Sea bottom parameter estimation by inversion of underwater acoustic sonar data

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Chapter 2

Seabed effects on underwater acoustic propagation

2.1 Introduction

In this chapter, the ocean environment is introduced from an acoustic point of view. In the scope of the thesis, devoted to estimating bottom properties through acoustic means, particular attention is paid to the interaction of the sound with the seabed. In addition, the model that is used for calculating the propagation of sound under water throughout this thesis is described.

In Section 2.2 the wave equation, governing the propagation of sound, is presented, and the parameters that influence the propagation of sound are identified. In Section 2.3 these parameters are considered in more detail and typical values for these parameters are given. In Section 2.4 the normal-mode technique for estimating solutions to the wave equation is described. The influence of bottom type on the acoustic propagation is illustrated in Section 2.5. To this end, two ocean environments are defined, which differ with respect to their bottom characteristics. The seabed reflection coefficient is introduced and calculated for the two environments. Further, the transmission loss, i.e., the difference between the transmitted and received signal level, is determined for these two environments. In addition, the effect of bottom type on the shape of the received signals is shown. As a last illustration, work carried out for model validation purposes is presented. This example from practice shows the impact of bottom type on the acoustic propagation using real acoustic data.

2.2 Propagation of sound through the water

The propagation of sound is governed by the wave equation\(^1\)

\[
\nabla \cdot \left( \frac{1}{\rho} \nabla P \right) - \frac{1}{\rho c^2} \frac{\partial^2 P}{\partial t^2} = 0
\]

(1)

with \(P\) the pressure, \(\rho\) the density, and \(c\) the sound speed.

From this equation it is seen that the propagation of sound is influenced by parameters of the medium, viz., the sound speed and the density. A parameter not yet introduced, but also influencing the propagation of sound, is the attenuation constant. When modeling the acoustic propagation, all these parameters need to be known both in the water column, and in the seabed underlying the water column. These parameters are considered in Section 2.3. In Section 2.4 a technique for determining solutions to Eq. (1) is described.
2.3 The ocean-acoustic environment

In this section the parameters, both in the water column and in the seafloor, influencing the propagation of sound are considered. In addition, this section presents guidelines for the typical values encountered for each of these parameters.

2.3.1 The water column

The ocean is an acoustic waveguide that is limited from above by the sea surface and from below by the ocean bottom. The sound speed in the waveguide plays the same role as the index of refraction in optics. It is a function of the temperature, the salinity, and the depth in the water column. A device that is often used for estimating the sound speed as a function of depth, the ‘sound speed profile’, is the Conductivity-temperature-Depth (CTD) sensor. Also expendable bathymetry temperature (XBT) measurements are sometimes carried out for measuring the sound speed profile (using independent measurements or a database for obtaining the salinity). Sound speed profiles show different behavior for different seasons and for different geographical positions. Figure 1 shows both a typical summer and a typical winter profile, respectively, both taken in the same shallow water area (~ 100 m water depth). The winter profile shows lower sound speeds due to the lower water temperature. The sound speed profile corresponding to the summer profile increases from the top to about 15 to 18 m water depth due to the increase of pressure with depth, with the water temperature remaining (almost) constant. The corresponding layer is denoted as the ‘surface duct’. Below 18 m there is a strong decrease in sound speed due to a decrease in temperature. The layer corresponding to this strong decrease is called the ‘thermocline’.

![Fig. 1 Typical winter (solid) and summer (dashed) sound speed profile.](image)

The sea surface is a rough surface due to the presence of sea surface waves. These waves result in scattering of sound, i.e., the sound is scattered away from the specular direction. The influence of the rough sea surface on the acoustic propagation is often modeled as a loss term. This loss term is dependent on the amount of roughness, i.e., the heights of the sea surface waves, and therefore on the speed of the wind above the water. Another effect of the rough sea surface is the presence of air bubbles in the upper part of the water column. These air bubbles result both in a scattering of the sound at the bubbles, and in a change of the sound speed in the bubble region. Typically, the entrainment depth of the bubbles equals 0.4 m for wind speeds less than 7 m/s, and increases, for example, to 0.8 m at 10 m/s wind speed.
2.3.2 The seafloor

The other boundary of the waveguide is the sea floor. For shallow water environments (water depth of several 100 m), with sound propagating over distances that are many times larger than the water depth, the sound experiences a considerable interaction with the sea floor. Consequently, the sea floor has a large influence on the propagation of the sound. This influence is dependent on the sea bottom type, and therefore, information on the sea bottom parameters is essential when modeling the propagation of sound through shallow waters. At the same time, bottom parameter estimation through acoustic means becomes feasible. The bottom parameters comprise, for example, the density, the attenuation constant, and the sound speed in the sea bottom. In what follows we will consider some of these parameters and indicate the range of values encountered. The articles that are referred to have a widespread use in the underwater community, and form the standard literature on this topic.

Two sediment parameters not yet introduced are the porosity and the grain size. They are denoted as geo-technical parameters. It is relatively easy to determine these parameters with standard techniques, and they can be used, by employing empirical relations as shown below, for determining the density, the sound speed, and the attenuation. These latter three parameters directly influence the acoustic propagation and are called geo-acoustic parameters. When determining their values using bottom samples, they can either be measured directly from the sample, or be estimated indirectly from the geo-technical parameters.

Figure 2 shows the sediment grain size plotted versus the sediment porosity.

\[ s = 20.8 + 9.43 \cdot \varphi - 0.334 \cdot \varphi^2 \]  (2)

A term often used in relation to porosity is the ‘void fraction’, which is the volume of voids divided by the total volume. The porosity is known to depend on a number of factors, the most important of which is the grain size, which explains the strong correlation between these two parameters in Fig. 2.\(^3\) In this figure also an empirical relation between these two parameters is presented, which is obtained as a fit through a large amount (> 500) of measurements.\(^4\) The expression for this relation is

\[ \text{Fig. 2 Grain size versus porosity.} \]
with \( \varphi \) being the \(-2\log\) of the grain size in mm, and \( s \) the porosity (\%). No error on this regression equation is provided since this error could not be obtained from (Bachman\(^4\)) unambiguously, but it roughly amounts to about 10 %. The regression equation is valid for porosities from 36.7 % to 85.8 %, and for grain sizes of \( \sim 1 \mu m \) to \( \sim 570 \mu m \).

Figure 3 shows for a set of sediment types the density \( \rho (g/cm^3) \) versus the porosity \( s (\%) \). Also shown is the theoretical relation \(^5\)

\[
\rho = \rho_{solid} \left(1 - n\right) + \rho_w n
\]  

(3)

In this expression \( n \) is the fractional porosity \( (n = s/100) \), and the subscripts \( w \) and \( solid \) denote water and mineral solids. The value for \( \rho_{solid} \) amounts to about 2.7 g/cm\(^3\).

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![Graph showing density versus porosity.](image)

**Fig. 3** Density versus porosity.

Figure 4 shows the compressional wave speed \( c_p \) versus the density. In addition, an empirical relation between these two is shown \(^3\)

\[
c_p = 2330.4 - 1257 \cdot \rho + 487.7 \cdot \rho^2 \pm 33 \quad m/s
\]  

(4)

This expression has been derived as a fit through several hundreds of measurements, and is valid for \( 1520 < c_p < 1840 \text{ m/s} \) and \( 1.25 < \rho < 2.10 \text{ g/cm}^3 \). Note that for the low densities (\( < 1.4 \text{ g/cm}^3 \)) the sound speed is relatively insensitive to density. In some sediments (see (Hamilton\(^3\)) and (Ors\(\)i\(^6\))) velocity might decrease with increasing density in this range. This behavior appears only for high porosity sediments with no rigidity, i.e., without strength. It is well known that longitudinal sound speed depends on the two medium parameters compressibility \( (\beta) \) and density according to

\[
c_p = \frac{1}{\sqrt{\beta \rho}}
\]  

(5)

Regarding the sediment as a suspension of mineral particles in water, and thereby regarding \( \beta \) and \( \rho \) in Eq. (5) as the total compressibility and density, \( c_p \) becomes
\[ c_p = \sqrt{\frac{1}{n\beta_w + (1-n)\beta_{solid}}} \sqrt{\frac{1}{n\rho_w + (1-n)\rho_{solid}}} \]  

which is known as Wood's equation. The relation between compressional wave speed and density as predicted by the Wood's equation is shown in Fig. 4 as well, clearly exhibiting a minimum in sound speed as a function of density. The presence of gas bubbles in the sediment can further reduce \( c_p \).

Fig. 4 Compressional wave speed versus density as obtained from measurements (squares). Also indicated is the empirical relation obtained as a fit through a large amount of measurements (solid line, and dotted below the densities for which the fit is valid). The dashed lines indicate the error. The (-) line indicates the relation as predicted by Wood's equation.

From measurements it was found that the attenuation approximately increases linearly with frequency\(^5\), and can thus be expressed in dB/\( \lambda \), with \( \lambda \) the acoustic wavelength. In (Hamilton\(^7\)) the values of the attenuation constant \( \alpha \) are related to porosity. Roughly the attenuation constant amounts to about 0.2 dB/\( \lambda \) for high porosity sediments (\( s > 60\% \)), whereas it has a value of about 0.8 dB/\( \lambda \) for sandy sediments (\( s < 40\% \)).

All seabed parameters considered above can vary with depth, both as a result of the increase in pressure and temperature\(^5\) when going deeper into the bottom, but also as a result of the fact that the sediment can consist of several layers on top of each other. The complete set of seabed parameters influencing the acoustic propagation is often denoted as the geo-acoustic model of the real seabed. The parameters for the geo-acoustic model need to be known up to the 'effective acoustic penetration depth'. At high frequencies (several kHz), bottom information is required only for the few upper meters, whereas at lower frequencies, information on bottom parameters is needed up to much deeper depths into the sediment (several tens of meters). A complete geo-acoustic model of the seabed requires information of all geo-acoustic parameters up to the effective acoustic penetration depth. Obtaining such a detailed description is not feasible under practical circumstances and approximations are necessary. Measurements roughly indicate that the attenuation increases with increasing depth for silt-clay sediments, whereas it decreases with depth for sand-silt sediments.\(^5\) However, the effect is relatively small and is generally neglected. Also in (Hamilton\(^5\)) measurements are presented that illustrate the effect of depth in the sediment on the density. The exact behavior depends on sediment type, but in general the density increases when going deeper into the
sediment. For example, the measurements indicate for a sediment with an upper sediment density of $\sim 1.52 \text{g/cm}^3$ an increase up to $\sim 1.55 \text{g/cm}^3$ over 20 m. The effect of this increase on the acoustic propagation can often be neglected. The effect of depth in the sediment on the sediment sound is dependent on sediment type. Velocity gradients in sediments are usually positive, with the velocity increasing linearly or parabolic. In the remainder of the thesis, the velocity is assumed to increase linearly. Typical values for the gradient are $1 \text{s}^{-1}$. For sand type sediments the gradient is often somewhat higher ($4 \text{s}^{-1}$). Although a gradient of $1 \text{s}^{-1}$ results in an increase of sound speed of only 20 m/s over 20 m depth, this can have an impact on the sound propagation, and often is accounted for.

The effect of the sediment layering can easily be taken into account. However, for layers that have thicknesses similar to the effective acoustic penetration depth the layering can be neglected. Also for thinner sediment layers the layering is often not taken into account when modeling the sound propagation. Justification for the single sediment layer assumption is obtained from literature. Here it was found that inversions of synthetic data, calculated for a multi-layer bottom, and using a two-layer model for the forward calculations with linearly varying sound speeds, resulted in properties of the two-layer bottom that fitted the properties of the actual multi-layer model reasonably well. The sediment is overlying a medium called the sub-bottom. This medium is modeled as a homogenous layer.

2.4 Normal-mode solution of the wave equation

When employing the matched field inversion technique (see Chapter 1), bottom properties are estimated by calculating the received acoustic signals for a large set of candidate environments. The candidate environment that results in modeled signals that show maximum similarity with the measured acoustic signals (as received on a dedicated receiving system) is taken to be the true environment. Since for each new unknown environment a large number of calculations are needed, a very strict requirement on the propagation model is that the calculations are fast. This prevents the use of models based on finite elements and finite differences. Also the model needs to be applicable to a wide range of frequencies, thereby excluding the use of ray based models which are only valid in the high frequency limit. Another requirement is that the model should be suitable for environments that vary both in the depth- and the range-direction. These environments are called 'range-dependent’, whereas environments that vary only with depth are called ‘range-independent’. The requirement that the model should be applicable to range-dependent environments, is not easily met by the wavenumber integration approach. The two remaining techniques are the parabolic equation method and the normal-mode method. Since for most situations the normal-mode technique is faster, the model applied in the succeeding chapters for calculating the propagation of the sound is based on the normal-mode technique. The current section presents the basics behind this normal-mode technique. For rigid sediments, part of the energy is transformed into a transversal or shear wave. Here, all layers are assumed to be fluids and, therefore, the seabed is not allowed to support shear waves.

2.4.1 The normal-mode technique

In this section the problem of determining the pressure field resulting from a point source in a horizontally stratified medium is considered. More detailed information can be found in (Jensen\textsuperscript{1}).

For a single frequency $\omega_0$, the wave equation has the following form
\[
\n\n\n\]

with \( P \) the acoustic pressure at position \( r \) and time \( t \), due to a source at position \( r_s \), and strength \( S \), \( c \) and \( \rho \) are the sound speed, and the density, both as a function of depth.

Assuming that the ocean environment is cylindrically symmetric, with the source position at zero range, and accounting for the fact that the received signal must have the same time dependence as the source, the following relation is valid

\[
P(x,y,z,t) = p(r,z)e^{-i\omega t} \quad \text{Eq. (8)}
\]

with \( r \) the horizontal distance, or range, between source and receiver. By substituting this expression into Eq. (7), and using cylindrical symmetry, one obtains

\[
\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} + \frac{\partial^2 p}{\partial z^2} - \frac{1}{\rho} \frac{\partial p}{\partial z} + k^2 p = -\frac{S}{2\rho} \delta(r) \delta(z-z_s) \quad \text{Eq. (9)}
\]

with \( k \) the total wavenumber

\[
k(z) = \frac{\omega}{c(z)} \quad \text{Eq. (10)}
\]

Eq. (9) is the so-called Helmholtz equation for a cylindrically symmetric medium.

In the following the ocean environment is simplified and assumed to consist of three layers (see Fig. 5):

- A water column of depth \( H_w \) with density \( \rho_w \) (= 1 g/cm\(^3\)), sound speed profile \( c_w(z) \), and attenuation constant \( \alpha_w \);
- A sediment layer of thickness \( H_s \), and density \( \rho_s \) sound speed profile \( c_s(z) \), and attenuation constant \( \alpha_s \);
- A semi-infinite homogeneous sub-bottom with density \( \rho_b \) sound speed \( c_b \) and, attenuation constant \( \alpha_b \).

Thus the densities and attenuation constants are assumed to be constant in each of the layers, whereas the sound speed in the water column and the sediment layer is allowed to vary with depth \( z \). The sub-bottom sound speed is however assumed to be constant. The total sound speed profile is

\[
c(z) = \begin{cases} 
c_w(z) \text{ for } 0 \leq z \leq H_w \\
c_s(z) \text{ for } H_w \leq z \leq H_w + H_s \\
c_b \text{ for } z \geq H_w + H_s 
\end{cases} \quad \text{Eq. (11)}
\]
Fig. 5  Schematic of the simplified range-independent ocean environment.

It is assumed that the sea surface is a pressure release boundary, corresponding to a reflection coefficient of $R = -1$ and a transmission coefficient $T = 0$, which means that there is no transmission of sound from the water to the air above the water, i.e.,

$$P(r,0,t) = 0$$

(12)

Further, it is assumed that at some sufficiently great depth $H_t = H_w + H_s + H_B$ (see Fig. 5), a perfectly rigid boundary exists, i.e.,

$$\frac{\partial P}{\partial z}(r,H,t) = 0$$

(13)

$H_B$ should be selected such that there is no contribution from below $H_t$ to the total acoustic field. Experience has learned that taking $H_B$ equal to 20 acoustic wavelengths is sufficient.

The Helmholtz equation can be solved by applying the technique of separation of variables, which implies substitution of

$$p(r,z) = R(r)\Psi(z)$$

(14)

in the homogeneous Helmholtz equation, i.e., Eq. (9) with the right-hand side equal to zero. One then obtains the following two differential equations

$$\frac{d^2 R}{dr^2} + \frac{1}{r} \frac{dR}{dr} + \mu R = 0$$

(15)
where $\mu$ is the separation constant. The boundary conditions are

\[
\begin{align*}
\Psi(0) &= 0 \\
\frac{d\Psi}{dz}(H_z) &= 0
\end{align*}
\] (17)

Eq. (16), the depth-dependent Helmholtz equation or modal equation, together with the boundary conditions, is a standard Sturm-Liouville eigenvalue problem. It has an infinite number of solutions $\Psi_n(z)$ (eigenfunctions or modes) for distinct real values $\mu_n = k_n^2$ (the eigenvalues) of the separation constant. For an arbitrary sound speed profile $c(z)$ this problem has to be solved numerically.

The eigenfunctions of a Sturm-Liouville problem are orthogonal and can be normalized, i.e.,

\[
\int_0^{H_z} \frac{\Psi_n(z)\Psi_m(z)}{\rho(z)} dz = \delta_{n,m}
\] (18)

with $\delta_{n,m}$ Kronecker's delta. Since the eigenfunctions form a complete orthonormal set, the solution of Eq. (9) can be written as

\[
p(r,z) = \sum_{n=1} R_n(r)\Psi_n(z)
\] (19)

The coefficients $R_n(r)$ are determined as follows: Substituting Eq. (19) into Eq. (9), multiplying by $\Psi_n(z)$, integrating over $z$, and using the orthonormality of the eigenfunctions (Eq. (18)) one obtains

\[
\frac{d^2 R_n}{dr^2} + \frac{1}{r} \frac{dR_n}{dr} + \mu_n R_n = -\frac{\delta(r)S(z)\Psi_n(z)}{2\pi \rho(z)}
\] (20)

This standard equation (Bessel's equation of order zero) has the following solutions

\[
R_n(r) = \frac{iS(z)}{4\rho(z)} \Psi_n(z_s) H_0^{(1,2)}(k_n r)
\] (21)

with $H_0^{(1)}$ and $H_0^{(2)}$ the zeroth order Hankel functions of the first and second kind, respectively. The radiation condition (energy must radiate outward as $r \to \infty$) and the choice of $e^{-\alpha r}$ for the time dependence of $P$ imply that the Hankel function of the first kind has to be taken.

Now the final solution becomes

\[
p(r,z) = \frac{iS(z)}{4\rho(z)} \sum_{n=1} \Psi_n(z_s)\Psi_n(z) H_0^{(1)}(k_n r)
\] (22)
being the normal-mode solution to the wave equation.

Using the asymptotic expression for the Hankel function (to a good approximation valid for $k_n r > 2$)

$$ H_0^{(1)}(k_n r) = \frac{2}{\pi k_n r} e^{-i(k_n r - \frac{\pi}{4})} \quad (23) $$

Eq. (22) can be written as

$$ p(r,z) = \frac{e^{i\omega/4}}{\rho(z_s) \sqrt{8\pi r}} \sum_{n=1}^{\infty} \Psi_n(z_s) \Psi_n(z) e^{ik_n r} \quad (24) $$

According to Eq. (8) and (24) the pressure field $P(r, z, t)$ can be regarded as a superposition of cylindrical waves

$$ P_{c,n}(r,z,t) = \frac{A_n(z)}{\sqrt{r}} e^{i(k_n r - \omega t)} \quad (25) $$

with phase speeds $c_n = \frac{\omega}{k_n}$. The eigenvalues $k_n$ can therefore be interpreted as horizontal wavenumbers, i.e., wavenumbers in the $r$-direction.

The modal equation, Eq. (16), has an infinite number of solutions $\Psi_n(z)$ for distinct values $k_n^2$ of the separation constant $\mu$. All eigenvalues $k_n^2$ are real and are all less than $\omega/c_{\text{min}}$, with $c_{\text{min}}$ being the lowest value in the total sound speed profile (Eq. (11)). Consequently, all corresponding phase velocities are greater than $c_{\text{min}}$, i.e., $c_n > c_{\text{min}}$, $\forall n$. The $n^{th}$ mode function $\Psi_n$ has $n$ zeros on the interval $[0, H]$. The phase speed spectrum can be divided in two different regions, comprising the discrete modes and the so-called leaky modes. The eigenvalues of the discrete modes satisfy

$$ \frac{\omega}{c_r} < k_n < \frac{\omega}{c_{\text{min}}} \quad (26) $$

assuming $c_b$ to be the highest sound speed value in the total profile, Eq. (11). The corresponding phase velocities satisfy

$$ c_{\text{min}} < c_n < c_b \quad (27) $$

Hence, for discrete modes to exist $c_b$ must be greater than $c_{\text{min}}$. The number of discrete modes ($L$) is finite. The eigenvalues of the leaky modes satisfy

$$ 0 < k_n < \frac{\omega}{c_b} \quad (28) $$

with corresponding phase velocities

$$ c_b < c_n < \infty \quad (29) $$
Fig. 6 illustrates the two regions of phase speeds.

\[
\begin{array}{c}
\text{discrete} \quad \text{leaky} \\
0 \quad c_{\text{min}} \quad c_b \quad \infty
\end{array}
\]

These leaky modes correspond to sound rays traveling at grazing angles greater than the critical angle at the sediment/sub-bottom interface, and therefore part of the energy carried by these modes leaks into the sub-bottom. Hence, the contribution of these modes to the pressure field becomes negligible at a sufficient distance from the source. This is only legitimate for 'long-range' propagation in shallow water for ranges that are an order of magnitude larger than the depth. This is the case for the remainder of this thesis. The leaky modes are also denoted by 'continuous modes'. If the leaky modes contribution is not taken into account, the solution becomes

\[
p(r, z) = \frac{e^{i\pi/4} S_\omega}{\rho(z) \sqrt{8 \pi \omega}} \sum_n \Psi_n(z) \Psi_n^*(z) \frac{e^{ik_n r}}{\sqrt{k_n}}
\]  

(30)

In the above discussion no losses due to attenuation, both in the water column and in the seabed, and due to the scattering of sound at the rough sea surface are taken into account. Their contribution is implemented through perturbation theory, where a small imaginary part is added to the total wavenumber. This leads to modal attenuation coefficients \( \alpha_n \) for each mode \( n \), where

\[
\alpha_n = \alpha_n^w + \alpha_n^s + \alpha_n^b + \alpha_n^{\text{scat}}
\]

(31)

with the superscripts \( w, s, b, \) and \( \text{scat} \) denoting water, sediment, sub-bottom, and scattering, respectively. The expressions for the water, sediment, sub-bottom, and scattering modal attenuation constants can be found in (Ingenito) and (Kuperman). For illustrative purposes we present the expressions for the water, sediment and sub-bottom modal attenuation constants:

\[
\alpha_n^{w,s,b} = \frac{\alpha_n^{w,s,b} f}{\tilde{c}_n^{w,s,b} 20^{10} \log e} \frac{\omega}{\rho_n^{w,s,b} k_n} \int \frac{\tilde{c}_n^{w,s,b} \Psi_n(z)^2}{c(z)} \frac{dz}{c(z)}
\]

(32)

with \( \alpha_n^{w,s,b} \), denoting the attenuation constants in the water (\( w \)), sediment (\( s \)), and sub-bottom (\( b \)), respectively. \( \tilde{c}_n^{w,s,b} \) denotes the mean sound speed in the water column, the sediment, and the sub-bottom layer, respectively. The integral has to be taken over the corresponding layer, i.e., for determining \( \alpha_n^w \) the integral is taken from 0 to \( H_w \), for determining \( \alpha_n^s \) the integral is taken from \( H_w \) to \( H_w + H_s \), and for determining \( \alpha_n^b \) the integral is taken from \( H_w + H_s \) to \( H_w + H_s + H_b \). The factor \( f / (\tilde{c}_n^{w,s,b} 20^{10} \log e) \) converts the units from dB/\( \lambda \) to 1/m. Including the loss terms the expression for \( p(r, z) \), Eq. (30), now becomes
\[ p(r,z) = \frac{e^{i\pi/4}S_\omega}{\rho(z_s)\sqrt{8\pi r}} \sum_{n=1}^{L} \Psi_n(z_s)\Psi_n(z) e^{(ik_n-\alpha_n)r} \sqrt{k_n} \]  

(33)

For obtaining \( \Psi_n \) and \( k_n \), the modal equation Eq. (16) has to be solved numerically. Finite-difference discretization is applied for transforming the modal equation, and the corresponding boundary conditions, Eq. (17), into an eigenvalue problem. This is considered in Appendix A. This resulting algebraic eigenvalue problem is solved by using EISPACK routines that determine the eigenvalues and eigenvectors (or eigenfunctions) for a real symmetric tridiagonal matrix in a specified interval.\(^{13}\)

### 2.4.2 Range-dependency through the adiabatic approximation

In Section 2.4.1 the environment is allowed to vary with depth, but is assumed to be constant in the range direction. In situations where the range-dependence of the environment is such that it cannot be neglected, still use can be made of the normal-mode solution. As a result of its relatively (compared to other techniques for calculating the sound propagation in range-dependent environments) short calculation times, the approach most commonly used is to employ the ‘adiabatic approximation’.

In the adiabatic approximation, the environment is divided in segments that all together span the entire source-receiver range. Figure 7 shows an example of such segmentation. Within each of these segments the environment is assumed range-independent.

![Fig. 7](image)

**Fig. 7** Example of segmentation applied for the adiabatic normal-mode approach. In each range segment the upper number indicates the amount of modes calculated for that particular range segment. The lower number indicates the amount of modes accounted for as a function of range \( r \).

For each of the segments the eigenvalues and eigenvectors are determined. The assumption made in the adiabatic approximation is that from one range segment to another, the modes couple without any transfer of energy to higher or lower order modes. This means that there is no energy transfer in between modes of different orders, i.e., mode \( n \) does not couple with modes \( n+1, n+2, \ldots, L \), and with modes \( n-1, n-2, \ldots, 1 \). Considering Fig. 7, a result of the approximation applied is observed: for the situation considered the amount of modes increases from 26 to 28 when going to larger water depths. This however is not accounted for in the adiabatic approximation, since the new modes (at larger water depth) have no
neighboring modes to couple with at the left side. Similarly, when going to smaller water depths, modes will disappear. For the example shown in Fig. 7 and for a receiver at range \( r = 7 \) km, 25 modes will be accounted for in all range segments.

For the cases considered in this thesis, the adiabatic approximation can be applied when accounting for range-dependency, since the slopes are small. However, up to what slopes the adiabatic approximation is valid is still a topic of research.

The derivation of the expressions for the adiabatic approximation is given in (Jensen\(^1\)) and here only the resulting expression is presented

\[
p(r, z) = \frac{e^{i\pi/4} S_{\omega}}{\rho(z_s) \sqrt{8\pi}} \sum_{n=1}^{L_{\min}} \Psi_n(0, z_s) \Psi_n(r, z) e^{-\int_0^r k_n(r') dr' - \int_0^r \alpha_n(r') dr'}
\]

with \( L_{\min} \) the minimum amount of modes encountered over all segments up to range \( r \). Note that employing the adiabatic approximation requires solving for the eigenvalues of the modal equation in all segments. The eigenfunctions are needed only for the segments that contain the source and the receiver. For calculating the received complex pressure (Eq. (34)) use is made of the averaged (over range) horizontal wavenumber and the averaged modal attenuation coefficient.

### 2.5 Examples

In this section the effect of the sea bottom properties on the sound propagation is illustrated through the use of two bottoms with different properties. The first is a sand-silt-clay like bottom, with geo-acoustic bottom parameters as shown in Fig. 8. The second is a muddy bottom, i.e., a mixture of silt and clay with a sound speed lower than that in the water column. The corresponding geo-acoustic model is shown in Fig. 9. The sub-bottom is taken the same for the two environments. Note that this model for the seabed corresponds to the model discussed at the end of Section 2.3.

![Fig. 8 Geo-acoustic model for the sand-silt-clay environment.](image)

![Fig. 9 Geo-acoustic model for the mud environment.](image)

First, the influence of bottom type on the bottom reflection coefficient is considered. The bottom reflection coefficient is the ratio of the amplitudes of the reflected plane wave to the plane wave incident on the water/sediment interface, and provides a measure for the effect of the sound interacting with the sea bottom. The practical applications of the reflection
coefficient are limited and it is not used for the forward modeling, but the concept is a means for illustrating the energy transport in and out of the ocean waveguide. By showing the influence of bottom type on the reflection coefficient, it is demonstrated that it is indeed feasible to estimate bottom properties through acoustic means. As a next example for the influence of bottom type the transmission loss, i.e., the amount of energy lost in between source and receiver, is calculated for the two environments.

At the end of this section results of a model validation exercise are presented, demonstrating the influence of sea bottom type on the underwater acoustics in real life.

2.5.1 Interaction of sound with the seafloor

Figure 10 shows the reflection of sound at an interface that separates two homogeneous fluid media. The environment considered is 2D, i.e., there is no variation in the y-direction.

![Fig. 10 Reflection and transmission.](image)

For deriving expressions for the reflected and transmitted waves we consider a plane harmonic wave

\[ p(x, z, t) = p(r, t) = e^{i(kr - ct)} \quad \text{with} \quad k \cdot r = k_x x + k_z z \]  

(35)

The wavenumber \( k \) is the absolute value of \( k \), i.e., \( |k| = k \).

Assuming the incident wave to have unit amplitude, and denoting the amplitudes of the transmitted and reflected waves by \( T \) and \( R \), respectively, one obtains

\[ p_r = R e^{i\theta_r (x \cos \theta_i - z \sin \theta_i)} \quad k_1 = \frac{\omega}{c_1} = \|k_1\| \]

(36)

\[ p_t = T e^{i\theta_t (x \cos \theta_i - z \sin \theta_i)} \quad k_2 = \frac{\omega}{c_2} = \|k_2\| \]

In these expressions, \( \theta_i \) is the grazing angle of incidence (which equals the grazing angle of reflection) and \( \theta_t \) is the grazing angle of transmission. The time factor \( e^{-\omega t} \) is omitted, since it is common for \( p_r, p_r, \) and \( p_r \). \( R \) and \( T \) are the amplitude reflection-coefficient and the amplitude transmission-coefficient, respectively). \( p_i, p_r, \) and \( p_t \) are the incident, reflected, and transmitted waves. In the above expressions, \( R, T \) and \( \theta_t \) are unknowns that have to be determined from the boundary conditions. The following two boundary conditions are valid for \( z = 0 \).
1) Continuity of pressure:

\[ p_i + p_r = p_f \]  

(37)

2) Continuity of particle velocity in the z-direction:

\[ \frac{1}{i \omega \rho_1} \frac{\partial (p_i + p_r)}{\partial z} = \frac{1}{i \omega \rho_2} \frac{\partial p_f}{\partial z} \]  

(38)

Employing the first boundary condition results in the following expression

\[
(1 + R) = T e^{i(k_1 \cos \theta_1 - k_2 \cos \theta_2) x}
\]  

(39)

Eq. (39) is only valid if the right side of the expression is independent of \(x\)

\[ k_2 \cos \theta_2 - k_1 \cos \theta_1 = 0 \]  

(40)

This expression is known as Snell’s law of refraction, and is often written as (using Eq. (36))

\[ \frac{\cos \theta_2}{c_2} = \frac{\cos \theta_1}{c_1} \]  

(41)

According to Eq. (39) also the following expression is valid, where both \(R\) and \(T\) can become complex,

\[
(1 + R) = T
\]  

(42)

which together with the second boundary condition, gives

\[
1 - R = T \frac{\rho_1 c_1 \sin \theta_2}{\rho_2 c_2 \sin \theta_1}
\]  

(43)

Finally, this leads to the following expressions for \(R\) and \(T\)

\[
R = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad \text{and} \quad T = \frac{2Z_2}{Z_2 + Z_1}
\]  

(44)

with

\[
Z_1 = \frac{\rho_1 c_1}{\sin \theta_1} \quad \text{and} \quad Z_2 = \frac{\rho_2 c_2}{\sin \theta_2}
\]
In these above expressions is often referred to as the Rayleigh reflection coefficient; $Z_1$ and $Z_2$ are called the generalized acoustic impedances of the two media.

For illustrating the effect of bottom type on the reflection coefficient, $R$ is determined for the bottoms of Figs. 8 and 9. In the underwater acoustics community, often, instead of $R$ a quantity called bottom loss ($BL$) is employed, which is defined as $-20^{10} \log R$. In the following we will make use of both $R$ and $BL$. First, however, the critical angle, and the angle of intromission are considered.

### 2.5.1.1 The critical angle

If $c_2 > c_1$, then a critical angle $\theta_c$ for which perfect reflection occurs, exists. For $0 < \theta < \theta_c$, $|R| = 1$, and $R < 1$ for $\theta > \theta_c$. $\theta_c$ can be determined from Snell's law

$$\theta_c = \arccos \left( \frac{c_1}{c_2} \right)$$  \hspace{1cm} (45)

For $0 < \theta < \theta_c$, $\cos \theta > 1$, i.e., $\theta$ is purely imaginary. For these angles $|R| = 1$, i.e., perfect reflection, but with an angle dependent phase shift, i.e., $R$ is complex. For $\theta > \theta_c$, $R < 1$ and real.

### 2.5.1.2 Angle of intromission

The angle of intromission, $\theta_0$, is the angle at which all energy is transmitted into the bottom, i.e., $R = 0$. This requires that $\frac{\rho_2 c_2}{\sin \theta_2} = \frac{\rho_1 c_1}{\sin \theta_1}$ (see Eq. (44)). Applying Snell's law, this results in

$$\theta_0 = \arctan \left( \sqrt{1 - \left( \frac{c_2}{c_1} \right)^2} \right)$$  \hspace{1cm} (46)

This expression has a solution only if

1) $\rho_2 c_2 > \rho_1 c_1$ and $c_2 < c_1$;
2) $\rho_2 c_2 < \rho_1 c_1$ and $c_2 > c_1$.

Condition 1 occurs in muddy media. Condition 2, however, does not occur for real ocean sediments.

### 2.5.1.3 The influence of attenuation

Plane wave attenuation $\alpha'$ is defined through the following expression

$$\frac{dA}{dx} = -\alpha' A \Rightarrow A = A_0 e^{-\alpha'x}$$  \hspace{1cm} (47)
with $A_0$ the rms amplitude at $x = 0$. The unit of $\alpha'$ is in $\text{m}^{-1}$ if $x$ is in $\text{m}$. A plane wave in free space with sound speed $c$, and angular frequency $\omega$, will be of the following form

$$e^{(ikx-\alpha x)}$$

(48)

with $k$ the wavenumber, i.e., $k = \omega c$.

Attenuation is accounted for by including an imaginary part to the sound speed, i.e.,

$$c(z) = c_r(z) - ic_i(z).$$

Now the plane wave takes the form

$$e^{i\omega x/c} = e^{ikx/c} e^{ic_i/z}$$

(49)

Comparing Eq. (49) with Eq. (48), and assuming that $c_i^2 \ll c_r^2$ leads to the following relation

$$c_i = \frac{\alpha'}{\alpha} c_r^2$$

(50)

In Figs. 8 and 9 the attenuation $\alpha$ is given in dB/$\lambda$. The relation between $\alpha$ and $\alpha'$ is obtained by considering the ratio of the amplitudes in dB between points that are a wavelength apart

$$\alpha = -20\log \frac{e^{-\alpha(x+\lambda)}}{e^{-\alpha x}} = \alpha'20\log e = 8.686\alpha'\lambda$$

(51)

Figure 11 shows the effect of the attenuation on $BL$, for the two bottoms, with the upper plot corresponding to the sand-silt-clay bottom and the lower plot corresponding to the mud bottom, respectively. All sediment properties are taken constant and equal throughout both the sediment and the sub-bottom, with the values equal to those at the top of the sediments of Figs. 8 and 9. $c_p$, $\rho$, and $\alpha$ are the seabed compressional wave speed, the density, and the attenuation constant, respectively.

Fig. 11 $BL$ for a high speed (upper subplot) and a low speed (lower subplot) bottom, both with and without the effect of attenuation.
In the upper plot of Fig. 11, for the situation with an attenuation constant of zero, clearly the critical angle is seen at an angle of 15 degrees. The effect of the non-zero attenuation is especially noticeable at angles below the critical angle, where $|R|$ no longer equals one. For the mud sediment, with a sound speed in the seabed that is lower than the water sound speed, the intromission angle is observed at 20 degrees, which is in accordance with Eq. (46).

2.5.1.4 Theory for layered structures

Theory of the reflection and transmission coefficients for layered media can be found in e.g. (Jensen). For example, expressions for $R$ and $T$, for a seabed consisting of a homogeneous top layer on a homogeneous half-space are

$$R = \frac{R_{12} + R_{23}e^{2i\varphi_2}}{1 + R_{12}R_{23}e^{2i\varphi_2}}$$

$$T = \frac{T_{12} + T_{23}e^{i\varphi_2}}{1 + R_{12}R_{23}e^{i\varphi_2}}$$

with the subscript $ij$ indicating that the propagation direction is from medium $i$ to medium $j$. $\varphi_2$ is the vertical phase delay for sound crossing the layer of thickness $h_2$, i.e., $\varphi_2 = kh_2\sin\theta_2$. Note that for this layered situation both $R$ and $T$ now depend on the frequency.

Figures 12 and 13 illustrate the effect of the increasing sediment sound speed on the reflection coefficient.

![Fig. 12. BL for the sand-silt-clay sediment, and a frequency of 500 Hz. The solid line corresponds to a varying sediment sound speed (see Fig. 8). The dashed line corresponds to a homogeneous sediment, with a sound speed of 1565 m/s.](image1)

![Fig. 13. BL for the mud sediment, and a frequency of 500 Hz. The solid line corresponds to a varying sediment sound speed (see Fig. 9). The dashed line corresponds to a homogeneous sediment, with a sound speed of 1435 m/s.](image2)

The figures show $BL$ as a function of grazing angle (but for a fixed frequency of 500 Hz) for the two environments with an increasing (see Figs. 8 and 9), and a constant sediment sound speed. The constant sediment sound speed was taken as the average of the corresponding varying sound speeds, i.e., 1435 m/s and 1565 m/s, respectively. The theory for estimating the
bottom reflection coefficient for sediments with varying sound speeds goes beyond the scope of this chapter. We limit ourselves to showing that for the bottoms of Figs. 8 and 9, with a thin sediment and a small sound speed variation, there is no need to account for the sound speed variation, since the two curves, both in Fig. 12 and Fig. 13, almost coincide. In the remainder of this Section 2.5.1 we will therefore use the expressions for a constant sediment sound speed, Eq. (52). It should be mentioned here, that the normal-mode solution, which is used for calculating the acoustic propagation in the remainder of the thesis, does account for the variation of the sound speed in the sediment. In Fig. 12 clearly the critical angle belonging to the water/sediment interface at an angle of 15 degrees is seen. Although the mud sediment has values for the density and sound speed such that an intromission angle is expected (at 20 degrees, see Fig. 11), this feature is not seen in Fig. 13 due to the influence of the sub-bottom. Note in both figures the interference patterns due to the layered structure.

Figures 14 and 15 show BL for the two bottoms as a function of grazing angle and frequency. Clearly BL is higher for the mud sediment than for the sand-silt-clay sediment.

![Figures 14 and 15](image)

2.5.2 Influence of bottom type on transmission loss and received signal shape

Transmission loss TL is defined as

\[
TL(r,z) = -20\log\left(\frac{p(r,z)}{p_0(r=1\text{ m})}\right)
\]

with \(p(r,z)\) given by Eq. (33) and \(p_0\) the solution to the Helmholtz equation for a homogeneous medium without boundaries

\[
p_0(r) = \frac{1}{4\pi r}e^{ikr}
\]

i.e., a spherical wave with wavenumber \(k = \omega/c\), with \(c\) being the sound speed of the homogeneous medium. For further illustrating the effect of bottom type on the acoustic
propagation, $TL$ is calculated for the two bottoms of Figs. 8 and 9. To this end, Eq. (33) is used for determining the received pressures for the two bottoms, for a large set of receiver depths (ranging from 0–190 m) and for a large set of distances from the source (ranging from 0–5 km). The sound source is positioned at 70 m of water depth. These pressures have been used to determine $TL$. Figs. 16 and 17 show again the two environments, but now the complete sound speed profile (in water column, sediment layer and sub-bottom) is indicated as well.

The sea is perfectly flat, i.e., a wind speed of zero m/s. Also indicated in Figs. 16 and 17 is the amount of modes corresponding to each of the sediments. Note that the amount of modes is higher for the mud sediment than for the sand-silt-clay sediment. This is due to the larger range of sound speeds encountered in the three layers of Fig. 16 (water column, sediment layer, and sub-bottom).

Figure 18 shows $TL$ for the sand-silt-clay sediment, whereas $TL$ for the mud sediment is shown in Fig. 19.
Clearly $TL$ is much higher for the mud sediment than for the sand-silt-clay sediment. This can be understood by looking at Figs. 20 and 21, showing all modes for the two sediments, respectively. For the mud sediment all modes are oscillatory in the sediment, thereby experiencing high losses due to the sediment attenuation, see Eq. (32). On the contrary, for the sand-silt-clay sediment there are modes, viz., modes 1-19, that are only oscillating in the water column and exponentially decaying in the sediment, and thus experience hardly any losses due to the sediment attenuation.

Fig. 20 All modes corresponding to the sand-silt-clay sediment. Depth is along the $y$-axis, mode amplitude is plotted along the $x$-axis. The horizontal dashed lines indicate the sediment layer.
Fig. 21  All modes corresponding to the mud sediment. Depth is along the y-axis, mode amplitude is plotted along the x-axis. The horizontal dashed lines indicate the sediment layer.

In the majority of the chapters to follow, the signal that is used in the analysis consists of the received complex pressures as a function of hydrophone position and frequency. For all experiments described in this thesis use is made of a vertical array of hydrophones as the receiving system. (In none of the succeeding chapters information on the receiving system equipment used during the experiments is provided. Therefore, Appendix B briefly describes this receiving equipment). The resulting complex pressures as a function of depth are referred to as ‘pressure fields’. Figure 22 shows, for the two sediments, the absolute values of the pressure fields as a function of depth for two frequency values and three range values. For the mud sediment, the pressure field becomes less oscillatory due to the sediment attenuation. Sediment attenuation has less influence for the sand-silt-clay sediment, see the discussion above.
2.5.3 An example from practice: a model validation exercise

In (Simons) results of an acoustic model validation exercise are presented. The acoustic data used for the validation were collected from shallow waters in the Firth of Clyde off the West Coast of Scotland in the summer of 1997. The received signals were compared with simulations using a normal-mode propagation model. Here part of the results of (Simons) are presented with the purpose of illustrating both the importance of knowing the geo-acoustic parameters with sufficient accuracy, and the effect of temporal oceanographic variability on the received signals.

Figure 23 shows the track along which the acoustic propagation experiments were performed. The water depth along the track amounts to about 70 m. The receiving system consisted of a vertical receiving array containing three hydrophones, located under water at approximately 10, 35, and 55 meters. The source depth amounted to ~30-40 m.
A frequency modulated (FM) signal (chirp) of bandwidth 1-8 kHz with a pulse duration of 1 s was transmitted. The source pulse was transmitted every 35 s over a period of 70 min, giving a total of 120 pulses for each configuration. The received signals were correlated with the transmitted signal. This technique is called 'matched filtering' and reveals the multipath arrivals as a function of time. This is illustrated in Fig. 24, showing the result of applying the matched filter technique to a signal that contains two FM’s, starting at 1 and 1.5 s, respectively.

Fig. 24 Illustration of the matched filter technique: the upper subplot line indicates the original signal, the lower subplot indicates the result after matched filtering (in dB).
In the following we will compare the measured multipath structure with modeled multipath structures, i.e., the model/data comparison is carried out using signals in the time domain, in contrast to the remainder of the thesis, where the model/data comparison is carried out in the frequency domain.

2.5.3.1 Input data for the normal-mode model

Certain inputs to the model are explicitly known. These include the geometrical configuration, viz., the ranges between the ships (as calculated from the GPS ship positions) and the source and receiver depths (as obtained from the depth sensors mounted on the source and receiving hydrophone string).

The water column sound speed profiles that were used for the model input were obtained from CTD casts carried out from the receive ship close to the times of transmission. A single profile was selected for each configuration, and therefore range-dependence of the sound speed in the water column was not accounted for.

The bathymetry of the track was measured by an echosounder and appeared to be fairly range-dependent. To run a normal-mode model employing the adiabatic approximation, the track has to be divided into a number of segments, each with a constant water depth (see Section 2.4.2). The division in segments is based on bathymetry changes along the track and is such that the jump in water depth between adjacent segments is a constant. The water-depth jump should be sufficiently small such that decreasing this jump, and thereby adding more segments, has no further influence on the received signal. From the received signals calculated as a function of depth jump (ranging from 0.5 to 10 m), it was concluded that a 4-m jump is sufficiently small.

For a range of 10 km the 4-m depth jump resulted in six range segments. Figure 25 shows the measured bathymetry and the applied segmentation. The echosounder track does not exactly coincide with the acoustic track, and therefore, the actual bathymetry along the acoustic track can deviate from the bathymetry shown in Fig. 25. This allows the bathymetry to be varied within a few meters when improving the match between modeled and measured signals.

![Figure 25](image)

Fig. 25 Measured bathymetry along the acoustic track and the corresponding constant water-depth segments (indicated by the vertical dashed lines).

The normal-mode model used assumes the bottom to consist of a single sediment layer overlying a homogeneous sub-bottom. The sound speed in the sediment is allowed to vary
with depth. The densities and attenuation constants in sediment and sub-bottom are assumed constant.

For obtaining information on the geo-acoustic bottom parameters, use has been made of a geological map of the British Geological Survey (BGS) as a guide.\textsuperscript{15} Figure 23 shows this geological map of the Clyde area, according to which the sediment type along the acoustic track is classified as type mud (i.e., silt and clay). According to (McCann\textsuperscript{16}) the majority of the sound speeds that were measured for mud sediments have values ranging from 1450 up to 1575 m/s. The average density of mud sediments\textsuperscript{5} amounts to 1.5 (± 0.2) g/cm\textsuperscript{3}. Measured attenuation constants in marine sediments are known to exhibit a large spread. A realistic start value for the attenuation constant in mud is taken to be 0.15 dB/\lambda.\textsuperscript{5} We assumed a linearly increasing sound speed in the sediment. The typical sound speed gradient found in mud sediments\textsuperscript{16} is about 1 s\textsuperscript{-1}. According to the BGS map the sediment thickness along the acoustic track varies between 20 and 40 m.\textsuperscript{15} At the given frequency band of interest (centered around 4.5 kHz) and for the given sediment attenuation, the penetration of sound in the sediment is less than about 10 m. For sediment thickness we therefore adopted a value larger than 10 m, viz. 20 m, being the minimum value according to the map. For this set of sediment parameters, all sub-bottom parameters were found to have no influence on the model output. In order to limit the number of normal modes and hence the computation time, the sub-bottom sound speed was set to 1600 m/s. Further, the sub-bottom density and the attenuation constant were set arbitrarily at 1.75 g/cm\textsuperscript{3} and 0.7 dB/\lambda, respectively. In contrast, sediment sound speed, sediment attenuation, and geometrical parameters such as the water depth can have a pronounced effect on the received signals.

\subsection{2.5.3.2 Comparison of measured and modeled signals}

In an attempt to model this complicated environment at the relatively high frequencies of interest, the following approach was taken. As a first step the 2-km range data were considered. From a preliminary set of model runs it was found that the sediment density and the sediment sound speed gradient have only a very small influence on the received signals. The sub-bottom parameters and sediment thickness (> 10 m) have no influence at all.

From all geo-acoustic parameters, the upper sediment sound speed has by far the most significant influence on the propagation: increasing or decreasing the upper sediment sound speed results in an increase or decrease, respectively, of the amount of multipaths, but not in a time shift of the individual multipath arrivals. From varying the upper sediment sound speed, using the nominal values for the geometrical parameters, a value of about 1525 m/s was found to result in modeled signals with a time dispersion comparable to that of the measured signals.

As a next step, to further improve the precise match of the multipath arrival structure the geometrical input parameters (source depth, receiver depths, source/receiver range, and bathymetry) were varied within acceptable limits. For the bathymetry the measured bathymetry as shown in Fig. 25 was used and an offset was applied to it. The justification for applying an offset to the echosounder measurements is that the echosounder track did not exactly coincide with the acoustic track. According to bathymetry maps the offset can be as large as 6 m.

As expected, water-depth offset turned out to have the greatest influence. Applying a water depth offset of −5 m resulted in the best model/data match for all three hydrophones simultaneously. This offset corresponds to a water depth at the source of 61 m, which is in accordance with the bathymetry maps. For this new bathymetry again the influence of the source depth, source/receiver range and receiver depth was considered. Adjusting the receiver depths from their baseline values resulted in a further improved model/data match.

For the obtained ‘optimized’ geometrical parameter set, a new search for upper sediment speed was carried out in the range 1505 m/s - 1565 m/s (in 10 m/s steps). A value of 1545 m/s resulted in the best model/data match.
Finally the sediment attenuation constant was ‘optimized’. From all values considered (0.15 dB/λ to 0.85 dB/λ in steps of 0.1 dB/λ) 0.55 dB/λ resulted in an improved match, compared to the start value of 0.15 dB/λ.

It is emphasized that although a large amount of model runs (several hundreds) were carried out, only a small subset of all possible parameter combinations was considered. Performing an exhaustive full inversion for all parameters of the geo-acoustic profile would require a huge amount of model runs, which at these high frequencies of interest is not practical. (A single normal-mode model run for the band 1-8 kHz requires about 30 min on a state-of-the-art workstation).

For the 2-km range (deep source), Figs. 26-28 show the 120 individual measured signals for the three hydrophones, respectively. Also plotted in each figure are the medians of the experimental data, and the modeled received signals employing the optimized parameter set derived above. The measured and modeled signals are time aligned to allow a direct comparison.

![Image](image_url)

**Fig. 26** The 120 individual received signals (showing the amount of time variability) as received during the 2-km experiment on the upper hydrophone. Also shown are the median signal of the experimental data (thick curve) and the modeled signal (lowest curve).
Fig. 27  The 120 individual received signals (showing the amount of time variability) as received during the 2-km experiment on the middle hydrophone. Also shown are the median signal of the experimental data (thick curve) and the modeled signal (lowest curve).

Fig. 28  The 120 individual received signals (showing the amount of time variability) as received during the 2-km experiment on the deepest hydrophone. Also shown are the median signal of the experimental data (thick curve) and the modeled signal (lowest curve).

It can be concluded that a set of parameters, comprising both geometrical and geo-acoustic parameters, is derived that, for the 2-km range, results in an acceptable match between data
and model. Especially the first two groups of multipaths (within the first 10 ms) are modeled quite well for all three hydrophones simultaneously. The locations of the later arrivals are less well modeled.

Figure 29 shows the received signals for the 5-km range (deep source, middle hydrophone). Although an extensive search was carried out for a set of both geometrical and geo-acoustic parameters in an attempt to model these signals, none of the tested input parameter sets has resulted in an acceptable match. Obviously, the strong time variability at this range precludes a deterministic modeling of the precise multipath arrival structure. However, the decreased time dispersion for the 5-km range, compared to that at 2 km, can only be explained with a lower sediment sound speed, i.e., sediment sound speed has to decrease with range. This is also in accordance with the expected transition from sandy mud to mud with increasing range (see Fig. 23). Ignoring range-dependence in the surficial sediment speed would have resulted in an increase in time dispersion with increasing range. On the contrary, measured time dispersion decreases with increasing range. A sediment speed of 1510 m/s at 5-km range from the source can explain the observed time dispersion.

![Fig. 29](image)

*Fig. 29* The 120 individual received signals for the 5-km, middle hydrophone, experiment.

Due to the very high time variability of the 10-km range signals, an acceptable model/data agreement could not be obtained. Further, it should be emphasized that we are considering propagation of relatively high frequency sound in only 70 meters of water over a range of 10 km (which is over 140 times the water depth). It is therefore postulated that at 10 km the applied modeling is too simple.

### 2.5.3.3 Relation with matched field inversion

The approach taken is actually the ‘matched field inversion’ approach described in the introduction, where the set of unknown parameters that results in modeled data that have a maximum match with the measured data is assumed to comprise the ‘true’ unknown parameter values. However, no use is made of an optimization method, and the optimization, in this case the maximization of the match, can be considered to be a search ‘by hand’ and using physical intuition. Only a limited amount of parameter values and parameter
combinations are considered. It can easily be understood that this procedure for obtaining estimates for the unknown parameters is not very practical. This is especially true for problems with a large number of unknowns, and a large number of parameter combinations containing parameter values that are significantly different from the ‘true’ parameters, but that nevertheless look like a good optimization result. These parameter combinations that correspond to an optimum within a part of the entire search domain containing all possible parameter combinations, are denoted by local optima. More sophisticated global search methods exist, and will be considered in the next chapters.

From the previous it can be concluded that the influence of the oceanographic time variability on the received signals increases with increasing range. At 5-km range the variability in received signals is such that for obtaining an acceptable match the temporal (and spatial) variation of the sound speed profile should be accounted for. Not only increasing the range, but also increasing the frequency is expected to hamper the matched field inversion performance. This is the result of the smaller wavelengths at higher frequencies. When modeling the propagation of sound, features as small as the wavelength have to be accounted for.

The applicability of matched field inversion as a function of range and frequency is schematically illustrated in Fig. 30.

![Fig. 30 Schematic of range/frequency combinations allowing for matched inversion, indicated by shaded areas.](image)
References
