Precision of the ATLAS muon spectrometer
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Chapter 4  Muon chamber measurement precision

I devoted myself to study and to explore by wisdom
all that is done under heaven.

Ecclesiastes 1:13

The precision of the track segment measurements in the individual chambers is one of the prime factors in the precision of the muon momentum measurement. In the precious chapter, it was shown that the wire positions of the BOL chambers are known with a precision of 15 μm compared to a regular grid. This chapter describes how the chamber track segments are reconstructed using this regular grid as input. After describing the test stand that is used for studying single chamber performance, the drift tube calibration procedure and the track reconstruction algorithm are explained in detail. Finally, as an example, we present the results for one BOL chamber.

4.1 The BOL cosmic ray test stand

After completion of a BOL chamber, it is tested by operating it in a dedicated test stand using cosmic muons. Figure 4-1 is a photograph of the test stand, showing three installed BOL chambers and four trigger units installed below the chambers. The stand became operational early 2002. The aims of the test stand are to find dead and noisy channels and to study the uniformity, efficiency and resolution of the tubes for quality control. In the future the wire positions will also be determined using the cosmic muons. The same analysis software is used as was used for analysis of the DATCHA data (paragraph 5.4), and has been adapted to cope with the new trigger configuration, the new MDT read-out hardware and the different raw data format.

4.1.1 The muon drift chambers

The test stand can accommodate five BOL chambers installed in horizontal position. Each BOL chamber is approximately 5 m long, 2.2 m wide and 0.5 m high. They are stacked vertically and are supported as they will be in ATLAS. Argon and carbon dioxide gases arrive separately at the set-up and are mixed locally using mass flow controllers. At the outlet of the gas system a pressure controller is installed to regulate the gas pressure. The high-voltage is generated by a 40 channel C.A.E.N. High Voltage System Model SY 127. One channel is used per multilayer and the high-voltage is distributed via on-chamber 'hedgehog' boards. On the read-out side, the chambers are equipped with hedgehog boards holding the fast shaping FBPANIC-04 pre-amplifier, which is a
modified version of the pre-amplifiers originally developed for the muon drift chambers of the LEP L3 experiment. A so-called datimizer card is installed onto each hedgehog board. This card houses the discriminators and a 32 channel TDC (Time to Digital Converter) with $25/32 \approx 0.78$ ns wide bins. A thick copper-clad ground plate is mounted between the pre-amplifier and datimizer boards to minimise electromagnetic interference. In the year 2002 only three chambers were equipped with read-out electronics. Two of the three chambers were equipped with twin-tube jumpers on the high-voltage boards (see appendix B). The MDT operation point is listed in table 4-1. A higher gas gain and lower threshold are chosen than foreseen in ATLAS. Each trigger, the digitised data is transferred from the datimizers to a few Nimrods (NIKHEF Muon Read-Out Driver), which are installed in a VME crate, and the raw data is written to a disk.

### 4.1.2 The muon trigger

Several scintillator counters are installed below the chambers to serve as a cosmic muon trigger. Each scintillator is 90 mm wide, 44 mm high and 2300 mm long with the longest side running in the z-direction. They are configured in four trigger units, which are located at several positions in x. Each unit consists of two sets of three scintillator counters with a block of 500 mm of iron in between, as illustrated in figure 4-2. The trigger signal of a unit is the coincidence between the two blocks of scintillators where the scintillators in each block are combined in a logical OR. The main trigger is the OR of the four trigger units, which results in a trigger rate of about 26 Hz. The iron serves as a muon momentum cut of about 0.75 GeV/c.

The scintillator counters are read out with photo multipliers on one end, and the hit times are measured with a dedicated datimizer card. Within each set of three scintillators, two counters are read out on one end and one counter on the opposite end. If a hit is registered in two scintillators of the same set with read-out on opposite sides, a ‘trigger pair’ is formed. If two pairs are found, one in the upper set and one in the lower set, the upper set is taken as the trigger pair. The mean time of the hits of the trigger pair is used as the time of passage of the muon: the main trigger time. All MDT hits are corrected for this time. This is essential since all measured times have an arbitrary common offset that changes from event to event. The mean time is the only time that has a fixed offset w.r.t. the time of passage of the muon. If no trigger pair can be formed (which is possible because of the geometry) the event is ignored in the off-line analysis. This reduces the used data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas mixture</td>
<td>Ar/CO$_2$ 93/7</td>
</tr>
<tr>
<td>Pressure</td>
<td>3 bar absolute</td>
</tr>
<tr>
<td>High voltage</td>
<td>3300 V</td>
</tr>
<tr>
<td>Gas renewal rate</td>
<td>1 volume / 10 days</td>
</tr>
<tr>
<td>Gas gain</td>
<td>$7 \times 10^4$</td>
</tr>
<tr>
<td>Discriminator threshold</td>
<td>13$^{\text{th}}$ electron (45 mV)</td>
</tr>
</tbody>
</table>

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**Table 4-1** MDT operation point in the cosmic ray test stand at NIKHEF.
sample by almost a factor of two and the effective trigger rate is about 14 Hz. The time difference of the hits in the trigger pair is used to calculate the position of the muon along the scintillator (z-direction) using an inverse signal propagation speed of 6.4 ns/m. This speed has been measured with the set-up itself by extrapolating the MDT tracks to the scintillators and comparing the extrapolated track z-positions to the measured scintillator hit time differences. A 'trigger track' is reconstructed in the x-y plane from the x-positions of the scintillators that are hit.

4.2 Calibration of the drift tubes

Before an MDT chamber can be used for an accurate measurement, its tubes need to be calibrated. The MDTs (including their read-out) only measure the times of arrival of hits in the TDCs compared to the time of the arrival of the trigger signal in the same TDC. These times need to be converted into drift distances in the corresponding MDTs. The total measured time includes, apart from the drift time in the MDT itself, the time of flight of the muon, the signal propagation time along the MDT and time delays in cables and electronics. All these timing effects need to be taken into account to get precise drift times, which are then converted into drift distances via the so-called space-time ($r$-$t$) relation. This relation is not known in advance and its determination is part of the calibration procedure. The main aspects of the calibration are explained in more detail in the following paragraphs.

4.2.1 Signal propagation along the drift tube

The signal propagation delay $\Delta t_{\text{prop}}$ is shown schematically in figure 4-3. This delay depends on the distance between the read-out end of the tube and the position along the tube where the muon passes. It also depends on the signal shape at the point of impact, the shaping properties of the tube and the front-end electronics. In particular, it depends on the discriminator threshold due to the finite rise-time of the signal.

The drift tube acts like a coaxial wave transmission line with an impedance of 382 $\Omega$, where the signal travels at the speed of light. However, the electrical resistance of the wire (44 $\Omega$/m) gives rise to a frequency dependent impedance and as a consequence the shape of the signal will change [41]. Notably the leading edge peak, which is relevant for the time measurement, loses height as it propagates through the tube. The signal changes will result in an extra delay in the time measurement on top of the delay purely due to the propagation distance.

Dedicated measurements were performed to study the effect of the drift tube signal shaping on the propagation delay. Raw muon signal pulses are measured with a digital oscilloscope at the pre-am-
Figure 4-4 Pre-amplifier output signals of twin MDTs for various impact points of a muon along the tube. The first arriving signals (solid lines) are from the tube traversed by the muon. The later arriving signals (dashed lines) are from the ‘twin-partner’ of that tube. The muons traversed at 0.75 m (a), 2.5 m (b) and 4.25 m (c) from the end where the tubes are interconnected. Here, the MDTs are operated with Ar / CO₂ 80 / 20 at 2 bar absolute pressure with a high-voltage of 3150 V, resulting in a maximum drift time of 1200 ns.

4.2 Calibration of the drift tubes
plifier outputs of one pair of twin-tubes (see appendix B). The muon signal current splits in two and travels to both ends of the tube. One half of the signal is registered at the read-out side, and the other half of the signal travels via the high-voltage end into its partner tube and reaches the read-out end there. Depending on the location of the muon hit along the tube, the two signal halves travel different propagation lengths. Some examples of both signal halves for muon hits at several locations along the tube are shown in figure 4-4.

The original signal (in the tube that was traversed by the muon) arrives first and the ‘twin’ signal (in the partner tube) arrives somewhat later. Focusing on the first signal peak, which is relevant for the time measurement, we make the following observations:

- The shape of the twin signal is very similar to the original signal.
- The amplitude of the leading edge peak of the twin signal is smaller than the original signal. This effect is stronger as the difference in propagation distance increases.

Figure 4-5 shows the ratio of the heights of the leading edge peaks of the twin and original signals as a function of the difference in propagation distance. Each measurement point is the average of about 12 muon pulses. The systematic errors are due to uncertainty in the amplification factor of the pre-amplifiers and are the dominant error. The measurements are consistent with the simplified theoretical expectation deduced from reference [41], which predicts an exponential loss with a characteristic length of 17 m.

The effective propagation speed can be determined by measuring the time difference between the original and twin signals as a function of the difference in propagation distance. The slope of this linear function gives the propagation speed. The leading edge time of a signal is simulated from the oscilloscope data as the time at which the signal crosses a threshold level. This way the speed can be determined for several threshold levels using the same raw input signals. Figure 4-6 shows the measured time difference (about 12 signals per point) versus the propagation distance for two values of the threshold. The intercept at about 11 ns is due to the extra delay inserted in between the two tubes. Figure 4-7 shows the inverse of the effective signal propagation speed determined in this way as a function of the threshold level. The inverse effective speed is close to the inverse speed of light at zero threshold, and increases with increasing threshold. The errors are highly correlated because the same data set is used for all measurement points.
The position along the tube (x-coordinate) is determined from the trigger track (see paragraph 4.1.2) using the nominal y-position of the wire. Due to the extrapolation of the trigger track, the resolution of the x-coordinate increases from about 20 mm for the tubes closest to the trigger units (i.e. the lowest layer of the lowest chamber) to about 100 mm for the tubes farthest away from the trigger units (i.e. the upper most layer of the fifth chamber). The x-coordinate is converted into a time delay using the effective propagation speed. For the cosmic ray test stand an inverse speed of 3.8 ns / m is used, which translates the x-coordinate resolution into a resolution of the propagation delay correction of 0.07 to 0.38 ns (depending on the tube position). This is adequate since it is well below the TDC timing step size (0.78 ns). A wrong speed would introduce systematic effects in the drift time, depending on x. Typically one would like to keep the systematic effect below about 0.5 ns. For a 5 m long tube this translates into a required precision on the propagation speed of about 5%, equivalent to about 25 mV on the discriminator threshold, which is easily achieved.

4.2.2 Time of flight of the muon

The time of flight delay $\Delta t_{ToF}$ depends on the distance between the point where the muon passed the MDT and the point where it passed the trigger pair of scintillators (see paragraph 4.1.2). The nominal y- and z-coordinates of the MDT wire are taken, and the x-coordinate is calculated from the trigger track at nominal wire y-position. The distance is converted into a time assuming the muon travels at the speed of light. This assumption is adequate since even for the slowest muons (0.75 GeV/c) the speed is 99.0% of the speed of light.
4.2.3 \( t_0 \) and \( t_{\text{max}} \) calibration

The value of a hit time as measured by the TDC has an arbitrary constant offset due to cable lengths, electronics, etc. This offset is in general different for each tube, which causes the TDC spectra for tubes to be shifted w.r.t. to each other. The maximum drift time can also vary from tube to tube due to variations in gas properties and high-voltage. These differences could all be absorbed in the \( r-t \) relations if a different one is used for each tube. This, however, requires many \( r-t \) relations and their determination through auto-calibration (see paragraph 4.2.4) would be difficult, if not impossible. To use the same \( r-t \) relation for more than one tube, the TDC times are shifted and scaled per tube before the times are converted into a distance via a common \( r-t \) relation:

\[
t = (t_i - t_{0,i} - \Delta t_{\text{prop}} - \Delta t_{\text{ToF}}) \frac{t_{\text{max, ave}}}{t_{\text{max, i}}}
\]

where \( t_i, t_{0,i} \) and \( t_{\text{max, i}} \) are the TDC time of a hit, the shift \((t_0)\), and the maximum drift time \((t_{\text{max}})\) respectively of tube \( i \). The scaling time \( t_{\text{max, ave}} \) is the average \( t_{\text{max}} \) of all tubes sharing the same \( r-t \) relation.

The value of \( t_0 \) \((t_{\text{max}})\) is determined from the lower (upper) edge of the TDC spectrum of a single tube by fitting this edge with a scaled Fermi function plus a constant background:

\[
N = \text{background} + \frac{\text{scale}}{1 + e^{\frac{\text{center} - t}{\text{width}}}}
\]

where \text{background}, \text{scale}, \text{center} and \text{width} are the parameters to fit and \( N \) the number of entries in a bin at time \( t \). At \( t = \text{center} \), \( N \) reaches half its maximum value (above background). The absolute value of \text{width} indicates how wide the transition is and its sign determines the direction of the transition, which is opposite for the \( t_0 \) and \( t_{\text{max}} \) fits. The two fits are done independently at their respective local part of the spectrum. Each fit is done twice with the MINUIT fitting package [36]. The first fit (using \( \chi^2 \) minimisation with Gaussian errors for reasons of execution speed) serves to get good estimates of the \text{center} and the \text{width} parameters. The second fit uses those parameter values to restrict its fitting region to the part that is most relevant for the \( t_0 \) \((t_{\text{max}})\) determination: \text{center} - 10 \times \text{width} \leq t \leq \text{center} + 10 \times \text{width} and uses the maximum likelihood method with Poissonian errors for maximum accuracy. Figure 4-8 shows the edges of the TDC spectrum and the two final fits for one single tube.

The value of \( t_0 \) \((t_{\text{max}})\) is not unambiguously defined from the fitted function. It can be chosen 'anywhere' along the function provided an extra time offset is allowed in the \( r-t \) relation. Two possible options are found in literature:

1. The \( t \) at half the maximum (i.e. \( t_0 = \text{center} \)) [42].
2. The \( t \) at which the line tangent at \( t = \text{center} \) crosses the background (i.e. \( t_0 = \text{center} - 2 \cdot \text{width} \)) [43].

We have chosen a solution based on the \( t \) at which its statistical uncertainty due to the fit of function 4-2 is minimal. An overall shift in \( t_0 \) for all tubes is absorbed in the \( r-t \) relation. The \( t_0^\text{fit} \) extracted from the fit and its error \( \sigma_{t_0}^{fit} \) can be expressed as:
Figure 4-8 Lower (a) and upper (b) edge of the TDC spectrum of a single tube, including the fitted functions to determine the $t_0$ (a) and $t_{max}$ (b) values. The lower edge of the spectrum has a bin size that is equal to the TDC bin size to get high accuracy. The upper edge of the spectrum has a bin size that is four times as large to increase the statistics per bin.

$$t_0^{fit} = center - \hat{t} \cdot width$$

$$(\sigma_0^{fit})^2 = \sigma_{center}^2 + \hat{t}^2 \sigma_{width}^2 - 2 \hat{t} \sigma_{center} \sigma_{width} \rho_{c,w}$$

$$= \sigma_{center}^2 (1 - \rho_{c,w}^2) + \sigma_{width}^2 \left( \hat{t} - \frac{\sigma_{center}}{\sigma_{width}} \rho_{c,w} \right)^2$$

where $\sigma_{center}$ and $\sigma_{width}$ are the errors on center and width, and $\rho_{c,w}$ their correlation. The minimum value for $\sigma_0^{fit}$ is reached for:

$$\hat{t}^{min} = \rho_{c,w} \sigma_{center} / \sigma_{width},$$

and has the value:

$$\sigma_{0, min}^{fit} = \sigma_{center} \sqrt{1 - \rho_{c,w}^2} \quad \text{for} \quad t_0^{fit} = center - \rho_{c,w} \sigma_{center} / \sigma_{width} \cdot width.$$  

For stability the same value $t^{ave}$ is used for all tubes of a chamber and is determined as the average value of the $t$'s of the individual tubes. The same procedure is applied to the $t_{max}$ fits. The times in the fitted TDC spectrum are already corrected for some $t_0^{old}$ and the final $t_0$ and $\sigma_0$ are:

$$t_0^{final} = t_0^{old} + center_0 - t_{0}^{ave} \cdot width_0$$

$$\sigma_0^{final} = \sqrt{\sigma_{center,0}^2 + (t_0^{ave})^2 \sigma_{width,0}^2 - 2 t_0^{ave} \sigma_{center,0} \sigma_{width,0} \rho_{c,w,0}}.$$

### 4.2 Calibration of the drift tubes
The final \( t_{\text{max}} \) is taken with respect to the fitted \( t_0 \) and its value and error are given by:

\[
\begin{align*}
t_{\text{max}}^{\text{final}} &= \frac{\text{center}_{\text{max}} - \text{width}^{\text{ave}} - (\text{center}_0 - \text{width}^{\text{ave}})}{\text{final}} \\
\sigma_{\text{max}}^{\text{final}} &= \sqrt{\left(\sigma_{0}^{\text{final}}\right)^2 + \sigma_{\text{center}, \text{max}}^{2} + (\text{width}^{\text{ave}})^2 - 2\sigma_{\text{center}, \text{max}} \sigma_{\text{width}, \text{max}} \sigma_{\text{width}, \text{max}} P_{c, w, \text{max}}}
\end{align*}
\]

This approach results in the smallest tube-to-tube variation in the \( t_0 \) and \( t_{\text{max}} \) due to the uncertainty of the fit. Table 4-2 shows the results of the \( t_0 \) and \( t_{\text{max}} \) calibration. The scale parameter column is added to give an indication of the statistics involved. The values of \( \tilde{t} \) indicate that our solution lies in between the two solutions quoted from literature. The last column gives the precisions of the \( t_0 \) and \( t_{\text{max}} \) determinations.

**Table 4-2** Typical values of parameters of \( t_0 \) and \( t_{\text{max}} \) calibration fits to TDC spectra of individual tubes. Data collected with 23 hours of cosmic rays.

<table>
<thead>
<tr>
<th></th>
<th>scale (1/bin)</th>
<th>width (ns)</th>
<th>( \sigma_{\text{center}} ) (ns)</th>
<th>( P_{c, w} )</th>
<th>( \tilde{t} )</th>
<th>( \sigma ) (ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_0 )</td>
<td>30</td>
<td>1.8</td>
<td>0.5</td>
<td>0.63</td>
<td>1.2</td>
<td>0.4</td>
</tr>
<tr>
<td>( t_{\text{max}} )</td>
<td>22</td>
<td>-8.1</td>
<td>2.3</td>
<td>0.64</td>
<td>1.0</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Figure 4-9 shows a typical TDC spectrum for all tubes in a BOL chamber after all timing corrections. The shape is typical for the Ar / CO\(_2\) drift gas in a radial electric field: steep rise at \( t = 0 \) from muons that hit the wire, with a finite slope due to the statistical nature of the signal generation and due to the pre-amplifier shaping; the maximum close to \( t = 0 \) due to the local high drift velocity (in turn due to the local high electric field); slowly decreasing after the initial maximum due to a slow decrease in the local drift velocity (in turn due to the decreasing local electric field). It can be shown that the TDC spectrum is proportional to the local drift velocity (apart from folding with the resolution and the distribution of the incoming particles) [44]. The ‘dip’ around 30 ns is specific for the chosen operating point and is less pronounced if the high-voltage is decreased by 100 - 200 V. Physically it means that the drift velocity is locally decreasing with increasing electric field (for the electric field value at the radius corresponding to the time of the dip).

### 4.2.4 \( r-t \) calibration and resolution determination

The space-time \( (r-t) \) relation is determined in an iterative procedure, one per chamber. This ‘autocalibration’ procedure [45] uses only the hits of the detector itself to find its calibration. A first estimate of the \( r-t \) relation is obtained by integration of the TDC spectrum with a correction for \( \delta \)-rays [44]. In each iteration the previous \( r-t \) relation is used to reconstruct the tracks per chamber (see paragraph 4.3). The next \( r-t \) relation is obtained by accumulating the measured times as a function of the predicted radius, where the latter is equal to the distance from the reconstructed track to the respective wire. A profile histogram in \( r \) from 0 to 14.6 mm in 0.2 mm bins is used to accumulate the measured times. Figure 4-10 shows the profile histograms of the initial and final \( r-t \) relations for three BOL chambers. The \( r-t \) relations are highly non-linear, which is typical for the Ar / CO\(_2\) drift gas in a radial electric field. The advantage of binning in \( r \) instead of in \( t \) (and accumulating the predicted radii) is that the histogram range is fixed and known in advance. Moreover the statistics is similar in each bin, provided the tubes are illuminated uniformly. Figure 4-11 shows the MDT drift velocity as a function of the track position. It is determined as the inverse of
Figure 4-9 A TDC spectrum for all tubes in a BOL chamber after all timing corrections.

Figure 4-10 Initial (grey) and final (black) r-t relations for three BOL chambers. The average measured time is determined as a function of the predicted radius from the track position. The vertical error bars indicate the r.m.s.

Figure 4-11 MDT drift velocity as a function of the track position for three BOL chambers. It is determined as the inverse of the derivative of the r-t relation.

the derivative of the final r-t relations, which is an approximation because it ignores the geometry of the primary ionisation cluster distribution. The shape is very similar to the TDC spectrum (figure 4-9), as expected.

To improve the reliability of the convergence of the procedure, a set of stringent cuts is applied to the hits and tracks that are included in the calibration:

4.2 Calibration of the drift tubes
• At least six hits on the track;
• Confidence level of the track fit greater than 0.1;
• At least one hit (i.e. wire position) on each side of the track, both in the bottom and top multilayers;
• The r.m.s of the measured hit radii greater than 2.5 mm;
• A hit must be closer to the track than three times the local resolution.

The procedure is considered converged if the change (w.r.t. the previous iteration) in the average of the track fit residuals is small at all radii.

The resolution $\sigma_r$ is determined alongside the $r$-$t$ relation for each $r$ bin. The resolution of a drift tube is defined as the r.m.s. of the residuals of the measured hit radii relative to the real track position. Due to absence of an external track position measurement, the track position is determined by fitting a track to the hits themselves. The resolution can be estimated from the distribution of the track fit residuals (figure 4-12). This distribution, however, is narrower than the resolution because the hit concerned is included in the fit, which gives a bias to smaller residuals. Another estimate for the resolution is the distribution of the hit residuals, where the hit concerned is left out of the track fit (‘fit-without-hit’, figure 4-13).

This distribution, however, is broader than the resolution because the inaccuracy of the reconstructed track position adds to the residuals. This effect can be corrected for by accumulating the estimated inaccuracies $\sigma_{track}^2$ due to the track position (and angle) and subtracting (in quadrature) at the end its average from the width $\sigma_{fwh}$ (of a Gaussian fit to the central $\pm 2\sigma$) of the track fit-without-hit residual distribution:

![Figure 4-12](image1.png)  ![Figure 4-13](image2.png)

*Figure 4-12* Distribution of the track fit residuals and a Gaussian fit to the central $\pm 2$ sigma, drawn over the full range.

*Figure 4-13* Distribution of the track hit residuals leaving out the hit concerned from the track fit. A Gaussian fit to the central $\pm 2$ sigma is drawn over the full range.
The value of $\sigma_{\text{track}}^2$ depends, apart from the spatial distribution of the hits, only on the input errors given to the track fit (see equations 4-22), and could therefore be larger than $\sigma_{\text{fwh}}^2$, which is non-physical (negative $\sigma^2$). To avoid this problem and improve the robustness of the method we scale equation 4-8 by the input error $\sigma_{\text{input}}^2$:

$$\frac{\sigma_r^2}{\sigma_{\text{input}}^2} = \frac{\sigma_f^2}{\sigma_{\text{input}}^2} - \frac{\sigma_{\text{track}}^2}{\sigma_{\text{input}}^2}. \quad 4.9$$

The $\sigma_{\text{track}}^2$ for a hit depends on the location of the hit along the track, and the $\sigma_{\text{input}}^2$ for a hit depends on the radius of the hit. Equation 4-9 assumes that $\sigma_{\text{input}}^2$ is the same for all hits, and is therefore an approximation. We now impose the equilibrium condition $\sigma_{\text{input}}^2 = \sigma_r^2$, but keep $\sigma_{\text{input}}^2$ in the $\sigma_{\text{track}}^2$ term. Rewriting then gives:

$$\sigma_r = \sqrt{\frac{1}{\frac{\sigma_f^2}{\sigma_{\text{input}}^2} - \frac{\sigma_{\text{track}}^2}{\sigma_{\text{input}}^2}}}, \quad 4.10$$

showing that the tube resolution is a scaling factor ($< 1$) times the width of the ‘fit-without-hit’ residual distribution. This procedure is applied separately for each $r$-bin of the $r$-$t$ relation, which validates the assumption of a constant $\sigma_{\text{input}}^2$ (within one bin). The flatness of the confidence level distribution (see figure 4-18 b) is a strong indication that this method works well. The scaling factor changes from close to 1 at $r = 0$ to about 0.75 at large $r$. To limit the contribution of the reconstructed track uncertainty, while allowing at the same time tails in the residual distribution, a number of cuts are applied to the tracks and hits that are included in the determination of the resolution:

- At least three hits in each multilayer on the track including the hit;
- Loose cut on the $\chi^2$ per degree of freedom ($< 10$) of the track including the hit;
- Stringent cut on the confidence level ($> 0.1$) of the track without the hit.

Figures 4-14 (a) and (b) show the average of the track fit residuals and the tube resolution respectively, as a function of the radial position of the track, using both the initial and the final $r$-$t$ relations. The improvement by the iterative procedure is evident, as both the absolute values and the chamber-to-chamber variations are reduced by a significant amount. The final averages are significantly smaller than the resolution at all radii. The remaining systematics in the averages, in particular below $r = 3$ mm, are possibly related to deviations in $t_0$s that are not fully absorbed by the auto-calibration procedure and more investigation is needed to reduce them. The shape of the resolution is rather typical for $\text{Ar} / \text{CO}_2$. The relatively bad resolution near the wire ($r = 0$) is due to the locally high drift velocities and due to the geometry of the primary ionisation clusters for particles passing close to the wire. The locally improved resolution at $r = 2$ mm is correlated to the locally lower drift velocity (compare figure 4-11). This ‘dip’ is absent for a lower high-voltage on the wire (see figure 2-6 and table 2-2), and is therefore related to the higher electric field.

4.2 Calibration of the drift tubes
The $r$-$t$ relation is stored as a table of $(r, t, \sigma_r)$ points. Linear interpolation between those points is applied by the analysis program to convert measured times into 'measured' radii.

One would like to summarise the resolution in a single number. Since the track fit uses $1/\sigma^2$, a good average resolution number is the ‘inverse quadratic average’:

$$\sigma_{\text{IQA}} = \sqrt{\frac{1}{r_{\text{max}}}} \int_{r_{\text{max}}}^{r_{\text{max}}} \frac{1}{\sigma^2(r)} dr .$$

Applying this formula to the resolution curves gives statistical uncertainties of 89 $\mu$m for the chambers with twin-tube jumpers and 95 $\mu$m for the chamber without twin-tube jumpers. Applying the same formula to the average residual curves, we find systematic uncertainties of 1 - 3 $\mu$m, which is well below the statistical uncertainty. This indicates that the auto-calibration procedure has converged. We conclude that the single MDT resolution is about 90 $\mu$m, which is close the aim of 80 $\mu$m [24]. This is acceptable for this set-up since there is still a contribution from the relatively large multiple scattering\(^1\) of the low momentum cosmic muons (1 GeV/c).

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\(^1\) This contribution is estimated to be 37 $\mu$m. This value is determined by comparing the width of the track fit residual distribution (figure 4-12) to the width of the same distribution, where a cut is made on the angle differences between the track segments in the three chambers. This way the muons with large scattering are removed from the sample. The estimated resolution without multiple scattering is therefore $\sqrt{90^2 - 37^2} = 82 \mu$m.
4.3 Track reconstruction

The muon track reconstruction proceeds along three steps:

1. Within a single chamber, patterns are defined by a collection of hits consistent with a trail left by a passing muon, within a certain distance. The hit topology cut requires a minimum number of hits per multilayer and per chamber;

2. Each pattern is subjected to a track fit. If the $\chi^2$ per degree of freedom is less than 5, it is accepted. Otherwise a track fit is applied to all sub-patterns with one hit less that still satisfy the hit topology cut. If the best $\chi^2$ of the fits to the sub-patterns is below the $\chi^2$ cut, the corresponding pattern is accepted as belonging to a valid track. Otherwise the procedure is repeated for sub-patterns with two hits less and so on. If no track is found that satisfies all cuts, the pattern is rejected;

3. Track segments in the individual chambers are combined into a ‘global’ track.

The details of the first two procedures are discussed in the following paragraphs. The treatment of global tracks is deferred to the next chapter (paragraph 5.5). A fully reconstructed cosmic muon event is shown in figure 4-15.

4.3.1 Pattern recognition

The pattern recognition starts with a pair of hits: one in the top multilayer and one in the bottom multilayer of a chamber. For this pair the four track ambiguities are determined (see figure 4-16). These four ambiguities serve as seeds for pattern candidates: for each ambiguity all hits within a certain distance (5 mm) of the trajectory are added to the list of hits for this ambiguity. Lists with a minimum total number of hits (4) and a minimum number of hits in each multilayer (2) are accepted as a pattern.

The procedure is repeated until the list of possible hit pairs is exhausted. At this stage duplications (real copies and proper subsets) are removed. On average about one pattern per chamber per event is found (excursions to more than ten patterns are observed in 0.5% of the events). Multiple usage of hits is not explicitly excluded.

**Figure 4-16** Principle of pattern recognition. For each pair of hits (the circles) with one hit in each multilayer (solid circles) the four track ambiguities (the lines) are used as seeds to collect hits on a pattern.
Figure 4-15 Event display of the analysis software showing a fully reconstructed event in the NIKHEF ATLAS muon cosmic ray test station. Two reconstructed muons are shown with zooms per multilayer of the best track segments. The three rectangles below $y = 1000$ mm represent the trigger scintillators that registered a hit. The small solid black box around $(z, y) = (100 \text{ mm}, 200 \text{ mm})$ represents the combined hits in the scintillator trigger pair. The lighter and darker tubes indicate the read-out front-end electronics cards. A tube with a cross is known to be dead (disconnected from high-voltage because of a broken wire, or not connected to read-out).
4.3.2 Track fit

The track is modelled as a straight line with reference point \((z_0, y_0)\) and angle \(\theta\) with the z-axis. The \(n\) hits are represented as circles where each circle \(i\) \((i = 1, 2 \ldots n)\) has its centre at the wire position \((z_i, y_i)\) and a radius \(r_i\) corresponding to the measured drift distance, which is determined from the measured time (equation 4-1) and the auto-calibrated \(r-t\) relation (figure 4-10). The starting values of the track parameters come from the pattern recognition. The track is fit to the hits by minimising the \(\chi^2\), which is given by:

\[
\chi^2 = \sum_{i=1}^{n} \frac{(\Delta_i - r_i)^2}{\sigma_i^2}
\]

where \(\Delta_i\) is the distance from the track to the wire and \(\sigma_i\) the error (assumed Gaussian) at the measured \(r_i\). The error is taken from the auto-calibration procedure (figure 4-14 b). The various parameters are indicated in figure 4-17. The wire positions are assumed to lie on a regular grid (see paragraph 3.2.3) and are calculated with equation 3-3 using the site-grid parameters as measured by the X-ray tomograph (table 3-9). The distance \(\Delta_i\) is equal to the absolute value of the \(y\)-coordinate of the wire position expressed in the coordinate system of the track. The origin of the track coordinate system is at \((z_0, y_0)\) and the z-axis coincides with the track. The wire coordinates in the track coordinate system \((z'_i, y'_i)\) are given by:

\[
\begin{align*}
z'_i &= z_i \cos(\theta) + y_i \sin(\theta) - e, \\
y'_i &= -z_i \sin(\theta) + y_i \cos(\theta) - d,
\end{align*}
\]

where

\[
e = z_0 \cos(\theta) + y_0 \sin(\theta) \quad \text{and} \quad d = -z_0 \sin(\theta) + y_0 \cos(\theta).
\]

The variable \(d\) runs perpendicular to the track and is the distance from the track to the origin of the global coordinate system. The variable \(e\) runs along the track and is the distance from the reference point \((z_0, y_0)\) to the point of closest approach to the origin of the global coordinate system. The angle \(\theta\) and the perpendicular shift \(d\) are the fit parameters and \(e\) will be chosen to make the correlation between \(\theta\) and \(d\) zero. The \(\chi^2\) now reads:

\[
\chi^2 = \sum_{i=1}^{n} \frac{\left(\left|y'_i\right| - r_i\right)^2}{\sigma_i^2} = \sum_{i=1}^{n} \left(\frac{\left|-z_i \sin(\theta) + y_i \cos(\theta) - d - r_i\right|}{\sigma_i^2}\right)^2.
\]

The \(\chi^2\) is minimised in an iterative procedure by setting the derivatives to the parameters to zero along the procedure described in appendix C. The matrix \(A\) and vector \(b\) are given by:
\[
A = \begin{bmatrix}
A_{\theta \theta} & A_{\theta d} \\
A_{d \theta} & A_{dd}
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{n} \frac{(z_i')^2}{\sigma_i^2} & \sum_{i=1}^{n} \frac{z_i'}{\sigma_i} \\
\sum_{i=1}^{n} \frac{z_i'}{\sigma_i} & \sum_{i=1}^{n} \frac{1}{\sigma_i}
\end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix}
b_{\theta} \\
b_{d}
\end{bmatrix} = \begin{bmatrix}
\sum_{i=1}^{n} \frac{z_i'(y_i' + r_i)}{\sigma_i^2} \\
\sum_{i=1}^{n} \frac{y_i'r_i}{\sigma_i^2}
\end{bmatrix},
\]

where the \(\mp\) is the opposite of the sign of \(y_i'\). We use the following definitions, which are all constants in the fit:

\[
S = \sum_{i=1}^{n} \frac{1}{\sigma_i^2}, \quad S_z = \sum_{i=1}^{n} \frac{z_i}{\sigma_i^2}, \quad S_y = \sum_{i=1}^{n} \frac{y_i}{\sigma_i^2},
\]

\[
S_{zz} = \sum_{i=1}^{n} \frac{z_i^2}{\sigma_i^2}, \quad S_{yy} = \sum_{i=1}^{n} \frac{y_i^2}{\sigma_i^2}, \quad S_{zy} = \sum_{i=1}^{n} \frac{z_iy_i}{\sigma_i^2},
\]

\[
S_{yy-zz} = \sum_{i=1}^{n} \frac{(y_i - z_i)(y_i + z_i)}{\sigma_i^2},
\]

\[
z_c = \frac{S_z}{S}, \quad y_c = \frac{S_y}{S},
\]

\[
R_i = \frac{r_i}{\sigma_i}, \quad R_i' = \frac{r_i'}{\sigma_i}, \quad R_i'' = \frac{y_i'r_i}{\sigma_i^2},
\]

where \((z_c, y_c)\) is the weighted average of the hit positions. \(S_{yy-zz}\) is calculated in this way instead of simply \(S_{yy} - S_{zz}\) for numerical stability. The matrix and vector elements read explicitly:

\[
A_{\theta \theta} = \cos^2(\theta)S_{zz} + \sin^2(\theta)S_{yy} + 2\sin(\theta)\cos(\theta)S_{zy} + eS(\cos(\theta)z_c + \sin(\theta)y_c)) ,
\]

\[
A_{dd} = S ,
\]

\[
A_{\theta d} = S(\cos(\theta)z_c + \sin(\theta)y_c - e) ,
\]

\[
b_{\theta} = \sin(\theta)\cos(\theta)S_{yy-zz} + (2\cos^2(\theta) - 1)S_{zy} + \cos(\theta)\sum_{i=1}^{n} \mp R_i^z + \sin(\theta)\sum_{i=1}^{n} \mp R_i^y
\]

\[-dS(\cos(\theta)z_c + \sin(\theta)y_c - e)S(\cos(\theta)y_c - \sin(\theta)z_c) - e\sum_{i=1}^{n} \mp R_i ,
\]

\[
b_{d} = S(\cos(\theta)y_c - \sin(\theta)z_c - d) + \sum_{i=1}^{n} \mp R_i .
\]

The value of \(e\) can be freely chosen, and we choose it to make the correlations zero by setting the diagonal elements \(A_{\theta d}\) to zero:

\[
e = \cos(\theta)z_c + \sin(\theta)y_c.
\]
For this value of $e$, the new values of the parameters are given by:

$$\theta^{\text{new}} = \theta + \frac{b_\theta}{A_{\theta \theta}} \quad \text{and} \quad d^{\text{new}} = d + \frac{b_d}{A_{dd}}. \quad 4-19$$

The track fit routine is called many times and execution speed is therefore an important issue. Equations 4-19 are coupled and the convergence of the fit can be sped up by decoupling them. This can be achieved by shifting the hits before the fit by $(z_c, y_c)$. For the shifted hits the values of $z_c, y_c$ and $e$ are zero, which also simplifies equations 4-17 and causes an additional gain in execution speed. After the fit, the track is shifted back. Indicating the $S$ and $R$ (equations 4-16) of the shifted hits by $S'$ and $R'$, equations 4-17 simplify to:

$$A_{\theta \theta} = S'_{yy} + \cos(\theta)(2\sin(\theta)S'_{zy} - \cos(\theta)S'_{yz} - zz) ,$$

$$A_{dd} = S ,$$

$$A_{\theta d} = 0 ,$$

$$b_\theta = -S'_{zy} + \cos(\theta) \left( \sin(\theta)S'_{yy} - zz + 2\cos(\theta)S'_{zy} + \sum_{i=1}^{n} \mp R'_i \right) + \sin(\theta) \sum_{i=1}^{n} \mp R'_i , \quad 4-20$$

$$b_d = -Sd + \sum_{i=1}^{n} \mp R_i .$$

The new $\theta$ and $d$ (per iteration) are given by:

$$\theta^{\text{new}} = \theta + \frac{b_\theta}{A_{\theta \theta}} \quad \text{and} \quad d^{\text{new}} = \sum_{i=1}^{n} \mp R_i . \quad 4-21$$

In each iteration the sign of $y'_i$, which depends on both $d$ and $\theta$, needs to be calculated for each hit to determine the sign of the $R'_i$'s. This still gives a coupling between the two parameters if a hit changes side of the track during the fit. The fit is considered converged if the angle step is smaller than 0.1 $\mu$rad and the position step is smaller than 0.1 $\mu$m. The fit converges in less than 4 iterations for 95% of the tracks and uses on average 2.3 iterations.

The errors on the parameters are given by:

$$\sigma_\theta = \frac{1}{\sqrt{A_{\theta \theta}}} \quad \text{and} \quad \sigma_d = \frac{1}{\sqrt{A_{dd}}} . \quad 4-22$$

After the fit, the track is shifted back to its original location:

$$z_0 = z_c - \sin(\theta)d , \quad 4-23$$

$$y_0 = y_c + \cos(\theta)d ,$$

which is the rotation point of the track for uncorrelated track parameters.
Figure 4-18 shows the distribution of the $\chi^2$ per degree of freedom and the confidence level of the track fit using the auto-calibrated $r-t$ relation. The flatness of the confidence level distribution evinces an excellent understanding of the errors.

\[ \text{Figure 4-18} \quad \text{Distribution of the } \chi^2 \text{ per degree of freedom (a) and the confidence level (b) of the track fit using the auto-calibrated } r-t \text{ relation.} \]

4.3.3 Drift tube detection efficiency and hit-on-track efficiency

When a muon traverses an MDT, the drift tube does not register a hit with 100% efficiency. Two different types of inefficiency occur:

1. No hit at all. This is due to the statistical nature of the signal generation in the gas. Since the discriminator threshold is at a finite signal level, there is a finite probability that the signal will be too low to generate a hit. In particular, this efficiency is lower close to the tube wall since the gas length traversed by the muon is rapidly decreasing. As a consequence, the average number of primary electrons becomes smaller, leading to a lower average signal height, and a smaller number of the statistically fluctuating signals will be above the discriminator threshold;

2. A hit at the ‘wrong’ place, where ‘wrong’ could for example be defined as more than five times the resolution away from the track:

   a. A hit can be too close to the wire if another particle ($\delta$-electron ejected by the muon) traverses the tube closer to the wire than the muon. This other hit will then mask the muon hit. This masking probability is zero for tracks that pass through the wire and increases with increasing distance from the track to the wire, to reach about 15% near the tube wall. This results in an overall inefficiency due to $\delta$-electrons of 4% [24];

   b. A hit can be too far away from the wire if the leading edge signal peak is below the threshold, but a later signal peak is above the threshold (see e.g. figure 4-4); or if the
complete muon signal is below the threshold and another particle (δ-electron) traverses the tube at a larger distance from the wire.

Figure 4-19 shows the efficiency of the MDTs as a function of the radial position of the track. The efficiency is defined as the number of found hits divided by the number of expected hits. A hit is expected in a tube if the reconstructed track passes within 14.6 mm from the wire. To avoid biases, only good quality tracks are included: at least five hits are required on the track, at least two in each multilayer, and the $\chi^2$ per degree of freedom should be less than 2. Two types of efficiency are shown: found hits assigned to the track, and any hits found in the tube. The ‘any hits’ efficiency is close to 100% and decreases rapidly close to the tube wall, as expected. The ‘hit-on-track’ efficiency decreases with increasing radius mainly due to the δ-electrons (case 2a above). The small decrease at $r = 0$ is probably due to the (ignored) asymmetry in the resolution in this region. The overall hit-on-track efficiency is 94.5%.

Figure 4-20 (a) shows the relative number of hits that are more than five times the local resolution closer to the wire than the track (case 2a above). These hits are most probably due to δ-electrons and this figure therefore gives an indication of the relative number of δ-electrons that obscure a muon hit. These measurements agree with

![Figure 4-19](image)/![Figure 4-20](image)

Figure 4-19 Efficiency of an MDT tube as a function of the track position. Both 'any-hit' efficiency (points) and 'hit-on-track' efficiency (line) are shown.

Figure 4-20 Relative number of hits that are more than five times the local resolution closer to the wire (a) and farther away from the wire (b) than the track.
the measurements done elsewhere [24]. As such, it is the complement of the hit-on-track efficiency (figure 4-19), except close to $r = 0$. As expected, is it zero close to the wire and then increases linearly towards the tube wall, simply because the available phase space increases. Below $r = 1$ mm the phase space has basically reduced to zero. The sharp rise near the tube wall is probably due to the increased amount of matter that is traversed by the muons that pass close to the wall, which increases the probability of a δ-electron. Figure 4-20 (b) shows the opposite: the relative number of hits that are more than five times the local resolution farther away from the wire than the track (case 2b above). It is below 0.5% with a small rise close to $r = 0$, which is probably due to the (ignored) asymmetry in the resolution in this region, and is in this region the complement of the hit-on-track efficiency (figure 4-19).

Most of the inefficiency of the hits that can be used on a track is due to δ-electrons. This is an important factor to be taken into account in simulations of MDTs, in particular for studies of muon track reconstruction efficiency and resolution.

4.4 Results on BOL chambers

The results presented in this section are obtained with data that was collected in 23 hours. After the software trigger cut (paragraph 4.1.2), 1.15 million events are included in the analysis. All results shown are from the same chamber (BOL-6, not equipped with twin-tube jumpers).

4.4.1 Dead and noisy channels

Figure 4-21 shows the total number of hits collected per tube for one BOL chamber. The typical distribution, which is repeated every layer, is due to the geometrical acceptance of the set-up. The tubes collect around 10k hits each. Dead channels, noisy channels and tubes with low efficiency are immediately evident from this histogram. Tube number 360 (top multilayer, middle layer, tube number 72) is not connected to the read-out. Tube numbers 239 (top multilayer, bottom layer, tube
number 23) and 352 (top multilayer, middle layer, tube number 64) have broken wires and are disconnected from the high-voltage. The hits present in tube 352 are therefore all noise hits. Known dead channels are also indicated on the event display of figure 4-15. The breaking of the wires is caused by high gas flow rates due to uncontrolled deflation of the chamber during gas leak tests, and will be avoided in the future. This chamber also has a few noisy channels with a noise hit rate of about 1 kHz, which is still negligible compared to the expected beam related background hit rate of 15 kHz in ATLAS (BOL tubes).

4.4.2 Uniformity of the operating point

The maximum drift time of an MDT is sensitive to the operating point (gas composition, pressure, temperature and high-voltage) and is therefore a good indicator for the uniformity of a chamber. Figure 4-22 shows the maximum drift times for all tubes in a BOL chamber. The large r.m.s. (16 ns) is due to systematics which emerge as clearly visible patterns in this figure. The repetitive structure with a period of three tubes is related to the gas connection scheme: within each layer, three tubes are connected in series to the gas manifold. The further downstream a tube is, the higher its maximum drift time. The r.m.s. of the maximum drift times per sub-group is around 11 ns, and the sub-groups are spaced by about 20 ns. This is significant compared to the precision of the $t_{max}$ determination (1.7 ns, see table 4-2). Within each sub-group another systematic is evident from figure 4-22: per layer the values decrease with increasing tube number, i.e. with increasing distance from the inlet of the gas distribution bar, and with decreasing distance from the outlet of the gas collection bar. These systematics are not caused by a pressure drop, because the gas flow is very low, and the effect hardly changes if the flow is stopped or if it is increased by a factor of 5. The effects are possibly related to the high gas leak rate of this chamber ($10^{-2} - 10^{-3}$ bar liter / s compared to $10^{-5}$ bar liter / s for a certified BOL chamber). It could also be residual impurities in the gas, which are not cleaned out due to the low gas flow. The cause is not really known and further tests are needed. The maximum drift time variations are to a large extent compensated by the rescaling procedure described in paragraph 4.2.3.

![Figure 4-22](image_url)  

*Figure 4-22* Maximum drift times for the BOL-6 chamber as a function of the (continuous) tube number in the chamber.
4.4.3 Efficiency

Figure 4-23 shows the efficiency per tube for one BOL chamber. To exclude the edge effects, hits are only counted if the track passes between 1 and 14 mm from the wire. Most tubes have excellent efficiency (99.97%). This is what is expected, because the discriminator threshold is very low compared to the signal height. As a cross-check, the average efficiency of the chamber is also calculated by counting the number of tracks with a hit in each of the 6 tubes that were expected to have a hit (again for $r = 1 - 14$ mm), and comparing that to the number of tracks that were expected to have 6 hits. The average single tube efficiency for this chamber calculated with this method is $99.533 \pm 0.006\%$, which is close to the expected efficiency including the 3 dead tubes (99.31%).

When the dead tubes are excluded, the efficiency goes up to $99.984 \pm 0.001\%$, which is close to the efficiency determined the other way. Tube numbers 239, 352 and 360 have zero efficiency because they are disconnected from either high-voltage or read-out (also see paragraph 4.4.1). The zero efficiency of tube number 352 confirms that the hits in this tube (figure 4-21) are not related to tracks, and are therefore noise hits.

![Figure 4-23](image)

Figure 4-23 MDT efficiency as a function of the (continuous) tube number for the BOL-6 chamber. Only radial track positions between 1 and 14 mm are included.

4.4.4 Resolution

Figure 4-24 shows the resolution for all tubes in one BOL chamber, calculated with equation 4-10, where the hits of all radii are mixed (per tube). This means that the assumption of a constant $\sigma_{input}$ is not valid, and the values need not necessarily be the same as the inverse quadratic average of the $r$-dependent resolution (deduced from figure 4-14 b). The tube-to-tube variations, however, can still be studied with this approximation. Most tubes have similar resolution, and a few are significantly worse. Those few could be examined in more detail, and possible problems could be found and maybe fixed. There seems to be a systematic worse resolution near the end of the chamber (tubes 1, 72, 73, 144, 145 etc.), which is probably an artifact of the geometry.
4.4.5 Track reconstruction precision

The precision with which the track segments are reconstructed in the individual chambers contributes to the precision of the muon momentum measurement in ATLAS and is therefore an important parameter. The precision of the track segment position depends on the number of hits on the track, and therefore on the number of layers in a chamber. The precision of the track segment angle depends moreover on the spacer height of the chamber. For a BOL chamber tested at NIKHEF with cosmic muons, the distribution of the precision of the track segments, calculated with equation 4-22, is shown in figure 4-25. The average values are somewhat larger than the goals (see paragraph 2.3.1), which is a direct result of the somewhat worse resolution (see paragraph 4.2.4), which in turn is due to the larger multiple scattering of the lower momentum cosmic muons (see footnote 1 on page 99). The values are also larger because the goals assume six hits on the track and this sample includes tracks with five hits (26%). When the resolution is corrected for the estimated contribution of multiple scattering, the average precision is 36 µm on the track position and 177 µrad on the track angle.

![Figure 4-24](image)

**Figure 4-24** Resolution as a function of the (continuous) tube number for the BOL-6 chamber. The horizontal line indicates the 80 µm resolution goal.

![Figure 4-25](image)

**Figure 4-25** Distribution of the precision of the track segment position (a) and track segment angle (b) in the BOL-6 chamber.
Figure 5-1 Photograph of the DATCHA set-up located in the former UA1 cavern at CERN.