A term structure model of interest rates and forward premia: an alternative monetary approach
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Appendix

A.1 The Bellman equation

Define the representative agent’s value function as

$$ J(W, X, Y, t) = \max_{\varphi_i} E_t \int_t^{\infty} e^{-\rho s} U(c(s), m_d(s), s) \, ds, $$

where $\varphi_i = [c, m_d, a, b, f] \, \text{denote the control vector. We incorporate all state variables in the agent’s value function, as they capture the technology evolution underlying the production of both consumption/investment goods and they determine the real rate of return on the financial assets. From Merton’s (1971) analysis we know that we can write agent’s i optimization problem in a discrete time setting as follows:}$

$$ J(W, Y, X_d, X_f, t) = \max_{\mathcal{C, m}} E_t \left\{ \int_t^{t+\Delta t} U(c(s), m_d(s), s) \, ds \right. $$

$$ \left. + J[W(t + \Delta t), Y(t + \Delta t), X_d(t + \Delta t), X_f(t + \Delta t), t + \Delta t] \right\} $$

(0.42)

Assume that $J$ is twice differentiable and its Hessian is negative definite. Expanding the $J(\cdot)$ term by Taylor’s theorem and taking expectations of this expression gives:

$$ E_t [ J(W_i(t + \Delta t), Y(t + \Delta t), X_d(t + \Delta t), X_f(t + \Delta t), t + \Delta t)] = $$

$$ J(W_i, X_d, X_f, t) + \frac{\partial J}{\partial t} \Delta t + \frac{\partial J}{\partial W_i} E_t [\Delta W_i] + \frac{\partial J}{\partial Y} E_t [\Delta Y] $$

$$ + \frac{\partial J}{\partial X_d} E_t [\Delta X_d] + \frac{\partial J}{\partial X_f} E_t [\Delta X_f] + \frac{1}{2} \frac{\partial^2 J}{\partial W_i^2} \text{var}(\Delta W_i) $$

$$ + \frac{1}{2} \frac{\partial^2 J}{\partial Y^2} \text{var}(\Delta Y) + \frac{1}{2} \frac{\partial^2 J}{\partial X_d^2} \text{var}(\Delta X_d) + \frac{1}{2} \frac{\partial^2 J}{\partial X_f^2} \text{var}(\Delta X_f) $$
where $\xi$ includes higher order terms. Substituting this expression into equation (0.42), whereby the approximation
\[
\int t+\Delta t \ U(c_i(s), m_d(s), s) \, ds = U(c_i(t), m_d(t), t) \Delta t + o(\Delta t)
\]
is used and $J(\cdot)$ is subtracted from both sides, gives us

\[
0 = \max_{E_i} \left\{ E_t \left[ U(c_i(t), m_d(t), t) \Delta t \right] + \frac{\partial J}{\partial t} \Delta t + \frac{\partial J}{\partial W_i} E_t [\Delta W_i] + \frac{\partial J}{\partial X_d} E_t [\Delta X_d] + \frac{\partial J}{\partial X_f} E_t [\Delta X_f] + \frac{\partial J}{\partial Y} E_t [\Delta Y] + \frac{1}{2} \frac{\partial^2 J}{\partial W_i^2} \text{var} (\Delta W_i) + \frac{1}{2} \frac{\partial^2 J}{\partial X_d^2} \text{var} (\Delta X_d) + \frac{1}{2} \frac{\partial^2 J}{\partial X_f^2} \text{var} (\Delta X_f) + \frac{\partial^2 J}{\partial W_i \partial X_d} \text{cov} [\Delta W_i, \Delta X_d] + \frac{\partial^2 J}{\partial W_i \partial X_f} \text{cov} [\Delta W_i, \Delta X_f] + \frac{\partial^2 J}{\partial W_i \partial Y} \text{cov} [\Delta W_i, \Delta Y] + \xi \right\},
\]

By taking the limits and the moments of the variables involved we obtain the Bellman equation (4.3).

A.2 Proposition 1.

Proof.

The marginal rate of substitution of the foreign good for the domestic good:

From the first order condition in equation (4.4) we obtain for the domestic and foreign good, respectively,

\[
U_{c_h} = J_w
\]

\[
U_{c_f} = \epsilon_h J_w
\]

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By substituting equation (0.45) in equation (0.46) the marginal rate of substitution between the two composite goods in the economy in equation (4.9) is obtained.

*The marginal rate of substitution of money balances for goods:*

From equations (4.4) and (4.5) of the first order conditions, we obtain \( U_c(t) = J_{w}(t) \) and \( U_m(t) = J_{w}(t)R(t) \). After substituting \( U_c(t) \) for \( J_{w}(t) \) in the marginal utility of money and dividing by \( U_c(t) \), we obtain the marginal rate of substitution in equation (4.10).

*The marginal rate of substitution of foreign money for domestic money:*

From the first order condition for the money demand we obtain

\[
U_{mh}(c, m_d) = J_{W_h}R_h(t) \quad (0.47)
\]

\[
U_{mf}(c, m_d) = J_{W_f}R_f(t). \quad (0.48)
\]

Substituting equation (0.47) in equation (0.48) we obtain the marginal rate of substitution between the two monies in this economy, equation (4.11).

**A.3 Theorem 1.**

**Proof.**

*The spot exchange rate level in equilibrium:*

By taking the partial derivatives of the log utility function with respect to each consumption good, and substituting this in equation (4.9) we obtain the relative demand for goods in equilibrium in the home country \( h \),

\[
\frac{\theta_{hf} \epsilon_{hh}^*(t)}{\theta_{hh} \epsilon_{hf}^*(t)} = \epsilon_{hf}(t). \quad (0.49)
\]

From equation (0.49) it can be observed that an increase in the spot exchange rate induces the local agent to consume more of the local good relative to the foreign good, since in
terms of domestic currency the foreign good has become more expensive. Substituting the market clearing condition for the goods/equity market, $c^*_{hh} = \zeta^0 h \eta_h$ and $c^*_{hf} = \zeta^0 f \eta_f$, in equation (0.49) we obtain the spot foreign exchange level in equation (4.13).

The demand for real balances:

By taking the partial derivatives of the log utility function with respect to the $i$-th good and the $i$-th money, and substituting in equation (4.10) we obtain the equilibrium demand for the $i$-th real money balances in equation (4.14).

The relative demand for real money balances:

By taking the partial derivatives of the log utility function with respect to both monies, and substituting this result in equation (4.11) we obtain the relative demand for real money balances in this economy in equilibrium, equation (4.15).

Solving for the price of money:

Under certain regularity conditions, it can be shown that the state-price deflator in a general equilibrium is given by $e^{-\rho U_{c^*_j}(t)}$. As pointed out by Bakshi and Chen (1996) and Basak and Gallmeyer (1999), in equilibrium the price of money should be equal to the expected present value of its future "dividends". The "dividends" in this case are the future marginal benefits from holding cash balances, which in equilibrium are equal to $\pi_i(\tau)R_i(\tau)$ for $\tau \geq t$. Using this basic economic principle and the state-price deflator results in the following expression for the price of money:

\[
\pi_i(t) = \frac{1}{U_{c^*_j}(t)} E_t \int_t^\infty e^{-\rho(s-t)} U_{c^*_i}(s) \pi_i(s) R_i(s) ds \quad j, i \in \{h, f\}. \tag{0.50}
\]

Substituting the marginal rate of substitution (equation 4.10) in this expression gives:

\[
\pi_i(t) = \frac{1}{U_{c^*_j}(t)} E_t \int_t^\infty e^{-\rho(s-t)} \pi_i(s) U_{m,i}(s) ds. \tag{0.51}
\]

Separability of the logarithmic preferences and the equilibrium condition $M^d_{ji} = \zeta^m_{ji} M_i$.

\footnote{For a detailed discussion of the necessary and sufficient conditions, see Duffie (1992).}
and $c^*_{ji} = \zeta^0_{ji} \eta_i$ implies:

$$
\pi_i(t) = \frac{\zeta^0_{ji} \delta_{ji} \eta_i E_t}{\zeta^m_{ji} \theta_{ji}} \int_t^\infty e^{-\rho(s-t)} \frac{1}{M_i(s)} ds.
$$  \hfill (0.52)

Using equation (4.2) obtains the following:

$$
M_i(s) = M_i(t) \exp \left[ \left( \mu^*_m - \frac{1}{2} S^2_{m_i} \right) (s-t) \right] + S_m \int_t^s d w_m (\tau), \quad \text{for } s > t
$$

Then based on the moment generating function for a gaussian distribution, we know that

$$
E_t \left[ \frac{1}{M_i(s)} \right] = \frac{1}{M_i(t)} \left[ \exp \left[ - \left( \mu^*_m - S^2_{m_i} \right) (s-t) \right] \right]
$$  \hfill (0.53)

By taking the expectation first and substituting the conditional expectation of the inverse of the money level, equation (0.53), in the equation of the price of money (0.52) results in:

$$
\pi_i(t) = \frac{\zeta^0_{ji} \delta_{ji} \eta_i(t)}{\zeta^m_{ji} \theta_{ji}} \frac{1}{M_i(t)} \int_t^\infty \exp \left\{ - \left( \rho + \mu^*_m - S^2_{m_i} \right) (s-t) \right\} ds.
$$  \hfill (0.54)

Performing the integration results in the explicit closed-form expression for the price of money in equation (4.16). \hfill \Box

### A.4 Proposition 2.

**Proof.**

Under the log utility function the first order conditions, equations (4.6) and (4.7), can be written as

$$
0 = (\alpha - 1) \bar{r}_i + (S_{m_i} a_i + S_{\pi,\eta_i} b_i) \frac{J_{W,\bar{w}_i}}{J_{W_i}}
$$  \hfill (0.55)

$$
0 = (\beta_i - 1) \bar{r}_i + (S_{\pi,\pi_i} b_i + S_{\pi,\eta_i} a_i) \frac{J_{W,\bar{w}_i}}{J_{W_i}}
$$  \hfill (0.56)
Given the specification for the production process, equation (4.1), and the process for the price of money, equation (4.21), we can state the following:

\[ \text{Cov} \left( \frac{d\eta}{\eta}, dW_i \right) = (S_{\eta\eta}a_i + S_{\eta\pi}b_i) \]

and

\[ \text{Cov} \left( \frac{dB}{B}, dW_i \right) = (S_{\pi\pi}b_i + S_{\pi\eta}a_i), \]

where \( \frac{d\eta}{\eta} = \left[ \frac{d\eta_{\eta}}{\eta_{\eta}}, \frac{d\eta_{\pi}}{\eta_{\pi}} \right] \) and \( \frac{dB}{B} = \left[ \frac{d\eta_{\pi}}{\eta_{\pi}}, \frac{d\eta_{\pi}}{\eta_{\pi}} \right] \). Therefore, the following expressions for the risk premiums on the real and nominal assets are obtained, respectively:

\[
(\alpha - 1r_i(t)) = -\frac{J_{W_i,W_i}}{J_{W_i}} \text{Cov} \left( \frac{d\eta}{\eta}, dW_i \right) 
\]

and

\[
(\beta - 1r_i(t)) = -\frac{J_{W_i,W_i}}{J_{W_i}} \text{Cov} \left( \frac{d\pi}{\pi}, dW_i \right). 
\]

Because money and nominal bonds have the same risk exposure, we know that \( \text{Cov} \left( \frac{d\pi}{\pi}, dW_i \right) \) represents the covariance between the nominal assets and the changes in the real wealth. Thus, the risk premiums on both real and nominal assets are determined by the covariance of their respective returns with the changes in the real wealth. This allows us to rewrite the risk premiums in equations (0.57) and (0.58) as one asset pricing equation:

\[
E[R_q] - r_i(t) = -\frac{J_{W_i,W_i}}{J_{W_i}} \text{Cov} (R_q, dW_i), \quad (0.59)
\]

where \( R_q \) is the rate of return on the \( q \)-th security in the economy. The separability of the indirect utility function, \( J(W_i, X_i, Y_i, t) \), implies that \( J_{W_i} = \frac{1}{W_i} \) and \( W_i = \frac{1}{J_{W_i}} \). Therefore,

\[ dW_i = -\frac{1}{J_{W_i}^2} dJ_{W_i}. \]

Using this results the covariance between the return on the \( q \)-th security and the flow
budget constraint can then be rewritten as

\[
\text{cov} (R_q, dW_i) \frac{J_{W_i}}{J_W} = \text{cov} \left( R_q, \frac{dW_i}{J_{W_i}} \right) \frac{J_{W_i}}{J_W}.
\]

Recall that \( W_i = \frac{1}{J_W} \) and \( \frac{W_i J_{W_i}}{J_W} = -1 \), then

\[
\text{cov} (R_q, dW_i) \frac{J_{W_i}}{J_W} = \text{cov} \left( R_q, \frac{dW_i}{J_{W_i}} \right) \frac{-W_i J_{W_i}}{J_W} = \text{cov} \left( R_q, \frac{dW_i}{J_{W_i}} \right).
\]

Substituting this result in equation (0.59) obtains the following expression for the risk compensation:

\[
E [R_q] - r_i(t) = -\text{cov} \left( R_q, \frac{dW_i}{J_{W_i}} \right).
\]

Employing the envelope conditions \( U_{\epsilon_i}(t) = J_{W_i}(t) \) and \( U_{m_i}(t) = J_{W_i}(t) R_i(t) \), and applying Itô's lemma leads to:

\[
E [R_q(t)] - r_i(t) = -\text{cov} (R_q, dM_i) \frac{U_{m_i}}{U_{m_i}} - \text{cov} (R_q, dx_i^*) \frac{U_{m_i^*}}{U_{m_i}}.
\]

The separability of the log utility specification \( (U_{m_i^*} = 0) \) yields:

\[
E [R_q(t)] - r_i(t) = -\text{cov} (R_q, dM_i) \frac{U_{m_i m_i}}{U_{m_i}}. \tag{0.60}
\]

Even in the case of state-independent constant drift and diffusion terms, the money supply process, \( \frac{dM_i}{M_i} \), in equation (4.2) incorporates both the common and local sources of uncertainties, namely \( w_x(t) \) and \( w_{x_i}(t) \), respectively. Using the money supply process and the indirect utility function in the case of logarithmic preferences one obtains:

\[
(E [R_q] - r_i) = \text{cov} (R_q, R_{X_h}) m_{x_h} - \text{cov} (R_q, R_{X_f}) m_{x_f} + \text{cov} (R_q, R_Y) m_y \tag{0.61}
\]

where
The equilibrium nominal interest rate:

Given a separable log utility specification over consumption and money, the first order conditions in equations (4.4) and (4.5) provide the following expression for the nominal interest rate:

$$ R_i(t) = \frac{\delta_{i1}}{\theta_{i1}} \frac{c^*_i(t)}{\pi_i(t) M_{ii}(t)}. \quad (0.62) $$

Substituting the solution for the price of money, equation (4.16), in this expression results in equation (4.29) in Theorem 3.

The equilibrium real interest rates.

Since the spot exchange rate adjusts instantaneously to restore the equilibrium on the equity market, the real rates of return on the domestic equity and the foreign equity expressed in domestic currency are equal. The optimization problem for the two equities is therefore the same. Given the equilibrium condition $f_1' = 0$ and $B_i = 0$, and the derivatives of the indirect utility function, $-\frac{J_{W,W}}{J_{W_i}} = 1$ and $-\frac{J_{W,X}}{J_{W_i}} = -\frac{J_{W,Y}}{J_{W_i}} = 0$, the first order conditions (4.6) and (4.7) can be written as:

$$ (\alpha_i - r_i(t)) = 2S_{\eta,\eta}a_i + S_{\eta,B}b_{i1} + S_{\eta,\bar{B}}b_{ij} $$
$$ (\beta_i - r_i(t)) = 2S_{\eta,B}a_{i1} + S_{B,B}b_{i1} + S_{B,\bar{B}}b_{ij} $$
$$ (\bar{\beta}_j - r_i(t)) = 2S_{\eta,\bar{B}}a_i + S_{B,B}b_{i1} + S_{\bar{B},\bar{B}}b_{ij}. \quad (0.63) $$
By using equations (4.21) and (4.29) we can write the equilibrium real rate of return of the domestic nominal bonds expressed in domestic currency as

$$\frac{dB_h}{B_h} = \beta_h dt + S_{x_h} dw_{x_h} (t), \quad (0.64)$$

where

$$\beta_h = \rho + \alpha_{\eta,h} - \gamma_h \left( \sigma_{\eta_h}^2 + \sigma_{\eta,h}^2 \right)$$

$$S_{x_h} dw_{x_h} (t) = (1 - \gamma_h) \left( \sigma_{\eta_h} dw_{\eta} (t) + \sigma_{x_h} dw_{x_h} (t) \right) - \sigma_{m_h} dw_{m_h} (t)$$

By using equations (4.21), (4.29), and (4.17) we can write the real rate of return of the nominal bonds expressed in domestic currency as

$$\frac{d\epsilon_h B_f}{\epsilon_h B_f} = \tilde{\beta}_f dt + S_{\tilde{B}_f} dw (t), \quad (0.65)$$

where

$$\tilde{\beta}_f = \beta_f + \mu_{\epsilon_f} + S_{x_f \epsilon_f}$$

$$= \rho + \alpha_{\eta,h} - \gamma_f \sigma_{\eta_h} \sigma_{\eta_f}$$

and

$$S_{\tilde{B}_f} = S_{x_f} dw (t) + S_{\epsilon_f} dw (t)$$

$$= \left( \sigma_{\eta_h} - \gamma_f \sigma_{\eta_f} \right) dw_{\eta} (t) - \gamma_f \sigma_{\eta_f} dw_{\epsilon_f} (t) + \sigma_{x_h} dw_{x_h} (t) - \sigma_{m_f} dw_{m_f} (t)$$

From equations (4.1), (0.64), and (0.65) we know that the co-variances are:

$$S_{\eta_{\eta,h}} = \sigma_{\eta_h}^2 + \sigma_{\eta_h}^2$$

$$S_{\eta,\theta_h} = (1 - \gamma_h) \left( \sigma_{\eta_h}^2 + \sigma_{\eta,h}^2 \right)$$

$$S_{\eta,\eta,\eta} = \sigma_{\eta_h}^2 + \sigma_{\eta_h}^2 \left[ \sigma_{\eta_h} - \gamma_f \sigma_{\eta_f} \right] = S_{\eta_h} - S_{\eta_h \eta_f}$$

$$S_{B,B} = (1 - \gamma_h)^2 \left( \sigma_{\eta_h}^2 + \sigma_{\eta,h}^2 \right) + \sigma_{m_h}^2$$

$$S_{\tilde{B},B} = (1 - \gamma_h) \left( \sigma_{\eta_h} - \gamma_f \sigma_{\eta_f} \right) \sigma_{\eta,h} + \sigma_{\eta,h}^2$$

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Using the equilibrium expression for the real rate of return on nominal assets, equations (0.64) and (0.65), and the co-variances determined above the system of first order equations (0.63) can be written as

\[
\begin{align*}
    r_s(t) &= \alpha_t - 2 \left( \sigma_{\eta_{y,h}^2} + \sigma_{\eta_{z,h}^2} \right) a_t \\
           &\quad - (1 - \gamma_h) \left( \sigma_{\eta_{y,h}^2} + \sigma_{\eta_{z,h}^2} \right) b_{ii} \\
           &\quad - \left[ \sigma_{\eta_{z,h}^2} + \sigma_{\eta_{y,h}} \left( \sigma_{\eta_{y,h}^2} - \gamma_f \sigma_{\eta_{y,f}} \right) \right] b_{ij} \\
    r_s(t) &= \rho + \alpha_{\eta,h} - \gamma_h \left( \sigma_{\eta_{y,h}^2} + \sigma_{\eta_{z,h}^2} \right) - 2 (1 - \gamma_h) \left( \sigma_{\eta_{y,h}^2} + \sigma_{\eta_{z,h}^2} \right) a_i \\
           &\quad - \left[ (1 - \gamma_h)^2 \left( \sigma_{\eta_{y,h}^2} + \sigma_{\eta_{z,h}^2} \right) + \sigma_{m_h}^2 \right] b_{ii} \\
           &\quad - (1 - \gamma_h) \left( \left( \sigma_{\eta_{y,h}^2} - \gamma_f \sigma_{\eta_{y,f}} \right) \sigma_{\eta_{y,h}} + \sigma_{\eta_{z,h}} \right) b_{ij} \\
    r_s(t) &= \rho + \alpha_{\eta,h} - \gamma_f \sigma_{\eta_{y,h}} \sigma_{\eta_{y,f}} - 2 \left[ \sigma_{\eta_{z,h}^2} + \sigma_{\eta_{y,h}} \left( \sigma_{\eta_{y,h}^2} - \gamma_f \sigma_{\eta_{y,f}} \right) \right] a_i \\
           &\quad - (1 - \gamma_h) \left( \left( \sigma_{\eta_{y,h}^2} - \gamma_f \sigma_{\eta_{y,f}} \right) \sigma_{\eta_{y,h}} + \sigma_{\eta_{z,h}} \right) b_{ii} \\
           &\quad - \left( \left( \sigma_{\eta_{y,h}} - \gamma_f \sigma_{\eta_{y,f}} \right)^2 + \gamma_f^2 \sigma_{\eta_{y,f}}^2 + \sigma_{\eta_{z,h}}^2 + \sigma_{m_f}^2 \right) b_{ij} \\
    1 &= 2a_h + b_{hh} + b_{hf}
\end{align*}
\]

Simultaneous solution of this system [equation (0.66)] results in the equilibrium real interest rate and the equilibrium portfolio demand for the assets in the economy in equations (4.30), (4.35), (4.36), and (4.34).

\section*{Proof. Theorem 2(b):}

The fundamental valuation equation:

Given our log utility specification, the first order condition for the contingent claims
(equation 4.8) and the market clearing conditions, \( f'1 = 0 \), provide the following for expected real rate of return on the contingent claim:

\[
\zeta = r_t(t)1 + S_{\eta,\tau}a_{\eta}^* + S_{\beta,\beta}b_{\beta}^*
\]  

(0.67)

Next, applying Itô's lemma to the \( k \)-th contingent claim, \( F^k(Y, X_h, X_f, t) \), leads to:

\[
dF^k(Y, X_h, X_f, t) = \left\{ F_Y^k \mu_Y(Y, t) + \sum_{i}^{h,f} F^k_{X_i} \mu_{x_i}(X_i, t) + F^k_t + \frac{1}{2} F_Y^k \sigma_Y^2 + \frac{1}{2} \sum_{i}^{h,f} F^k_{X_i, x_i} \sigma_{X_i}^2 \right\} dt
\]

\[
+ F_Y^k \sigma_Y dw_y(t) + \sum_{i}^{h,f} F^k_{X_i, x_i} dw_{x_i}(t),
\]

Therefore, the drift and diffusion terms of the conjectured process for the contingent claim in equation (3.10) must be, respectively:

\[
F^k \zeta^k = F_Y^k \mu_Y(Y, t) + \sum_{i}^{h,f} F^k_{X_i} \mu_{x_i}(X_i, t) + F^k_t
\]

\[
+ \frac{1}{2} F_Y^k \sigma_Y^2 + \frac{1}{2} \sum_{i}^{h,f} F^k_{X_i, x_i} \sigma_{X_i}^2,
\]

(0.68)

and

\[
F^k \sigma_{F_k} dw(t) = F_Y^k \sigma_Y dw_y(t) + \sum_{i}^{h,f} F^k_{X_i, x_i} dw_{x_i}(t).
\]

(0.69)

Using this diffusion term to reduce the covariances in equation (0.67) results in the following expression for the total expected return on the contingent claim:

\[
F^k \zeta^k = F^k r_t(t) + F^k s_{x_h} \left( S_{\eta_h} a_{\eta_h}^* + S_{\beta_h} b_{\beta_h}^* \right)
\]

\[
+ F_Y^k \left( S_{\eta_f} a_{\eta_f}^* + S_{\beta_f} b_{\beta_f}^* \right)
\]

\[
+ F_{X_f}^k \left( S_{\eta_f} a_{\eta_f}^* + S_{\beta_f} b_{\beta_f}^* \right),
\]

(0.70)

Substituting the expression for the drift term (0.68) in equation (0.70) obtains the fun-
B.1 Equilibrium nominal interest rates.

The money supply process in equation (5.3) can be written as:

\[ M_{s,t}(T) = M_{s,t}(t) \exp \left[ \int_t^T \left( \mu_{m}^*(X_i(s), Y(s), s) \right. \right. \]
\[ \left. \left. - \frac{1}{2} \sigma_m^2 - \frac{1}{2} \gamma_i \sigma_{\eta_{yi}}^2, Y(s) - \frac{1}{2} \gamma_i \sigma_{\eta_{zi}}^2, X_i(s) \right) ds \right. \]
\[ + \int_t^T S_{m_t}^* (X_i(t), Y(t), t) dw_{m_t}^* (s) \right], \text{ for } T > t, \tag{0.71} \]

where

\[ \mu_{m_t}^* (X_i(t), Y(t), t) = (\mu_m^* + \gamma_i \alpha_y Y + \gamma_i \alpha_x X_i) \]
\[ S_{m_t}^* (X_i(t), Y(t), t) dw_{m_t}^* (t) = \sigma_m dw_{m_t}(t) + \gamma_i \sigma_{\eta_{yi}} \sqrt{Y(t)} dw_y(t) + \gamma_i \sigma_{\eta_{zi}} \sqrt{X_i(t)} dw_z(t). \]

As shown in Proposition 1, the demand for real money balances, equation (4.14), implies the following for the equilibrium price of money:

\[ \pi_i(t) = \frac{\delta_{m t} c_i(t)^*}{\theta_{m t} M_i(t) R_i(t)} \tag{0.72} \]

As presented in (0.52), the price of money must also satisfy the following condition in equilibrium:

\[ \pi_i(t) = \frac{\sigma_{\eta_{yi}} \delta_{ji} \eta_i E_t \int_t^\infty e^{-\rho(s-t)} \frac{1}{M_i(s)} ds.} {\gamma_{ji} \theta_{ji}} \tag{0.73} \]

From these expressions for the price of money [equations (0.72) and (0.73)], the money supply process in equation (0.71), and the equilibrium condition for money \( M_{d,t} = M_{s,t} = \)
\[ M_t, \text{ we know that} \]
\[ \frac{1}{R_t(t)} = \int_t^\infty e^{-r(s-t)} E_t \left[ \exp \left( Q_x(s) + Q_y(s) \right) \right] ds \quad (0.74) \]

where
\[ Q_x(t) = -\int_t^T \left( \mu_m + \gamma_i \alpha_x, X_i(s) - \frac{1}{2} \sigma_m^2 - \frac{1}{2} \gamma_i^2 \sigma_{\eta_x,i}^2 X_i(s) \right) ds \]
\[ -\int_t^T \sigma_m dw_m(s) - \int_t^T \gamma_i \sigma_{\eta_x,i} \sqrt{X_i(t)} dw_{X_i}(s) \]

and
\[ Q_y(t) = -\int_t^T \left( \gamma_i \alpha_y, Y(s) - \frac{1}{2} \gamma_i^2 \sigma_{\eta_y,i}^2 Y(s) \right) ds - \int_t^T \gamma_i \sigma_{\eta_y,i} \sqrt{Y(s)} dw_y(s). \]

Making use of the law of iterated expectations obtains:
\[ \frac{1}{R(t)} = \int_t^\infty e^{-r(s-t)} E_t \left[ E_X \left[ \exp \left( Q_x(s) \right) \right] E_Y \left[ \exp \left( Q_y(s) \right) \right] \right] ds \quad (0.75) \]

We know that, conditional on \( X_i(t) \) and \( Y(t) \), \( M_{s,t}(t) \) is just a geometric Brownian motion. Thus, we can use the moment generating function of a normal to evaluate the expectation conditional on the stochastic processes \( X_i(t) \) and \( Y(t) \). Therefore,
\[ \frac{1}{R(t)} = \int_t^\infty e^{-r(s-t)} E_t \left[ \exp \left( -\int_t^T (\tilde{Q}_x(s) + \tilde{Q}_y(s)) ds \right) \right], \quad (0.76) \]

where
\[ \tilde{Q}_x(t) = \mu_m - \sigma_m^2 + \gamma_i \left( \alpha_x, - \gamma_i \sigma_{\eta_x,i}^2 \right) X_i(t) \text{ and} \]
\[ \tilde{Q}_y(t) = \gamma_i \left( \alpha_y, - \gamma_i \sigma_{\eta_y,i}^2 \right) Y(t). \]
Define

\[ Z \left( \tilde{Q}_x(t), \tilde{Q}_y(t), t, T \right) = E_t \left[ \exp \left( - \int_t^T \left( \tilde{Q}_x(s) + \tilde{Q}_y(s) \right) ds \right) \right]. \]

where \( Z \left( \tilde{Q}_x(T), \tilde{Q}_y(t), T, T \right) = 1 \). Intuitively, \( Z \) can be interpreted as a claim that pays one dollar at maturity and \( \left( \tilde{Q}_x(s) + \tilde{Q}_y(s) \right) \) as the stochastic discount factor. It is obvious that after discounting with \( \left( \tilde{X}(s) + \tilde{Y}(s) \right) \) that \( Z \left( \tilde{Q}_x(t), \tilde{Q}_y(t), t, T \right) \) is a martingale. Consider a function of this claim,

\[ U(t, Z) = \exp \left( - \int_t^T \left( \tilde{Q}_x(s) + \tilde{Q}_y(s) \right) ds \right) \times Z \left( \tilde{X}(t), \tilde{Y}(t), t \right) \quad (0.77) \]

then we know that \( U(t, Z) \) is a martingale and the following holds

\[ E [dU] = 0 \quad (0.78) \]

By applying Itô’s lemma on equation (0.77) and using condition (0.78) we obtain the following partial differential equation:

\[ 0 = \frac{1}{2} \tilde{\sigma}_x^2 X(t) Z_{\tilde{Q}_x} \tilde{Q}_x + \frac{1}{2} \tilde{\sigma}_y^2 Y(t) Z_{\tilde{Q}_y} \tilde{Q}_y + \left( \tilde{\theta}_x - \tilde{\kappa}_x X(t) \right) Z_{\tilde{Q}_x} \]
\[ + \left( \tilde{\theta}_y - \tilde{\kappa}_y Y(t) \right) Z_{\tilde{Q}_y} + Z_t - \left[ \mu_{\alpha_1} - \sigma_{\alpha_1} \right] \]
\[ + \gamma_i \left( \alpha_{\alpha_1} - \gamma_i \sigma_{\alpha_1}^2 \right) X_i(t) + \gamma_i \left( \alpha_{\gamma_i} - \gamma_i \sigma_{\gamma_i}^2 \right) Y(t) \right] Z, \quad (0.79) \]

where

\[ \tilde{\theta}_x = \gamma_i \left( \alpha_{\alpha_1} - \gamma_i \sigma_{\alpha_1}^2 \right) \theta_{x_1} \]
\[ \tilde{\kappa}_x = \gamma_i \left( \alpha_{\alpha_1} - \gamma_i \sigma_{\alpha_1}^2 \right) \kappa_x \]
\[ \tilde{\sigma}_x = \gamma_i \left( \alpha_{\alpha_1} - \gamma_i \sigma_{\alpha_1}^2 \right) \sigma_x \]
\[ \tilde{\theta}_y = \gamma_i \left( \alpha_{\gamma_i} - \gamma_i \sigma_{\gamma_i}^2 \right) \theta_y \]
\[ \tilde{\kappa}_y = \gamma_i \left( \alpha_{\gamma_i} - \gamma_i \sigma_{\gamma_i}^2 \right) \kappa_y \]
\[ \tilde{\sigma}_y = \gamma_i \left( \alpha_{\gamma_i} - \gamma_i \sigma_{\gamma_i}^2 \right) \sigma_y \]

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Application of the separation of variable technique on equation (0.79) we obtain the following solution for \( Z(t, \tau) \),

\[
Z(t, \tau) = A_{x_i}(\tau) A_{y_i}(\tau) \exp \left[ -\mu_m \tau - B_x(t, \tau) X - B_y(t, \tau) Y \right], \tag{0.80}
\]

where

\[
A_{x_i}(\tau) = \left( \frac{2\varphi_{x_i} e^{(\kappa_{x_i} + \varphi_{x_i}) \tau/2}}{(\kappa_{x_i} + \varphi_{x_i}) (e^{\varphi_{x_i} \tau} - 1) + 2\varphi_{x_i}} \right)^{\frac{2\varphi_{x_i}}{\sigma_{x_i}^2}}
\]

\[
A_{y_i}(\tau) = \left( \frac{2\varphi_{y_i} e^{(\kappa_{y_i} + \varphi_{y_i}) \tau/2}}{(\kappa_{y_i} + \varphi_{y_i}) (e^{\varphi_{y_i} \tau} - 1) + 2\varphi_{y_i}} \right)^{\frac{2\varphi_{y_i}}{\sigma_{y_i}^2}}
\]

\[
B_x(t, \tau) = \frac{2\gamma_i (\alpha_{x_i} - \gamma_i \sigma_{x_i}^2)}{(\kappa_{x_i} + \varphi_{x_i}) (e^{\varphi_{x_i} \tau} - 1) + 2\varphi_{x_i}}
\]

\[
B_y(t, \tau) = \frac{2\gamma_i (\alpha_{y_i} - \gamma_i \sigma_{y_i}^2)}{(\kappa_{y_i} + \varphi_{y_i}) (e^{\varphi_{y_i} \tau} - 1) + 2\varphi_{y_i}}
\]

\[
\varphi_{x_i} = \sqrt{\kappa_{x_i}^2 + 2\gamma_i (\alpha_{x_i} - \gamma_i \sigma_{x_i}^2) \sigma_{x_i}^2}
\]

\[
\varphi_{y_i} = \sqrt{\kappa_{y_i}^2 + 2\gamma_i (\alpha_{y_i} - \gamma_i \sigma_{y_i}^2) \sigma_{y_i}^2}
\]

Substituting this solution (0.80) in equation (0.76) and taking the integral we obtain the equilibrium nominal interest rate in country \( i \), equation (5.4).

B.2 Theorem 3.

To derive the equilibrium nominal price at time \( t \) of a pure discount nominal bond that matures at \( T \) that is traded on the local securities markets of country \( i \), \( P_i(t, T) \) we first conjecture the solution in equation (5.16). Substituting this conjectured solution in the PDE in equation (5.15), subject to its boundary condition, \( P_i(t, T) = 1 \), results in
the following partial differential equation

\[
0 = \left\{ B_{y,t} - \lambda_i^* + \gamma_i^* \kappa_y B_{y_i} + \frac{1}{2} \lambda_i^* \sigma_y^2 B_{y_i}^2 \right\} Y \\
+ \left\{ B_{x,t} - \gamma_i^* + \gamma_i^* \kappa_x, B_{x_i} + \frac{1}{2} \gamma_i^* \sigma_x^2 B_{x_i}^2 \right\} X_i \\
- \left\{ \lambda_i^* \kappa_y \theta_y B_{y_i} + \gamma_i^* \kappa_x, \theta_x, B_{x_i} + \mu_i + \rho \right\}
\]

(0.81)

In order for this to hold independently of \( Y \) or \( X \), the following three ordinary differential equation must hold, for \( i = h, f \),

\[
\frac{\partial}{\partial t} B_{y_i} = \lambda_i^* - \lambda_i^* \kappa_y B_{y_i} - \frac{1}{2} \lambda_i^* \sigma_y^2 B_{y_i}^2 \\
\frac{\partial}{\partial t} B_{x_i} = \gamma_i^* - \gamma_i^* \kappa_x B_{x_i} - \frac{1}{2} \gamma_i^* \sigma_x^2 B_{x_i}^2 \\
\frac{\partial}{\partial t} A_i = - \left( \lambda_i^* \kappa_y \theta_y B_{y_i} + \gamma_i^* \kappa_x, \theta_x, B_{x_i} + \mu_i + \rho \right)
\]

Application of the standard Ricatti technique and re-arranging terms obtain the equilibrium value of the nominal bond in equation (5.16)

**B.3 Theorem 4.**

From Equations (5.4), (5.6), and (5.7) we can express the state variables in terms of the nominal interest rate differential and the volatilities of the nominal interest rates in both countries. The local state variable can be written as

\[
X_h = \xi_{x_h} \left( \lambda_i^* \sigma_y^2 \left[ - (R_h - R_f) + \left( \mu_{n_h} - \mu_{m_f} \right) \right] - \left( \sigma_{m_h}^2 - \sigma_{m_f}^2 \right) \right) \\
- \left( \lambda_f^* - \lambda_h^* \right) V_d R_h + \gamma_f^* \sigma_y^2 \xi_{x_h} \left( V_d R_h \lambda_f^2 - V_d R_f \lambda_h^2 \right),
\]

(0.82)

where
\[ \xi_{X_1} = \frac{1}{\gamma_f \gamma_h (\lambda_f^2 \sigma^2_{x_1} + \gamma_f \sigma^2_{x_2} (\lambda_f - \lambda_f^2))} \]
\[ \xi_{X_2} = \gamma_f^2 \sigma^2_{z_f} \xi_{X_1} \]
\[ \xi_{X_3} = \gamma_h^2 \sigma^2_{z_h} \xi_{X_1}. \]

The foreign state variable can be expressed as

\[ X_f = \xi_{X_3} \left( \lambda_f^* \sigma^2_y \left[ - (R_h - R_f) + \left( \mu_{m_h} + \mu_{m_f} \right) - \left( \sigma^2_{m_h} - \sigma^2_{m_f} \right) \right] 
+ \left( \lambda_f^* - \lambda_h^* \right) V_{dR_f} \right) + \gamma_h^* \sigma^2_y \xi_{X_1} \left( V_{dR_h} \lambda_f^* - V_{dR_f} \lambda_h^* \right). \] (0.83)

The common/international state variable in this two-country world economy can be written as

\[ Y_h = \gamma_f^* \gamma_h^* \xi_{X_1} \left( \gamma_f^* \sigma^2_{y_f} \gamma_h^* \sigma^2_{y_h} \left[ R_h - R_f - \left( \mu_{m_h} + \mu_{m_f} \right) + \left( \sigma^2_{m_h} - \sigma^2_{m_f} \right) \right] 
- \left( \gamma_f^* \sigma^2_{z_f} V_{dR_h} - \gamma_h^* \sigma^2_{z_h} V_{dR_f} \right) \right). \] (0.84)

By substituting these expressions for the state variables in equation (5.19) and rearranging terms we obtain equation (5.26).
Table 1
Properties of Interest rates and Exchange Rates

The data are sampled monthly over the period of January 1981 to December 2001, that is 251 observations for each series. $R_i$ denotes country $i$ interest rate, on a annualized basis. The rates of depreciation of the currency prices, $s_{t-1} - s_n$, are defined as the change in the logarithm of the spot exchange rate. The instantaneous forward foreign exchange premiums, $f_{t-1} - s$, are defined as the difference between the logarithm of the forward foreign exchange and the logarithm of the spot exchange rate. $\mu_t$ denotes the sample mean, $\sigma_t$ the sample standard deviation, and $\rho_{ij}$ the j-th order autocorrelation. $PP_t$ is the Phillips and Perron (1981) unit roots test statistics against the critical value of -3.99 at one percent and -3.43 at five percent. The covered and uncovered interest rate parity regression equations are of the form: $f_{t-1} - s = \beta_1 + \beta_2(R_{w} - s_t) + \epsilon_t$ and $s_{t-1} - s_t = \beta_1 + \beta_2(R_{w} - s_t) + \epsilon_t$, respectively. Numbers in parentheses are Newey-West standard errors.

Panel I: Summary Statistics

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Panel II: The CIP and UIP regressions

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### Table 2
Parameter Estimates of the GARCH and EGARCH specification for the volatility of the Short-Term Interest rates Dynamics

*(z-statistics are indicated in brackets)*

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### Table 3
The sign and size test for asymmetric effects in the interest rates.

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<th>F-stat**</th>
<th>P-value</th>
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<tr>
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<td>(-0.49)</td>
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<tr>
<td><strong>UK-rate</strong></td>
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<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>-0.000</td>
<td>0.000</td>
<td>-0.003</td>
<td>0.024</td>
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<td>16.72</td>
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<td>(6.87)</td>
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</tbody>
</table>

* The $t$-statistics are indicated in brackets

** Wald test for the joint hypothesis of zero on the sign and size parameters ($a_0=a_1=a_2=a_3=0$).
Table 4
GMM-estimates of the interest rates and the exchange rate dynamics

The variables included in the test are the one-month US eurocurrency interest rate (\( R_{US} \)), the UK one-month eurocurrency interest rate (\( R_{UK} \)), and the rate of depreciation of the dollar price of UK-pound (\( s_{UK} \)). The estimation period is from January 1981 to December 2001 (that is 252 observations). The sample consists of monthly data expressed in annualized form. The parameters are estimated by the Generalized Methods of Moments, with \( t \)-statistics in brackets. The \( \chi^2 \) test statistics are reported for six over-identifying restrictions. The parameter estimates are from the following discrete-time system of equations:

\[
x_{t+1} - x_t = a_0 + a_1 R_{diff,t} + a_2 V_{R,t} + a_3 V_{R,t} + \varepsilon_{t+1}
\]

\[
E[e_{t+1}] = 0, \quad E[e_{t+1}^2] = b_0 + b_1 R_{diff,t} + b_2 V_{R,t} + b_3 V_{R,t}
\]

<table>
<thead>
<tr>
<th></th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( \chi^2 )-Test</th>
</tr>
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<tr>
<td>( R_{US} )</td>
<td>9.5*10^{-5}</td>
<td>0.024</td>
<td>-0.003</td>
<td>0.002</td>
<td>-2*10^{-5}</td>
<td>0.001</td>
<td>3*10^{-5}</td>
<td>1.4*10^{-4}</td>
<td>1.75</td>
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<td>(0.15)</td>
<td>(3.23)</td>
<td>(-6.39)</td>
<td>(1.99)</td>
<td>(-1.49)</td>
<td>(2.15)</td>
<td>(2.14)</td>
<td>(2.09)</td>
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<tr>
<td>( R_{UK} )</td>
<td>-0.002</td>
<td>0.080</td>
<td>-0.001</td>
<td>0.007</td>
<td>-4*10^{-5}</td>
<td>0.001</td>
<td>-1*10^{-5}</td>
<td>1.6*10^{-4}</td>
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<td>(-1.93)</td>
<td>(5.85)</td>
<td>(-3.65)</td>
<td>(3.04)</td>
<td>(-4.93)</td>
<td>(4.98)</td>
<td>(-2.94)</td>
<td>(8.87)</td>
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<tr>
<td>( s_{UK} )</td>
<td>-0.005</td>
<td>-0.196</td>
<td>-0.005</td>
<td>0.007</td>
<td>-1*10^{-4}</td>
<td>-0.012</td>
<td>-1*10^{-4}</td>
<td>0.002</td>
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<tr>
<td></td>
<td>(-1.97)</td>
<td>(-2.26)</td>
<td>(-4.03)</td>
<td>(1.36)</td>
<td>(-0.81)</td>
<td>(-2.21)</td>
<td>(-1.75)</td>
<td>(3.29)</td>
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</table>
GMM-estimates of the Cross-sectional Restrictions on Monthly Yield Changes

The variables included in the test are the one-, three- and six-month and one-year US and UK eurocurrency interest rates. The estimation period is from January 1981 to December 2001 (that is 252 observations). The sample consists of monthly data expressed in annualized form. The parameters are estimated by the Generalized Methods of Moments, with t-statistics in brackets. The parameter estimates are from the following discrete-time system equations of the term structure of interest rates:

\[ \Delta x_t = a_1 \Delta R_{\text{diff},t} + a_2 \Delta V_{R_k,t} + a_3 \Delta V_{R_f,t} + e_t \] and \( E[e_t] = 0. \)

### Table 5

<table>
<thead>
<tr>
<th></th>
<th>( a_1 )</th>
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<th>( a_3 )</th>
<th>( R )</th>
</tr>
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<tr>
<td><strong>US-yields</strong></td>
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<tr>
<td>One-month</td>
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<td>0.026</td>
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<tr>
<td>(5.99)</td>
<td>(3.28)</td>
<td>(4.88)</td>
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<tr>
<td>Three-month</td>
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<td>0.255</td>
<td>0.34</td>
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<td>(4.77)</td>
<td>(-0.06)</td>
<td>(4.01)</td>
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</tr>
<tr>
<td>Six-month</td>
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<td>0.305</td>
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<td>(5.75)</td>
<td>(-0.01)</td>
<td>(3.99)</td>
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<tr>
<td>One-year</td>
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<td>-0.006</td>
<td>0.266</td>
<td>0.33</td>
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<tr>
<td>(6.25)</td>
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<td>(4.45)</td>
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<td></td>
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<tr>
<td><strong>UK-yields</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One-month</td>
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<td>0.029</td>
<td>0.369</td>
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<td>(-4.21)</td>
<td>(3.65)</td>
<td>(3.69)</td>
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</tr>
<tr>
<td>Three-month</td>
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<td>0.022</td>
<td>-0.261</td>
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</tr>
<tr>
<td>(-3.65)</td>
<td>(3.03)</td>
<td>(-2.54)</td>
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<tr>
<td>Six-month</td>
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<td>0.013</td>
<td>0.196</td>
<td>0.29</td>
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<tr>
<td>(-2.67)</td>
<td>(1.33)</td>
<td>(2.73)</td>
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<tr>
<td>One-year</td>
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<td>0.026</td>
<td>0.014</td>
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<tr>
<td>(-2.86)</td>
<td>(3.87)</td>
<td>(0.14)</td>
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</tr>
</tbody>
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### Table 6

Parameter Estimates of EGARCH specification for the volatility of the spot US-$ price of the UK-£

\( (z\text{-statistics are indicated in brackets}) \)

<table>
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<tr>
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<th>( a_2 )</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
<th>( b_4 )</th>
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<tr>
<td></td>
<td>0.453</td>
<td>-0.078</td>
<td>-0.318</td>
<td>-0.298</td>
<td>0.013</td>
<td>0.963</td>
<td>0.187</td>
<td>-0.008</td>
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<td></td>
<td>(4.85)</td>
<td>(-4.92)</td>
<td>(-0.90)</td>
<td>(-1.86)</td>
<td>(1.10)</td>
<td>(33.56)</td>
<td>(2.02)</td>
<td>(-0.24)</td>
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Table 7
GMM-estimates of the Term Structure of foreign exchange returns
and the calibrated model parameter values

The variables included in the test are the one- and six-month and one-year compounded US- and UK-yields \((K_{i,r})\) and the logarithm of the spot dollar price of the UK-pound, \(e\). The estimation period is from January 1981 to December 2001 (that is 252 observations). The sample consists of monthly data expressed in annualized form. The parameters are estimated by the Generalized Methods of Moments, with t-statistics in brackets. The parameter estimates are from the following discrete-time system of equations:

\[
s_{t+1} - s_t = a_1 + a_2 (K_{t+1} - K_{t+1}^*) + e_{t+1} \quad \text{and} \quad E[e_t] = 0,
\]

where \(r\) denotes time to maturity and (*) represents the UK-yields. \(R_{US}, R_{UK}, \Delta s, V_{SR,US}, V_{SR,UK}\), and \(V_{SS}\) denotes the US and UK one month eurocurrency interest rates, the changes in US-dollar price of the UK-pound, the volatility of the dynamics of the US- and UK-one month interest rates and the dollar price of the UK-pound, respectively. The unconditional mean, variance, and co-variance are represented by \(E(.), \text{Var}(.), \text{and Cov}(.)\), respectively. These moments are estimated by the Generalized Methods of Moments, with t-statistics in the last column.

Panel III contains the calibrated values for the model parameters based on the Panel I Fama-regression results and the Panel II moment estimates.

<table>
<thead>
<tr>
<th>Panel I: Regression results</th>
<th>(a_1)</th>
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<th>(R^2)</th>
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<tr>
<td>Maturities</td>
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<tr>
<td>One-month</td>
<td>-0.006</td>
<td>-2.631</td>
<td>0.04</td>
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<tr>
<td></td>
<td>(-3.12)</td>
<td>(-2.06)</td>
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</tr>
<tr>
<td>Six-month</td>
<td>-0.029</td>
<td>-2.009</td>
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<tr>
<td></td>
<td>(-3.83)</td>
<td>(-2.21)</td>
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<tr>
<td>One-year</td>
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<td>-1.394</td>
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<tr>
<td></td>
<td>(-3.09)</td>
<td>(-1.99)</td>
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<table>
<thead>
<tr>
<th>Panel II: Moment estimates</th>
<th>Coefficient</th>
<th>(t)-statistics</th>
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<tr>
<td>(E(R_{US}))</td>
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<td>16.42</td>
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<tr>
<td>(E(R_{UK}))</td>
<td>0.092076</td>
<td>20.78</td>
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<tr>
<td>(E(V_{SR,US}))</td>
<td>0.000081</td>
<td>2.60</td>
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<tr>
<td>(E(V_{SR,UK}))</td>
<td>0.000060</td>
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</tr>
<tr>
<td>(E(V_{SS}))</td>
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<td>15.62</td>
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<tr>
<td>(E(\Delta s))</td>
<td>-0.025677</td>
<td>-1.01</td>
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<tr>
<td>(\text{Var}(R_{US}))</td>
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<tr>
<td>(\text{Var}(R_{UK}))</td>
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</tr>
<tr>
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<td>(\text{Cov}(R_{US},V_{SR,US}))</td>
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<tr>
<td>(\text{Cov}(R_{US},V_{SR,UK}))</td>
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<tr>
<td>(\text{Cov}(R_{US},V_{SS}))</td>
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<td>1.86</td>
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<tr>
<td>(\text{Cov}(R_{UK},V_{SR,US}))</td>
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<tr>
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<td>(\text{Cov}(V_{SR,US},V_{SS}))</td>
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<td>$\mu_{US}$</td>
<td>$\mu_{UK}$</td>
<td>$\gamma_{US}$</td>
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<tr>
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<td>----------</td>
<td>-------------</td>
</tr>
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<td>0.0207</td>
<td>0.0579</td>
<td>0.0131</td>
</tr>
<tr>
<td>$\theta_{\xi_{US}}$</td>
<td>$\theta_{\xi_{UK}}$</td>
<td>$\theta_{\eta}$</td>
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<tr>
<td>0.0626</td>
<td>0.1544</td>
<td>0.3392</td>
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Table 7 (Continued)
Figure 1. The short-term US interest rate. The solid line represents the level of the one-month euro-dollar interest rates. The dotted line denotes the changes in this interest rate.
Figure 2. The short-term UK interest rate. The solid line represents the level of the one-month euro-pound interest rates. The dotted line denotes the changes in this interest rate.
Figure 3. The rate of depreciation of the UK-pound. The currency is defined as the dollar price of the UK-pound.
Figure 4. Volatility of the US short-term interest rate changes. The dotted line is the absolute changes in the one-month euro-dollar rate and the solid line is the standard deviation of the changes in the US-rate as implied by the EGARCH volatility-model.
Figure 5. Volatility of the UK short-term interest rate changes. The dotted line is the absolute changes in the one-month euro-pound rate and the solid line is the standard deviation of the changes in the UK-rate as implied by the EGARCH volatility-model.