Sea bottom parameter estimation by inversion of underwater acoustic sonar data

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Chapter 4

Multi-frequency matched field inversion of benchmark data using a genetic algorithm

Abstract

For a selected number of shallow water test cases of the 1997 Geo-acoustic Inversion Workshop we have applied matched field inversion to determine the geo-acoustic and geometric (source location, water depth) parameters. A genetic algorithm has been applied for performing the optimization, whereas the replica fields have been calculated using a standard normal-mode model. The energy function to be optimized is based on the incoherent multi-frequency Bartlett processor. We have used the data sets provided at a few frequencies in the band 25-500 Hz for a vertical line array positioned at 5 km from the source. A comparison between the inverted and true parameter values is made.

4.1 Introduction

For the Geo-acoustic Inversion Workshop (Vancouver, 24-26 June, 1997) broadband acoustic field data were generated for a series of range-independent shallow water environments. The SAFARI model was used to calculate the acoustic fields for all these test cases. Participants of the workshop were instructed to invert the simulated data to estimate the geo-acoustic model parameters. These parameters were unknown to the participants, except for a calibration test case. For some cases the source location and water depth were unknown as well. The sound speed profile in the water column is downward refracting for all cases and known to the users. The data were generated for a total of seven test cases.

For the first five of the test cases (including the calibration test case), the geo-acoustic model consists of a single fluid sediment on top of a fluid homogeneous half-space. The unknown parameters for the sediment are layer thickness, sound speed at the top and bottom of the sediment, density and attenuation. It is hereby assumed that the sound speed profile in

the sediment is linear. For the half-space the unknown geo-acoustic parameters are sound speed, density and attenuation.

For the sixth test case the geo-acoustic model is a multiple fluid layer sediment. Each layer is homogeneous and the number of layers is unknown. The geo-acoustic model of the seventh test case again consists of a single sediment and half-space, but now the layers are supposed to be elastic.

For each test case there are three realizations (denoted by a, b, and c), i.e., field data were generated for three sets of parameter values.

The bounds from which the values for each parameter were selected, i.e., the search ranges, were known to the participants, whereas the specific values were unknown (except for the calibration test case).

The acoustic field data were provided as the complex pressure at specific vertical and horizontal slices in the water column, so that participants could design their own ‘experiments’ to invert the data (e.g. using either vertical or horizontal line arrays). At the ranges 1, 2, 3, 4, and 5 km the data were provided at 1 m depth intervals from 1 m to 100 m throughout the water column. At depths of 75 m and 100 m the data were provided at 50-m range intervals from 50 m to 5000 m. Further, at each grid point the complex pressure was provided from 25 to 199 Hz in 1-Hz intervals, and from 200 to 500 Hz in 5-Hz intervals.

For a selected number of test cases, we have applied matched field inversion (MFI) to determine the unknown parameters. In MFI the unknown parameters are determined by minimizing an energy function $E$. In this study $E$ should provide a measure for the difference between the pressure field calculated by SAFARI and the pressure field calculated by a model (the replica field) for a set of values for the unknown parameters. As such, the unknown parameters are determined through an optimization procedure, which involves finding a set of parameter values that minimizes the discrepancy between the two pressure fields. The number of possible parameter value combinations is extremely large, as the number of unknown parameters is in the order of ten. In addition, the parameter search space can have a large number of local minima. Finding the global minimum of the energy function requires modern global optimization methods, such as simulated annealing or genetic algorithms.

The replica pressure fields in the inversion were calculated using a standard normal-mode model. A brief description of this model is given in Section 4.2. For the calibration test case a direct comparison of the pressure fields generated by SAFARI and those generated by our normal-mode code has been performed, the results of which are also presented in Section 4.2.

The test cases for which inversion was performed are described in Section 4.3. The choice for the test cases taken in consideration is partly based on the specific capabilities of the normal-mode model that has been used.

We have used a genetic algorithm as the global search method. Section 4.4 provides a description of the basic principles of a genetic algorithm. It also provides the specific setting of the algorithm for the current inversion, including the type of energy function used.

Results are presented and discussed in Section 4.5.

4.2 The forward acoustic model

The model used for the forward replica calculations is a standard normal-mode model, which has been developed in our group in MATLAB. For this model the ocean environment is assumed to consist of three layers: water column, sediment layer and homogenous half-space. Densities and attenuation constants in all layers are assumed to be independent of depth. The density in the water column is $1 \text{g/cm}^3$. Of course, the attenuation can be dependent on frequency. The sound speed in the water column and in the sediment is allowed to vary with depth, whereas it is supposed to be constant in the half-space.
The numerical technique for solving the depth-dependent Helmholtz or modal equation and its boundary conditions is a finite-difference discretization. The resulting algebraic eigenvalue problem is solved using routines of the well-known EISPACK package to compute the eigenvalues and eigenvectors of a real symmetric tridiagonal matrix in a specified interval. This eigenvalue interval is chosen such that only the modes corresponding to the discrete eigenvalue spectrum are calculated, thereby omitting the continuous spectrum. Loss effects due to volume attenuation in the water column, sediment and half-space are taken into account by first order perturbation theory. Shear in the bottom layers is not accounted for.

Range-dependent ocean environments are handled by using the adiabatic approximation. This is, however, not needed for the present study, as all test cases are range-independent.

For the calibration test case (see Fig. 1) the pressure fields generated by SAFARI have been compared directly with the fields calculated using our normal-mode model.

![Fig. 1 The calibration test case.](image)

This has been done in the following way: let \( p_{\text{obs}} \) (obs = observed, or true), and \( p_{\text{calc}} \) (calc = calculated) be the pressure fields calculated using SAFARI and the normal-mode model, respectively. When using the data at the fixed ranges (vertical line arrays consisting of 100 hydrophones with 1-m spacing), both \( p_{\text{obs}} \) and \( p_{\text{calc}} \) are complex vectors of length 100. Now, as a measure of the agreement between the two models, we have used an energy function \( E \) based on the single-frequency linear or Bartlett processor, which is given by

\[
E = 1 - \left| p_{\text{obs}} \cdot p_{\text{calc}}^* \right|^2 
\]

with the suffix * denoting the complex conjugate. Here it is assumed that \( p_{\text{obs}} \) and \( p_{\text{calc}} \) are normalized, i.e., \( \| p_{\text{obs}} \| = \| p_{\text{calc}} \| = 1 \). The difference in propagation convention between SAFARI \( e^{+i\omega t} \) and the normal-mode code \( e^{-i\omega t} \) is accounted for.
$E$ has been calculated as a function of frequency for the data provided at the fixed ranges. The results for the 1-km and 5-km data are presented in Fig. 2. (The curves for the 2, 3 and 4 km data lie in between these two curves).

![Graph showing $E$ as a function of frequency for data at 1 km and 5 km.](image)

**Fig. 2** Agreement between the two models for the calibration test case for the data at 1 and 5 km.

It is observed that the model outputs agree very well as the energy function is quite low for all frequencies and all ranges. For the 1-km data this agreement is slightly worse than that for the data at all other ranges. This is due to the fact that the leaky modes (continuous spectrum), which become more important at shorter ranges, are not taken into account by the normal-mode model.

The generally excellent agreement between the two models for the calibration test case gives much confidence for the inversion work.

### 4.3 The test cases selected for inversion

The normal-mode model used precludes inversion of the data for test case 6 (more than one sediment layer) and test case 7 (elastic layers). For the inversion we have selected the test cases that were denoted AT and WA. From the remaining four test cases that can be handled by our acoustic model these two cases are the most difficult as the number of unknown parameters is largest. Moreover, the two cases selected are quite different: for AT all unknown parameters are geo-acoustic parameters, whereas for WA also geometrical parameters (source location and water depth) are unknown. The selected test cases are depicted schematically in Fig. 3. In this figure the unknown parameters are indicated by squares.
Fig. 3  The test cases called AT and WA. The unknown parameters are those in boxes.

The search bounds for the unknown parameters are given in Table I. The data for all test cases were generated under the constraints

\[ c_{1,\text{sed}} < c_{2,\text{sed}} < c_b \]  

(2)

and

\[ \rho_{\text{sed}} < \rho_b \]  

(3)

These constraints were accounted for in the optimization procedure.

Table I  Unknown parameters and their search bounds for test cases AT and WA.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Test case</th>
<th>Symbol</th>
<th>Search interval</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sediment thickness</td>
<td>AT, WA</td>
<td>( h_{\text{sed}} )</td>
<td>[10, 50]</td>
<td>m</td>
</tr>
<tr>
<td>Sediment density</td>
<td>AT, WA</td>
<td>( \rho_{\text{sed}} )</td>
<td>[1.4, 1.7]</td>
<td>g/cm(^2)</td>
</tr>
<tr>
<td>Speed at top of sediment</td>
<td>AT, WA</td>
<td>( c_{t,\text{sed}} )</td>
<td>[1500, 1600]</td>
<td>m/s</td>
</tr>
<tr>
<td>Speed at bottom of sediment</td>
<td>AT, WA</td>
<td>( c_{b,\text{sed}} )</td>
<td>[1550, 1750]</td>
<td>m/s</td>
</tr>
<tr>
<td>Density of half-space</td>
<td>AT, WA</td>
<td>( \rho_b )</td>
<td>[1.6, 2.0]</td>
<td>g/cm(^2)</td>
</tr>
<tr>
<td>Sound speed of half-space</td>
<td>AT, WA</td>
<td>( c_b )</td>
<td>[1600, 1800]</td>
<td>m/s</td>
</tr>
</tbody>
</table>
| Sediment attenuation               | AT        | \( \alpha_{\text{sed}} \) | [0.05, 0.5]    | dB/\(
\lambda\) |
| Half-space attenuation              | AT        | \( \alpha_{\text{h}} \) | [0.05, 0.5]    | dB/\(
\lambda\) |
| Source range (5 km data)            | WA        | \( f_s \)       | [5.0, 5.4]     | km   |
| Source depth                        | WA        | \( z \)         | [10, 30]       | m    |
| Water depth                         | WA        | \( H_w \)       | [100, 120]     | m    |

4.4 The genetic algorithm

As mentioned in the introduction we have used a genetic algorithm (GA) as the global search method. The algorithm, which has been developed at TNO-FEL in MATLAB\(^8\), is described below. Gerstoft was the first to apply GAs to inverse problems in underwater acoustics.\(^2\)

The first step in a genetic algorithm is to create an initial population consisting of \( q \) members. Each member represents a possible parameter value combination, i.e., a possible solution to the optimization problem. This first generation is created randomly. The
population size \( q \) should be large enough to ensure that the problem space can be searched thoroughly. On the other hand the population size should be not too large, thereby limiting the amount of energy function evaluations (i.e., the number of forward acoustic model calculations).

At this creation stage the members are in their binary encoded form, i.e., the parameter value combinations are represented by a string of zeros and ones. In the following these strings are denoted as chromosomes. Each parameter is represented by a certain part of the chromosome. These parts are called genes. The encoded form of the parameter value combinations is needed when applying certain operators as will be explained later.

After decoding, the values for the energy function \( E \) can be calculated for all members of this first population. This is also referred to as assigning a fitness value to each member. When the energy function is normalized \((0 < E < 1)\), the fitness \( \varphi \) is given by

\[
\varphi = 1 - E
\]  

(4)

i.e., a low value for the energy function means a high value for the fitness.

The energy function we have selected is based on the incoherent multi-frequency linear or Bartlett processor\(^7\) and is given by

\[
E(m) = 1 - \frac{1}{K} \sum_{k=1}^{K} \left| p_{\text{obs}}(f_k) \cdot p_{\text{calc}}^*(f_k, m) \right|^2
\]  

(5)

with

- \( m \) the vector containing the parameters for which the inversion is performed
- \( K \) the number of frequencies
- \( p_{\text{obs}}(f_k) \) the (normalized) pressure field at frequency \( f_k \) calculated by SAFARI
- \( p_{\text{calc}}(f_k, m) \) the (normalized) pressure field at frequency \( f_k \) calculated by the normal-mode model

We have used the data provided at fixed ranges, thereby inverting the pressure field across a vertical line array consisting of 100 hydrophones with 1-m spacing. From these vertical array data, we have selected the 5-km data. This is a somewhat arbitrary choice as the models agree very well at all ranges, although it is somewhat worse at 1 km (see Fig. 2).

For the creation of the next generation, first a parental population is selected from the initial population. This selection is based on the fitness values obtained for the different chromosomes: a higher fitness implies a larger probability of being selected, thus resulting in a parental population with a higher proportion of fit members. The selection criterion should be such that, on the whole, more opportunities to reproduce are given to the population members that are the most fit. However, at the beginning the selection criterion should not be chosen too strict, as that would force the algorithm to converge to a local minimum. On the other hand a criterion that allows nearly all members to reproduce will result in slow convergence. In our application the probability \( p_j \) for the member \( m_j \) to be selected is taken as

\[
p_j = \frac{e^{-E(m_j)}}{\sum_{j=1}^{q} e^{-E(m_j)}}
\]  

(6)
The temperature $T$ is chosen equal to the lowest value of the energy function found in the entire current population. This choice results in a flat probability distribution at the beginning, but as the optimization process continues, the temperature will decrease, resulting in a more peaked probability distribution and therefore more emphasis will be put on the most fit members in later generations.

The following step is to create a population of $q$ children. This is done by applying different operators to the members of the parental population. These operators are crossover and mutation, and they are applied to the members when they are in encoded form. In order to apply crossover the members of the parental population are paired randomly. Crossover results in the exchange of corresponding chromosome parts between the two chromosomes of each set of parents. We have applied multiple point crossover: a crossover point is selected at each gene, i.e., the number of crossover points is equal to the number of parameters for which the optimization is performed.

Consider for example the following two genes, representing different values for the same parameter (encoded using $N$ bits):

$$ (a_0, a_1, ..., a_{N-1}) \text{ and } (b_0, b_1, ..., b_{N-1}) $$

with $a_i$ and $b_i = 0, 1$. Applying crossover at location $i$ results in the creation of the following two genes:

$$ (a_0, ..., a_{i-1}, b_i, ..., b_{N-1}) \text{ and } (b_0, ..., b_{i-1}, a_i, ..., a_{N-1}) $$

Crossover is applied with crossover probability $p_c$. Using a value of $p_c$ less than one will allow genes to be passed on to the next generation without the disruption of crossover (usually $0.6 < p_c < 1.0$). The crossover point, i.e., the location on the gene at which it is cut, is selected at random.

After crossover another operator called mutation is applied to the chromosomes. Mutation changes each bit of the chromosome with a certain probability $p_m$.

Crossover is considered to be a mechanism for rapid exploration of the search space. More crossover points or a higher crossover probability imply a more thorough search, but also more disruption. On the other hand, mutation is a process that provides a small amount of random search, ensuring that no point in the search space has zero probability of being explored. However, the mutation probability should not be chosen too high as then the search becomes effectively random (in general $p_m < 0.1$). At the start of the algorithm (i.e., for the first generations) crossover is the more productive operator, but as the population converges, mutation becomes increasingly important.

A new population (again consisting of $q$ members) is established by taking at random $f_r \cdot q$ ($0 < f_r < 1$) members of the children population and the $(1-f_r) \cdot q$ most fit members of the original population. $f_r$ is called the reproduction size. The choice for $f_r$ is an important research item. For values of $f_r$ close to one, or even equal to one (generational replacement), convergence of the algorithm to the global minimum might be slow. On the other hand, low values of $f_r$ might promote the algorithm to converge rapidly to a local minimum. In a recent study we have investigated the performance of a GA for different values for $f_r$. It should be noted, however, that such a study is very time consuming and that the performance of a certain value for a GA parameter can be very dependent on the particular values chosen for other GA parameters.

The new population is used as the next generation onto which the same procedure is applied as described above. This process is continued for a certain amount of generations, which should be chosen large enough to allow convergence of the optimization process.

Most of the values for the GA parameters were taken equal to those used in (Gerstoft):
- population size $q = 64$
- crossover probability $p_c = 0.8$
- mutation probability $p_m = 0.05$
- reproduction size $f_r = 0.5$

The number of generations is taken to be 400, hence the number of forward model calculations per GA run amounts to approximately 13000 per frequency.

A diagram summarizing the different steps in the optimization process using a GA is given in Fig. 4.

![Flowchart](image)

Fig. 4 Flowchart of the optimization process in a genetic algorithm.

### 4.5 Results

From experience\(^8\) we know that broadband processing outperforms single-frequency processing. For this reason we have selected the (incoherent) multi-frequency Bartlett processor (see Section 4.4). It is difficult to know in advance the set of frequencies needed for a successful inversion of shallow water environments in general. For instance, the frequencies required to successfully estimate the half-space parameters will definitely depend on sediment thickness and sediment attenuation. However, in general, one can argue that the high frequencies are useful for the sediment properties near the water/sediment interface, whereas
lower frequencies will be better for probing deeper into the bottom. We have selected the frequencies 25, 30, 50, 100, 200, 400 Hz, thereby exploiting the full frequency band available, still using the data at a limited number of frequencies.

As mentioned previously we have chosen the widely used experimental configuration of a vertical array that spans the whole water column, i.e., the classical approach. We have selected the data at the vertical line array at 5 km using the data provided at all depths.

For the choices made concerning frequencies and experimental configuration, we have performed the inversion for all three realizations of the two test cases selected.

Estimates for the values of the parameters for which the optimization is performed have to be derived from the members of the final GA population. Increasing the probability of finding the global optimum, the GA has been run five times independently for each of the six inversions (two test cases times three realizations). At the same time, the parameter space close to the global optimum is explored more thoroughly, thereby improving the accuracy of the parameter estimates.

As an example, the energy function values for the parameter values in the final populations are shown in Fig. 5 for test case WA, realization (b). The dashed lines denote the true parameter values. The total number of parameter combinations shown in this figure amounts to $5 \times 64 = 320$.

Estimates for the unknown parameters can be obtained by simply taking the parameter combination with the lowest energy function value. This solution to the inverse problem is referred to as $GAB_{\text{best}}$.

An alternative method to obtain estimates for the unknown parameters from the final populations is to calculate the so-called $a posteriori$ mean values. These are given by

$$\sum_{j=1}^{nq} m_j \sigma(m_j)$$

with

$$\sigma(m_j) = \frac{-E(m_j)}{T} \left[ \frac{e^{-E(m_j)} \sum_{i=1}^{nq} e^{-E(m_i)}}{T} \right]$$

with $n$ the number of independent GA runs for each inversion (being 5). Following (Gerstoft\textsuperscript{2}), the temperature parameter $T'$ is taken equal to the average value of $E$ of the 50 best members. This solution to the inverse problem is referred to as $GAM_{\text{mean}}$.

Generally, it is useful to calculate both the $GAB_{\text{best}}$ and $GAM_{\text{mean}}$ solution, since a significant difference between the two solutions for a particular parameter indicates that the acoustic field is hardly sensitive to corresponding changes in that parameter. This corresponds to a flat or at least ambiguous distribution of energy function values for this parameter, see Fig. 5. (This is only valid for a temperature $T'$ that is not too low as then both solutions will coincide automatically).

From the final populations it is also possible to calculate the so-called $a posteriori$ covariance\textsuperscript{2}, which is usually used as an estimate of the uncertainty on the solutions. However, this covariance is not an objective measure for the uncertainty as it will depend on the weighting applied (in this case the value for $T'$) and the type of energy function used.

For all six inversions the GA estimates are compared with the true values in Figs. 6 and 7. Both the $GAB_{\text{best}}$ and the $GAM_{\text{mean}}$ results are displayed in these figures. It is observed that the
inverted values for the geometric parameters (source location and water depth), the sediment parameters ($c_{1,sed}$, $\rho_{sed}$ and $\alpha_{sed}$) and the half-space sound speed $c_h$, are in excellent agreement with the true values. The agreement between the estimated and true values for the remaining parameters is somewhat worse, but still very good: even for the least sensitive parameter $\rho_b$ the estimated and true values are highly correlated. Note that for $\rho_b$ significant differences in the $GA_{best}$ and $GA_{mean}$ solution occur indicating that this parameter is less well determined. This is clearly observed from the flat energy function distribution for $\rho_b$ (see Fig. 5).

![Energy vs Parameter Values](figure5.png)

**Fig. 5** Energy versus parameter values in the final populations for test case WA, realization (b). The dashed lines indicate the true parameter values.
Fig. 6  GA estimates and true parameter values for all six inverted environments. The GA_{best} and GA_{mean} solutions are denoted by stars and crosses, respectively, whereas the circles denote the true parameter values.
Fig. 7  GA estimates and true parameter values for all six inverted environments. The GA_best and GA_mean solutions are denoted by stars and crosses, respectively, whereas the circles denote the true parameter values.

We also investigated the correlation between the different parameters. According to (Gerstoft9), correlations can cause problems in finding the global optimum when using search algorithms that are based on perturbing one parameter at a time. This is especially true when in the energy function a long valley exists that is orientated obliquely to the parameter axes. However, our experience is that despite of correlation a genetic algorithm still manages to find the global optimum, probably because it changes all parameters at the same time. Further, we have observed that, in general, the use of multiple frequencies results in much sharper peaks in the energy function, i.e., the parameters are better determined, and hence a considerable reduction in the correlation.

Finally, we have investigated the inversion performance for array configurations with fewer hydrophones than that for the full array (100 hydrophones with a spacing of 1 m). For this investigation we used the 5-km data of test case WA(b). Conventional beamforming
requires the sampling to be done at half-wavelength for the highest frequency, i.e., about 2 m at 400 Hz. Using multiple frequencies probably allows for a larger spacing.

Figure 8 presents the $G_{A_{\text{best}}}$ and $G_{A_{\text{mean}}}$ solutions as a function of hydrophone spacing. One observes that the inversion performance does not degrade up to hydrophone spacings of at least 20 m (corresponding to only five hydrophones). Note that the estimates for the geometric parameters ($z_s$, $r_s$ and $H_w$) do not deviate from the true values for hydrophone spacings as high as 38 m (corresponding to only 2 hydrophones at depths of 37 and 75 m, respectively).

The minimum required hydrophone spacing when using a broadband processor (coherent or incoherent) is subject to further research.

![Fig. 8](image)

Fig. 8  GA estimates as a function of hydrophone spacing for the test case WA(b). The $G_{A_{\text{best}}}$ and $G_{A_{\text{mean}}}$ solutions are denoted by stars and crosses, respectively, whereas the horizontal dashed lines denote the true parameter values.

### 4.6 Summary and conclusions

Matched field inversion has been applied to part of the broadband data of the benchmark exercise to determine geo-acoustic bottom parameters, water depth and source location. Use has been made of a genetic algorithm as optimization method and a standard normal-mode code as forward acoustic model. The geo-acoustic model of the test cases selected consisted
of a single fluid sediment on top of a fluid homogeneous half-space. For these type of range-independent shallow water environments we have clearly demonstrated that all unknown parameters can successfully be inverted when use is made of a vertical line array configuration and a broadband processor. For the latter we have selected the incoherent multi-frequency Bartlett processor using only a few frequencies that are more or less evenly distributed over the band 25-500 Hz.

4.7 Acknowledgments

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References
