Optimal Strategic Timing of Financial Decisions
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Chapter 2

Strategic Advantage and the Optimal Exercise of Entry Options

2.1 Introduction

This paper studies the process of entry following corporate investment aimed at gaining a strategic advantage. We analyse the timing of entry in continuous time, adopting the context of the strategic real option methodology of valuation to price growth opportunities (Lambrecht (2001) and Grenadier (1996)). For illustration of the strategic advantage which produces an enhanced entry capacity, we adopt the notion of platform investment, a concept which has achieved mainstream interest in management science (for a survey see Kogut and Kulatilaka (1994)). An alternative interpretation is the valuation of the acquisition of a company active in a related sector with significant growth opportunities in the future.

Platform investment refers to the establishment of a broad logistic infrastructure, which allows easier entry in related product segment. There are well-documented examples of platform investment, which resulted in significant market advantages for the investing firms. However, to date there are hardly any theoretical treatment of the implied strategic and valuation issues.

A classic example is the information and distribution infrastructure built by Wal-Mart, targeted at rapid collection of information on customer purchases at individual shops connected with a rapid-response retail distribution system (Khanna and Tice (2000)). Other examples are the creation of business-to-business communications networks that integrate suppliers and manufacturing plants with designers, considerably facilitating new product development, e.g. in the airspace and the car industry. Another possible, if rather special, example of a platform is a computer operating system. A company, which controls the dominant operating system, is in an advantageous position to develop subsequent software applications compatible

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1 A logistic infrastructure may not only incorporate physical assets but also brand name market knowledge and customer information systems.

2 An example which has been extensive studied is, for instance, Boeing's productive process which was closely integrated via a common communication and productive platform with its suppliers.
with the platform, and enjoy an immediate advantage with users.

Internet sites are a final example. Owners of frequently visited sites (network hubs, or "portals") potentially have access to large distributional capacity for information, services and product sales. Successful horizontal Internet portals, either in closed systems (such as AOL) or open access (search engines such as Yahoo, or general retail outlets such as Amazon) have become focal points on the Internet by offering an inexpensive service, thus building a strong brand name, while learning about their customers' characteristics. This may create a basis for offering new services, lowering the cost of reaching customers and thus the cost of entry in new market segments.

This paper does not attempt to model in detail the nature of the strategic advantage gained by the "platform" investment. These are large differences between proprietary and open platforms, and between those created to coordinate production versus those directed at distribution. Some platforms grant strong advantages, others can be more easily replicated. We simply recognize that the establishment of a platform requires extraordinary initial expenditures relative to conventional investment, but grants easier entry opportunities in related segments.\(^3\)

Specifically, we study the value of strategic entry advantages relative to conventional entry under demand uncertainty.

Our analysis is related to the literature on strategic real options. The commitment of irreversible investment may confer strategic advantages as a result of lower capacity costs (Dixit (1980)), operating costs (Kulatilaka and Perotti (1998)) or faster timing-to-market (Kulatilaka and Perotti (forthcoming)). In all of these models, the investing firm strengthens its post-entry position, gaining market share, and may discourage entry by competitors (Dixit (1980) and Lambrecht and Perraudin (1997)). On the other hand, investment destroys the value of the option to wait until new information on demand is available, which is valuable under market uncertainty (McDonald and Siegel 1986). An important contribution that includes both entry and exit decisions in continuous time offered by Lambrecht (2001).

In our approach we model the advantage gained by a platform investment as a greater ease of entry in related products. The construction of a logistic platform can thus be described as the acquisition of a set of strategic entry options (see Baldwin and Clark (2000) and Kogut and Kulatilaka (1994)).\(^4\) We therefore explicitly treat entry in related segment, which require us to consider cross-effects.

The basic model we develop is as follows. A firm considers a platform investment, which leads to produce a new differentiated product. This affects demand in a related market segment in which there is also a traditional incumbent. Demand for both products is uncertain, strictly correlated and evolves stochastically over time. The

\(^3\)It is also uncertain whether initial strategic advantage on Internet may not be easy replicated, given the ease of entry. The huge cost of offering free services (search, storage or information services) to anyone, with great uncertainty over the size of the future demand, has caused many of such investments to fail.

\(^4\)Another possible advantage of a platform is the lower costs of access to customers, and thus a lower marginal cost. Our focus on fixed cost allows us to concentrate more on the timing of entry rather than market share.
2.1. Introduction

Platform investment cost is greater than conventional entry but lowers the cost of subsequent entry in the related market segment.

The value of the platform depends on the value of the entry advantage, which is a strategic option in the sense that it can be exercised at an optimal timing, taking into account its impact on market share and margins as well as the possible entry by the competitor. As the market expands, the attractiveness of being first increases, until the value of exercising the entry option exceeds the value of the waiting-to-invest option. See Grenadier (1996) for a sophisticated application to the real estate industry.

In an entry game with symmetric firms, there is an ambiguity as to which firm will enter as first. In our case, due to the different investment costs, the platform firm enjoys a period of de facto monopoly on the choice to cross-enter as leader, when it knows that the competitor firm would not yet find it profitable to enter. In this case, the innovator exercises optimally this entry option as leader by comparing the value of waiting to enter (and thus avoiding losses) to the higher profits after entry.\(^5\)

We show that the optimal entry strategy depends on the degree of strategic advantage gained by the platform investment.\(^6\) If the entry costs advantage is small, the platform firm enters just before the competitor would exercise its entry option before its optimal timing in order to pre-empt its competitor. If the comparative advantage is significant, the platform firm knows that the traditional firm would never enter as first (because the period of time in which it would enjoy higher profits is not enough long to compensate for its high investment cost). In this case the platform firm can choose the optimal entry timing without fearing pre-emption, trading off the value of the waiting-to-invest option against the higher profits from cross-entry. There is finally a third case, in which creating a platform allows to credibly suppress unprofitable cross-entry, and allows to sustain two more profitable parallel monopolies. Thus a platform may serve to reduce competition rather than enhancing it.

The model allows studying the impact of market uncertainty over the optimal timing of exercise of the entry option, and therefore its valuation relative to the waiting to invest option. The ex-ante effect of uncertainty on the strategic advantage and thus of the platform value is not immediately obvious: the higher is uncertainty, the higher is option value to wait, but at the same time the higher the additional expected profits from cross-entry. This last effect is due to the fact that in a static oligopolistic market, firms increase their margins as demand rises; therefore, profits are endogenously convex in demand. Because of this convexity, a mean-preserving increase in uncertainty favours investment in the strategic growth option over the waiting option.\(^7\) In addition, in a dynamic context with learning, uncertainty affects

\(^5\)The approach is related to Smets (1991).

\(^6\)Note the difference with the classic "monopolistic" real option of McDonald and Siegel (1986), is that in our context entry affects prices and market shares.

\(^7\)This result cannot be directly compared to the apparently contrary implications on the relative value of the waiting-to-invest option McDonald and Siegel (1986). In the context of imperfect competition, investment has an impact on market structure and marginal profitability (Kulatilaka
the optimal timing of entry. Our main result is to show that the general result is unambiguous: greater uncertainty increases the strategic advantage of the platform and thus its value relative to traditional entry.

The next section introduces a basic model of Cournot competition in continuous time.\textsuperscript{8} In the third section we determine the cross-entry strategies of the platform firm. Section fourth analyses the uncertainty effect on the entry strategy of the platform. In section 5 we compute the platform value and the effect of higher uncertainty. At the end we offer some concluding thoughts and ideas for further work.

2.2 The basic model

We consider two firms $i$ and $j$, competing in real time over $t = [0, \infty)$ in two related products. The first one has invested in a traditional, specialized investment ($T$); the other firm, the innovator, has the possibility to invest in a platform infrastructure ($I$).\textsuperscript{9}

There are only two differentiated products, indexed by 1 and 2. The inverse market demand functions for the two products are:

\begin{align}
  p_1 &= \gamma - (q_{1i} + q_{1j}) - a(q_{2i} + q_{2j}) \\
  p_2 &= \gamma - a(q_{1i} + q_{1j}) - (q_{2i} + q_{2j})
\end{align}

Note that the two product are partial strategic substitutes; the parameter $a$ represents the intensity of substitution, and ranges from $0$ (no substitutability) to $1$ (identical products).

We assume that firms have complete information about the demand and each other's cost structures.\textsuperscript{10} Conventional entry in any market requires the investment of an amount $I$; investment in a platform costs $K$, with $K \gg I$. In addition, a platform firm would pay a lower cost $I_P$ as the incremental investment to enter in any market segment, with $I_P < I < I_P + K$. Thus there is an extra cost to build a platform, but this allows easier cross-entry thereafter.\textsuperscript{11}

Firms that compete in the same market engage in Cournot competition. When each firm produces a differentiated product, they maximize their profits as monopolists in their own market, though their margins are influenced by the substitution effect among products.

\textsuperscript{8}We have considered price competition in another draft. In our context neither a platform firm nor a traditional firm ever cross-enters under price competition in each other's market segment, so the platform has no value.

\textsuperscript{9}More generally, platform may grant entry options in multiple market segments. We set this number to one for simplicity.

\textsuperscript{10}Strategic entry under imperfect information is analysed in Lambrecht and Perraudin (1999).

\textsuperscript{11}This assumption sets our analysis apart from most of the literature on entry in strategic entry such as Smets (1991), Dixit (1980) and Huisman and Kort (1999).
2.2. The basic model

The intercept of the demand function, $\gamma$, represents the size of market demand. Demand size is uncertain and evolves stochastically over time. We assume that it follows a Geometric Brownian Motion:

$$\frac{d\gamma}{\gamma} = \frac{1}{2} \left( \mu - \frac{1}{2} \sigma^2 \right) dt + \frac{1}{2} \sigma dz \quad (2.3)$$

In the specification, $\sigma$ appears as parameter both in the uncertainty and in the drift term of the demand process, in order to isolate the effect of demand uncertainty from its drift. This will allow us later to carry out a mean-preserving-spread analysis in the sense of Rotschild and Stiglitz (1970), by varying its demand uncertainty while adjusting the drift so that there is no change in expected demand. To this goal, we define $\alpha = \frac{2\mu - \sigma^2}{2}$ so that we can rewrite the market process, (2.3), as:

$$\frac{d\gamma}{\gamma} = \frac{1}{2} \alpha dt + \frac{1}{2} \sqrt{2(\mu - \alpha)} dz \quad (2.4)$$

We next define $\gamma^2 = \theta$. We can express the evolution of $\theta$ through Ito's Lemma as a Geometric Brownian motion:

$$\frac{d\theta}{\theta} = \mu dt + \sigma dz \quad (2.5)$$

The reason for introducing thus quadratic transformation is that oligopolistic profits under Cournot competition are convex in demand. We see later that $\theta$ is linearly related to firm profits, and $\sigma$ is closely related to profit uncertainty.

To ensure finite valuations, we assume that $\mu < r$, where $r$ is the discount factor. We also assume that $\alpha < \mu( < r)$ to ensure $\sigma^2 > 0$.

We first evaluate the value of a "platform" firm and then compare it with the value of the traditional firm.

2.2.1 Product Market Interactions

Initially the platform firm operates in a different product segment from the competitor. This leads to a parallel differentiated monopoly. The associated profit flow (see the Appendix 2.A.1) is:

$$\prod_{i,t} = \prod_{j,t} = \frac{1}{(a + 2)} \theta_t \equiv M \theta_t \quad (2.6)$$

and the associated valuation of the firm is:

$$E \left[ \int_0^\infty M \theta_t e^{-rt} dt \right] = \frac{M}{r - \mu} \theta_t \quad (2.7)$$

This is the discounted stream of expected profits when the underlying expected profit growth is equal to $\mu$ and there is no change in market structure.

Both firms have the possibility to enter in the competitor's market segment: for the traditional firm the cross-entry will cost $I$, while for the platform firm the cost
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will be \( I_P \) (with \( I_P < I \)). If each firm cross-enters in the other’s market (parallel duopoly), the profit flow and the expected present value are (see Appendix 2.A.1):

\[
\prod_{i,t} = \prod_{j,t} = \frac{2}{9} \alpha + 1 \theta_t \equiv D \theta_t \tag{2.8}
\]

\[
E \left[ \int_0^\infty D \theta_t e^{-rt} dt \right] = \frac{D}{r - \mu} \theta_t \tag{2.9}
\]

When a firm cross-enters as first (we term this situation an "asymmetric duopoly"), the rival may prefer at first to wait to enter, if the waiting to invest option is more valuable than the higher profits from symmetric duopoly.\(^{12}\) In this case the first firm to cross-enter enjoys higher asymmetric profits (being a monopolist in its own segment and a duopolist in the second). These extra profits are just temporary: as demand increases the other firm will find it convenient to exercise the option to enter, and the market outcome will be a parallel duopoly in both products. However, the option to enter first is clearly valuable. As we will see, this is one of the advantages of building a platform.

We define as asymmetric duopoly the case in which one firm operates in both segments and the other in just one). The profits flow and its expected present value for the firm present in both markets are (see Appendix 2.A.2):

\[
\prod_{i,t} = \frac{1}{18} \left( 2 - a \right) \theta_t + \frac{1}{12} \left( 3 - a \right) \theta_t = \frac{1}{36} \left( 13 - 5a \right) \theta_t \equiv L \theta_t \tag{2.10}
\]

\[
E \left[ \int_0^\infty L \theta_t e^{-rt} dt \right] = \frac{L}{r - \mu} \theta_t \tag{2.11}
\]

while the other firm earns:

\[
\prod_{i,t} = \frac{1}{9} \theta_t \equiv F \theta_t \tag{2.12}
\]

with an expected present value of:

\[
E \left[ \int_0^\infty F \theta_t e^{-rt} dt \right] = \frac{F}{r - \mu} \theta_t \tag{2.13}
\]

Note that these profits are equal to profits under Cournot competition, as the entrant produces less in its original market due to the cross-effect on demand.

Because \( 0 < a < 1 \), a firm always prefers to be in both market segments if its competitor produces just in one, while it would prefer to be monopolist in just one market than being duopolist in both. However, the option to cross-enter grants

\(^{12}\)In a static framework firms would end up in a mutual forbearance (this concept has been first presented in Edwards (1955)): since profits are higher under parallel monopoly (\( M > D \)), both firms would prefer to maintain a collusive arrangement avoiding entry under a threat of an immediate cross-entry in case the competitors enters as first. However, in a dynamic framework this is often not sustainable (see Fudenberg and Tirole (1985)).
temporarily higher returns as long as the competitor does not follow immediately.\textsuperscript{13} This can be summarized with the following relations: $F \leq D \leq M \leq L$.

Notice that since $0 < a < 1$, $L - M$, the incremental gain from entry as leader, $L - M$, and the incremental gain from entry as follower, $D - F$, are decreasing in $a$. Moreover, $L - M < D - F$: the profit increment as a result of cross-entry is never larger than the profit increment of a follower cross-entrant.\textsuperscript{14} Another result is that $M < L - M$: entering the second market leads to less than double monopoly profits.

2.3 The dynamic game

We first analyse the optimal entry strategies of the two firms. We can then compute the value of the platform and traditional investment strategy.

For low levels of demand, there is no incentive for immediate cross-entry in the other product, so both firms remain as monopolists in their own market segment (although their profits are influenced by the substitution effect among products).

However, as demand rises, the attractiveness of entry in the second segment increases. Since the innovating firm controls a platform, it can enter in the first product with a lower incremental cost than the traditional producer. Intuitively, it will have a lower threshold for entry.

We show later that in equilibrium the traditional firm is always a follower in the entry game. As a result, for a certain time interval the platform firm enjoys a sort of monopoly on the option to enter as leader, knowing that the competitor would not move first. This is the crucial strategic entry advantage for the platform firm.

We define the entry threshold of the platform firm as $\theta^*_T$. At this threshold its profits flow will be by the sum of profits in the shared market plus monopoly profits in its market segment (see equation (2.11)), while the traditional producer earns duopolistic profits in its segment (2.13)).

The traditional firm will also cross-enter (see equation (2.8)) as demand rises enough. We define this subsequent entry threshold as $\theta^{**}_T$. This value affects the value of the strategic entry option by the platform firm, which is a function of the expected time of follower entry. We will see that this expected entry time is not trivial to compute.

The time structure of the game is sketched in Figure 2.1.

\textsuperscript{13}Under some parameters values, this creates a sort of prisoner's dilemma: firms would like to mutually commit not to enter in each other's market, but such a commitment is not credible. For other parameter values, the platform can act as a threat supporting equilibrium without cross-entry, as we show later.

\textsuperscript{14}This difference at first increases as $a$ rises (diminishing the differentiation) and subsequently it diminishes, vanishing when $a = 1$ (no product differentiation). This relation will be useful later for our conclusions.
2.3.1 Optimal Entry of the Follower

We solve the game backward. We first assume that the innovator (the platform) has already cross-entered, and compute when the traditional firm would follow.\textsuperscript{15} Subsequently, we check that the entry strategy for the innovator leads it to enter first.

Let $\theta = \theta_T^{**}$ be the threshold level of demand such that when $\theta \geq \theta_T^{**}$ the traditional firm is willing to cross-enter in the other market. At that point the present value of the traditional firm is worth $\frac{D}{r-\mu} - I$, where $I$ is its own cost of entry. For $\theta < \theta_T^{**}$, the traditional firm waits until the first passage in time of the threshold value; during this interval it earns $\frac{F}{r-\mu}$. Thus the expected present value of its cross-entry investment is given by:

\[
E \left[ \int_0^{T^{**}} e^{-rt} F \theta_t dt \right] + E \left[ e^{-rT^{**}} \right] \left[ \frac{D}{r-\mu} \theta_T^{**} - I \right]
\]

(2.14)

where $T^{**}$ is the expected time to reach the point $\theta_T^{**}$ starting from $\theta$. Following Dixit and Pindyck (1994) and Harrison (1985), we solve this optimal crossing problem. $T^{**}$ equals:

\[
T_T^{**} (\theta_t) = -\frac{\beta}{r} \ln \frac{\theta_t}{\theta_T^{**}}
\]

(2.15)

where $\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + 2 \frac{\tau}{\sigma^2}} > 1$. $\beta$ is thus negatively related to market and profit uncertainty.\textsuperscript{16} As $\beta$ is inversely related to both $\sigma$ and $\mu$ (respectively profit and demand uncertainty). Note that $\beta$ does not depend on $\alpha$.

\textsuperscript{15}We will show later that this is always the case in any equilibrium with entry, due to the entry cost difference.

\textsuperscript{16}More precisely when uncertainty tends to 0, $\beta$ tends to infinity while when uncertainty tends to infinity, $\beta$ tends to 1.
2.3. The dynamic game

The expected value of the traditional firm after entry:

\[ V_T(\theta_t) = \begin{cases} \frac{F\theta_t}{r-\mu} \left[ 1 - \left( \frac{\theta_t}{\theta_T^*} \right)^{\beta-1} \right] + \left( \frac{\theta_t}{\theta_T^*} \right)^\beta \left[ \frac{D}{r-\mu} \theta_T^{**} - I \right] & \text{if } \theta_t < \theta_T^{**} \\ \frac{D}{r-\mu} \theta_t - I & \text{if } \theta_t \geq \theta_T^{**} \end{cases} \]

while the optimal entry threshold \( \theta \) is given by:

\[ \theta_T^{**} = \frac{\beta}{\beta - 1} \frac{I}{D - F} = \frac{\beta}{\beta - 1} \frac{I}{\eta(1+\alpha)(r-\mu)} \quad (2.16) \]

As demand uncertainty increases the follower tends to enter in the second market at higher demand values; however the expected time of follower entry is shorter.

The entry threshold as follower for the platform firm is given by the same expression (2.17), with \( I_P \) substituting for \( I \). We call this entry threshold \( \theta_T^{**} \). The effect of demand uncertainty on this entry threshold and the associated expected time is the same.

2.3.2 Optimal Entry as Leader

The next step is to determine the cross-entry point for the platform firm. We will demonstrate that depending on the investment cost advantage, \( \frac{I_P}{I} \), we can have three kind of different equilibria: we will classify them as weak strategic advantage, strong strategic advantage with entry and strong strategic advantage without entry.

In order to define these equilibriums we identify for both the traditional and the platform firm the critical demand levels at which they would enter as first in the second market. We define these points respectively \( \theta_T^* \) and \( \theta_T^** \).

This first entry threshold, the entry threshold for the traditional firm, \( \theta_T^* \), is determined comparing the payoff of entry as leader and entry as follower for the traditional firm (as in Smets (1991) and Grenadier (1996)). We will show that in equilibrium the traditional firm will never enter as first, but this potential entry point may affect the entry threshold by the platform firm. Due to the investment cost advantage, the payoff of being a leader relative of being a follower, is always higher than this payoff for the traditional firm. In some cases the periods of extra profits gained by entry as leader is too short for the traditional firm to ever wish to be first, since the platform will follow soon after. This is the case under Strong Strategic Advantage.

In other cases, the traditional firm may wish to invest as first if the platform waited for too long(Weak Strategic Advantage). We first focus on this case.

A traditional firm would enter in the other segment as first, as soon as the payoff of being a leader is higher than the payoff of being a follower, that is when:

\[ \frac{F\theta_t}{r-\mu} \left[ 1 - \left( \frac{\theta_t}{\theta_T^*} \right)^{\beta-1} \right] + \left( \frac{\theta_t}{\theta_T^*} \right)^\beta \left[ \frac{D}{r-\mu} \theta_T^{**} - I \right] \geq \left( \frac{\theta_t}{\theta_T^*} \right)^\beta \left[ \frac{D}{r-\mu} \theta_T^{**} - I \right] \quad (2.18) \]
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where:

$$\theta_p^{**} = \frac{\beta}{\beta - 1} \frac{I_p (r - \mu)}{D - F}$$

(2.19)

which is the entry threshold of the platform firm as follower. The expression for \(\theta_p^{**}\) differs from (2.17) only by the different investment costs.

**Proposition 2.1 (Weak strategic advantage)**

When the platform firm enters just in time to pre-empt the traditional firm, that is, before the traditional firm would cross-enter as first. The waiting-to-cross-enter option is given by:

$$\Delta_T = (L - F) \theta_t - \left( \frac{L - D}{\theta_p^{** \beta - 1}} + \frac{D - F}{\beta \theta_T^{** \beta - 1}} \right) \theta_t^{\beta} - I (r - \mu)$$

(2.21)

**Proof.** To demonstrate this proposition we first compute when and for which demand value a traditional firm would try to pre-empt a platform firm and subsequently we show that for these demand values for the platform firm it is always convenient to pre-empt the traditional firm an instant before.

Rearranging equation (2.18) we obtain:

$$\Delta_T = (L - F) \theta_t - \left( \frac{L - D}{\theta_p^{** \beta - 1}} + \frac{D - F}{\beta \theta_T^{** \beta - 1}} \right) \theta_t^{\beta} - I (r - \mu) \geq 0$$

(2.22)

\(\Delta_T\) is a concave function\(^{17}\), is negative when \(\theta = 0\) and has a maximum for positive values of \(\theta\). Hence, the level of demand at which a traditional firm prefers to be a leader rather than follower is given by the \(\theta\) that solves \(\Delta_T (\theta) = 0\) and for which \(\frac{\partial \Delta_T}{\partial \theta} > 0\). This equation cannot be solved explicitly. However, we can define some features of \(\Delta_T (\theta) = 0\). \(\Delta_T (\theta) = 0\) has solutions when its maximum is positive, which requires that:

$$I_p \frac{I}{I} > \left( \frac{5}{4} \frac{g^{-\beta} \beta}{4^{-\beta} - 9^{-\beta}} \right)^{\frac{1}{\beta - 1}}$$

(2.23)

Since \(\Delta_T (\theta)\) is concave, if the above condition is satisfied, \(\Delta_T (\theta) = 0\) has two real roots; the smallest one is the one for which \(\frac{\partial \Delta}{\partial \theta} > 0\). We indicate this \(\theta\) as \(\theta_T^*\).

When this condition is not satisfied \(\Delta_T (\theta)\) is negative: the traditional firm prefers to be a follower rather than a leader for any demand values.

When instead \(I_p > \left( \frac{5}{4} \frac{g^{-\beta} \beta}{4^{-\beta} - 9^{-\beta}} \right)^{\frac{1}{\beta - 1}}\), the traditional firm would enter as soon as \(\theta > \theta_T^*\) such that \(\Delta (\theta_T^*) = 0\).

To demonstrate that the platform firm would cross-enter at \(\theta_T^*\) we have to take into consideration the following elements:

\(^{17}\)The concavity is due to the fact that the advantage created by the platform leads to temporary market dominance. Note that in our model, the platform does not create eternal monopoly.
2.3. The dynamic game

- due to the lower inferior investment cost, a platform firm prefers already to be a leader rather than a follower.

- when the traditional firm cross-enters as leader, it must enjoy asymmetric profits for a certain period, that is \( \theta_T^* < \theta_P^* \) otherwise the traditional firm would not cross-enter; since the platform firm would immediately follow.

- given \( L - M < D - F, \theta_P^* < \theta_m^* \).

It follows that \( \theta_T^* < \theta_m^* \), that is, the traditional firm would enter before it is optimal for the platform firm to cross-enter as a monopolist. Hence, the platform firm is forced to cross-enter at \( \theta_T^* \), a level of demand at which it would prefer to retain the option to wait to invest a bit longer. ■

The economic interpretation of condition (2.20) is that if the strategic entry advantage is large the traditional entry firm has no incentive ever to cross-enter as leader; the platform would subsequently cross-enter too soon for it to be able to recover the entry cost.

Under weak strategic advantage, for demand values immediately below \( \theta_T^* \), the payoff of being a leader for the platform firm is higher than the payoff of being a follower, but not higher than the value of the waiting-to-enter option. In this interval, the platform firm can still choose to wait because the threat of the competitor's cross-entry is not credible. However, because at \( \theta_T^* \) the traditional firm would certainly cross-enter, the platform firm will cross-enter just before \( \theta_T^* \), since it prefers to be a leader rather than a follower. In Fig. 2.2, the traditional firm for some demand values has a positive payoff being a leader relatively to being a follower: it would enter at \( T \), the point where the payoff of being leader (LP) crosses the payoff of being a follower (FO).

Hence when the platform has a Weak Strategic Advantage, it would prefer to wait longer if there was no entry threat by the traditional firm. However, the innovator firm is forced to enter at \( \theta_T^* \) to pre-empt its competitor.

Intuitively, the traditional firm wants to enter when the payoff of being a leader is higher than of being a follower, that is when \( \Delta_T(\theta) > 0 \). However when the investment difference is large, it never gets the chance to be a leader, because for any demand levels \( \Delta_T(\theta) < 0 \), the traditional firm would enjoy leadership profits, \( L \), for too short a time (or no time at all) given the early subsequent entry by the platform firm.

When condition (2.20) is not verified, the platform does not need to fear cross-entering of the traditional firm as leader because the pay-off would be negative. In this case the platform firm enjoys a de facto monopoly on the entry-as-leader option and it can decides if and when to exercise it. In some cases it will choose not to exercise it at all, which allows to preclude any cross-entry and to support parallel monopoly. We first focus on when the platform firm does prefer to enter.

Proposition 2.2 (Strong strategic advantage with entry) When

\[
\frac{I_P}{I} < \left(\frac{1}{\beta} \frac{L - D}{L - D} + 1\right) \left(\frac{L - M}{D - F}\right)^{\frac{1}{\beta - 1}}
\]  

(2.24)
Figure 2.2: Entry strategies under Weak Strategic Advantage (PV=Platform Value, PP=Platform Pay-off, LP=Traditional firm Payoff as Leader, FO=Value of Entry as Follower for the traditional firm, FP=Follower Payoff of the traditional firm, E=Entry Threshold for the Platform firm, T=Entry Threshold for the traditional firm as leader, M=Entry Threshold for the Platform in case of Monopoly, F=Traditional Firm Entry Threshold as Follower), \((\sigma = 0.3, \mu = 0.01, r = 0.05, a = 0.125, I = 100, I_P = 80)\)

and

\[
\frac{I_P}{I} < \left( \frac{5}{4} \right)^{\frac{1}{\beta-1}} \left( 5^{\frac{1}{\beta-1}} - 9^{-\beta} \right) \tag{2.25}
\]

the platform firm can behave de facto as a monopolist on the entry as leader option. It enters at \(\theta_m^* = \frac{\beta}{\beta-1} \frac{I_P(r-\mu)}{L-M}\) and the waiting-to-cross-enter option is given by:

\[
V_{\theta,2} = O_M \theta^2 = \left[ \frac{L-M}{\theta_m^{\beta-1} \beta (r-\mu)} - \frac{L-D}{\theta_T^{*\beta-1} (r-\mu)} \right] \theta^2 \tag{2.26}
\]

**Proof.** See Appendix 2.C for a demonstration. ■
2.3. The dynamic game

Hence in case of a de facto monopoly the platform firm will choose to cross-enter when the expected profits of cross-entering are higher than the wait-to-cross-enter option. When the strategic advantage is strong enough, that is when condition (2.24) is verified, a market threshold value exists such that it is optimal to exercise the option. For this market value the profits of the temporary asymmetric duopoly profits plus the subsequent parallel duopoly profits are higher than the waiting-to-cross-enter option. When instead this condition is not verified, this threshold value does not exists: the waiting-to-cross-enter option is always more valuable than the expected profits that derive from cross-entering.

**Proposition 2.3 (Strong Strategic Advantage with no entry)** When

\[
\frac{I_P}{I} > \left( \frac{L - M}{\beta L - D} \right)^{\frac{1}{\beta - 1}} L - M \quad D - F \tag{2.27}
\]

and

\[
\frac{I_P}{I} < \left( \frac{5 \cdot 9^{-\beta} \beta}{44 - 9^{-\beta} - 9^{-\beta}} \right)^{\frac{1}{\beta - 1}} \tag{2.28}
\]

the platform firm, which can behave de facto as a monopolist on the entry as leader, prefers not to enter at all. As a result, the market remains in a parallel monopoly.

**Proof.** Under \( \frac{I_P}{I} < \left( \frac{5 \cdot 9^{-\beta} \beta}{44 - 9^{-\beta} - 9^{-\beta}} \right)^{\frac{1}{\beta - 1}} \), the platform firm can behave de facto as a monopolist. But when \( \frac{I_P}{I} > \left( \frac{L - M}{\beta L - D} \right)^{\frac{1}{\beta - 1}} L - M \) it does not find it convenient to cross-enter.

At the same time entry by the traditional firm would also be not convenient as it would lead to an early cross-entry by the platform firm, for which now entry is a dominating strategy.

It follows that the platform firm does not enter and both firms prefer to remain in parallel monopoly. 

Under conditions (2.27) and (2.28) the platform firm can still de facto behave as a monopolist, but at the same time has no incentive to invest in the other market because the expected profits would be lower than in the case of parallel monopoly.

As depicted in Fig. 2.3, a traditional firm would never cross-enter under the threat of a subsequent entry of the platform as follower: his payoff of being a leader rather than a follower \((LP)\) is always negative. Hence the platform can be seen as a tool to avoid aggressive cross-entry, which is in some cases less desirable than parallel monopoly.

In conclusion, with platform investment we can have 3 equilibria:

- **Weak Strategic Advantage:** when the strategic entry advantage is not so large, the platform firm faces a cross-entry threat of the traditional firm before its optimal timing of entry, \( \theta_m^* \). Hence, the platform firm enter much earlier than it would like: it enters just before \( \theta \) reaches \( \theta_T^* \), inducing a later cross-entry by the competitor at \( \theta_T^* \).
Figure 2.3: Strong Strategic Advantage entry strategies. (PV=Platform Value, PP=Platform Pay-off, LP=Leader Pay-off of the traditional firm, FO=_FOLLOWer Option of the traditional firm, FP=Followe r Pay-off of the traditional, E=Entry of the platform, M= Entry of the platform in case of Monopoly, F=Traditional Firm Entry as Follower), \((\sigma = 0.3, \mu = 0.01, r = 0.05, I = 100, I_p = 10, a = 0.125)\).
2.4. Uncertainty analysis

- Strong Strategic Advantage with no entry: the platform firm has the option to behave as a monopolist on entry, but at the same time has no incentive to cross-enter because the payoff is not high enough. The outcome is a parallel monopoly with no entry.

- Strong Strategic Advantage with entry: when the strategic entry advantage is large enough and uncertainty is low, the platform firm gains the option to behave as a monopolist on the cross-entry option and it can disregard the risk of an entry as leader by the competitor. Hence the platform firm cross-enters at $\theta_m^*$, trading off the waiting-to-invest option against the gain from entry and the traditional firm at $\theta_T^{**}$.

A general conclusion from these results is that the conclusions from the real option approach to investment do not apply trivially to circumstances of imperfect competition. When firms do not have a monopoly on the option to enter, they cannot time their entry optimally except when they have acquired a significant entry advantage. However, even acquiring a weak strategic advantage does effect the timing of potential competitor entry.

We focus our comparative static analysis to study a second claim of the real option literature, namely the higher uncertainty induces firms to delay new investment. Kulatilaka and Perotti (1998) have already shown in a static model that the effect is ambiguous under imperfect competition. The next section investigate the optimal timing of entry in continuous time.

2.4 Uncertainty analysis

We now investigate the impact of demand uncertainty on the timing of entry and on the value of the platform at the point of entry. Note that because of convexity of profits in demand, expected profits increase with demand uncertainty. The uncertainty may encourage entry as it offers convex payoffs from the capture of market share in the other segment, whose expected value increases with volatility. However, the value of the waiting to invest option also increases with uncertainty.\footnote{We do not analyse the effect of an increased risk premium as systematic risk increases, which reduces the value of entry relative to waiting to enter (Kulatilaka and Perotti (1998)).}

Uncertainty also affects the competitor entry behaviour. As we have shown, whatever is the strategic advantage of the platform firm, for higher uncertainty levels the follower decides to cross-enter at higher demand values. However, given the higher volatility, this point is reached sooner and so the expected time of cross-entering of the follower is ultimately shorter.

We analyse first the impact of uncertainty on the condition (2.24) that distinguishes between Strong (with and without entry) and Weak Strategic Advantage. We next analyse its impact on the expected profits in case of Strong Strategic Advantage with entry, with no entry and in case of Weak Strategic Advantage. Later we solve for the value of the platform investment and the impact of uncertainty.
Chapter 2. Entry with Strategic Advantage

The two conditions that determines the three entry conditions of the platform firm are represented as function of demand uncertainty in Figure 2.4.

The condition for the Strong Strategic Advantage (see equation (2.24)) has a monotonic behaviour in respect to market uncertainty. Ceteris paribus, an increase in uncertainty increase the range of parameters under which the platform investment leads to Strong Strategic Advantage. This is because the value of cross-entry as leader for the traditional firm declines, as it would enjoy a shorter period of asymmetric profits before the subsequent cross-entry by the platform firm.\textsuperscript{19}\textsuperscript{20} Thus in these cases, greater uncertainty enhance the value of the platform.

Under Strong Strategic Advantage, the higher is demand uncertainty, the more the no entry case is attractive: the gain for the platform firm to cross-enter is lower than the value of a parallel monopoly. For high demand uncertainty there is no risk that the traditional firm would enter as leader; at the same time also cross-entering for the platform firm is not very attractive because the period in which it could enjoy asymmetric duopoly profits is too short. Hence it may prefer no-entry parallel monopoly to direct competition.

In case of Strong Strategic Advantage, we show that an increase in demand uncertainty induces a platform firm to postpone cross-entry. Formally:

\textbf{Proposition 2.4} Under Strong Strategic Advantage a mean-preserving increase in

\textsuperscript{19}The period of higher profits for the platform firm is longer, as its competitor has a higher entry cost.

\textsuperscript{20}The only values of the investment ratio for which there can be a switch between Strong and Weak Strategic Advantage are between 0.44 and 0.63; for investment cost ratio higher than 0.63 the platform firm has a Weak Strategic Advantage; otherwise it enjoys a Strong Strategic Advantage.
demand uncertainty (defined as an increase in increase in \( \mu \) holding the market drift constant) leads to an entry point at higher market values by the platform firm.

**Proof.**

\[
\frac{\partial \theta_m^*}{\partial \mu} = \frac{2rI_P - (2\mu - \alpha) \sqrt{\alpha^2 + 4r(\mu - \alpha) + 2(r - \mu)(\mu - \alpha) + \alpha^2}}{(L - M) \left( \sqrt{\alpha^2 + 4r(\mu - \alpha)} \left( \sqrt{\alpha^2 + 4r(\mu - \alpha) + \alpha - 2\mu} \right) \right)}
\]

which can be seen after some tedious calculations to be always positive. ■

From this Proposition it results clear that under Strong Strategic Advantage with entry, the comparative static effect of greater uncertainty is similar to the case of the traditional real option theory: the higher is uncertainty, the longer the platform firm waits to cross-enter (in fact an increase in demand uncertainty also induces the traditional firm to wait longer as follower to cross-enter). The platform firm prefers to wait longer for two reasons. The first one is in line with classic real option theory: higher uncertainty induces the platform firm to wait longer in order to avoid the risk of losses due to weak demand. At the same time, the traditional firm delays its timing of entry as follower; given the convexity of the post-entry extra-profits, waiting for the platform firm becomes more attractive than immediate asymmetric profits at low market demand levels.

The value of the platform firm at the point of entry reflects the present value of the expected profits during the period of asymmetric profits plus those during the subsequent period of parallel monopoly. Both these expected profits increases with market uncertainty. Hence as higher uncertainty induces an increase of the underlying payoff and an increase in option value (holding constant the payoff), we can conclude that the platform firm value at the point of entry increases for higher uncertainty values (See Fig. 2.5).

Under **Strong Strategic Advantage with no entry**, the platform value is given by the present value of the profit in parallel monopoly: \( \frac{M}{r - \mu} \). As an increase of uncertainty determines an increase in \( \mu \), we have that the higher is uncertainty, the higher is platform value.

Under **Weak Strategic Advantage**, a change in uncertainty can have either a positive or negative effect on the cross-entry threshold.

There are three elements that affect this decision. The higher the uncertainty, the higher is the option value to be a follower; at the same time, also the payoff to be a leader is higher (due to the endogenous convexity of the profits). The third element is the timing of the follower entry: the higher the uncertainty, the higher the entry threshold of the follower, which allows the first cross-entrant to enjoy asymmetric profits for a longer time.

In this case the total effect of uncertainty on the demand threshold of entry and thus the present value of the profits of the platform firm is not monotonic. In particular, the effect depends on the investment cost ratio, our measure of the strategic advantage gained by the platform investment.

Recall that under weak strategic advantage, the timing of entry of the platform firm is exactly at the point of entry of the traditional firm as leader. When the
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Figure 2.5: Entry threshold and platform value before and after entry for different uncertainty values ($J_p = 10, I = 100, r = 0.05, \alpha = 0, \mu = 0.01$ or $0.02$)
2.4. Uncertainty analysis

Figure 2.6: Entry threshold for a platform firm as a function of market uncertainty for different investment cost ratios (Weak Strategic Advantage).

strategic advantage is not very high, the cross-entry threshold of the traditional firm increase with demand uncertainty: thus the higher the uncertainty, the earlier the platform firm is forced to enter pre-emptively. Ceteris paribus, because of the higher expected profit from entry, (since the potential losses avoided by waiting to invest are less convex than the potential higher gains under entry) by Jensen’s inequality, a mean-preserving increase in uncertainty favors exercising the entry option over the waiting option.²¹

The total effect of uncertainty on entry behavior is not monotonic. When uncertainty is low, as uncertainty increases, the relative stability of the demand level induces the traditional firm to wait longer to enter. At a high uncertainty level, the expected period of higher profits for the traditional firm increases and so it would enter earlier (see Fig. 2.6).

This reasoning can also explain the non-monotonicity of the present value of the platform firm with respect to demand uncertainty: for low uncertainty levels the present value increases, but it decreases for very high uncertainty. Figure 2.7 show that the platform value at different cost advantage is a direct consequence of the expected period of asymmetric profits after cross-entry as leader.²²

²¹This behavior recalls the results in a static setting: the higher profit convexity leads to a greater increase in the value of the entry option relative to the waiting to invest option (Kulatilaka and Perotti (1998)).

²²Note that as a direct consequence of the entry strategy of the traditional firm, the expected period of asymmetric profit for the platform firm is constant with respect to the investment ratio.
2.5 The Ex-ante Value of the Platform

Until now we have assumed that the incumbent has a traditional technology, while the platform is developed by an innovating firm at a higher cost.

When would it be convenient for an innovative firm to build a platform rather than making a conventional entry? What is the ex-ante value of a platform?

We first compute the value of the option to acquire the platform (i.e. the value of the underlying technology). Later we compute for which platform cost the innovative firm finds it convenient to buy the platform (i.e. the right to produce with the platform). Finally, we consider when it would start production in the first segment immediately, or wait for a later optimal time to start production.

2.5.1 Strong Strategic Advantage

We analyze first at what demand value a new firm with a platform would enter in the first market segment, and subsequently when it would acquire a platform. As the traditional firm is assumed to be the incumbent in one market segment, the platform firm will find convenient to avoid direct competition and enter initially on the other market segment.\(^{23}\)

We first compute the platform value in case of strong strategic advantage.

\(^{23}\)Either in strong or weak strategic advantage the platform firm would never choose to enter first in the same market segment of the incumbent. Subsequent cross entry would be more attractive as the profit jump would be higher; however, this does not compensate for the lower profit flow during the first part of the game when the two firms compete in the same market.
2.5. The Ex-ante Value of the Platform

In case of strong strategic advantage with entry, computing the platform value and when the innovative firm would enter in the first market segment means computing the value of an American compound option and its exercise threshold. The calculations of this follows the similar steps of Appendix 2.C. The compound option value is of the type:

\[ V_{S,1} = O_{S,1} \theta^3 \]  

(2.30)

The constant term, \( O_{S,1} \), is given by:

\[ O_{S,1} = O_M + \theta_{S,1}^\beta \frac{M}{\beta (r - \mu)} = O_M + \theta_{S,1}^\beta \frac{I_P}{\beta - 1} \]  

(2.31)

where \( O_M \) is given by (see Appendix 2.C for the relative calculations):

\[ O_M = \frac{L - M}{\theta_m^{\sigma+1} (r - \mu)} - \frac{L - D}{\theta_T^{\sigma+1} (r - \mu)} \]  

(2.32)

It follows that optimal entry threshold will be in correspondence of the following \( \theta \), that we define \( \theta_{S,1} \):

\[ \theta_{S,1} = \frac{I_P (r - \mu)}{M \beta + 1} \]  

(2.33)

It is now possible to calculate the demand threshold to acquire a platform. We also rule out the possibility that the traditional incumbent firm is capable of building the platform. We assume that a platform can be built only by one firm; we do not describe the initial "race to the platform".\(^{24}\)

If there are many firms with the opportunity to invest in the unique technology to build the platform, the option to acquire the platform will be exercised as soon as the expected profits are above the investment cost, that is when \( \theta \) reaches \( \theta_{S,0} \).\(^{25}\)

\[ \theta_{S,0} = \left( \frac{K}{O_{1, S}} \right)^{\frac{1}{\beta}} \]  

(2.34)

where \( K \) is the cost of the platform.

The innovative firm will acquire the platform as soon as the initial market demand reaches a threshold \( \theta_{S,0} \) (which is smaller than \( \theta_{S,1} \)), such the investment cost is less than the platform value:

\[ K < O_M \theta_{S,1}^\beta + \frac{I_P}{\beta - 1} \]  

(2.35)

\(^{24}\)An additional argument is that an incumbent firm would not build a platform just to enter in the second market, as traditional entry would be cheaper. A new firm can use the platform to enter both segments so it finds convenient to acquire it for lower demand values than the incumbent firm.

\(^{25}\)This is a different situation than in a classical real option investment environment where the firm holds a monopoly on the investment opportunity.
At the point when the firm invests in the platform, it will not necessarily choose to enter in a market segment.

When demand parameters (e.g., low drift, low uncertainty, high discount rate, low product differentiation) are such that condition (2.35) is not verified, a new firm will invest in the platform and in the first product simultaneously.

As one can easily see, the higher is the demand uncertainty, the less stringent is this condition. In a highly volatile market, expected margins are high; the firm then prefers to guarantee for itself a strategic advantage by acquiring the platform at the same time it waits longer (i.e., for a better level of market demand) to become active in the market. The impact of uncertainty on the platform value (measured by the reciprocal of $\beta$) is unambiguously positive.

2.5.2 Weak Strategic Advantage

In case of weak strategic advantage, the entry cost advantage gained by the platform firm is not large. The platform value can be computed following the same steps for the other case as:

$$V_W = O_{W,1} \theta^3$$

(2.36)

where:

$$O_{W,1} = \frac{I_P}{\beta - 1} \theta_{W,w}^\beta + \left( \frac{1}{\theta_T^*} \right)^\beta \left( \frac{L - M}{r - \mu} \theta_T^* - I_P \right) - \frac{L - D}{r - \mu} \left( \frac{1}{\theta_T^{**}} \right)^{\beta-1}$$

(2.37)

The entry threshold to enter in the first product for a firm which has already acquired the platform is:

$$\theta_{W,1} = \frac{I_P (r - \mu)}{M} \frac{\beta}{\beta - 1}$$

(2.38)

that is, at the same demand threshold as $\theta_{S,1}$ (see equation (2.31)).

We can now calculate when it is convenient to invest in the platform. The threshold value is given by:

$$\theta_{W,0} = \left( \frac{K}{O_{W,0}} \right)^{\frac{1}{\beta}}$$

(2.39)

The platform firm will invest in the platform before entering in the first market segments, when $\theta_{S,0}$ is smaller than $\theta_{S,0}$, that is when:

$$K < \frac{I_P}{\beta - 1} + \left( \frac{\theta_{W,0}}{\theta_T^*} \right)^\beta \left( \frac{L - M}{r - \mu} \theta_T^* - I_P \right) - \frac{L - D}{r - \mu} \theta_{W,0} \left( \frac{\theta_{W,0}}{\theta_T^{**}} \right)^{\beta-1}$$

(2.40)

26This constant is positive when $\frac{I_P}{\beta - 1} > \left( \frac{M}{L - D} \right)^{\frac{1}{\beta}} \frac{M}{D - F}$. This is always true since $1 > \left( \frac{M}{L - D} \right)^{\frac{1}{\beta}} \frac{M}{D - F}$ and given the assumption on the investment costs.
2.6. Conclusions

otherwise it will invest in the platform and in the first product simultaneously.\textsuperscript{27}

This condition is more stringent than under Strong Strategic Advantage, when early entry is more profitable.

After tedious calculations, it can be shown that the higher is the uncertainty, the higher is the value of the platform. The more demand is volatile, the more it becomes more attractive to secure control of the platform for itself even without the immediate profits of the first product. At the same time, the timing of platform acquisition will occur earlier.

2.6 Conclusions

We have considered a strategic logistic investment under dynamic uncertainty on future consumer demand. An innovative firm acquires a productive and distributive platform (perhaps via an acquisition) to gain superior entry advantages. We study the subsequent entry decision in specific products and compute the value of the option to wait in favor against the (temporarily) higher profits of immediate entry. We can rationalize a much greater value for platform firms relative to traditional producers. Platform investment turns out to have several strategic effects. It may grant a competitive advantage to enter earlier in competitors’ market segments; but it also eliminates the possibility of simultaneous entry and may help avoiding undesirable "excessive" competition equilibria.

Investing in a platform means acquiring a "strategic cross-entry option", a concept related to the "strategic growth option" (Kulatilaka and Perotti (1998)). Platform investment involves absorbing significant demand uncertainty; therefore, the value of not committing funds to platform investment increases with demand uncertainty. Yet we obtain that in general, in a regime of higher uncertainty of demand, \textit{platform strategic advantage increases more than the value of the option to wait. In fact, a platform firm may ends up "controlling" the entry options, and behaving more as a de facto monopolist.}

Our surprising results in the positive effect of uncertainty on the value of investment are driven by the oligopolistic market structure. Firms with market power respond to higher demand by increasing both output and prices; therefore, profits are endogenously convex in demand. High uncertainty gives more weight to potential high profits than to the greater risk of lower profits under low demand. Since the potential losses avoided by waiting to invest are less convex than the potential gains from entry, more uncertainty (in the sense of a mean-preserving spread in demand) reduces the threshold of expected future demand at which the firm finds attractive to enter, and increases the value of the entry options relative to the waiting to invest option.

In our context, greater uncertainty means greater risk but also greater opportunities thanks to the early (cross-)entry advantage gained. A limit of the model is the very simplified notion of strategic advantage we have adopted. In future we plan to

\textsuperscript{27}We abstract here from the possibility of the traditional firm entering both market segments in order to pre-empt entry.
model more exactly the nature of platform investments well as the initial decision on the size of the platform. A broader platform may allow entrance in unrelated market segments without cross-effects.

Intuitively, the optimal platform size (which is related to the potential range of differentiated products) depends on the fact that the greater is the initial differentiation, the weaker are the cross-price effects. The outcome of this decision is not trivial, as it depends on product features and expected market demand evolution.
2.A. Competition analysis

Appendix

2.A Competition analysis

2.A.1 Parallel monopoly

Consider first the case when both firms are monopolist in one market. They will choose their output taking into account the impact of the output by the other firm on its own marginal profitability. We assume that firm $i$ is active in market 1 and firm $j$ in market 2. The profit maximization problem for firm $i$ can be summarized as follows:

$$\text{Max}_{q_{1i}}(p_1q_{1i}) = \text{Max}_{q_{1i}} \left[ \sqrt{\theta} - (q_{1i} + q_{1j}) - a(q_{2i} + q_{2j}) \right] q_{1i} \quad (2A.1)$$

Firm $j$ has a symmetric profits maximization problem. The outcome of the game is symmetric:

$$q_{1i} = q_{2j} = \frac{\sqrt{\theta}}{a + 2} \quad (2A.2)$$

$$p_1 = p_2 = \frac{\sqrt{\theta}}{a + 2} \quad (2A.3)$$

$$\Pi_i = \Pi_j = \frac{\theta}{(a + 2)^2} \equiv M\theta \quad (2A.4)$$

The expected present value of all future profits as $t$ goes to infinity is given by:

$$E \left[ \int_0^\infty M\theta e^{-rt}dt \right] = \frac{M}{r - \mu} \quad (2A.5)$$

Notice that this coincide with the classic constant growth valuation model.

2.A.2 Duopoly in both markets (parallel duopoly)

We consider now the case when both firms are active in both markets. The outcome of the game in this case is symmetric:

$$q_{1i} = q_{1j} = q_{2i} = q_{2j} = \frac{1}{3} \frac{\sqrt{\theta}}{a + 1} \quad (2A.6)$$

$$p_1 = p_2 = \frac{1}{3} \sqrt{\theta} \quad (2A.7)$$

Note that prices are the same, as in the classic Cournot outcome; however, quantities are lower as firms take into account the demand cross-effect (which is increasing in $a$).

Profits are also symmetric and decreasing in $a$:

$$\Pi_i = \Pi_j = \frac{2}{9} \frac{\theta}{a + 1} \equiv D\theta \quad (2A.8)$$
for an expected present value equal to:

\[ E \left[ \int_0^\infty D\theta e^{-rt}dt \right] = \frac{D}{r - \mu} \quad (2A.9) \]

### 2.A.3 Asymmetric competition

The last case is where in market 1 we have a duopoly while in market 2 we have a monopoly; firm \( j \) is in both products while (firm \( i \)) is only in the first one.

The strategic output choice of this asymmetric competition is:

\[ q_{i1} = \frac{1}{3} \sqrt{\theta} \quad (2A.10) \]
\[ q_{ij} = \frac{1}{6} \left( 2 - a \right) \sqrt{\theta} \quad (2A.11) \]
\[ q_{2j} = \frac{1}{2} \frac{\sqrt{\theta}}{a + 1} \quad (2A.12) \]

which results in differentiated prices:

\[ p_1 = \frac{1}{3} \sqrt{\theta} \quad (2A.13) \]
\[ p_2 = \frac{\sqrt{\theta}}{2} \left( \frac{1 - a}{3} \right) \quad (2A.14) \]

Note that prices in the monopolized product (product 2) equal the monopoly price even, though there is a cross-effect. The reason is that the firm \( j \) which is monopolist in the other product chooses to produce less than the Cournot amount of product 1 to keep up profits in the segment it controls. The resulting profit levels for the firm \( j \) in both product equal:

\[ \Pi_{1j} = \frac{1}{9} \theta \quad (2A.15) \]
\[ \Pi_{2j} = \frac{1}{36} \theta \left( a - 3 \right) \frac{a - 2}{1 + a} \quad (2A.16) \]

For the firm present in both markets, the sum of the profit flow from the two markets is given by:

\[ \Pi_N = \frac{1}{36} \frac{13 - 5a}{1 + a} \theta \equiv L\theta \quad (2A.17) \]

and its present value is:

\[ E \left[ \int_0^\infty A\theta e^{-rt}dt \right] = \frac{L}{r - \mu} \quad (2A.18) \]

The other firm earns:

\[ \Pi_i = \frac{1}{9} \theta \equiv F\theta \quad (2A.19) \]
Thus the expected present value is:

\[
E \left[ \int_0^\infty L \theta e^{-r t} dt \right] = \frac{F}{r - \mu} \quad \text{(2A.20)}
\]

Notice also that the equilibrium value of \( p_i \) is the same as in the classical Cournot duopoly. This means that in terms of expected profit margins firm \( i \) is indifferent whether the other firm is present in the other market or not.

Note finally that when \( a \) tends to 1 (minimum differentiation) the results tends to correspond to the classical Cournot case, while if it tends to 0 (maximum differentiation) they tend to the classical monopoly case.

2.B Timing of entry

We first demonstrate that the firm with the platform is entering before if the competitor has not invested in the platform.

If the curve of the threshold value for the asymmetric case is always above the one for the asymmetric case where both firms have invested in the platform, this implies that the leader with the platform in the asymmetric case enters before than in the symmetric case.

Analytically this means that the following relation has to always verified:

\[
\frac{L - F}{r - \mu} \theta - \frac{L - D}{r - \mu} \left( \frac{\theta}{\partial \theta^*} \right)^{\beta - 1} - \frac{D - F}{r - \mu} \theta \left( \frac{\theta}{\partial \theta^*} \right)^{\beta - 1} - \left[ 1 - \left( \frac{\theta}{\partial \theta^*} \right)^{\beta - 1} \right] I_P > 0
\]

Rearranged:

\[
\frac{L - F}{r - \mu} \left( \frac{1}{\theta_1^{**}} - \frac{1}{\theta_1^{**}} \right) > 0
\]

This is always negative because \( \theta_1^{**} > \theta_1^{**} \).

Due to the difference in the investment costs between the platform and the non platform investment when the firms have invested both in the platform the leaders enters before than when they have invested in non platform. This implies that the firm with the platform will be the leader in the when the two firms have invested differently and will invests before than in the case both firms have platform or have no platform.

2.C Strategic entry value of a monopolist entry option

We want to demonstrate that when the firms cross-enters as soon as it prefers to be a leader rather than a follower, it enters always before than a firm with similar
Chapter 2. Entry with Strategic Advantage

features but with the monopoly on the option to cross-enters. To demonstrate this we first calculate the option value and the threshold \( \theta \) of the option if the firm with platform has the monopoly to enter in the second market as first. Second we see when it has economic sense for the firm with the monopoly to ever cross-enters and finally we compare this with the case in which the firm cross-enters as soon as it is convenient for her to be a leader.

As the firm has the monopoly to enter in the second market, the solution is found solving a similar differential equation as in Dixit and Pindyck (1994), but with slightly different value matching and smooth pasting condition. In this case in fact we have to take into account the fact that at \( \theta_1^* \) the other firm cross-enters. So the new differential equation and its conditions are:

\[
\frac{1}{2} \sigma^2 \theta^2 G''(\theta) + \mu \theta G'(\theta) - rG(\theta) + M\theta = 0
\]  
\[\text{for } G(0) = 0 \]  
\[G(\theta_0^*) = O_M \theta_0^\beta + M \frac{\theta_0^{\beta-1}}{r-\mu} = L \frac{\theta_0^{\beta-1}}{r-\mu} \left[ 1 - \left( \frac{\theta_0^{q_\beta}}{q_\beta} \right)^{\beta-1} \right] + \left( \frac{\theta_0^{q_\beta}}{q_\beta} \right)^{\beta-1} \frac{D}{r-\mu} - I_P \]  
\[G'(\theta_0^*) = \beta O_M \theta_0^\beta - M \frac{\theta_0^{\beta-1}}{r-\mu} = L \frac{\theta_0^{\beta-1}}{r-\mu} \left[ 1 - \beta \left( \frac{\theta_0^{q_\beta}}{q_\beta} \right)^{\beta-1} \right] + \beta \left( \frac{\theta_0^{q_\beta}}{q_\beta} \right)^{\beta-1} \frac{D}{r-\mu} \]

It follows that the option value is given by \( O_A \theta^\beta \) and the threshold value is given by \( \theta_0^* \) where:

\[
\theta_0^* = \frac{\beta I_P (r - \mu)}{\beta - 1} \frac{L - M}{r - \mu}
\]

\[
O_M = \frac{L - M}{\theta_0^* \beta^\beta (r - \mu) - \theta_0^{\beta-1} (r - \mu)} \]

\( O_M \) can be negative. Substituting the entry values and rearranging \( O_M \) becomes:

\[
O_M = \frac{I_P}{\beta - 1} \left[ (L - M)^\beta - \beta (L - D)(D - F)\beta^{-1} \left( \frac{I_P}{I} \right)^\beta \right] \left[ I_P (r - \mu) - \frac{\beta}{\beta - 1} \right]^{-\beta}
\]

\( O_M \) is then positive when the second term is positive, that is when:

\[
\frac{I_P}{I} < \left( \frac{1}{\beta} \frac{L - M}{L - D} \right)^{\frac{1}{\beta - 1}} \frac{L - M}{D - F}
\]

The second term is always smaller than 1 the above condition is not always verified. However, this is not relevant for our model because for the interval of \( \gamma \) we are interested in, the option plus the discounted profit give a positive value.

When \( \frac{I_P}{I} < \left( \frac{1}{\beta} \frac{L - M}{L - D} \right)^{\frac{1}{\beta - 1}} \) the platform firm can behave de facto as a monopolist on the entry option because for the traditional firm entry as leader is not convenient.

When \( \frac{I_P}{I} < \left( \frac{1}{\beta} \frac{L - M}{L - D} \right)^{\frac{1}{\beta - 1}} \frac{L - M}{D - F} \), the constant term of the option is positive thus the platform firm finds convenient to cross-enter because its payoff is higher than the payoff of remaining in a parallel monopoly.

Hence the platform firm enter at \( \theta_0^* = \frac{\beta}{\beta - 1} \frac{I_P (r - \mu)}{L - M} \).
2.D Pre-emptive entry by the platform firm

In this Appendix we want to demonstrate that when the conditions for the strategic advantage are verified, \( \theta^m = \frac{\theta I^p (r - \mu)}{\pi - M} \) is always smaller than when the traditional firm would enter as leader.

The traditional firm would enter when it prefers to be leader rather than a follower, that is when:

\[
\Delta_T (\theta) = (L - F) \theta - \left[ \frac{L - D}{\theta^*_T} + \frac{D - F}{\theta^*_T} \right] \theta^3 - \left[ 1 - \left( \frac{\theta}{\theta^*_T} \right)^3 \right] I (r - \mu) = 0 \quad (2D.29)
\]

We want then to see when \( \Delta_T (\theta_m) \) is positive or negative. We express the above condition in terms of the investment ratio:

\[
\Delta_T (\theta_m) = \frac{\beta}{\beta - 1} \left[ \frac{L - F}{L - M} \right] \left[ \frac{D - F}{L - M} \right] \left[ \frac{L - D}{D - F} + \frac{1}{\beta} \left( \frac{I}{I^p} \right)^{\frac{1}{\beta - 1}} \right] - \frac{I}{I^p} \quad (2D.30)
\]

First of all, when \( \frac{L}{F} \) tends to zero, \( \Delta_T (\theta_m) \) tend to minus infinity and the platform firm would enter before the traditional firm.

Second it can be noticed that the first two parts of \( \Delta_T (\theta_m) \) is always smaller than 1 while the second is always bigger than 1. It follows that \( \Delta_T (\theta_m) \) is always negative. Since there are no values of the ratio \( \frac{I}{F} \) for which \( \Delta_T (\theta_m) \) is positive, platform firm when it has the monopoly to enter is entering always before the traditional firm would enter strategically.