that $\sigma_{\eta_z,f}^2 > \alpha_{z,f}$ and $\gamma_f^* > 0$. In this latter case the puzzle is explained when the expected real rate of return in production of the foreign good is relatively lower than its local risk factor and the monetary response in that country is relatively small. This shows that our model can explain the puzzle independent of the common factor and with plausible parameter values.\(^9\)

To illustrate the dynamics of the relation between the expected rate of depreciation of the currency and the nominal interest rate differential we examine the case of the asymmetrical monetary response, namely $(\alpha_{y,h} > \alpha_{y,f}$ and $\lambda_h^* < \lambda_f^*)$. Note that the same arguments hold for $\alpha_{y,h} < \alpha_{y,f}$ and $\lambda_h^* > \lambda_f^*$. To show that this relation between the nominal interest rate differential and the expected rate of depreciation of the domestic currency is consistent with the equilibrium conditions on all the markets in this two-country world economy consider a negative shock in the common state in the economy. As a result the foreign nominal interest rate decreases relative to the nominal interest rate $[\lambda_h^* < \lambda_f^*$ in equation (5.4)] and the nominal interest rate differential increases. Given the assumption in this particular case that $\alpha_{y,h} > \alpha_{y,f}$, the real rate of return of domestic production decreases relatively more than that of foreign production. Ceteris paribus, rational investors will shift their portfolio investments towards the foreign productive assets. Under these circumstances the exchange rate appreciates to restore the equilibrium.

\(^9\)The parameter values do not violate the Feller condition that guarantees the positivity of the nominal interest rates.
between the real rate of return on domestic equity and that on foreign equity expressed in domestic currency [see equations (4.13) and (5.19)]. The decrease in the value of the foreign currency vis-à-vis the domestic currency decreases the real rate of return on the foreign equity denominated in the domestic currency such that it equates the real rate of return on the domestic physical capital. As a result the representative investors are indifferent between purchasing any of the two equities in this two-country world economy. This result shows that an increase in the nominal interest rate differential is consistent with an appreciation of the currency. It is also consistent with the equilibrium conditions on the other markets in equilibrium.

Under these conditions (i.e. the monetary response parameters are smaller in the domestic country) the decrease in the foreign nominal interest rate relative to the domestic nominal interest rate increases the relative opportunity cost of holding domestic money, i.e. $R_h/R_f$. Ceteris paribus, the increased relative opportunity cost induces rational investors to shift away from domestic nominal money balances towards foreign nominal money balances [see Theorem 1]. Note that in terms of domestic currency the relative opportunity of holding domestic money increases more, as the appreciation of the spot exchange rate decreases the opportunity of holding foreign currency expressed in domestic currency, $\epsilon_{hf}(t)R_f(t)$. As can be observed from equation (4.16) a decrease in domestic production relative to foreign production is consistent with a reduction of the price of domestic money relative to the price of foreign money. In terms of the price level, the price of the domestic good increases more than the price of the foreign good. Thus, the equilibrium on the money market, equation (4.20), is restored through the adjustment of the exchange rate and the price of money. An increase of the nominal interest rate differential in this economy is consistent with an appreciation of the spot exchange rate, a relative decrease in the price of money, and a relative decline in real demand for domestic money balances.

Similar explanation applies to the more general case of the whole term structure of forward premia. We can decompose the covariance between the expected rate of

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depreciation and the forward premium in two components, namely, the covariance of the forward risk premia with the expected rate of depreciation and the variance of the expected rate of depreciation:

\[ \text{cov} [q(t, T), rp(t, T) + q(t, T)] = \text{cov} [q(t, T), rp(t, T)] + \text{var} [q(t, T)]. \]

By using equations (5.34) and (5.35) we obtain the covariance between the term structure of forward risk premia and the term structure of expected rate of depreciation and the variance of the expected rate of depreciation, respectively, as:

\[ \begin{align*}
\text{cov} [q(t, T), rp(t, T)] &= B^*_y(T, t) \left[B^*_{y_h}(t, T) - B^*_{y_f}(t, T)ight] \text{var} [Y] \\
&+ B^*_{x_h}(T, t) \left[B^*_{x_h}(t, T) - B^*_{x_f}(T, t)ight] \text{var} [X] \\
&+ B^*_{x_f}(T, t) \left[B^*_{x_f}(t, T) - B^*_{x_f}(T, t)ight] \text{var} [X], \\
\end{align*} \tag{5.40} \]

and

\[ \begin{align*}
\text{var} [q(t)] &= \left[B^*_{x_h}(T, t)\right]^2 \text{var} (X) + \left[B^*_{x_f}(T, t)\right]^2 \text{var} (X) \\
&+ \left[B^*_{y}(T, t)\right]^2 \text{var} (Y). \\
\end{align*} \tag{5.41} \]

This gives us the covariance between the expected rate of depreciation and the forward premium and the variance of the forward premium, respectively, as

\[ \begin{align*}
\text{cov} [q(t, T), rp(t, T) + q(t, T)] &= B^*_y(T, t) \left[B^*_{y_h}(t, T) - B^*_{y_f}(t, T)ight] \text{var} [Y] \\
&+ B^*_{x_h}(T, t) B^*_{x_h}(t, T) \text{var} [X] \\
&+ B^*_{x_f}(T, t) B^*_{x_f}(t, T) \text{var} [X], \\
\end{align*} \tag{5.42} \]

and

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\[
\text{var} (rp(t,T) + q(t,T)) = B^2_{rf}(t,T) \text{var} (X_f) + B^2_{zh}(t,T) \text{var} (X_h) \\
+ [B_{yh}(t,T) - B_{yf}(t,T)]^2 \text{var} (Y). 
\] (5.43)

The tractable expression obtained in equation (5.42) shows that our framework is capable of explaining the forward premium puzzle [Nielsen and Saá-Raquejo (1993), Saá-Raquejo (1994), Ahn (1995), Bakshi and Chen (1997), and Backus et al. (2001)] even for longer horizons based on the equilibrium quantities, since the covariance of the expected rate of depreciation with the forward premium can take both positive and negative values.

A negative covariance is obtained for instance for \(B^*_y(T,t) > 0\) and \(B_{yh}(t,T) < B_{yf}(t,T)\) or for \(B^*_y(T,t) < 0\) and \(B_{yh}(t,T) > B_{yf}(t,T)\). Note that \(B^*_y(T,t) > 0\) when \(\alpha_{yh} > \alpha_{yf}\) and \(B_{yh}(t,T) < B_{yf}(t,T)\) when \(\gamma_h \alpha_{yh} < \gamma_f \alpha_{yf}\). Therefore, for \(\alpha_{yh} > \alpha_{yf}\), the puzzle is explained when \(\frac{\gamma_h}{\gamma_f} < \frac{\alpha_{yh}}{\alpha_{yf}} < 1\). Under these conditions the covariability of the risk premia with the expected depreciation is larger than the variability of the expected depreciation.

In economic terms the puzzle is explained when for instance international shocks increase the expected real rate of return on productive investment relatively more in the domestic economy, while a much stronger monetary response abroad results in relatively higher return on the foreign nominal bond. Recall that the real rate of return of production affects the spot exchange rate and the log-expected exchange rate at time \(T\). A relatively large rate of return on domestic productive investment depreciates the exchange rate and log-expected spot exchange rate at time \(T\). A smaller monetary response in the home country (i.e. for instance a relative stronger countercyclical), implies a relative smaller decrease in the domestic bond price relative to the foreign bond \([B_{yh}(t,T) < B_{yf}(t,T)]\). The depreciation [increase in \(q(t,T)\)] and the stronger countercyclical monetary in the home country (i.e. a relative smaller increase in the return on the domestic nominal bond, \(R_h\)) decreases the expected excess return on the currency, \(rp(t,T)\). This explains the negative co-variability between the expected exchange rate depreciation and the expected excess return on the currency. Thus the puzzle is explained when the two markets, equity market and bond market, do not move together. That is
\( \alpha_{y,h} > \alpha_{y,f} \) while \( R_h < R_f \) (this is for short maturities, for the entire term structure this is \( B_{y_h}(t,T) < B_{y_f}(t,T) \)). The reverse holds for \( B_{y}^{*}(T,t) < 0 \) and \( B_{y_h}(t,T) > B_{y_f}(t,T) \).