Chapter 4

The Price of Rapid Exit in Venture-backed IPOs

4.1 Introduction

Recent years have witnessed an impressive increase both in the number of the Initial Public Offerings (IPOs) and in the level of underpricing. These twin phenomena are the distinctive elements of what is called "hot issue" market. In the same period, venture capitalist backed IPOs resulted more underpriced than non venture capital backed ones (Ljungqvist (1999) and Franzke (2001)). This event is not common as in general the opposite occurs, i.e. venture capitalist backed firms are less underpriced than non venture capital backed ones. This paper sees these phenomena as interrelated and offers an explanation. The decision to take a firm public is a consequence of the decision to exit from the actual firm to enter in a new and more productive investment opportunity. Hence the higher the profitability, the more the financiers are available to pay to disinvest; the different underpricing strategy of venture capital and non venture capital backed firms is related to the different roles that the financiers plays in the firm.

The existence of "hot issue" cycles was already discussed in Ibbotson and Jaffe (1975): Ibbotson and Ritter (1995) reports examples of IPOs clustering with higher underpricing for the last 40 years in different countries.

Figure 4.1 and 4.2 report the IPO volume and the degree of underpricing in 1960-2001 for the US. The IPO cluster of the early '80 according to Ritter (1984) was due to an exceptional issue wave in the natural resource industry: in that period high oil prices caused an extraordinary favourable situation in the oil sector, and many start-up companies related to the natural resources sector were taken public in highly underpriced IPOs. In the second half of the nineties, the diffusion of the use of Internet and new communication services triggered an impressive IPO wave, coupled with strong underpricing. A "cold issue" market occurred at the end after the market crash of '87, the IPOs volume fell by almost 80% and average underpricing halved relative to the previous year.

Lowry and Schwert (forthcoming) and Hoffmann-Burchardi (2001) have shown that periods of heavier underpricing and higher IPO volume over the last 40 years
Figure 4.1: IPO monthly numbers from Jan. 1960 to Aug. 2001 (Source: http://bear.cba.ufl.edu/ritter/pool.htm)
4.1. Introduction

Figure 4.2: IPO Average Initial Returns from Jan. 1960 to Aug. 2001 (Source: http://bear.cba.ufl.edu/ritter/ipol.html)
were not simultaneous, but rather positively correlated. Usually, periods of heavy underpricing are followed by periods of high IPO volume.

Ritter (1984) does not find evidence that the "hot issue" market anomaly is due to a difference in risk, namely whether the underpricing is higher in periods of predominantly high-risk IPOs.

Benveniste and Spindt (1998) and Loughran and Ritter (forthcoming) see the underpricing as the cost the underwriter has to pay to collect information about the true value of the firm. While these papers offer an explanation of the variation over time of underpricing, they do not explain the variation in IPO volume and its positive relation with underpricing.

Benveniste, Busaba and Wilhelm (forthcoming), Benveniste, Ljungqvist, Wilhelm and Yu (2001) and Lowry and Schwert (forthcoming) elaborate the idea of Benveniste and Spindt (1998) and suggest that information spillovers in the learning process associated with an IPO induce firms from the same sector to go public in the same period. Underpricing is the cost to be paid to gather information from investors about the sector. Each time a firm is taken public, more information is revealed about the sector, reducing uncertainty over other sector firms' value. This reduces the cost to go public and hence increases the number of firms that are willing to go public.1

Other papers interpret these anomalies as the result of irrationality of overoptimism among investors and analysts (Rajan and Servaes (1997) and Ljungqvist, Nanda and Singh (2001)). In periods of over optimism, high post-IPO primes induce more firms to take advantage of the favourable situation and go public.

This paper offers a new explanation to the hot issue puzzle. We model the IPO as an exit decision of an investor seeking to exploit other investment opportunities. While in most models of the IPO process the entrepreneur decides to bring the firm public for either diversification or fund rising, we interpret the IPO mostly as exit channel (Black and Gilson 1998). In the basic model, an early investor or the entrepreneur himself brings the firm public to raise funds to invest in a new opportunity. In order to be able to sell, the investor is willing to incur some underpricing. Both the stake sold and its pricing are needed to signal the value of the present and the future investment. Though the new investment opportunity return does not affect directly the payoff of the outside investors, this information is needed to perceive fully the trade-off faced by the early stage investor and thus to interpret correctly the signal on the value of the IPO firm. We obtain a unique equilibrium where the fraction of shares retained signals the present firm value, while the underpricing signals the investment opportunity profitability.

Allen and Faulhaber (1989), Grinblatt and Hwang (1989) and Welch (1989) interpret the underpricing as a signal of the firm value. However, no convincing empirical support has been found (Garfinkel (1989), Jegadeesh, Weinstein and Welch (1993) and Michaely and Shaw (1994)). In our model, the underpricing signals the

1There is uncertain evidence on sector clustering. While Ritter (1984) can point to the boom in the oil sector to explain the '80 hot market, Helwege and Liang (2001) finds less industry clustering during hot periods. Their conclusions could be influenced by the definition of hot issue market adopted.
expected profitability of a distinct firm (the new investment opportunity).

The model predicts that, the better the new opportunities, the more eager is the investor to sell, and then the heavier the underpricing. Hence, periods of particularly profitable new investment opportunity create periods of heavy underpricing. Subsequently, as soon as the new investments become mature, they are taken public and so a higher volume of IPOs occurs. This implies that, in periods of economic expansions with many new investment opportunities, there is more underpricing followed by more IPOs. Vice versa, in periods when there are few and poorly profitable investment opportunities, exiting investors retain a larger fraction of shares and underpricing less.

"Hot issues" periods may thus not be triggered by optimism on the demand side, the outside investors, but rather by the supply side, when there is a stronger incentive for early stage investors to exit in order to enter new ventures.

Empirically, there is no firm relationship between the presence of venture capital and underpricing or rather it differs across periods.

In early studies, venture capital backed IPOs were found to be less underpriced than non-venture capital backed IPOs (Barry, Muscarella, Peavy and Vestuypens (1990), Meggison and Weiss (1991) and Lin and Smith (1998)). However, during hot issue markets, both the proportion of venture capital backed IPOs and their relative underpricing increase; in some cases they are significantly more underpriced than the average IPO (Francis and Hasan (2001), Franzke (2001), Ljungqvist (1999) and Smart and Zutter (2000)).

The certification role of the venture capitalist, who cares about his reputation as he returns often to the IPO market, should imply lower underpricing of venture backed firms. There is no current argument why venture backed firms should be more underpriced during hot issue market.

Venture capital finance represents a small fraction of the total financing, but is recognized to be critical in the financing of innovative ventures. Figure 4.3 and 4.4 show how this activity increased heavily in the last years until the slow down due to the Internet and telecommunications sectors crises.

Venture capital activity is mainly characterized by two elements: the investment in unlisted securities of recently formed firms and its advising and monitoring function that enhance to increase the chance of success of new enterprises (Bartlett (1999) and Gompers and Lerner (1999)). Thus venture capital investment requires a medium term prospective term, so the venture needs to be highly profitable to compensate the high illiquidity, the business risk and the services supplied. Thus the venture capital industry is oriented towards potentially marketable innovative ideas, often emerging from advances in science and technology.

Most of the work on venture capital concentrates attention separately on either feature.\(^2\)

\(^2\)In absolute terms, the length of relationship of the venture capital with the financed firm varies according to the activity of the firm itself (for a sector specification of the average length, see Cumming and MacIntosh (2001)).

\(^3\)The advising and monitoring capability have been studied among others by Repullo and Suarez (2000) and Cornelli and Yosha (1997); empirical evidence is offered by Gompers and Lerner (1999),
Figure 4.4: Quarterly Number of Venture Capital financed firms and Total and Average Venture Capital Investment (Source: http://www.nvca.org/ffax.html)
Many empirical papers report the exit behaviour of a venture capitalist, but recently only Berglöf (1994) and Schwienbacher (2002) model it theoretically. Berglöf (1994) describes how control rights of venture capital are allocated as compensation for his capability to sell to a larger company that operates in the same or in a similar business. Schwienbacher (2002) derives the optimal capital structure when there is a the agency conflict between the entrepreneur that prefers to go public to preserve his independence and the venture capitals who prefers to exit through IPO or a trade sale depending on the level of innovation.

Exit is a very important phase of the capital finance process as it is the moment when the venture capitalist realizes his returns to distribute them to its investors. The IPO is usually the most profitable exit route for the venture capitalist (Berglöf 1994).

We explicitly model the monitoring role of the venture capitalist as well as its exit strategy. This allows identifying some important effects, such as the difference in underpricing and fraction of shares sold relative to non VC backed firms.

In the second part of the paper, we study closely the exit of a venture capitalist. As in the basic model, exit is triggered by the arrival of a new investment opportunity through. The difference in underpricing by venture capitalists depends in whether the market is hot or not. The IPO of a venture capitalist differs from the one of a traditional investor (or of entrepreneurs) because of his role in controlling entrepreneurial moral hazard. We show that the contractual solution to the moral hazard issue of the new opportunity influences the modality of the disinvestments through IPO of the current investment.

The venture capitalist cannot freely choose his stake in the new investment. While the traditional investor has no constraint on the amount of capital to invest in the new firm, a venture capitalist facing the moral hazard problem has an optimal share in order to have the maximum effort exerted by the entrepreneur and the maximum profits.

As for the early stage investor, the venture capitalist prefers to incur a signalling cost selling the older investment rather than missing the new opportunity. Depending on the degree of moral hazard in the new investment opportunity, the venture capitalist will underprice more or less than a traditional financier.

This may explain the different empirical results concerning underpricing behaviour of venture and non-VC backed firms.

During economic booms, when there are many new and profitable investment opportunities, the higher is the expected profitability of the new investment opportunity, the lower are the constraints due to the moral hazard problems. In such periods the venture capitalist is willing to invest more and thus to liquidate quickly the current mature investments (and at higher discount).


4 Though IPOs constitute a small fraction of the total venture capitalist portfolio (between 20 and 35 percent, according to Cumming and Machintosh (2001)), they contribute the highest the returns (from 30% to more than 50% of total return).

5 The contrary does not apply: the venture capitalist-entrepreneur contracting is not influenced by the modality of exit of the venture capitalist from the previous contract.
4.2. The "hot issue" market model

4.2.1 The time structure

In this section we model a two period exit strategy by an early stage investor through an IPO aimed at funding a new investment. We will consider later the case of exit by a venture capitalist.

The early stage investor holds a fraction, $\alpha_1$, in a firm whose value is $V \in [L, \infty)$. He also has the opportunity to invest in a new venture whose expected return is $\pi \in [\pi_L, \infty)$. To invest in the new venture, he needs to exit from the old investment. We assume that the investor has limited wealth or limited capacity in managing companies. Hence, at period 0 the early stage investor decides to bring the first firm public in order to reinvest the IPO proceeds.

We assume that in the new opportunity, he does not face any entrepreneurial moral hazard.

Both the older firm value and the expected returns of the new investment opportunity are known to the early stage investor, but not to outside investors. Thus, the early stage investor faces a double asymmetric information problem when he sells his stake in the firm. The signalling of the new investment opportunity profitability is necessary to let outside investors perceive fully the trade off of the early stage investor and then to decode the signal on the firm value.

Hence, the early stage investor needs to signal this information. Because there are two different pieces of information asymmetries that have to be conveyed, he needs two signals. We assume that he can signal only via the fraction of shares sold during the IPOs, $\beta$, and the underpricing discount, $D$.

At period 1, once the market opens, the firm value has become known to all investors and so the final fraction can be sold at the true firm value without any further discount.

The time structure of the model is sketched in Fig. 4.5.

---

6 Also financial constraint considerations could be considered though they are believed to be less common nowadays.

7 This can be due to a lack of moral hazard problems, or the fraction of financing in the new investment opportunity is too small to induce moral hazard, or because the traditional investor is himself going to be the entrepreneur of the new firm.

8 In this model we do not allow for overpricing, that is, we do not allow for a negative $D$. 
Chapter 4. Venture-backed IPOs

4.2.2 IPO exit with uncertainty only on the IPO firm value

At time 0 the early stage investor sells through an IPO all or a part of his stake in order to be able to invest in the new opportunity. The amount of capital available in the new opportunity is determined by the proceeds of the IPO: $\beta \alpha_1 P$, where $\beta$ is the fraction of the firm sold during the IPO, $\alpha_1$ is the fraction hold by the early stage investor and $P$ is the IPO price. The new investment opportunity is perfectly divisible and has expected profitability $\pi$. So the total outcome deriving from the IPO exit and the subsequent investment in the new opportunity is given by $\pi \beta \alpha_1 P$.

As in Grinblatt and Hwang (1989), to solve the double signalling problem, we first solve a simpler case where there is only one source of asymmetry of information. Thus, initially we assume the outside investors know the expected profitability of the new investment opportunity, but not the value of the firm sold during the IPO.

The early stage investor is risk neutral and maximizes his expected wealth $E[W]$, given by 2 components: the return from investing the IPO proceeds at time 0 in the new firm, $\pi \beta \alpha_1 P$, and the return from selling on the market at time 1 the remaining share, $(1 - \beta) \alpha_1 V$. Assuming a zero interest rate, the objective function for an early stage investor is:

$$\max_{\beta, D} E[W] = \max_{\beta, D} E[\pi \beta \alpha_1 P + (1 - \beta) \alpha_1 V]$$

(4.1)

The fraction of the stake hold in the firm, $\alpha_1$, is irrelevant for the maximization problem.

As the outside investor is a price taker, the price settled by the early stage investor is equal to the value inferred by outside investors given the signals, minus
4.2. The "hot issue" market model

the underpricing, that is \( P = E [V | \beta, D] - D \), where \( V (\beta, D) \) is the inferred firm value by outside investors given the two signals. So the maximization problem of equation (4.1) becomes:

\[
\max_{\beta, D} E[W] = \max_{\beta, D} E[\beta \pi [E[V | \beta, D] - D] + (1 - \beta)(V + \varepsilon_1)] \\
= \max_{\beta, D} \beta \pi (E[V | \beta, D] - D) + (1 - \beta)V
\]

(4.2)

(4.3)

Expected wealth, given by equation (4.3), decreases with underpricing as long as \( \pi \geq 1 \) and \( 0 < \beta < 1 \), the maximum expected profit is given by no underpricing, that is \( D = 0 \). Hence, the best way to signal the firm value is through the fraction of shares sold during the IPO, \( \beta \). The maximization problem reduces to:

\[
\max_{\beta} E[W] = \max_{\beta} \beta \pi E[V | \beta] + (1 - \beta)V
\]

(4.4)

where \( E[V | \beta] \) is the value of the firm as perceived by the outside investors given the signal, \( \beta \).

Solving this maximization problem we obtain a signalling schedule that gives the unique Pareto-dominant separating equilibrium (Riley (1979) and Salanié (1999)).

Proposition 4.1 The signalling schedule that gives the unique Pareto-dominant separating zero profit equilibrium is:

\[
E[V | \beta] = \beta^{-1+\frac{1}{\pi}}L
\]

(4.5)

where \( L \) is the minimum possible firm value.

Proof. See Appendix 4.A

As in Leland and Pyle (1977), when the early stage investor sells all his stake, \( \beta = 1 \), the firm value signalled to the outside investor is at its minimum, \( L \).

Indeed, any schedule for which when all the shares are sold, the firm value perceived by the outside investors is smaller than the possible minimum firm value, that is \( E[V | 1] < L \), is for the early stage investor always Pareto dominated by the price schedule with \( E[V | 1] = L \).

At the same time, if the signalling schedule were such that the firm value perceived by the outside investor, when all the stake is sold, is higher than the liquidation value, \( E[V | 1] > L \), the early stage investor would have no incentive to liquidate the firm: he would find more attractive to sell all his stake to outside investors for \( E[V | 1] \) rather than liquidating it at \( L \). In this case he would gain the extra \( E[V | 1] - L \) even when the firm value is at its minimum, \( L \).

Furthermore, as long as \( \pi \geq 1 \), the first derivative of the signalling schedule with respect to \( \beta \) is negative and the second derivative is positive. This implies that a lower fraction sold signals a higher firm value and the marginal effect of the signal is greater for high firm value than for low firm values.

The proceeds from the IPO available for the new investment opportunity are given by \( \beta^\frac{1}{\pi}L \). Hence, given the signalling schedule, the early stage investor of a
low value firm has more capital available to invest in the new opportunity than the early stage investor of a high value firm.

Intuitively, this derives from the fact that the higher the firm value, the higher opportunity costs of a fast sale. Hold fixed the selling price, a higher fraction of firm sold, would imply a higher capital available for the new investment opportunity. However, as a higher fraction sold signals a low firm value, the early stage investor prefers to invest less in the new opportunity and to give up some profit rather than to sell a high value firm at the low value firm price.

Empirically, this means that high value firms will be sold with a very low free float.

4.2.3 Double asymmetric information

Starting from the previous results, we now derive the signalling schedule when there is asymmetric information both on the value of the firm taken public and on the new investment opportunity. Though the profitability of the new opportunity does not influence directly the outside investors pay-offs, it is a necessary information for the outside investors to know the objective function of the early stage investor and hence to interpret correctly the signal on the firm taken public. A second signal, the share pricing is needed.

We indicate with \( E[\pi | \beta, D] \), the profitability of the investment opportunity inferred by outside investors given the observed underpricing and the fraction of shares sold.

When the new opportunity profitability is the lowest possible, \( \pi = \pi_L \geq 1 \), the early stage investor has no reason to incur any cost to signal its profitability, i.e. \( D = 0 \). In this case the solution of the signalling problem can be reduced to the case of known profitability. Applying the results of Proposition 4.1, the signalling schedule in this case is:

\[
E[V_L | \beta] = \beta^{-1+\frac{1}{L}}L
\]  

(4.6)

**Proposition 4.2** When both the new investment opportunity and the firm value are unknown to the outside investors, there exists a two parameter separating signalling equilibrium that is given by:

\[
E[V | \beta, D] = E[V_L | \beta] + D
\]

(4.7)

\[
E[\pi | \beta, D] = \pi_L \left(1 + \frac{\beta^{1-\frac{1}{L}} - D}{L}D\right)
\]

(4.8)

**Proof.** See Appendix 4.B
4.2. The "hot issue" market model

<table>
<thead>
<tr>
<th>Low $\pi$</th>
<th>High $\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low $V$</td>
<td>Big fraction sold</td>
</tr>
<tr>
<td></td>
<td>Low underpricing</td>
</tr>
<tr>
<td>High $V$</td>
<td>Small fraction sold</td>
</tr>
<tr>
<td></td>
<td>Low underpricing</td>
</tr>
</tbody>
</table>

Figure 4.6: The four main signaling behaviors of the traditional investor

These signalling schedules are such that the retained share $(1 - \beta)$ signals the firm value, while the underpricing $D$, or better the percentage of initial return $\frac{D}{V_L}$, signals the new investment opportunity profitability.

When there is no underpricing, by construction the inferred value of the new investment is the lowest possible, $\pi_L$, increases in a convex way as $D$ increases and is infinite as $D$ equals the total firm value, $V$.

The level of the discount $(D = \left(\frac{\pi}{\pi_L} - 1\right)V_L)$ is proportional to the expected extra gain (above the minimum expected profitability $\pi_L$) of the new investment. More precisely, an increase in the expected profitability of the new investment profitability induces the early stage investor to increase proportionally the percentage of first day initial returns.

Given these features, we can identify 4 cases, as sketched in Fig. 4.6:

- **Low firm value and low profitability of new investment.** In this case the investor prefers to sell a large stake of the firm rather than underpricing. It is more attractive to sell more of the old firm to invest it in the new investment, rather than selling less to mimic a high value firm: the lost gain from investing less in the new investment would be higher than the overpricing. On the other side underpricing is not attractive, as the discount cannot be recovered investing in a poorly profitable new investment.

- **Low firm value and high profitability of new investment.** The early stage investor will sell a large stake in the old firm and will underprice heavily in order to have enough capital to invest in the new investment opportunity.

- **High firm value and low profitability of new investment.** In this case it is costly for the early stage investor to sell quickly the old firm to invest in the new investment. Thus, he retains a large fraction of the old firm and underprices for a small amount.

- **High firm value and high profitability of new investment.** The early stage investor will sell a small fraction of his stake at a very low price and will invest then a small amount of capital in the new investment opportunity.
Given these results, we can now interpret the variation of IPO underpricing over time. When there are good investment prospects, due for example to technological innovation in some sectors, investment opportunities are many and highly valuable. This triggers a "hot issue" market: the early stage investors will take public the existing firms, selling many shares at low price, and invest the proceeds in new ventures. Subsequently, the early stage investors will take the new ventures public in order to invest; if the "opportunity wave" has past at the IPO fewer shares will be sold and will be under-priced less. The prospects created, for example, by the "dot com" boom, were many and highly positive and this has induced an increase on both the underpricing and the volume of IPOs.

As a final remark, it is interesting to notice that the early stage investor maximizes not only over the level of underpricing and the fraction of shares to hold, but indirectly also over the optimal level of capital to invest in the new investment opportunity. This element together with the moral hazard issue is the driving element that explains the different IPO behaviour of a venture capitalist in respect to a traditional investor, as we show in the next section.

### 4.3 The Venture Capitalist

In this section we present a variation of the above model to show how the venture capitalist signalling behaviour differs from the one of a traditional investor as he faces moral hazard problem.

As we said, a venture capitalist differs from an early stage investor because he cannot freely choose the amount of capital to invest due to the moral hazard problem with the entrepreneur.

The time structure of the model changes slightly. We have now a cycle subdivided into 3 periods with 4 different players: the venture capitalist, the entrepreneur, the outside investors and the financiers of the venture capitalist activity.

At period 1, the venture capitalist is interested in financing a new firm whose initial investment need is $K \in [K_L, \infty)$, where $K_L$ can be considered the minimum capital need of any investment. The expected value of this firm is partly stochastic and partly depends on the amount of effort exerted by the entrepreneur, $e \in [0, \varepsilon]$. The effort exerted is unobserved by the venture capitalist and implies some sort of personal cost that for simplicity we pose it equal to $e$.

Hence, the objective functions of the two players, the venture capitalist and the entrepreneur, do not coincide and a moral hazard problem arises. In order to overcome this problem the venture capitalist finances the firm with equity for a certain fraction equal to $\alpha_2 \in [\alpha_L, 1]$. The characteristics of the contract between the entrepreneur and the venture capitalist are not known to the outside investors: the outside investors know only that the venture capitalist finances the new venture through equity, but they do not know the size of the investment, $K$, the share

---

9We are aware of the relevance on the firm value of the venture capitalist effort, as consultant and monitor. However, for simplicity, we ignore this component.
participation of the venture capitalist, \( \alpha_2 \), and the quality of the entrepreneur, that is more technically said the marginal productivity of the entrepreneur effort, \( \pi'(e) \).

At period 2 the firm value, \( V_2 \), is realized and can range from 0 to infinite. This firm value can be observed by both the venture capitalist and the entrepreneur, but not by the outside investors and the financiers.

At period 2 the venture capitalist wants to take the firm public for two reasons. First, at period 2 the venture capitalist has a new investment opportunity whose features are known only to the venture capitalist. To take advantage from it, the venture capitalist is forced to disinvest the firm because by assumption he has a limited capacity in managing the financing activity.\(^\text{10}\)

Second, IPO can be a tool for the venture capitalist to ease the subsequent profits distribution to his own financiers: as Gompers and Lerner (1999) has shown, profits distribution usually happens through shares distribution from the venture capitalist portfolio and an IPO constitutes a prerequisite to ease the subsequent financiers exit.

As for the early stage investor, the venture capitalist, taking a firm public, incurs a double asymmetric information problem: on the firm value and on the profitability of the new venture. This time, however, the profitability of the new venture is determined by the terms of the venture capitalist-entrepreneur contract conditions (i.e. capital invested and shares distribution) that are not observable to others. Again, we assume that the venture capitalist can use as signals only pricing and quantity of the share sold, i.e. the underpricing discount, \( D \), and the stake sold, \( \beta \).

At period 3, after the IPO, the firm value is revealed to all players and the venture capitalist transfers the remaining shares, \( 1 - \beta \), to the his own financiers.

At this point, the cycle repeats itself: with the selling of the stake of the new firm at period 2 and 3, the venture capitalist has the possibility to finance further investment opportunities, etc.

Studying this cycle allows us to see the influence of the past and future decisions of the venture capitalist on the investment and disinvestments decisions.

The new time structure is sketched in Fig. 4.7.

In what follows we first study the contracting between the venture capitalist and the entrepreneur; subsequently we see how this influences (and is influenced by) the IPO signalling behaviour.

### 4.3.1 The Contract between Venture Capitalist and the Entrepreneur

After the venture capitalist has invested in the firm in period 1, in period 2 he disinvests through an IPO in order to be able to invest in the new opportunity and to later distribute the remaining share to the financiers. The amount of capital that the venture capitalist is investing in the new investment opportunity is not

\(^\text{10}\)For more details on the determinants of the venture capitalist portfolio size see Cumming (2001).
Chapter 4. Venture-backed IPOs

VC signs contract with E

Moral Hazard problem on the effort exerted by E

IPO VC knows $V_2$ and $\pi' (\bar{e})$

Outside investors do not Signal through D and $\beta$

VC sells $1-\beta$ to F

Figure 4.7: Time Structure of the Venture Capital Model

determined by the outcome of the IPO, but is chosen so as to take into account the entrepreneurial moral hazard in the new venture.

The degree of moral hazard depends on the productivity of entrepreneurial effort, which may reflect either the quality of the entrepreneur or of his idea. This entrepreneur quality is a function of the effort exerted by the entrepreneur himself and is defined as the change in profitability of the firm on the base of the effort exerted. More formally:

$$\frac{\partial \Pr (\pi_i \mid e)}{\partial e} > 0$$  \hspace{1cm} (4.9)

where $\Pr (\pi_i \mid e)$ is the probability for the $i-$ venture return to be $\pi_i$, given the entrepreneur effort, $e$.

We assume that the venture capitalist has all the bargaining power, but has to give enough incentives to the entrepreneur to exert effort, $e$. This effort satisfies the monotone likelihood property (MLRP) relative to firm value (Salanié 1999).\footnote{This requires that $\frac{\partial}{\partial \pi_i} \left[ \frac{\partial \Pr (\pi_i \mid e)}{\partial e} \frac{1}{\Pr (\pi_i \mid e)} \right] \text{ is positive.}$}

The venture capitalist can induce the entrepreneur to exert effort by increasing his stake, i.e. diminishing his own equity stake, $\alpha_2$. The higher is the stake of the entrepreneur, the greater is his incentive to increase profits.

We assume that income is verifiable and the firm enjoys limited liability. All agents are risk neutral.
Proposition 4.3 The optimal contract offered by the venture capitalist is such that the entrepreneur exerts the maximum effort, $\bar{e}$, which requests that the venture capitalist holds a stake equal to:

$$\alpha_i = 1 - \frac{1}{\int_0^\infty \partial \Pr(\pi_i | \bar{e}) / \partial \bar{e} \, \pi_i \, K \, d\pi_i}$$

(4.10)

and the expected return of the investment is given by:

$$\int_0^\infty \Pr(\pi_i | \bar{e}) \, d\pi_i = 1 + \frac{\bar{e}}{(1 - \alpha_i)K}$$

(4.11)

Proof. See Appendix 4.C ■

This result offers some interesting observations.

The venture return depends on the effort exerted from the entrepreneur relative to his capital involvement. The entrepreneurial stake, $1 - \alpha_2$, depends on his marginal productivity: the higher the entrepreneur quality, the smaller the necessary financial participation of the entrepreneur to profits.\(^\text{12}\)

The expected discount on the subsequent IPO affects the profitability that the venture capitalist derives from each investment. This profitability increases with the expected fraction of the firm sold during the IPO, with the expected value of the future investment opportunity and decreases with the percentage of underpricing.

Interestingly, the IPO structure does not influence the contract between the venture capitalist and the entrepreneur. Intuitively, there is no conflict between the venture capitalist objective during the creation and the IPO phase: the higher the firm value in the first period, the more valuable the subsequent IPO. Thanks to this feature, there is no interaction among the different phases. Thus, we can study the IPO issue as a repeated static stage game.

We now move to study how IPO structure is affected by moral hazard in the subsequent investment. For consistency in the notation, we indicate with $\alpha_2$ the participation of the venture capitalist in the subsequent venture, with $\pi_2(e) = \int_0^\infty \Pr(\pi_i | e) \, \pi_i \, d\pi_i$ its expected return, and with $\pi'_2(e)$, the marginal productivity of the entrepreneurial effort of the new venture, $\int_0^\infty \partial \Pr(\pi_i | e) / \partial e \, \pi_i \, K \, d\pi_i$. Then, the results of Proposition (4.3) can be expressed as:

$$1 - \alpha_2 = \frac{1}{\pi'_2(e) K}$$

(4.12)

$$\pi_2(e) = 1 + \frac{\bar{e}}{(1 - \alpha_2)K}$$

(4.13)

4.3.2 Venture capitalist and entrepreneur contracting and the IPO

In the last section we have shown that how the venture capitalist addresses the moral hazard problem with the entrepreneur is not influenced by the structure

\(^{\text{12}}\)This result depends on the assumption that the venture capitalist has all the bargaining power. Modifying this assumption would not affect the quality of our results about the IPO structure.
of the subsequent IPO. In this section we study the opposite: how the venture capitalist-entrepreneur contracting over a future investment opportunity influencing the current IPO exit of the venture capitalist through the moral hazard constraint. In Section 4.2, we allowed the early stage investor to invest in the new investment opportunity an optimal amount, subject to a maximum funding, which depends on the IPO outcome. Here instead the venture capitalist, when deciding its investment has also to take into account the incentive constraint.

As before, the level of profitability of the new venture (which is now endogenous) is not known to outside investors. Specifically, we assume that outside investors do not know the marginal productivity of the entrepreneur, and cannot observe the stakes held by the entrepreneur and the venture capitalist.\(^{13}\)

In the presence of entrepreneurial moral hazard the IPO outcome changes. This constraint affects the IPO in two ways. First, the return of the new investment opportunity is now endogenous (expressed by equation (4.13)) and so the VC maximization problem becomes:

\[
\max_{\beta,D} E[W] = \max_{\beta,D} \beta \left( 1 + \frac{\bar{e}}{(1 - \alpha_2) K} \right) P + (1 - \beta) V
\]  

(4.14)

Second, the capital invested by the venture capitalist in the new firm, \(\alpha_2 K\), cannot be more than the capital raised during the IPO, \(\beta P\).

As we did in the basic model, we initially solve the IPO structure by assuming that the quality of manager and then the contracting conditions between venture capitalist and entrepreneur are common knowledge. In this case the value of the new venture is known and the maximization problem gives the following result.

**Proposition 4.4** When the contract between the entrepreneur and the venture capitalist in the new venture is common knowledge, the venture capitalist does not underprice. The unique equilibrium schedule to signal the firm value is given by:

\[
E[V | \beta] = \frac{\bar{e}}{L \beta (1 - \alpha_2) \text{ProductLog} \left[ \frac{\exp^{\frac{\alpha_2}{1 - \alpha_2}}}{L \beta (1 - \alpha_2)} \right]} 
\]  

(4.15)

where ProductLog \([z]\) is the function that gives the principal solution for \(w\) in \(z = w \exp^w\)

**Proof.** See Appendix 4.D

This signalling schedule is the unique Pareto dominating equilibrium for the venture capitalist. It is such that the lower is the fraction of shares sold, the higher the firm value is, just as in Leland and Pyle (1977). In fact the first derivative of the signalling schedule has to satisfy the following condition:

Note that the schedule is not affected by the total amount of capital needed for the new investment, but only by the stake acquired by the venture capitalist, \(\alpha_2\).

\(^{13}\)If we allow the proceeds to be directed to more than one venture, the VC share can be considered as a weighted average of the VC participation in each venture where the proceeds are invested.
4.3. The Venture Capitalist

When this information is not known to outside investors, the venture capitalist has to signal it through underpricing. If the venture capitalist is acquiring the minimum stake in the new venture, \( \alpha_L \), he is not willing to incur in any underpricing: outside investors cannot infer any inferior stake with no underpricing. Hence, the optimal signalling schedule, Pareto dominating for the venture capitalist, in case of no underpricing should indicate the minimum possible VC stake in the venture.

For this reason we are going to rename equation (4.15) \( E[V_L | \beta] \) where \( \alpha_2 \) is substituted with \( \alpha_L \).

**Proposition 4.5** When both the IPO firm value and the characteristics of the next investment are unknown to outside investors, the optimal signalling schedule are given by:

\[
E[V | D, \beta] = E[V_L | \beta] + D \tag{4.16}
\]

\[
E[\alpha_2 | D, \beta] = 1 - \frac{\bar{e} (1 - \alpha_L) V_L (\beta)}{\bar{e} \alpha_L D + (\bar{e} + (1 - \alpha_L) D \beta) V_L (\beta)} \tag{4.17}
\]

**Proof.** See Appendix 4.E.

The resulting double signalling schedule is such that the stake sold fraction signals the IPO firm value, while underpricing is signalling indirectly the marginal profitability of the new venture. We can identify four basic situations:

- **Low firm value and low marginal profitability of the new venture.** The venture capitalist has no incentive to retain a large stake in the old firm, as it is not very valuable; at the same time he has no incentive to underprice since the new venture is not particularly valuable. So at the IPO he sells a large fraction of the firm with low underpricing.

- **Low firm value and high marginal profitability of the new venture.** The venture capitalist is eager to liquidate the old investment to invest in the new venture. He thus disinvests a large fraction of the old investment at the cost of high underpricing.

- **High firm value and low marginal profitability of the new venture.** The venture capitalist has no incentive to sell out the old investment for the new one. He thus sells a small stake at modest underpricing: he tries to minimize both the fraction of disinvestments and the underpricing in order to raise the funds to invest in the new venture. In this case the venture capitalist has to invest a small stake to keep incentive strong.

- **High firm value and high marginal profitability of the new venture.** The venture capitalist faces a new investment in which it is optimal to accept a small stake. He thus prefers to disinvest a small fraction and accept a high underprice.
4.4 Analysis of the results and empirical implications

We have distinguished two types of IPO exits. In the basic model, an early stage investor wants to see to invest in a new venture without moral hazard (see Section 4.2). In the extended model a venture capitalist faces a moral hazard problem in the new venture. This determines their different IPO structure: the early stage investor is signalling the firm value and the expected return in the new investment opportunity. In contrast, a venture capitalist must signal the characteristic of the moral hazard in the new venture, which indirectly identifies its profitability.

In this section we analyse the difference in the signalling behaviour of these two inside investors, and identify some empirical implications as a function of external circumstances. We indicate with a subscript $TI$ what refers to the traditional investor signalling and with a subscript $VC$ what refers to the venture capitalist. We first study the difference in the signalling schedules of the old firm value, and subsequently compare the signal for the profitability of the new investment.

4.4.1 Signalling the IPO firm value

In order to make comparable the signalling schedules, we substitute $\alpha_2$ within equation (4.13) and rewrite the expressions for the retained stake:

$$\beta_{TI} = \left( \frac{L}{V_L} \right)^{\frac{\pi_L}{\pi_L - 1}}$$

(4.18)

$$\beta_{VC} = \frac{L}{V_L} \left( 1 + \frac{\ln \frac{V_L}{1 - \pi_L}}{1 - \pi_L} \right)$$

(4.19)

Comparing these two equations, we see that for constant firm values, the amount of shares sold during the IPO by the traditional investor is always higher.

4.4.2 Signalling the value of the new venture

With the underpricing traditional investors signal the expected return of the new investment opportunity, while the venture capitalists signal indirectly its stake in the new venture.

In order to compare the two levels of underpricing, we express the signalling schedule of the value of the new venture for the venture capitalist as:

$$E[\pi_{TI} | \beta, D] = \pi_L \left( 1 + \frac{D}{E[V_L | \beta]} \right)$$

(4.20)

$$E[\pi_{VC} | \beta, D] = \frac{\beta (\pi_L - 1) V_L^2}{\bar{e} V_L + (V_L - 1) \beta [(\pi_L - 1) V_L + \pi_L D]}$$

$$+ \frac{(\beta + \bar{e} \pi_L - \beta \pi_L + \beta \pi_L D) V_L + \pi_L (\bar{e} - \beta) D}{\bar{e} V_L + (V_L - 1) \beta [(\pi_L - 1) V_L + \pi_L D]}$$

(4.21)
This level of underpricing is higher for the venture capitalist than for the early stage investor when the underpricing is below $D^*$, where:

$$D^* = -\frac{(V_L - 1) \beta (1 - 2\pi_L + 2\pi_L) - \sqrt{(V_L - 1)^2 \beta^2 (1 - 2\pi_L)^2 + 4\epsilon^2 \pi_L^2}}{2\pi_L [\epsilon + \beta (V_L - 1) (\pi_L - 1)]} V_L$$

(4.22)

Hence, when the new venture is highly profitable so that the level of underpricing needs to be high, the venture capitalist underprices more than the traditional investor to signal the same value. When instead, the new investment opportunity is poorly profitable, the venture capitalist underprices less.

The intuition is simple. When the venture capitalist is investing in a very profitable new venture, the entrepreneurial moral hazard is less significant. The venture capitalist can then invest more, thus sells more and so he must underprice more. When the firm is not very profitable, the venture capitalist has to accept a small stake to induce the entrepreneur to exert effort.

This result can explain the contradictory empirical evidence of the literature on venture capital backed IPOs underpricing. On one side, Barry et al. (1990), Meggison and Weiss (1991) and Lin and Smith (1998) find that Venture Capital backed IPOs are less underpriced than non Venture Capital backed IPOs. The opposite happens in Francis and Hasan (2001), Smart and Zutter (2000) and Franzke (2001).

According to our model, these empirical results can be the result of different economic conditions. When the economy is expanding and there are many new investment opportunities requiring funding by venture capitalist, the underpricing is heavier. The opposite occurs when there are fewer or less attractive investment opportunities.

If we observe in fact the years studied in the empirical papers, we find those that find less underpricing for venture capital backed firm, refers to periods of relatively stable markets or very long periods where the hot issue markets are not disentangled. Barry et al. (1990) and Meggison and Weiss (1991) study the IPOs between '83-'87; in those years there has been no hot issue markets and the venture capitalist was underpricing less the the early stage investor (7% instead of 8%)

Lin and Smith (1998) considers a long time series ('79-'90) without distinguishing the hot market periods and finds that the venture capital underpricing is 12% while the early stage investor one is 17%.

The seconds instead focus on hot issue market periods and find more underpricing for venture capital backed firms. For example Ljungqvist (1999) for the US and Franzke (2001) for Germany find that at the end of the nineties during the Internet hot issue market venture backed firms were more underpriced than non venture capital backed firms.14

This is in line with our results: during boom periods the venture capitalist wants to exit quicker though incurring in heavier losses on order to exploit the new investment opportunities.

14In Ljungqvist (1999) we have a 18% underpricing for the VC and 17% for early stage investor and in Franzke (2001) we have respectively 64% and 61%.
4.5 Extension: the optimal timing for the venture capitalist

So far we have studied the IPO exit by traditional investors and venture capitalists in a static setting.

In this section we concentrate on the venture capitalist behaviour, and study a dynamic version of the model. We wish to identify some implications on the optimal timing of IPO issues in order to relate our results to the concentration of IPOs in hot markets. We also wish to examine the effect of uncertainty. Specifically, we investigate when it will be optimal for the venture capitalist to bring the firm public in a continuous time setting, by applying the real option methodology.\(^{15}\)

The profitability of the new investment opportunity is given by:

\[
\pi = \frac{S - K}{K}
\]  

(4.23)

where \(S\) is the value of this investment. \(S\) evolves continuously over time following a Geometric Brownian motion:

\[
\frac{dS}{S} = \mu dt + \sigma dt
\]  

(4.24)

Furthermore, we assume that the new investment can be taken at any point in time, and that the venture capitalist can decide to bring the current firm public at each instant. However, no new investment arrives until the current new venture is undertaken.

The value, \(S^*\), for which it is optimal to bring the first firm public can be found applying the classical steps of real option methodology. The critical element in this case is given by the underpricing derived from equation (4.15) and (4.17).

**Proposition 4.6** The optimal value of the second firm for which it is optimal for the venture capitalist to bring the first investment public is given by:

\[
S^* = \frac{\gamma_1}{\gamma_1 - 1} (\bar{e} + K - \beta V_L)
\]  

(4.25)

where

\[
\gamma_1 = \frac{1}{2} - \frac{\mu^2}{2\sigma^2} + \sqrt{\left(\frac{\mu^2}{2\sigma^2} - \frac{1}{2}\right) + 2\frac{\sigma^2}{\sigma^2}}
\]

**Proof.** See Appendix 4.F

The optimal new investment value for which it is optimal to perform the IPO of the old firm is a function of the fraction of capital supplied by the venture capitalist: the higher this fraction, the lower the investment opportunity for which it is optimal to sell the old and invest in the new investment. The participation level of the venture capitalist is determined by the entrepreneurial moral hazard problem: the higher the marginal productivity of the entrepreneur, the lower the moral hazard

\(^{15}\)For a review see Dixit and Pindyck (1994).
problem, and the lower the optimal new investment opportunity at which to perform the IPO.

In practice, this means that the more attractive is the new investment opportunity, the higher can be the venture capitalist involvement and the earlier the venture capitalist is willing to sell at the IPO and invest in it.

When instead the entrepreneurial project is not that attractive, or some managerial qualities are missing, the venture capitalist prefers to wait longer to give up the first investment opportunity.

4.6 Conclusions

This paper has modelled explicitly the exit strategy by investors who have a reinvestment strategy. Our model can contribute to explain some features of "hot markets". It can explain also some contradictory results of the empirical literature on the IPO underpricing behavior of venture capital and non-venture capital backed firms.

In our model a firm is taken public to exploit new investment opportunities. In expansion periods, when there are many new investment opportunities, the model predicts faster exit, thus more IPOs, and more under-priced. In periods when few investment opportunities, there would be fewer IPOs and less underpricing.

This model distinguishes between two kind of investors that take the firm public: a traditional investor, who does not face moral hazard conflicts, and the venture capitalist, that specializes in firms with moral hazard problems. We show that this difference may be crucial to explain different IPOs underpricing behaviour of the venture capital backed firms. The contractual solution to the moral hazard problem induces the venture capitalist to sell a higher fraction of valuable firms than the traditional investor and underprice more when the new opportunity is very profitable. When instead it is not very profitable, the venture capitalist underprices less than a traditional investor.

This is consistent with some evidence where during boom periods venture capitalists underprice more than other investors; while in other periods the VC backed firms are underpriced less.

We believe that studying the exit strategy explicitly can also help further study on hot issue markets. In particular, we aim at developing a full-fledged continuous time version of the model to better explain the clustering in hot issue market.

It would be also interesting to verify empirically the implications of the model on the level of underpricing and the fraction sold during hot and cold issue market periods, and distinguish between venture capital and non venture capital backed firms.
Appendix

4.A Proof Proposition 4.1

In order to proof proposition 4.1, we first find the informationally consistent Pareto dominating schedule and subsequently we demonstrate that this is the unique reactive equilibrium.

The informationally consistent Pareto-dominant price schedule must satisfy the following first order condition:

$$\frac{\partial E[W]}{\partial \beta} = \pi E[V | \beta] + \pi \beta E'[V | \beta] - V = 0$$

(4A.1)

To obtain an informationally consistent price function, we impose the self fulfilling belief condition, that is we impose that the belief on the firm value of the outside investors is correctly formed (Salanié (1999)), reflecting the true firm value:

$$E[V | \beta] = V$$

(4A.2)

Substituting equation (4A.2) in equation (4A.1) and rearranging, the first order condition becomes:

$$(-1 + \pi) E[V | \beta] + \beta \pi E'[V | \beta] = 0$$

(4A.3)

It follows that the family of optimal candidate as signallling schedule is the equation that solves the above differential equation.

The generic solution is given by:

$$E[V | \beta] = \beta^{-1+\frac{1}{\pi}} A$$

(4A.4)

where $A$ is an arbitrary constant.

In order to find the Pareto optimal schedule, the constant of integration has to be such that when the traditional investor sells all his holding the firm value is equal to the minimum firm value of the investment, $L$. Indeed, the schedules where $E[V | 1] < L$ allow for arbitragers and those where $E[V | 1] > L$ are Pareto dominated by $E[V | \beta] = L$

$$E[V | 1] = A = L$$

(4A.5)

So the Pareto-dominant price schedule is:

$$E[V | \beta] = \beta^{-1+\frac{1}{\pi}} L$$

(4A.6)

This price schedule verifies also the second order condition\textsuperscript{16} and its first and second

\textsuperscript{16}The second derivative given the informative consistent price schedule is given by:

$$\frac{\partial^2 E[W]}{\partial \beta^2} = 2\pi V''(\beta) + \beta \pi V'''(\beta) = V'(\beta) < 0$$

(4A.7)
derivatives are respectively given by:

\[ \frac{\partial E[V | \beta]}{\partial \beta} = \beta^{-2 + \frac{1}{\pi}} \left( -1 + \frac{1}{\pi} \right) L < 0 \]  
\[ \frac{\partial^2 E[V | \beta]}{\partial \beta^2} = \beta^{-3 + \frac{1}{\pi}} \left( -1 + \frac{1}{\pi} \right) \left( -2 + \frac{1}{\pi} \right) L > 0 \]

Now that we found the informationally consistent Pareto dominating schedule we demonstrate that this is the unique reactive equilibrium.

Riley (1979) demonstrates there is a unique reactive equilibrium if 6 conditions are satisfied.\(^{17}\) Using the notation of this paper we restate the 6 hypothesis and we verify that they are satisfied:

i. The first hypothesis is satisfied as by assumption \( V \in [L, \infty] \);

ii. \( E[W] \) and \( E[V | \beta] \) are differentiable in all the arguments;

iii. The maximand function, \( E[W] \), is strictly increasing in the market price of the firm;

iv. Hypothesis 4 is satisfied because:

- Given the condition on the integration constant the firm value has always positive value \( (E[V | \beta] > 0) \).
- See equation (4A.8)

v. Single-crossing property is satisfied as:

\[ \frac{\partial}{\partial V} \left( -\frac{\partial W}{\partial \beta} \frac{\partial W}{\partial P} \right) = \frac{1}{\beta \pi} > 0 \]

vi. From how we constructed the price schedule it derives that the maximand function in terms of firm value has a unique \( \beta \) that maximizes \( W \).

## 4.B Proof Proposition 4.2

If the firm value is signalled via underpricing we have that the optimal schedule has to satisfy the first order condition and the constraint given by the case of no underpricing:

\[ \frac{\partial W}{\partial D} = \beta \pi (E[V' | D] - 1) = 0 \]  
\[ V(0) = E[V_L | \beta] \]

\(^{17}\)Alternatively we could verify the uniqueness of the reactive equilibrium verifying the satisfaction of the 6 conditions of Engers and Fernandez (1987), as they demonstrate the uniqueness with less stringent assumptions.
Chapter 4. Venture-backed IPOs

Solving this differential equation we obtain equation (4.7).

Applying this result to the max function, it becomes:

$$\max_\beta E[W] = \max_\beta (E[V_L \mid \beta] + D - D) + (1 - \beta) V$$  \hspace{1cm} (4B.13)

The first order condition together with the self fulfilling belief condition, $V = E[V \mid \beta]$ and $\pi = E[\pi \mid \beta, D]$, is:

$$\frac{\partial W}{\partial \beta} = -\beta \pi_L D - \frac{\pi_L}{\beta} (E[\pi \mid \beta, D] - \pi_L) E = 0$$  \hspace{1cm} (4B.14)

Solving this equation and rearranging we obtain equation (4.8).

Applying the same reasoning as in Grinblatt and Hwang (1989), as this equilibrium maximizes the early stage investor utility, it is by construction for the early stage investor Pareto efficient. It is also unique as the other separating equilibria make the investor worse-off.

4.C Proof Proposition 4.3

In this Appendix we are going to find the optimal share distribution given the moral hazard problem between the entrepreneur and the venture capitalist. We rewrite the maximizing problem of the wealth of equation (4.1) is subject to different kind of constraint: the maximization of the entrepreneur utility in terms of effort exerted, the incentive compatibility constraint, the individual rational constraint of the entrepreneur (that is always satisfied as the incentive compatibility constraint is more stringent). Finally there is the individual rational constraint of the venture capitalist himself that is relevant only in terms of venture selection: the venture capitalist wants to invest only if the opportunity return is greater than 1. Formalizing this, we obtain:

$$\max_{\alpha_i} \int_0^\infty \Pr(\pi_i \mid e) \alpha_i [\pi_{i+1} \beta (\pi_i K - D) + (1 - \beta) \pi_i K] \, d\pi_i - \alpha_i K$$  \hspace{1cm} (4C.15)

s.t. \hspace{1cm} \max_{\epsilon} \int_0^\infty \Pr(\pi_i \mid e) (1 - \alpha_i) \pi_i K \, d\pi_i - e - (1 - \alpha_i) K$$  \hspace{1cm} (4C.16)

$$\int_0^\infty \Pr(\pi_i \mid e) (1 - \alpha_i) \pi_i K \, d\pi_i \geq (1 - \alpha_i) K + e$$  \hspace{1cm} (4C.17)

$$\int_0^\infty \Pr(\pi_i \mid e) \alpha_i [\pi_{i+1} \beta (\pi_i K - D) + (1 - \beta) \pi_i K] \, d\pi_i \geq \alpha_i K$$  \hspace{1cm} (4C.18)

$$0 \leq \alpha_i \leq 1$$  \hspace{1cm} (4C.19)

where $\pi_{i+1}$ is the profitability of the subsequent venture.

Letting $\mu$ and $\lambda$ denote the (nonnegative) multipliers of the constraints and transforming the first constraint with the first order condition, the relative Lagrangian
becomes:

\[ L = \int_0^\infty \Pr(\pi_i | e) \alpha_i [\pi_{i+1} - (1 - \beta) \pi_i K d\pi_i] - \alpha_i K + \rho \int_0^\infty \frac{\partial \Pr(\pi_i | e)}{\partial e} (1 - \alpha_i) \pi_i K d\pi_i - \lambda + \mu \int_0^\infty \Pr(\pi_i | e) (1 - \alpha_i) \pi_i K d\pi_i - \mu (1 - \alpha_i) K - \mu e \] (4C.20)

The solution of the maximization problem is given by the solution of the following system in respect of equations:

\[ \frac{\partial L}{\partial \alpha_i} = \int_0^\infty \Pr(\pi_i | e) \pi_{i+1} - (1 - \beta) \pi_i K d\pi_i - \rho \int_0^\infty \frac{\partial \Pr(\pi_i | e)}{\partial e} \pi_i K d\pi_i - \lambda + \mu \int_0^\infty \Pr(\pi_i | e) \pi_i K d\pi_i + \mu K = \Phi(4C.21) \]

\[ \frac{\partial L}{\partial \lambda} = \int_0^\infty \frac{\partial \Pr(\pi_i | e)}{\partial e} (1 - \alpha_i) \pi_i K d\pi_i - 1 = 0 \] (4C.22)

\[ \frac{\partial L}{\partial \mu} = \int_0^\infty \Pr(\pi_i | e) (1 - \alpha_i) \pi_i K d\pi_i - (1 - \alpha_i) K - e = 0 \] (4C.23)

As the venture capitalist wants the maximum effort exerted, \( \bar{e} \), the solution is then given by:

\[ \alpha_i = 1 - \frac{1}{\int_0^\infty \frac{\partial \Pr(\pi_i | e)}{\partial e} \pi_i K d\pi_i} \] (4C.24)

\[ \int_0^\infty \Pr(\pi_i | e) d\pi_i = 1 + \frac{\bar{e}}{(1 - \alpha_i)K} \] (4C.25)

### 4.D Proof of Proposition 4.4

In this Appendix we are going to find the optimal signalling schedule of the firm of the venture capitalist when the new investment opportunity features are known.

As for the early stage investor, it is more attractive for the venture capitalist to signal through the fraction of share sold rather than with underpricing.

So the first order condition is given by:

\[ \frac{\partial E[W]}{\partial \beta} = -V + \left(1 + \frac{\bar{e}}{K (1 - \alpha_2)}\right) E[V | \beta] + \left(1 + \frac{\bar{e}}{K (1 - \alpha_2)}\right) \beta E[V' | \beta] = 0 \] (4D.26)

Adding the self full-filling condition, \( V = E[V | \beta] \), and the financement condition, \( K = \frac{\beta E[V | \beta]}{\alpha_2} \), the above equation becomes:

\[ \bar{e} \alpha_2 E[V | \beta] + \beta^2 (1 - \alpha_2) E[V | \beta] E[V' | \beta] + \bar{e} \alpha_2 \beta E[V' | \beta] = 0 \] (4D.27)
The solution of this differential equation with the constraint that

$$E[V | 1] = L$$

is given by:

$$\frac{\beta E[V | \beta] - L}{L \beta E[V | \beta] - \alpha_2 \bar{v}} + (1 - \alpha_2) \ln \frac{E[V | \beta]}{L} = 0$$

(4D.29)

After some transformations, we obtain that:

$$E[V | \beta] = \bar{v} - \frac{\bar{v} \alpha_2 E[V | \beta]}{\bar{v} \alpha_2 + \beta [(1 - \alpha_2) \beta E[V | \beta]]}$$

(4D.30)

As for the proof of Proposition 4.1 we verify that the six conditions of Riley (1979) are satisfied. The first three conditions and the sixth are naturally satisfied. The fourth condition is satisfied, as from equation (4D.27):

$$E[V' | \beta] = \frac{\bar{v} \alpha_2 E[V | \beta]}{\bar{v} \alpha_2 + \beta [(1 - \alpha_2) \beta E[V | \beta]]} < 0$$

(4D.31)

The proof of the fifth condition, the single crossing one, is always satisfied, as:

$$\frac{\partial}{\partial V} \left( - \frac{\partial W}{\partial \beta} \right) = \frac{1}{\left( 1 + \frac{\bar{v}}{(1 - \alpha_2) K \beta} \right)} > 0$$

(4D.32)

4.E Proof of Proposition 4.5

The maximising problem can be simplified to:

$$\max_{\beta, D} E[W] = \max_{\beta, D} \beta \left( 1 + \frac{\bar{v}}{(1 - \alpha_2) K} \right) (E[V | D] - D) + (1 - \beta) V$$

(4E.33)

As we did in the first part of the paper we solve the optimisation problem in respect of $D$ considering the constraint that when there is no underpricing, the level of participation of the venture capitalist is at the minimum, $\alpha_2 = \alpha_L$, and the signalling of the firm value is given by the solution of equation (4.15). The maximum is then equal to:

$$V(D, \beta) = E[V_L | \beta] + D$$

(4E.34)

Inserting this in the maximization problem, we obtain:

$$\max_{\beta} E[W] = \beta \left( 1 + \frac{\bar{v}}{(1 - \alpha_2) K} \right) E[V_L | \beta] + (1 - \beta) V$$

(4E.35)
4.F. Proof of Proposition 4.6

Adding the conditions that $V = V_L - D$ and that $K = \frac{\partial V}{\partial \alpha}$, the first order condition becomes:

$$\frac{\partial W}{\partial \beta} = -D\tilde{e}(1-\alpha_2)\alpha_L + (\tilde{e}(\alpha_2 - \alpha_L) - (1-\alpha_2)(1-\alpha_L)\beta D)E[V_L | \beta] = 0$$

This brings to:

$$\alpha_2 = 1 - \frac{\tilde{e}(1-\alpha_L)E[V_L | \beta]}{\tilde{e}\alpha_LD + (\tilde{e} + (1-\alpha_L)D\beta)E[V_L | \beta]}$$

Given the construction of the two schedules, this is the unique Pareto dominating equilibrium.

4.F Proof of Proposition 4.6

In order to find the optimal timing of IPO for the venture capitalist, we solve the Bellman equation that equalizes the expected return of the option to invest in the new opportunity with the expected rate of capital appreciation for the small interval of time, $dt$.

The Bellman equation is given by:

$$\frac{1}{2}\sigma^2S^2F''[S] + \mu SF'[S] - rF[S] = 0$$

(4F.38)

Its general solution is given by:

$$F[S] = A_1S^{\gamma_1} + A_2S^{\gamma_2}$$

(4F.39)

where:

$$\gamma_{1,2} = \frac{1}{2} - \frac{\mu}{\sigma^2} \pm \sqrt{\left(\frac{\mu^2}{\sigma^2} - \frac{1}{2}\right) + 2\frac{r}{\sigma^2}}$$

(4F.40)

The optimal solution has to satisfy the following conditions:

$$F[0] = 0$$
$$F[S^*] = S^* - K - \beta D$$
$$\frac{\partial F[S^{\alpha}_{st}]}{\partial S} = 1 - \frac{\partial \beta}{\partial S} \frac{\partial D}{\partial S}$$

The first condition indicates that when the second investment opportunity is valueless, also the option to invest on it is valueless. From this follows that $A_2 = 0$.

\textsuperscript{18}for simplicity we do not consider the value of the first investment as through simplifications we would obtain the same results.
The second and third conditions are the value matching and the smooth pasting condition at the optimal stopping firm value. In the payoff of the option, we have introduced the underpricing that the venture capitalist is incurring at the moment of the IPO.

From equations (4.11), (4.15), (4.17) and (4.23):

\[
\beta = \frac{\bar{\alpha}_L L}{V_L (\bar{\alpha}_L + L (1 - \alpha_L) \ln \frac{V_L}{L})},
\]

\[
D = \frac{(1 - \alpha_L) (2K - S - \bar{\epsilon}) (\bar{\alpha}_L + L (1 - \alpha_L) \ln \frac{V_L}{L})}{\bar{\alpha}_L (\bar{\alpha}_L + L (1 - \alpha_L) (1 + \ln \frac{V_L}{L}))} V_L
\]

Solving the system of equations, we obtain:

\[
A_1 = \frac{S_{1}^{* \gamma_1 - 1}}{\gamma_1} \frac{\bar{\alpha}_L + L (1 - \alpha_L) \ln \frac{V_L}{L}}{\bar{\alpha}_L + L (1 - \alpha_L) (1 + \ln \frac{V_L}{L})}
\]

\[
S_{1}^{*} = \frac{\gamma_1}{\gamma_1 - 1} \frac{(\bar{\alpha}_L + L (1 - \alpha_L) (\ln \frac{V_L}{L} - 1)) K - \bar{\epsilon}}{\bar{\alpha}_L + L (1 - \alpha_L) \ln \frac{V_L}{L}}
\]