Measurements of the W-pair production rate and the W mass using four-jet events at LEP
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Chapter 4

Event Reconstruction

The reconstruction of data taken with the L3 detector is done in several steps. First the raw data are processed. In this phase the digitized output signals of all detectors are converted to physically meaningful quantities, like energies and locations of hits, using the most recent calibrations available. Next, the individual subdetector signals are combined into higher level objects, for instance clusters or tracks. From these, quantities relevant for the detection and reconstruction of $W^+W^-$ events are calculated. This procedure is briefly summarized in this Chapter.

4.1 Track Reconstruction

To construct tracks, the hits in the SMD, TEC and $z$-chambers have to be combined into a number of patterns. In events with multiple charged particles there can be hundreds of hits, making this a difficult task. The procedure adopted is to start the pattern recognition by analyzing the hits in the TEC using a Minimum Spanning Tree [84] algorithm. In short, the algorithm starts by combining hits on adjacent wires into a doublet. Then, doublets having a hit in common are added together to form trees. If more than one doublet can be added, the one that gives the smallest increase in the tree length is taken. Once no more doublets can be added, a circle is fitted to the tree. If the fit is good, the tree is accepted as a valid track segment. When all track segments have been found, the compatible ones are combined, in such a way that the longest possible tracks are formed.

After the tracks in the TEC have been found, each track is extrapolated to the $z$-chambers. With the combined $z$ and $r\phi$ information obtained using the $70^\circ$ angle between the second and third $z$-chamber plane and the $z$-axis, hits in the $z$-chamber can be matched to the tracks. Similarly, tracks are extrapolated to the SMD, and matching hits are again added. Once it is determined which hits are assigned to a track, a circle fit is performed to all hits to determine the optimal track parameters.
4.2 Cluster Formation in the Calorimeters

To form clusters, nearby hits in the individual calorimeter components (crystals for the ECAL, towers for the HCAL) have to be combined. For the ECAL, clusters are defined as continuous regions of crystals where at least 10 MeV is deposited in each crystal. For the HCAL, the minimum energy of a tower is nine MeV. As the HCAL segmentation is three dimensional, the clustering is in this case done in three dimensions. The principle behind the algorithm is again the association of nearby hits.

In the next phase, ECAL and HCAL clusters are combined into so-called “smallest resolvable clusters”. This is also done on basis of proximity. Three dimensional information is used, for instance ECAL clusters are never combined with HCAL clusters which only have energy deposits in the outermost part of the detector.

4.3 Energy Determination

Finally, the energy of the cluster formed as described above has to be determined from the energy deposits in the individual detector components associated to the cluster. For this it is important to realize that the cluster can reflect a single particle, but this is not always the case. For instance, for a high energy tau lepton in the process $\tau^- \to \pi^- \pi^0 \nu_\tau$, all decay products are frequently reconstructed as one cluster, reflecting the original tau lepton instead of its decay products. In practice, the L3 detector has insufficient precision to distinguish between all different cluster types and perform a complete energy flow analysis, so that for most clusters no attempt has been made to make a particle identification. The only exceptions are isolated photons, electrons and muons. The first two, electromagnetic, particles can usually be recognized by a narrow shower in the ECAL, with most energy concentrated in the central crystal and no or relatively low energy deposit in the hadronic calorimeter. In this case the energy of the particle is estimated by the energy measured in the ECAL. Muons can be recognized by a number of hits in the muon chambers, consistent with a track coming from the vertex where the electron positron pair collides. The momentum of the muon is estimated by measuring the curvature of the track. For all clusters not explicitly identified a pragmatic approach is taken instead: these clusters are considered to be massless, and the energy measurement is done as described below, without any assumptions regarding the particle identity.

In the schematic layout of the L3 detector in Figure 4.1 the subdetectors are shown and grouped into twelve classes (for historical reasons no detector is associated with the number five, the energy here is set to zero). For the energy measurement, the energies measured in the subdetectors have to be combined. The energy of a cluster can be written as

$$E_{clus} = \sum_{i=1}^{12} g_i E_i,$$  \hspace{1cm} (4.1)
where the $E_i$ are the energies measured in subdetector $i$ and $g_i$ are the so called $g$-factors. In principle one would expect that the best measurement would be obtained with all $g_i$ equal to one, assuming all subdetectors have been calibrated correctly. For a variety of reasons this is not the case. The ECAL, for example, has been calibrated under the assumption that the particle loses energy in the detector due to an electromagnetic shower, as is the case for photons and electrons. For particles with an electromagnetic signature, the $g$-factors corresponding to the ECAL are therefore indeed set to one. For hadronic particles in the ECAL the interaction mechanism is different, with sizable energy deposits mainly arising from nuclear interactions. The standard ECAL calibration will in this case not give the best estimate for the energy deposition, which is one of the reasons why the $g_i$ deviate from one for particles without electromagnetic signature. Other reasons can be energy absorption in front of the detector, for example by cables or support structures, so that only a part of the particle energy can be measured. In this case, even a perfect measurement of the energy deposited in the detector would still underestimate the energy of the original particle. This can be partially compensated by a $g$-factor larger than one. Likewise, the $g$-factors can compensate for energy leakage due to holes in the detector, or undetected particles like

Figure 4.1: Schematic view of the L3 detector with the twelve regions used for the energy measurement. For historical reasons no detector is associated with region five.
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neutrinos.

A separate issue is the reconstruction of clusters which have an energy deposition in the calorimeter that has been matched to one or more tracks in the central drift chamber, the TEC. In this case, the energy of the charged particles associated with the cluster is measured by the TEC, again assuming the particles are massless, as well as by the calorimeters. Simply adding both measurements would clearly lead to some double counting. For clusters reflecting just one charged particle one would expect the optimal energy resolution to be obtained by taking a weighted average of the TEC and calorimeter measurements. However, in practice it is not always possible to distinguish between clusters with only charged particles and clusters containing both charged and neutral particles. A more pragmatic approach is taken, and two sets of $g$-factors are created: one for clusters with energy measured in the TEC, and one without. The first set will then correct for the double counting by lowering the $g$-factors, while the second set will give an optimal energy measurement using the calorimeters alone.

To determine the values for the $g_i$ giving the best energy resolution, a high statistics sample of hadronic events, $e^+e^- \rightarrow q\bar{q}$, is selected. For this channel a high efficiency and purity are obtained. The total visible energy per event can be defined as the sum of all $N_{\text{clus}}$ cluster energies, i.e. $E_{\text{vis}} = \sum_{j=1}^{N_{\text{clus}}} E_j$. If all energies are measured perfectly, the visible energy should be equal to twice the beam energy $E_{\text{beam}}$. This provides an excellent way of determining the $g$-factors using the data by minimizing the visible energy resolution

$$\sigma_{E_{\text{vis}}}(g_i) = \sqrt{\frac{1}{N_{\text{events}}} \sum_{j=1}^{N_{\text{events}}} (E_{\text{vis},j}(g_i) - 2E_{\text{beam}})^2}$$

with respect to the $g$-factors $g_i$. Here $N_{\text{events}}$ denotes the number of selected hadronic events. This procedure ensures the proper energy scale as well as the optimal resolution on the visible energy. The data used are the calibration data at $\sqrt{s} = m_Z$, taken prior to each high energy run, where the high cross section ensures sufficient statistics.

If the detector response would be described perfectly in the Monte Carlo simulation, the $g$-factors obtained from the data could be used in the reconstruction of Monte Carlo events. In practice some differences between data and Monte Carlo unfortunately remain. For instance effects like noise, calibration effects or the exact amount of materials in front of a detector are difficult to model accurately.

To some extent it is possible to improve on the situation by using a different set of $g$-factors for the Monte Carlo. In the implementation used for this analysis, the average energy deposition per detector region $i$, $E_i^{\text{av}}$, is determined for the calibration data and an equivalent Monte Carlo sample. The $g$-factors applied for the Monte Carlo are then those as determined for the data, but scaled according to

$$g_i^{MC} = \frac{E_i^{\text{av, data}}}{E_i^{\text{av, MC}}} g_i^{\text{data}}.$$
4.3. Energy Determination

An alternative procedure to derive $g$-factors, not using the TEC for the cluster energy measurements, is described in Reference [85]. The $g$-factors for the data are obtained as described above, except that the data used is not the calibration sample taken at $\sqrt{s} = m_Z$, but the high energy data sample. The $g$-factors for the Monte Carlo are now also obtained by minimizing equation 4.2, except now using a Monte Carlo sample corresponding to the high energy data. A comparison of the energy measurement with the standard and the alternative $g$-factors will be used to assign a systematic error on cross section and mass measurements due to uncertainties in the energy measurement in Chapters 5 and 6.

4.3.1 Related Analysis Variables

In the selection of $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$ events, use is made of a number of variables related to the cluster formation and energy reconstruction:

- $E_{\text{vis}}$: the total visible energy, calculated from all clusters after application of $g$-factors;

- $E_{\text{long}}$: the “longitudinal” energy, calculated from all clusters after application of $g$-factors, by projection of the cluster energy on the $z$-axis: $E_{\text{long}} = \sum E_{\text{clus}} \cdot \cos \theta_{\text{clus}}$. $E_{\text{long}}$ should be close to zero for a balanced event;

- Cluster multiplicity: the number of reconstructed clusters with an energy of 300 MeV or more, after application of $g$-factors;

- $\text{max}(E_{\text{BGO}})$: the BGO energy of the cluster with the largest energy deposit in the ECAL. The cluster is assumed to be produced by an electron or photon, and no $g$-factors are applied;

- Sphericity [86]: let $Q_1$, $Q_2$ and $Q_3$ be the ordered, $Q_1 \leq Q_2 \leq Q_3$, eigenvalues of the momentum tensor $s^{ij}$ defined as

$$s^{ij} = \frac{\sum_a p_{ai}^i p_{aj}^j}{\sum_a p_a^2} \quad i, j = 1, 2, 3;$$

where the sum runs over all particles labeled “a” in the event, and where $p_{ai}$ is the $i^{th}$ component of the momentum vector $p_a$. The sphericity $S$ is then defined as

$$S = \frac{3}{2}(Q_1 + Q_2).$$

Values of $S$ are close to zero for pencil-like events, and close to one for spherical events.
4.4 Jet Reconstruction

After the clusters have been formed, the next step of the analysis is to combine them into groups which give a good representation of the underlying event structure, i.e. in the context of this analysis the four primary quarks. This procedure assumes that each cluster represents a particle that can be unambiguously assigned to one of the original quarks. A procedure without this assumption has been suggested [87, 88]. While being a simplification, the approach adopted here gives a more intuitive picture and the assumption made proves to be an adequate approximation of the more complicated physical reality.

The DURHAM algorithm [89] is used for the combination of clusters. Out of the list of the reconstructed particles, the pair \((i,j)\) with the minimal distance \(y_{ij}\) is chosen. The DURHAM algorithm defines the distance as

\[
y_{ij} = \frac{2 \min(E_i^2, E_j^2)(1 - \cos \theta_{ij})}{\sum_{k=1}^{N_{clus}} E_k},
\]

where \(E_i, E_j\) are the energies of clusters \(i, j\) and \(\theta_{ij}\) is the angle between the clusters \(i\) and \(j\). These two, “closest”, particles are replaced in the list by a single object obtained by adding their four-momenta. The procedure is then repeated until a predefined number of objects is reached. The resulting objects are called jets, assumed to correspond to the original quarks. It should be noted that adding the four-momenta of two particles that have a non-zero angle between them introduces a mass for the jet, even if the two original particles are massless.

For this analysis, four jets are constructed with this algorithm for each event. The smallest distance between any two of those jets, calculated with Formula 4.4, is denoted as \(y_{34}\). As described in Chapter 5, this quantity provides an important criterion for separating four-jet events from two- and three-jet events.

Other jet algorithms that have been studied are JADE [90], LUCLUS [91], DICLUUS [92], angular ordered DURHAM and CAMBRIDGE [93]. The performance expected from Monte Carlo studies for these algorithms is comparable to the one obtained using the DURHAM scheme. As an example of this, the jet-jet invariant mass after a constrained fit in Monte Carlo \(e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}\) events in shown for two algorithms, LUCLUS and DURHAM, in Figure 4.2. The figure shows that there is no significant difference in the overall performance (a), but that there can be large differences per event for a fraction of all events (b): for some events the jet clustering is unstable, an infinitesimal difference in input can lead to totally different jets. For an overview of these jet algorithms see also Reference [94].

4.4.1 Related Analysis Variables

In the selection of \(e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}\) events, use is made of a number of variables related to the jet reconstruction:
4.5. Constrained Fitting

Figure 4.2: Left: jet-jet invariant mass after a constrained fit in Monte Carlo $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$ events for the DURHAM algorithm (full histogram) and the LU-CLUS algorithm (dashed histogram, almost identical to the full histogram). Right: event-by-event difference between the two algorithms.

- $\max(E_{ASRC}/E_{jet})$: the maximum fraction of jet energy taken by a single cluster in all reconstructed jets. This should be relatively small for a true hadronic jet, and is typically large for a jet formed by an isolated particle;
- $\log(y_{34})$: the logarithm of the value of $y$ in the DURHAM jet finder at which the event changes from a 4-jet to a 3-jet topology;
- $\min(E_{jet}), \max(E_{jet})$: minimum and maximum of the energies of the reconstructed jets;
- $\min(\theta_{jet-jet})$: smallest angle between any two of the reconstructed jets;
- $m_{hemisphere}$: average mass of the jets if the event is forced into a 2-jet topology.

4.5 Constrained Fitting

In genuine $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$ events typically a very small fraction of energy is carried away by undetected initial state radiation photons, while most of it is distributed in the multi-hadron system resulting from the fragmentation of the four quarks. Given an ideal detector, the four-momenta of the reconstructed jets should in good approximation sum up to the total four-momentum of the initial electron-positron system. This is not realized in practice due to the finite resolution of the detector. However, due to the known four-momentum conservation, constraints can be put on the sum of the measured jet energies and momenta. By exploiting these constraints one can improve the measurement of the reconstructed jet...
parameters. This is done by varying the measured jet energies and angles in such a way that the four constraints:

\[ \sum_{i=1}^{4} E_i = \sqrt{s} \]  
\[ \sum_{i=1}^{4} \tilde{p}_i = 0 \]

are satisfied by the new, fitted, jet parameters. As there are more jet parameters that can be varied than equations to satisfy, many sets of parameters can fulfill the constraints. The solution minimizing:

\[ \chi^2 = \sum_{i=1}^{4} \frac{(E_i - E_{i,0})^2}{\sigma_E^2(E_i, \theta_i)} + \frac{(\theta_i - \theta_{i,0})^2}{\sigma_{\theta}^2(E_i, \theta_i)} + \frac{(\phi_i - \phi_{i,0})^2}{\sigma_{\phi}^2(E_i, \theta_i)} \]

is chosen, where \( E_{i,0}, \theta_{i,0} \) and \( \phi_{i,0} \) denote the measured parameters of jet \( i \) and \( E_i, \theta_i, \phi_i \) are the fitted parameters of jet \( i \). The measurements are assumed to be uncorrelated, and to have Gaussian errors. The jet velocities \( \beta = \frac{p_i}{E_i} \) are kept constant at their measured values during the minimization. This reduces the number of free parameters in the fit without compromising the resolution on the fitted parameters.

An important element of the \( \chi^2 \) calculation are the estimates of the jet measurement errors. The resolutions depend on jet energy \( E \) and polar angle \( \theta \) and are parametrized as:

\[ \sigma_E = \sqrt{E} \sqrt{a_E + \frac{b_E}{E}} \left( 1 + \frac{c_E}{\min(\theta, \pi - \theta)} + d_E \left| \theta - \frac{\pi}{2} \right|^2 \right) \]
\[ \sigma_\theta = \frac{1}{\sqrt{E \sin \theta}} \sqrt{a_\theta + \frac{b_\theta}{E}} \left( 1 + \frac{c_\theta}{\min(\theta, \pi - \theta)} + d_\theta \left| \theta - \frac{\pi}{2} \right|^2 \right) \]
\[ \sigma_\phi = \frac{1}{\sqrt{E \sin \theta}} \sqrt{a_\phi + \frac{b_\phi}{E}} \left( 1 + \frac{c_\phi}{\min(\theta, \pi - \theta)} + d_\phi \left| \theta - \frac{\pi}{2} \right|^2 \right), \]

where the parameters \( a_x, b_x, c_x, \) and \( d_x \) are determined from studies of \( e^+ e^- \rightarrow W^+ W^- \rightarrow q\bar{q}q\bar{q} \) Monte Carlo events [95].

Technically the constrained fit is performed by a numerical minimization of the \( \chi^2 \) defined in equation 4.7, using the gradient descent method implemented in the MINUIT [96] software package. A penalty contribution:

\[ \Delta \chi^2_{4C} = \frac{(\sum_{i=1}^{4} E_i - \sqrt{s})^2}{\sigma_1^2} + \frac{(\sum_{i=1}^{4} p_i^4)^2}{\sigma_2^2} + \frac{(\sum_{i=1}^{4} p_i^4)^2}{\sigma_3^2}, \]

with values of \( \sigma_{1,2,3} \) of the order of 100 MeV, is added to the \( \chi^2 \) to impose the constraints. This value is chosen to combine a fast convergence of the fit with the fulfillment of the constraints well within the experimental resolutions. The fit described above is referred to as a four-constraints, 4C, fit.

If the assumptions implicit in the definition of the \( \chi^2 \) and the constraints are correct, the resulting \( \chi^2_{4C} \) should have a \( \chi^2 \) distribution with four degrees of freedom. The constrained
fit probability $P_{\chi^2_{4C}}$, defined as the probability of drawing a value higher than the observed $\chi^2$, should then be uniformly distributed in the [0,1] interval. As shown in Figure 4.3c and d, the resulting distribution for $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$ events is indeed reasonably flat apart from the peak of events with very small probability. The peak is mostly due to non-Gaussian tails in the jet parameter resolutions.

The effect of the constrained fit on the jet energy estimates is illustrated in Figure 4.3a using $W^+W^-$ Monte Carlo events. The resolution can be seen to be improved by a factor of $\sim 2.6$ by the fit. In Figure 4.3b a similar plot is shown for the average reconstructed $W$ mass. For this quantity the resolution is improved by a factor 4.

The experimental statistical error on a reconstructed $W$ mass is of the order of 10 GeV. This is much larger than the intrinsic width of a $W$ boson, which is of the order of $\Gamma_W \approx 2$ GeV. Therefore it is reasonable to assume that both $W$ bosons in the event have identical mass and apply this as an additional constraint in the constrained fit. The fifth constraint improves the resolution on the reconstructed average $W$ mass by about 5%, contradicting the findings reported in [97].

In order to calculate two $W$ masses, the four jets have to be combined into two jet pairs, each assumed to correspond to a $W$ boson. There are three ways to perform this combination, and it is not possible to determine, on an event-by-event basis, which combination is the correct one. A five-constraints, 5$C$, fit is therefore applied to each of the three combinations. Technically the fifth constraint is implemented by adding a penalty contribution:

$$\Delta \chi^2_{5C,i} = \Delta \chi^2_{4C} + \frac{(m_{W1} - m_{W2})^2}{\sigma^2_4},$$

(4.9)

where $\sigma_4$ is of the order of $\Gamma_W$. The $\chi^2$ of the fit should be distributed according to the $\chi^2$ distribution with five degrees of freedom and can again be converted into a probability $P_{\chi^2_{5C}}$. If the assumptions made for the 4$C$ fit, as well as for the equal mass constraint, are correct, $P_{\chi^2_{5C}}$ should have a flat distribution in [0,1]. The probability $P_{\chi^2_{5C}}$ for the dijet combination with the highest probability is shown in Figure 6.1 in Chapter 6.

### 4.5.1 Related Analysis Variables

The selection of $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$ events and the $W$ mass analysis make use of the following event variables related to the constrained fit:

- $P_{\chi^2_{5C}}$ and all fitted jet quantities. If not mentioned otherwise, jet parameters are 4$C$-fitted.

- For the cross section determination, use is made of the two $W$ masses $m_{W_1}$ and $m_{W_2}$, as determined after a 4$C$ constrained fit. Three pairings of jets into $W$'s are possible, chosen is the combination with the smallest mass difference between the two $W$'s after first rejecting the combination with the smallest sum of masses. From Monte
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Carlo events at $\sqrt{s} = 189$ GeV we estimate this choice to be correct for approximately 72% of the events. $W_1$ is defined as the $W$ with the most energetic jet.

- For the $W$ mass analysis, the masses resulting from a 5C fit are used, ordered in probability $P_{\chi^2_{5C},\delta}$. 
Figure 4.3: *a:* Difference between reconstructed and generated jet energies in Monte Carlo $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$ events before the constrained fit (dotted histogram), after a 4C constrained fit (full histogram), and after a 5C constrained fit (dashed histogram). *b:* The same for the average reconstructed and generated $W$ masses. *c* and *d:* Distribution of the probability of the $\chi^2$ of the 4C constrained fit, $P_{\chi^2_{4C}}$, for $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$ events after preselection, with a logarithmic vertical scale for all events in *c*, and with a linear vertical scale for $P_{4C} > 0.05$ in *d*. Dots are data, open histogram is WW signal, dashed histogram is background.
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