Measurements of the W-pair production rate and the W mass using four-jet events at LEP
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Chapter 5

Event Selection and Cross Section Results

At LEP2 many processes generate final states that are observed with the L3 detector. In Figure 5.1 the calculated Standard Model cross sections of some of these processes are shown as a function of $\sqrt{s}$, the total energy in the center-of-mass system. Clearly it is important to select $W^+W^-$ events with a high efficiency while at the same time rejecting most of the events coming from other final states, which are background for W-physics purposes. As the $W^+W^-$ event signatures are dependent on the decay modes of both W bosons the event selection is different for the following cases:

- both W bosons decay to a charged lepton and a neutrino or anti-neutrino (leptonic decay mode);
- one W boson decays to a quark and an anti-quark, the other to a charged lepton and a neutrino or anti-neutrino (semi-leptonic decay mode);
- both W bosons decay to a quark and an anti-quark (hadronic decay mode).

For the event selection the emphasis of this thesis is on the latter, the fully hadronic, decay mode. The analysis of the data taken at $\sqrt{s} = 189$ GeV is presented in detail, for the data taken earlier the results are given.

5.1 Introduction

For the case where both W bosons decay to a quark–anti-quark pair, the experimental signature of the event is the observation of four hadronic jets, in general well separated. As discussed in Chapter 4, the jets can be combined to form two jet pairs with each a dijet mass approximately equal to the W boson mass. The main backgrounds are the processes $e^+e^-\rightarrow q\bar{q}$, $e^+e^-\rightarrow ZZ$, and the $W^+W^-$ events in which one of the W's decays to a lepton and an anti-neutrino.
The process $e^+e^- \rightarrow q\bar{q}$ can be split into a “high energy” and a “radiative return” class. In the latter case an initial state radiation photon is emitted with an energy such that the $Z$ in the intermediate state is on or near its mass shell, therefore enhancing the production cross section considerably. An $e^+e^- \rightarrow q\bar{q}$ event from either class can be mistaken for a four jet event as gluon radiation, initial state radiation and/or misreconstruction can form or fake the third and fourth jet. As can be seen in Figure 5.1, and taking into the account the fully hadronic $W^+W^-$ branching fraction given in Chapter 2, the cross section for $e^+e^- \rightarrow q\bar{q}$ is significantly larger than the cross section for $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$. At the $W$ production threshold it is in fact larger by two orders of magnitude. A powerful way of rejecting this background is therefore important.

For the process $e^+e^- \rightarrow ZZ$ the situation is different. The signature for $ZZ$ events where both $Z$'s decay to hadrons is similar to the $W^+W^-$ signature, making it very difficult to efficiently reject this background without cutting too much of the signal away. Fortunately the cross section for this process is small compared to the signal cross section, and although the $ZZ$ cross section rises with $\sqrt{s}$ the $q\bar{q}$ background will remain dominant.

The background from non-four jet $W^+W^-$ events arises mostly from events where one of the $W$ bosons decays into $\tau\nu_{\tau}$ and the other one into a quark–anti-quark pair. In these
cases two genuine jets exist from one W, and the lepton can be interpreted as a third. A hard gluon radiated from one of the quarks or misreconstruction can then fake a fourth jet. The accepted background due to these events is small, but as the cross section is proportional to the signal cross section the fraction of accepted events with respect to the signal will remain roughly constant with $\sqrt{s}$.

Other possible background sources can be distinguished from the signal rather well and are rejected almost completely. The largest remaining background contribution arises from so called two photon events, where the initial state electron and positron both radiate a virtual photon. These photons subsequently collide and form hadrons which can be detected. Usually the final state electron and positron have a low angle with respect to the beam pipe, and the total invariant mass of the final state hadrons is of the order of a few GeV. These events therefore have a signature rather different from the signal, but as the cross section of this process is several orders of magnitude larger than the signal cross section, a few events could eventually pass the selection criteria.

The event selection is focused on the rejection of the dominant background source, $q\bar{q}$ events. For optimal performance the selection is done in two steps. First, a loose selection is applied to obtain a set of promising four jet candidates. In this phase the events clearly incompatible with a hadronic four jet structure are rejected, while almost all the signal is preserved. Next, a neural network trained to separate $q\bar{q}$ events and $W^+W^-$ events is used for the final selection.

### 5.2 Selection

To obtain a sample of good $W^+W^-$ candidates a selection is performed using the variables described in Chapter 4. In Figure 5.2 the $\sqrt{s} = 189$ GeV data and corresponding Monte Carlo expectations are plotted for the important quantities used in the selection. The following cuts are applied:

- $E_{\text{vis}}/\sqrt{s} > 0.7$. The visible energy of the signal events is expected to be around $\sqrt{s}$. This cut suppresses events from background sources where energy is lost, such as two photon or “radiative return” events, as well as detector noise.

- Cluster multiplicity $> 30$. Many background processes have a multiplicity much lower than expected for the signal, as the fragmentation of four quarks produces many particles scattered through phase space. Almost all signal events pass this criterion, whereas background from processes like $e^+e^- \rightarrow e^+e^-$ is practically completely removed. As can be seen in Figure 5.2c the multiplicity in the Monte Carlo is shifted with respect to the multiplicity in the data. This problem is related to the behavior of electromagnetic clusters with relatively low energy, which is not well simulated. Possible systematic
Figure 5.2: The data collected in 1998 and corresponding Monte Carlo expectations for variables used in the preselection. All cuts except the one on the variable shown have been applied. The dots denote the data, the open histogram represents the total Monte Carlo expectation and the hatched histogram the sum of the background Monte Carlo expectations. Shown is respectively the visible energy scaled to $\sqrt{s}$ (a), the longitudinal energy (b), the cluster multiplicity (c), the maximum energy deposition of a cluster in the BGO part of the ECAL (d), the maximum energy fraction of a jet contained in a single cluster (e) and the logarithm of the jet resolution parameter $y_{34}$ (f).
errors due to this shift are expected to be small as this cut is effective mainly for background sources with a typical number of clusters much smaller than the cut value, whereas the typical multiplicity for the signal is much larger.

- $|E_{\text{long}}|/E_{\text{vis}} < 0.25$. Requiring a low value for the longitudinal energy imbalance selects balanced events. As most signal events are balanced events without missing energy this cut rejects almost no signal. The signal events that are rejected are mostly events where a jet is missing in the beam pipe. Background events are much more likely to miss energy in the forward or backward direction. For instance, events with a radiative return to the $Z^0$ often have a high energy photon lost in the beam pipe, while two photon events usually have an electron and/or positron with a low angle.

- $y_{34} > 0.001$. As explained in Chapter 4, this criterion selects events with a four jet like topology.

- $\max (E_{\text{BGO}}) < 45 \text{ GeV}$. For a small fraction of the “radiative return” events the radiated photon is emitted with an angle to the beam pipe sufficiently large to be measured with the L3 detector. In the reconstruction of such an event, the photon, typically with a high energy, can be interpreted as an additional jet. To prevent this, events where a single cluster has a sizable energy deposition in the BGO part of the electromagnetic calorimeter are rejected.

- $\max (E_{\text{ASRC}}/E_{\text{jet}}) < 0.8$. If more than 80% of the energy of any jet is contained in a single cluster the event is rejected. Usually this type of jets are formed by a lower energy initial state radiation photon that passed the cut on the maximum BGO energy.

- $\max |p_\mu| < 20 \text{ GeV}$, where $p_\mu$ is the momentum of a muon in the event. This cut mainly rejects $W^+W^-$ events where one of the $W$'s decays to a muon and a muon (anti-)neutrino, or events where one of the $W$'s decays to a tau and a tau neutrino and the tau decays to a muon and two neutrinos.

- Noise rejection. Occasionally correlated noise in the hadronic calorimeter may be serious enough to fake a four jet event. As this HCAL noise is uncorrelated with other detector elements these events can be rejected by requiring at least ten good tracks (ATRK's) in the TEC. For events with exactly zero good tracks it is assumed the TEC is switched off and the noise rejection criterion is replaced by the requirement that the total energy deposition in the BGO barrel exceeds 20 GeV.

The efficiency of the preselection for the signal and various background sources is given in Table 5.1. For the background the accepted cross section that is expected from the Standard Model is also given.
Even Selection and Cross Section Results

<table>
<thead>
<tr>
<th>Process</th>
<th>Efficiency (%)</th>
<th>Accepted Cross Section (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^+W^- \rightarrow q\bar{q}q\bar{q}$</td>
<td>92.3</td>
<td></td>
</tr>
<tr>
<td>$W^+W^- \rightarrow q\bar{q}q\bar{q}$</td>
<td>2.0</td>
<td>0.18</td>
</tr>
<tr>
<td>$q\bar{q}$ background</td>
<td>4.2</td>
<td>4.13</td>
</tr>
<tr>
<td>ZZ background</td>
<td>40.4</td>
<td>0.39</td>
</tr>
<tr>
<td>Two photon background</td>
<td>$9 \times 10^{-4}$</td>
<td>0.04</td>
</tr>
</tbody>
</table>

Table 5.1: Preselection efficiency and accepted background cross section for signal and background Monte Carlo for $\sqrt{s} = 189$ GeV.

Figure 5.3: Schematic layout of a three layer, feed forward neural network with one output node. The values of the input nodes are denoted by $x_k$, the values of the nodes in the hidden layer by $h_j$ while the final value for the output node is $n_{\text{out}}$. The weights of the connections between the input and hidden layers are denoted by $w_{jk}$ and those of the connections between the hidden layer and the output node by $W_j$.

5.3 Neural Network

As can be seen in Figure 5.2, there is no single variable which provides satisfactory separation between signal and background. As also many of the variables which can be used are correlated, it seems natural to exploit these correlations by using a multidimensional discriminator function $d = d(\vec{x}; \vec{p})$. Here $\vec{x}$ denotes a vector of input variables and $\vec{p}$ a vector of parameters which can be optimized to obtain good separation. In this analysis a function like this has been constructed using a neural network.

A simple neural network, similar to the one used, can be visualized by the architecture shown in Figure 5.3. Here the lowest layer of nodes represents the input parameters chosen to separate the signal from the background. For each input parameter $k$ a value $x_k$ between zero and one is calculated. These values are then used to calculate function values $h_j$ for the nodes in the middle, hidden, layer. The neural net output $nn_{\text{out}}$ is subsequently calculated.
5.3. Neural Network

Figure 5.4: Neural net activation function \( g(z) = \left[ 1 + \exp(-2z) \right]^{-1} \). The characteristic sigmoid shape is visible, the function \( g(z) \) is only sensitive to the value of \( z \) for small values of \(|z|\).

from the nodes in the hidden layer. This type of neural net is called a three layer, feed forward neural network with a single output node.

To calculate the function value of a node from the values \( x_k \) of the layer below a single parameter \( z \) is first calculated using the formula \( z = \sum_k w_k x_k + \theta \). The value of the node is then obtained using the “neural activation function” \( g(z) = \left[ 1 + \exp(-2z) \right]^{-1} \). This function has a characteristic sigmoid shape, as shown in Figure 5.4, and is responsible for the non-linear response of the network. In these formulas \( w_k \) denotes the weights with which the input values are combined, while \( \theta \) shifts the resulting sum to keep the average value of \( z \) in the region where the activation function is sensitive. For a neural network of this type with \( N_{in} \) input nodes and \( N_{h} \) hidden nodes the behavior of the total network can be summarized in a single formula:

\[
n_{out} = g \left[ \sum_{j=1}^{N_h} W_{j} g \left( \sum_{k=1}^{N_{in}} w_{jk} x_k + \theta_j \right) + \Theta \right].
\] (5.1)

Here the weights and offsets between the input and hidden layer are denoted by \( w_{jk} \) and \( \theta_j \), and those between hidden and output layer are \( W_j \) and \( \Theta \).

The crucial point when using a neural network is to obtain good values for the free parameters in formula 5.1, the weights and offsets. This is done using a sample of simulated events called the training sample. For these events it is known whether they were generated as signal or as background, so that an error measure can be defined as \( \sum_i (n_{out,i} - t_i)^2 \). Here the sum is taken over the entire training sample (both signal and background), and the target values \( t_i \) are one for signal events and zero for background events. By minimizing this error measure with respect to the weights and offsets proper values can be obtained. There are many algorithms available for this minimization. To obtain the parameters for the
neural network used in this thesis a so called back-propagation algorithm has been used, as implemented in the JETNET package [98].

**The Neural Net Input Variables**

For this analysis a neural network with three layers has been chosen. The input variables that were chosen are listed below, and a plot with data and Monte Carlo expectations for the variables that were not yet shown can be found in Figure 5.5. Note that for the calculation of $x_k$ the input variables have, where necessary, been rescaled to fall in the range $[0, 1]$.

- **Sphericity.** As described in Chapter 4, this event shape variable characterizes how spherical the event is. The background, mainly two jet events, can clearly be seen to peak at low values, while the signal events, with at least four jets, are more spherical.

- **$m_{W_1} - m_{W_2}$**, the difference of the two W masses as calculated after a four-constraints fit, as explained in Chapter 4. Note that the most energetic jet is by definition included in the first W, which explains why the distribution is not symmetric around zero. The asymmetry is larger for the background, as the energy of the most energetic jet is usually higher for the background (see Figure 5.5d).

- **$\min(E_{\text{jet}})$**. The main background essentially has a two-jet topology, but has been reconstructed assuming that there are four jets. This often results in two jets reflecting the original two jets, which have most of the energy, and two lower energy jets due to misreconstruction or a hard fragmentation. The energy of the least energetic jet is therefore in general lower for the background than for the signal. The energies are those calculated after a four-constraints fit.

- **$\max(E_{\text{jet}})$**. For similar reasons as for the least energetic jet, the energy of the most energetic jet is a variable with a good separating power between signal and background and is thus used as an input variable for the neural net.

- **$\min(\theta_{\text{jet-jet}})$**. The fourth jet in a background event will often be a gluon jet. As an emitted gluon typically has a low angle with respect to the original quark the minimum angle between two jets in the event will generally be smaller for background events.

- **$m_{\text{hemisphere}}$**. In this case, the event is reconstructed under the assumption that it is a two-jet event and consequently the clusters are grouped into two jets. If the assumption is correct the event should have two narrow, low mass jets, while for a signal event the four jet structure of the event will result in two broad, high mass jets. The average of the two jet masses is therefore a good discriminating variable. This average mass can be interpreted as the average hemisphere mass.
• log \( y_{34} \). As can be seen in Figure 5.2, the jet resolution parameter \( y_{34} \) provides a good separation between signal and background.

Most input variables exploit the difference between the true four jet topology of the signal and the underlying two jet structure of most background. The only exception to this is the mass difference of the two assumed W bosons. For this reason it is understandable that the neural network will not be able to distinguish ZZ events from \( W^+W^- \) events if both \( Z^0 \)'s decay to quarks: the event has a true four jet structure, and both reconstructed bosons have, within the experimental resolution, the same mass. As in any case, up to \( \sqrt{s} = 189 \) GeV, the \( q\bar{q} \) background is dominant, no attempt has been made to improve on this situation.

Note that the sum of the reconstructed masses has not been used as a neural net input variable. Although this would have added some separating power between \( W^+W^- \) events and ZZ events as well as two jet background the resulting gain is relatively small. At the same time the selection would have become explicitly dependent on the W mass. This could lead to an undesired systematic error for the cross section measurement as well as complications when the W mass will be fitted using the selected events.

### 5.3.1 Training the Neural Network

As mentioned above, the neural net used for the final event selection is a three layer neural network as described by equation 5.1, where the desired output is one for signal events and zero for background events. As the training is focused on the dominant two jet background, the training sample consists of a mixture of background \( e^+e^- \rightarrow q\bar{q} \) and signal \( e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q} \) Monte Carlo events. A total of \( 3 \times 10^6 \) \( q\bar{q} \) and \( 1.3 \times 10^5 \) signal events are available. Only half of these events have been used in the training of the neural network. This is necessary to be able to test the so called generalization performance of the neural net: the ability to distinguish signal and background on an independent sample, i.e. a sample that has not been used in he training. The importance of this generalization performance can be understood by looking at the large number of free parameters in formula 5.1: a neural network with seven input nodes and ten nodes in the hidden layer has a total of 91 free parameters to be determined. Especially with a small training sample, or a training sample containing only a small number of events that are difficult to classify, the risk exists that the network focuses on accidental features of the training patterns. As this so called overlearning could lead to an overly optimistic estimate of the performance, it is important to determine efficiencies and purities from an independent sample.

The training is performed in an iterative procedure in which the total training sample is used many times by the package performing the minimization, in this case JETNET. The number of cycles through the training sample is called the number of epochs. During the training the performance of the neural net can be monitored. Here the performance is characterized by \( \sqrt{\varepsilon \pi} \), the square root of the final selection efficiency \( \varepsilon \) for signal events times
Figure 5.5: The data collected at $\sqrt{s} = 189$ GeV and corresponding Monte Carlo expectations for variables used as input for the neural network. All cuts have been applied. The dots denote the data, the open histogram represents the total Monte Carlo expectation and the hatched histogram the sum of the background Monte Carlo expectations. Shown are respectively the sphericity (a), mass difference between the two reconstructed W masses (b), energy of the least energetic jet (c), energy of the most energetic jet (d), minimum angle between two jets (e) and the average mass of the two jets if the event is forced into two jets (f).

62
the fraction of all selected events that are signal events, the purity $\pi$. As this number is, in good approximation, inversely proportional to the expected statistical error of a cross section measurement, it is a convenient way to express the performance in a single number. An example of the performance of the neural network used for this analysis as a function of the epoch number is shown in Figure 5.6a. The performance of the training and of the testing sample are both shown. At first, the performance of both samples improves with the epoch number: the network is being trained. Then the performance can be seen to stabilize when the optimal configuration has been found. Also, it is clear that the training sample is large enough to prevent overlearning: the performance of the training and testing sample is, within the statistical precision, identical.

In Figure 5.6b the final performance, after sufficient training, is plotted as a function of the number of hidden nodes. As it is clear that adding more than seven hidden nodes is no longer useful, the number of hidden nodes for the net used in the analysis has been fixed at seven.

### 5.4 Results and Cross Section Determination

The most straightforward way to determine the signal cross section would be to cut on the neural net output $nn_{out}$. However, this way not all available information is exploited: events
with a neural net output close to one have a much higher purity than events that just pass the cut, and should therefore get more weight in the determination of the $W^+W^-$ cross section to obtain a better statistical sensitivity. The method chosen is to perform a fit to the full neural net output spectrum, i.e. to determine for which value of the signal cross section the predicted neural net output spectrum agrees best with the data. The exact procedure followed is described in [99, 100] and summarized below.

### 5.4.1 Fitting Method

First the shape of the neural net output is determined for the signal and various backgrounds. For this purpose, the neural net output spectrum is divided in $N_b$ bins, giving $d_i$ data events per bin $i$. For the signal and background Monte Carlos sample the number of events per bin is $a_{ji}$ for Monte Carlo sample $j$. As the total luminosity $L$ and the selection efficiency $\epsilon_j$ are known, the expected number of events $n_i$ in each bin can be calculated using

$$n_i = \sum_{j=1}^{N_{MC}} L \epsilon_j \sigma_j \frac{a_{ji}}{s_j}, \quad (5.2)$$

where the sum extends over the number of Monte Carlo samples $N_{MC}$, and $\sigma_j$ is the cross section of the corresponding process $j$. Here $s_j$ is the sum of the events in Monte Carlo sample $j$, i.e. $s_j = \sum_{i=1}^{N_b} a_{ji}$. The probability $P(d_i)$ to measure $d_i$ events when $n_i$ events are expected is given by Poisson statistics:

$$P(d_i) = e^{-n_i} \frac{n_i^{d_i}}{d_i!}. \quad (5.3)$$

Using equation 5.3 and omitting the constant factorials, the log likelihood $L$ can be written as

$$\log L = \sum_{i=1}^{N_b} (d_i \log n_i - n_i). \quad (5.4)$$

As the $n_i$ are a linear sum of the cross sections $\sigma_j$ of the various Monte Carlo processes one wants to measure, the cross sections can be obtained by maximizing the likelihood of equation 5.4 with respect to the $\sigma_j$, thus performing a standard binned log likelihood fit. The errors on the cross sections can as usual be found by looking at the contour $|L(\sigma_i) - L_{max}| = 0.5$, where $L_{max}$ is the value of the likelihood evaluated at the point where it is maximal. This explains why the logarithm of $d_i!$ can be left out of equation 5.4, as this term does not depend on the cross sections one wants to measure it will not influence the shape of the likelihood curve.

As the maximization cannot be performed analytically, a numerical procedure has to be followed. Often the MINUIT package [96] is used, giving fast and reliable estimates for the cross sections that are to be determined and their errors.
However, a drawback of the use of likelihood 5.4 is that it implicitly assumes that the Monte Carlo samples used are of infinite size, as otherwise the statistical fluctuations on the ratio $a_{ji}/s_j$ need to be taken into account. Since the size of the Monte Carlo samples actually used are too small to justify this assumption, the method has to be modified. This can be achieved by observing that the numbers $a_{ji}$ are nothing else but stochastic variables, depending on the expected number of Monte Carlo events in a particular bin. It is this, unknown, number of expected events $A_{ji}$ that should be used in formula 5.2:

$$N_i = \sum_j \lambda \epsilon \sigma_j \frac{A_{ji}}{S_j},$$

(5.5)

with $S_j = \sum_{i=1}^{N_b} A_{ji}$. If $A_{ji} << S_j$ it can be assumed safely that the $a_{ji}$ are generated from a Poisson distribution with mean $A_{ji}$. The improved log likelihood, again leaving out the constant factorials, can then be written as

$$\log L = \sum_{i=1}^{N_b} (d_i \log N_i - N_i) + \sum_{i=1}^{N_b} \sum_{j=1}^{N_{MC}} (a_{ji} \log A_{ji} - A_{ji}).$$

(5.6)

This likelihood now depends not only on the cross sections $\sigma_j$, but also on the $A_{ji}$. This means that to obtain values for the cross sections, the likelihood 5.6 has to be maximized with respect to $N_{MC} \cdot (N_b + 1)$ unknowns. This is obviously a more complicated task as the just $N_{MC}$-dimensional maximization necessary when using the likelihood from formula 5.4. As it turns out, however, the problem can be simplified considerably [99]. When a numerical maximization with respect to the $N_{MC}$ cross sections $\sigma_j$ is performed, for which one can again use the MINUIT package, the values for $A_{ji}$ that maximize equation 5.6 for a given set of $\sigma_j$ can quickly be found. This can be seen by taking the derivative of equation 5.6 with respect to the $A_{ji}$ and setting all derivatives to zero, giving $N_{MC} \cdot N_b$ equations. When substituting

$$x_i = 1 - \frac{d_i}{N_i}$$

(5.7)

this gives the equations

$$A_{ji} = \frac{a_{ji}}{1 + \lambda \epsilon \sigma_j a_{ji} S_j^{-1} x_i} \quad \forall i, j.$$  

(5.8)

This is a considerable simplification as the $N_{MC} \cdot N_b$ unknowns $A_{ji}$ can be calculated from just $N_b$ unknowns $x_i$. The $x_i$ can be calculated by combining the equations 5.5, 5.7 and 5.8 to form

$$\frac{d_i}{1 - x_i} = \sum_{j=1}^{N_{MC}} \frac{\lambda \epsilon \sigma_j S_j^{-1} a_{ji}^2}{1 + \lambda \epsilon \sigma_j S_j^{-1} a_{ji} x_i} \quad \forall i.$$  

(5.9)

As the equations 5.9 are just $N_b$ uncoupled equations which can easily be solved numerically, values for the $x_i$ can be found, which in turn give values for all $A_{ji}$ using equation 5.8.

This method has been implemented in the HBOOK package [100], which is the package used for this analysis.
5.4.2 Results

For this analysis the backgrounds taken into account are, as mentioned before, the $qq$ background, the $ZZ$ background, the background from $W^+W^-$ events decaying to different final states and the two photon events. For the last three background sources, the values for the cross sections are not determined from the data but are fixed at their Standard Model expectation values. For the largest, $qq$, background, this is not done: it is determined simultaneously with the signal cross section from the neural net output spectrum. This is done as it is known from LEPI that the number of four jet events predicted by the $qq$ Monte Carlo is not in satisfactory agreement with the number observed in the data. By leaving the $qq$ cross section free in the analysis, the dependence on the four jet cross section is diminished.

The results of the fits with the statistical errors are given in Table 5.2. For comparison the Standard Model expectation values are listed for the $qq$ background.

<table>
<thead>
<tr>
<th>Energy</th>
<th>Process</th>
<th>Cross Section</th>
<th>Measured</th>
<th>Standard Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s} = 161$ GeV</td>
<td>$qqqq$ signal</td>
<td>0.98$\pm$0.51 pb</td>
<td>142$^{+19}_{-18}$ pb</td>
<td>147 pb</td>
</tr>
<tr>
<td></td>
<td>$qq$ background</td>
<td>128$^{+18}_{-17}$ pb</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sqrt{s} = 172$ GeV</td>
<td>$qqqq$ signal</td>
<td>5.48$^{+0.92}_{-0.85}$ pb</td>
<td>121 pb</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$qq$ background</td>
<td>105$\pm$6 pb</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sqrt{s} = 183$ GeV</td>
<td>$qqqq$ signal</td>
<td>8.35$\pm$0.46 pb</td>
<td>107 pb</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$qq$ background</td>
<td>112$\pm$5 pb</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5.2: Cross sections obtained by the fit for signal and $qq$ background and their statistical error, for the data taken at $\sqrt{s} = 161 - 189$ GeV. For the background the Standard Model expectation value is also given. The statistical error includes the uncertainty due to finite Monte Carlo statistics.

The neural net output plots for the various energies are shown in Figure 5.7. In this plot the signal and $qq$ Monte Carlo have been scaled using the measured cross sections. Event displays of two selected (i.e. high neural network output) events are shown as an example in Figure 5.8.

5.5 Systematic Error Analysis

A common way of evaluating systematic errors is by varying the cuts within limits thought to be "reasonable" and assigning any change in the obtained cross-section to the systematic
Figure 5.7: Neural net output $n_{n_{out}}$ for the data collected at $\sqrt{s} = 161$ GeV (a), $\sqrt{s} = 172$ GeV (b), $\sqrt{s} = 183$ GeV (c) and $\sqrt{s} = 189$ GeV (d). All cuts have been applied. The dots denote the data, the open histogram represents the total Monte Carlo expectation and the hatched histogram represents the sum of the background Monte Carlo expectations. The signal and $q\bar{q}$ Monte Carlo are scaled according to the cross sections derived from the fit to the measured neural net output spectrum.
Event displays of two selected four-jet events at $\sqrt{s} = 189$ GeV, as an example of two typical events. Both events have a high value of the neural network output and are thus likely to be WW events.

The approach adopted here is to investigate possible uncertainties in the Monte Carlo modeling. For each possible effect, a new Monte Carlo sample is obtained with modeling parameters varied within the uncertainty. The analysis is redone using this Monte Carlo sample instead of the original sample, and a possible difference in the end result is used as an estimate of the systematic error due to this effect. This procedure ensures that the error is evaluated correctly even if an effect influences several analysis variables, and is independent of the data statistics. Below the possible effects that have been studied are listed. The total systematic error is obtained by summing up the individual error estimates in quadrature.

5.5.1 Modeling of the Detector Response

Systematic errors due to possibly incorrect modeling of the detector are described here. For this, the existing Monte Carlo samples have been re-reconstructed using different assump-
tions about the detector response, as described below.

**HCAL Energy Calibration**

The distribution of the energy deposited in the HCAL is shown for data and the Monte Carlo expectations in Figure 5.9a. The average energy deposit agrees between data and Monte Carlo up to $1.6 \pm 0.6\%$ on the selected event sample. During the re-reconstruction of all Monte Carlo samples used, all HCAL energies were changed by 2% to determine a possible error on the $W^+W^-$ cross-section. The result of the fit changed by 0.7%, which was taken as the systematic error due to the HCAL energy scale.

**ECAL Energy Calibration**

The total energy deposition in the electromagnetic calorimeters is shown in Figure 5.9b. The difference between the average energy deposit in the data and the Monte Carlo expectation is $0.0 \pm 0.5\%$. To estimate a possible systematic error due to miscalibration of the ECAL, the Monte Carlo samples were re-reconstructed after changing all BGO energies by 1% and SPACAL energies by 5%. The resulting change in the measured $W^+W^-$ cross-section was found to be 0.3%.

**Jet Angular Resolution**

To study the effect of possible mismatch between jet angular resolutions in data and Monte Carlo, the measured jet directions in the Monte Carlo have been changed by $0.5^\circ$ in a random direction. As this is approximately equal to the angular resolution, this change is considered to be conservative. The systematic error assigned due to this effect is 0.1%. The jet angular resolutions have been studied on two-jet events selected from data and Monte Carlo by comparing the acolinearity and acoplanarity distributions.

**Cluster Simulation**

The multiplicity distribution, as shown in Figure 5.2c, has traditionally been a difficult variable to model in the Monte Carlo simulation. The multiplicity has therefore not been used as an input variable for the neural network. Also, the selection cut on the total number of clusters is made at an especially low value. The events just passing the cut will then be mostly obvious background events that can be recognized by the neural network, thus limiting the sensitivity to the multiplicity distribution.

The mean of the Monte Carlo and data multiplicity distributions agree within the statistical precision when counting the multiplicity of clusters with at least 300 MeV. For clusters with more than 100 MeV, however, the difference is about four clusters. As this reflects imperfect Monte Carlo simulation, the systematic error has been evaluated by shifting the
Event Selection and Cross Section Results

Monte Carlo multiplicity distribution by a conservative amount of three clusters. After this the analysis has been redone. The change in the measured cross section is found to be negligible.

\textbf{g-factors}

As described in Section 4.3, the g-factors compensate for part of the discrepancies between the data and the Monte Carlo. However, there is no unique way of doing this, and some differences will remain. To determine the influence of this the Monte Carlo and data have been reconstructed using a set of g-factors determined in a different way, as explained in Section 4.3 [85], instead of using the g-factors used for the rest of this analysis. The resulting $W^+W^-$ cross section differs by 1.2%. The effects of incorrect detector modeling in the Monte Carlo, which have already been estimated above, will again contribute to this shift. For this reason, only half of the shift has been assigned as a systematic error.

\section{5.5.2 Luminosity Determination}

The luminosity used in the analysis has been measured using Bhabha events, as described in Section 3.2. The experimental systematic uncertainties originate from the event selection criteria, 0.10% and from the limited knowledge of the detector geometry, 0.05%. The limited Monte Carlo statistics results in an uncertainty of 0.07%, yielding a total experimental
systematic uncertainty of 0.13%. In addition, a theoretical uncertainty of 0.12% is assigned, originating from the uncertainty in the calculations of the Bhabha cross section[101]. The total error on the luminosity results in 0.18%[102]. As an uncertainty on the luminosity translates directly to an identical uncertainty on the measured cross section, a 0.2% systematic error has been assigned.

5.5.3 Modeling of the $W^+W^-$ Signal

Apart from imperfections in the modeling of the detector response, systematic errors can also arise from an imperfect simulation of the $W^+W^-$ signal in the Monte Carlo. Below the main uncertainties from this source are described.

W Mass and Width

Ideally, one would like to measure the $W^+W^-$ cross section without making any assumptions about the W mass and width. Unfortunately, one needs to choose values for the W mass and width in order to be able to produce the necessary $W^+W^-$ Monte Carlo events. In order to evaluate the dependence of the measured cross section on these parameters, several signal Monte Carlo samples have been generated. For each sample a different value for the W mass and/or width has been used. In Figure 5.10 the cross sections measured using these samples are compared to the one obtained using the standard Monte Carlo, where a W mass and width of 80.5 GeV and 2.11 GeV have been used, respectively. As can be seen, there is no significant dependence on either the W mass or width. Conservatively, a 0.3% systematic error has been assigned, as a smaller effect could not have been observed due to finite Monte Carlo statistics.

Four-fermion versus CC03 Monte Carlo

The KORALW Monte Carlo events used in this analysis were generated with only the CC03 diagrams switched on, whereas actually many more diagrams contribute to the $q\bar{q}q\bar{q}$ final state at LEP2, as explained in Section 2.2. Some of these final states can only be generated by ZZ-like (NC) diagrams; in this analysis these have been treated as background since we are only interested in the CC03 WW cross section. Nevertheless, the CC03 diagrams are in principle not enough to describe non-ZZ $q\bar{q}q\bar{q}$ final states; in addition there is interference between the CC and NC diagrams for those final states that can be created by both types of diagrams, like $ud\bar{u}d$. Fortunately, these effects are small for events without an electron or positron in the final state. In this thesis, the four-fermion effects are estimated by repeating the analysis by reweighting each $q\bar{q}q\bar{q}$ Monte Carlo event with a weight $w_i$ calculated as:

$$w_i = \frac{M_{i,AF-ZZ}^2}{M_{i,CC03}^2},$$

(5.10)
Event Selection and Cross Section Results

Figure 5.10: Shift in measured cross sections when Monte Carlo samples generated with different values for the W mass (a) and width (b) are used in the analysis. The errors are due to finite Monte Carlo statistics. When the W mass has been varied, the W width has been fixed at the Standard Model value. For the variation of the W width, a mass of 80.5 GeV has been used.

where $\mathcal{M}_{i,4f-ZZ}$ is the matrix element for event $i$ taking into account all four-fermion diagrams except the ZZ-production diagrams, and $\mathcal{M}_{i,CC03}$ is the matrix element for event $i$ taking into account the CC03 diagrams only. The resulting difference in cross section of 0.4% is taken as a systematic error.

ISR/FSR Simulation in $W^+W^-$ Events

The uncertainties related to the simulation of ISR and FSR in $W^+W^-$ events are estimated with the YFSWW3 Monte Carlo [34]. The difference in resulting $W^+W^-\rightarrow q\bar{q}q\bar{q}$ cross section between YFSWW3 and KORALW is $+0.03 \pm 0.4\%$. Since the actual theoretical uncertainty on the ISR/FSR simulation is larger than simply the YFSWW3-KORALW difference, a systematic uncertainty of 0.4% will be assigned. Removing ISR and FSR photons from the event and repeating the analysis gives consistent results.

Fragmentation

The uncertainties on the cross section measurement due to fragmentation are estimated by exchanging the standard baseline Monte Carlo using JETSET for baseline Monte Carlo’s using ARIADNE or HERWIG, or by variation of the JETSET parameters around their tuned values. The tuning of these programs is described in Section 3.3.1.
With ARIADNE, a change in cross section of 0.1% is observed. When HERWIG is used, the change is 3.1%. As described in Section 3.3.1, however, HERWIG does a significantly worse job in describing the Z data, even after tuning.

As an alternative to comparing different models, within the JETSET model the tuned parameters $\Lambda_{LLA}$, $b$ and $\sigma_q$ were varied within their errors resulting from their tuning [46]. This was done for all three parameters with a fast detector simulation [103], and for $\Lambda$, which gave the largest effect, with full Monte Carlo simulation as well. The three JETSET parameters were varied by $\pm 2$ and $\pm 3$ standard deviations; one standard deviation equals 34 MeV for $\Lambda$, 34 MeV for $\sigma_q$, and 0.12 GeV$^{-2}$ for $b$. Changes in cross section of 0.17%, 0.04% and 0.16% respectively were observed for each one standard deviation change of JETSET parameter. The parameter $\Lambda$ was also varied in a full simulation Monte Carlo sample, resulting in a 0.4% change in cross section per standard deviation change of $\Lambda$.

The results above indicate fairly small effects, with the exception of HERWIG. Taking into account HERWIG's deficiencies (see Section 3.3.1) we do not quote the full effects observed with this generator as a systematic uncertainty. Instead, we assign a systematic error on the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ cross section due to fragmentation uncertainties of 0.1 pb, which translates to 1.3%. This is significantly larger than the variations seen with ARIADNE or JETSET; and covers 40% of the variation seen with HERWIG.

**Bose-Einstein Correlations**

Correlations between identical bosons, so called Bose-Einstein correlations, affect the fragmentation of the $W^+W^-$ decay products. As has been described in section 2.5.3, several ways have been suggested to incorporate these correlations in the fragmentation model. Unfortunately it is up to now not possible to determine, using the data, whether any of these models describes the final state topology with satisfactory precision. For the standard Monte Carlo events, the LUBOEI variants BE$_{32}$ and BE$_0$ as implemented in the PYTHIA 6.1 package have been used [104]. In this routine, particles (to be more precise: bosons in the final state, such as pions) are reshuffled such as to reproduce phenomenologically the two-particle enhancement at low $Q$ for like-sign particles. Both models have two free parameters corresponding to the correlation strength and the source radius; these parameters have been tuned by L3 to be: $\text{PARJ}(92) = 1.5$ and $\text{PARJ}(93) = 0.33$ GeV for BE$_0$, and $\text{PARJ}(92) = 1.68$ and $\text{PARJ}(93) = 0.38$ GeV for BE$_{32}$ [81].

In the Monte Carlo used for the quoted result, only correlations between final state bosons originating from the same $W$ have been allowed. Alternatively, one can use the same model but allow correlations between all bosons, regardless of their original $W$ parent. In this case, the measured cross section changes by -0.25%, both for BE$_{32}$ and BE$_0$.

To study an extreme situation, one can also use JETSET without taking into account any BE correlations at all. In this case, the fragmentation model has been changed significantly, and JETSET parameters have been retuned [80]. With this model, a change of +0.15% in the
cross section is observed.

Bose-Einstein correlations in WW events have been studied by L3 in a dedicated study with data taken at $\sqrt{s} = 189$ GeV [49], as well as at higher energies [105, 50]. The conclusions from these studies are that correlations are observed within the same W with a strength compatible with those observed in light-quark Z decays, but that correlations between different W's are not observed in the data, and that their implementation in BE$_{32}$ and BE$_{0}$ is excluded, by more than 4 standard deviations. In fact, similar studies of all four experiments are now consistent and observe no signs of correlations between different W's [105].

At first look, the absence of inter-W Bose-Einstein correlations seems surprising. However, models of Bose-Einstein correlations have been constructed in the framework of the Lund model [51, 52]. In these models, Bose-Einstein correlations follow as a coherent effect related to the symmetrization of particle production from the Lund string. In fact these models reproduce Bose-Einstein correlations results measured in LEP1 data, but intrinsically predict no Bose-Einstein correlations between different W's, as these decay into different strings, unless color reconnection takes place. In addition there could be incoherent Bose-Einstein correlations, corresponding to the original Hanbury-Brown-Twiss (HBT) effect [106], but these typically have large length scales, corresponding to small $R$, and thus only small effects on inter-W correlations at LEP2. Further theoretical discussion can be found in Reference [50]. Other models of Bose-Einstein correlations, based on global event reweighting techniques, all predict that inter-W correlations give only very small observable effects, in agreement with our data [53, 54].

Given the results of the experimental studies of BEC in WW events, a systematic error of 0.1% is assigned on the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ cross section due to Bose-Einstein correlations.

**Color Reconnection**

As has been described in Section 2.5.4, it is unclear how a possible color rearrangement during the fragmentation of the four-quark system should be described. In the Monte Carlo used to obtain the cross section it is assumed that no such color reconnections takes place. To investigate the dependence of the analysis on this assumption, several models with different treatment of color reconnection have been studied, and Monte Carlo events have been generated for each model. In Table 5.3, the changes in the measured cross section is shown. A longer description of each model can be found in Section 2.5.4. Due to an error in the color reconnection model of HERWIG 5.9, this model is not used. Where it has been used, effects were consistent with zero.

The $W^+W^-\rightarrow q\bar{q}q\bar{q}$ data has also been used to directly search for effects of color reconnection [60]. The most sensitive way to study color reconnection has been found to compare the energy and particle flow between jets from the same W, and between jets from different W's. These studies show a good sensitivity to the predictions of the SK I model, and the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ data excludes very large reconnection probability but is not inconsistent
5.5. Systematic Error Analysis

<table>
<thead>
<tr>
<th>Model</th>
<th>Cross section shift (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PYTHIA SK I</td>
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</tr>
<tr>
<td>PYTHIA SK II</td>
<td>0.3</td>
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<tr>
<td>PYTHIA SK II'</td>
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<tr>
<td>ARIADNE 1</td>
<td>-0.16</td>
</tr>
<tr>
<td>ARIADNE 2</td>
<td>0.09</td>
</tr>
</tbody>
</table>

Table 5.3: Shift in measured cross section when using Monte Carlo samples generated using different assumptions regarding color reconnection. All models are briefly described in Section 2.5.4.

with the 30% of reconnected events predicted by the authors of the SK models, nor with zero. Similar conclusions are reached when studying the charged particle multiplicity in $q\bar{q}q\bar{q}$ and $q\bar{q}\ell\nu$ events.

The largest of the observed shifts, 0.4%, is assigned as a systematic error on the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ cross section due to color reconnection uncertainties.

5.5.4 Modeling of the Backgrounds

In this section the systematic errors arising from a possible misdescription of the backgrounds are discussed.

Four-jet Description in $q\bar{q}$

For the dominating background, $e^+e^-\rightarrow q\bar{q}$, the total cross section known from the Standard Model has not been used in the determination of the signal cross section. Instead, the background cross section has been fitted to the neural network output distribution. This is done as the measured number of multi-jet events in the $q\bar{q}$ data is not described satisfactorily by the Monte Carlo model [46]. Although the vulnerability to this problem is diminished by leaving the cross section of preselected $e^+e^-\rightarrow q\bar{q}$ events free, it is still quite possible for the shape of the neural net output spectrum to be influenced. In order to investigate this, a relatively high statistics data sample at $\sqrt{s} = m_2$ has been studied. As a function of $y_{34}$, the ratio of measured and expected events has been determined. For events with a high value for $y_{34}$, indicating a multi-jet topology, an excess in the data is found. Assuming this effect is similar at higher energy, the background Monte Carlo has been reweighted using the ratio described above. As events with a higher $y_{34}$ typically have a higher neural net output, the output shape obtained via reweighting has a larger number of events in the signal region. The ratio of the two spectra is shown in Figure 5.11. As expected, the ratio increases with increasing neural net output. Using the reweighted distribution yields a shift of -1.6% on the cross section.
Event Selection and Cross Section Results

Figure 5.11: The ratio of the reweighted neural net output spectrum and the one used to obtain the central cross section value. The normalization is such that the average weight of a qq Monte Carlo event is one.

This shift has been applied to the result, half of the shift is assigned as systematic error due to uncertainties in the QCD four-jet simulation.

**ISR Simulation in qq**

The Monte Carlo generator used for the production of the qq background is PYTHIA [41]. This multi-purpose generator does not describe the hard part of the initial state spectrum up to the desired precision, as it generates too many photons with high transverse momentum. This can be seen in Figure 5.12, where the energy of the most energetic bump has been shown. The events selected for this plot are required to pass all cuts described in Section 5.2 except the ones designed specifically to reject high energy photons measured in the detector. The peak of detected high energy photons is clearly visible, in both data and Monte Carlo. The Monte Carlo predictions are overestimating the data by approximately 20%. It is expected that this is not a serious problem, as those events are easy to reject. This hypothesis has been checked by reweighting all qq Monte Carlo such that events with an ISR photon with an angle to both initial leptons of at least ten degrees and more than ten GeV of energy get a 20% lower weight. In this case, the fitted W⁺W⁻ cross section changes by 0.2%. This shift has been applied to the result, half of the shift is assigned as systematic error due to uncertainties in the ISR simulation of the background.
5.5. Systematic Error Analysis

Figure 5.12: Maximum energy deposition in the BGO part of the ECAL. Cuts have been applied to select high energy, balanced, high multiplicity events but no effort has been made to reject events where a high energy photon has been detected.

ZZ Background Scale

In the cross section determination, the background from ZZ events is taken into account by fixing the expected number of events from this source to the Standard Model expectation value, which can be can be calculated with a 2% precision, and has been measured with about 20% precision [107, 108]. To obtain a systematic error estimate, the ZZ background estimate has been varied by the theoretical precision, giving a 0.1% uncertainty.

WW Non-four-quark Background Scale

For the background from $W^+W^-$ events decaying to other final states than $q\bar{q}q\bar{q}$, the Standard Model cross section for these processes have been used in the fit. In this case the cross sections have been varied by $\pm 50\%$, giving a 0.2% error on the measured signal cross section. As such a large variation of the background cross section results in a change in the measured signal cross section that is small compared to the other systematic errors and the statistical error, it is in the remainder of this analysis assumed that the cross section for $e^+e^-\rightarrow W^+W^-\rightarrow q\bar{q}q\bar{q}$ decaying to four jets is measured independent of the cross sections for other $W^+W^-$ decay chains.
### Event Selection and Cross Section Results

<table>
<thead>
<tr>
<th>Error source</th>
<th>Systematic error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HCAL Energy Scale</td>
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</tr>
<tr>
<td>ECAL Energy Scale</td>
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</tr>
<tr>
<td>Jet Angular Resolution</td>
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</tr>
<tr>
<td>Cluster Simulation</td>
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</tr>
<tr>
<td>$g$-Factors</td>
<td>0.6</td>
</tr>
<tr>
<td>Luminosity Measurement</td>
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</tr>
<tr>
<td>Fragmentation</td>
<td>1.3</td>
</tr>
<tr>
<td>WMass/Width Dependence</td>
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</tr>
<tr>
<td>Misdescription of ISR/FSR in Signal</td>
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</tr>
<tr>
<td>4-Fermion vs CC03 Effects</td>
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</tr>
<tr>
<td>Color Reconnection</td>
<td>0.4</td>
</tr>
<tr>
<td>Bose-Einstein Correlations</td>
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</tr>
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<td>$y_{34}$ Reweighting of $q\bar{q}$ Monte Carlo</td>
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</tr>
<tr>
<td>Misdescription of ISR in $q\bar{q}$ Monte Carlo</td>
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</tr>
<tr>
<td>ZZ Cross Section</td>
<td>0.1</td>
</tr>
<tr>
<td>$W^+W^-$ Background Cross Section</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2.0</strong></td>
</tr>
</tbody>
</table>

Table 5.4: Summary of the contributions to the systematic error on the cross section measurement at $\sqrt{s} = 189$ GeV.

### 5.5.5 Systematic Error Summary

The systematic error estimates from all sources considered have been summarized in Table 5.4, and add up to a total systematic error of 2.0%. The measured cross section for the process $e^+e^- \rightarrow W^+W^- \rightarrow q\bar{q}q\bar{q}$ at $\sqrt{s} = 189$ GeV then becomes $7.40 \pm 0.26$ (stat) $\pm 0.15$ (syst) pb.

The results of the $W^+W^- \rightarrow q\bar{q}q\bar{q}$ cross section measurements between $\sqrt{s} = 161$ and $\sqrt{s} = 189$ GeV, as presented in this thesis, are plotted graphically in Figure 5.13. Systematic errors on the $W^+W^- \rightarrow q\bar{q}q\bar{q}$ cross section at 161, 172 and 183 GeV were estimated in the same way as for the 189 GeV sample. Due to the fact that these samples are smaller, many systematic errors can be determined with less precision, and are conservatively assigned larger values: 5% at 161 GeV, 3.1% at 172 GeV, and 2.8% at 183 GeV. Figure 5.13 also shows the Standard Model prediction, the theoretical expectation if the WWZ vertex would not exist, and the theoretical prediction if only the neutrino exchange diagram existed. These
latter two predictions disagree with the data, whereas the Standard Model prediction agrees well with the data.

## 5.6 W Mass from WW Cross Section

Around threshold, $\sqrt{s} \approx 2m_W$, the WW production cross section is sensitive to the W mass. Therefore, the measured WW production cross section can be transformed into a measurement of the W mass. In 1996, L3 has taken data corresponding to an integrated luminosity of 11 pb$^{-1}$ at a center-of-mass energy $\sqrt{s} = 161.34 \pm 0.06$ GeV. In this section, the W mass will be derived from the WW production cross section measured in that data sample.

The GENTLE [109] program has been used to calculate the dependence of the CC03 WW production cross section on the W mass at this value of $\sqrt{s}$. The cross section for $WW \rightarrow q\bar{q}q\bar{q}$ is derived from this calculation by multiplication with the Standard Model branching fraction $\text{Br}(WW \rightarrow q\bar{q}q\bar{q}) = 45.6\%$. The result is graphically shown in Figure 5.14. The uncertainty of this calculation is estimated to be 2% [40].

As shown in Table 5.2, the CC03 cross section for $WW \rightarrow q\bar{q}q\bar{q}$ at $\sqrt{s} = 161$ GeV was measured to be $\sigma_{WW \rightarrow q\bar{q}q\bar{q}} = 0.98^{+0.61}_{-0.40}$ pb. For this sample, the systematic error on the measured cross section was estimated to be 5%, evaluated as explained earlier in this chapter, and dominated by the uncertainty in the description of the Monte Carlo of the neural network input parameters. Using the GENTLE calculation and the measured cross section for $WW \rightarrow q\bar{q}q\bar{q}$, the W boson mass is measured to be:

$$m_w = 81.33^{+1.17}_{-0.72} \pm 0.03 \text{ GeV}$$

(5.11)

where the first error includes statistical and systematic errors from the cross section measurement as well as the uncertainty on the GENTLE calculation, and the second error arises from the uncertainty on the LEP beam energy.

At $\sqrt{s} = 161$ GeV, the WW production cross section was also measured in the other decay modes $q\bar{q}\ell\nu$ and $\ell\nu\ell\nu$ ($\ell = e, \mu, \tau$). Combining all these measurements, the total WW production cross section was measured to be $\sigma_{WW} = 2.89^{+0.83}_{-0.72}$ pb, combining statistical and systematic errors. From this measurement, and the GENTLE calculation for the dependence of the total cross section on $m_w$, the W mass is derived to be

$$m_w = 80.80^{+0.48}_{-0.42} \pm 0.03 \text{ GeV}$$

(5.12)

At center-of-mass energies well above threshold, the dependence of the WW production cross section on the W mass is significantly reduced. This is shown graphically in Figure 5.15. As can be seen from the Figure, it is not useful to derive a W mass from the WW cross sections at these higher center-of-mass energies. Instead, in the next chapter the W mass will be derived directly from kinematical information in selected WW events.
Event Selection and Cross Section Results

Figure 5.13: Results of the $W^+W^-\rightarrow q\bar{q}q\bar{q}$ cross section measurements as presented in this thesis, for $\sqrt{s} = 161, 172, 183$ and 189 GeV. Errors shown include statistical and systematic errors. Also shown are the Standard Model prediction, the theoretical expectation if the WWZ vertex would not exist, and the theoretical prediction if only the neutrino exchange diagram existed.
5.6. W Mass from WW Cross Section

\[ \sqrt{s} = 161.34 \pm 0.06 \text{ GeV} \]

\[ \sigma_{WW \rightarrow q\bar{q}q\bar{q}} = 0.98^{+0.51}_{-0.40} \text{ pb} \]

\[ M_W = 81.33^{+1.17}_{-0.72} \pm 0.03 \text{ GeV} \]

Figure 5.14: Dependence of the CC03 cross section for \( WW \rightarrow q\bar{q}q\bar{q} \) on the W mass \( (m_W) \) at \( \sqrt{s} = 161.34 \text{ GeV} \), as calculated with GENTLE. Our measurement of this cross section, including its combined statistical and systematic error, is shown as a band. The error of 0.03 GeV on \( m_W \) is due to uncertainties in the LEP beam energy.
Figure 5.15: Dependence of the total CC03 cross section for WW production on the W boson mass at various values of $\sqrt{s}$. On the left side, an indication is given for the experimental accuracy reached at each $\sqrt{s}$, with the centers of the error bars at arbitrary position.