Precision of the ATLAS muon spectrometer
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Chapter 5  Muon chamber alignment precision

What is twisted cannot be straightened;
what is lacking cannot be counted.

Ecclesiastes 1:15

The relative alignment of the three chambers (one in each muon station) traversed by a muon is one of the prime factors in the precision of the muon momentum measurement. The previous chapter dealt with the precision of the track segments as measured by the individual chambers. This chapter deals with how the three segments are combined to determine the track ‘sagitta’, which is a direct measure for the muon momentum. Any misalignment of chambers will introduce a fake sagitta, and the alignment needs to be known to high accuracy to compensate for that. Expressed in terms of the sagitta, the accuracy of the alignment of the muon chambers is aimed to be 30 µm. The experimental set-up called ‘DATCHA’ was built to test whether this goal can be achieved, and is shown on the photograph in figure 5-1. It was assembled and operated at CERN in the period 1996-1999. The main results of ‘DATCHA’ have been published [46]. This chapter deals with the description of the set-up, the method used for the test and will show that the goal is achieved.

5.1 Method of verification

To verify the precision of an alignment system, an independent measurement is needed to compare it to. For our test, three prototype muon chambers are used in a full-size set-up in the configuration in which the ATLAS experiment will measure the muon tracks. Here, cosmic muons detected by the muon chambers are used for the independent measurement. Because of the absence of a magnetic field the muons go straight on average. In the test, the average sagitta is calculated from the cosmic muon tracks and from the alignment systems. This is done for several controlled displacements of the MDT chambers. The alignment sagitta values are then compared to the muon sagitta values. This gives the precision of the alignment system smeared with the precision of the muon tracks, which is dominated by the multiple scattering in detector material. Since the alignment systems in the DATCHA set-up were not calibrated, this test only shows on how well the alignment system can monitor changes in the geometry.
5.2 Description of the DATCHA set-up

The DATCHA\(^1\) set-up constitutes one full-size large muon barrel tower, which is about 0.5\% of the ATLAS muon spectrometer, including three Monitored Drift Tube (MDT) chambers, three Resistive Plate Chambers (RPCs) and 16 Rasnik alignment monitors. Figure 5-2 shows a schematic view of the set-up.

![Figure 5-2](image)

Figure 5-2 Schematic front-view (a) and side-view (b) of the DATCHA set-up.

As in ATLAS, the MDT chambers (here BIL, BML and BOL) measure the precision coordinate \((z)\) of the muons. The geometry of the MDT chambers in DATCHA is taken from the ATLAS geometry at the time of construction of the set-up and is somewhat different from the final layout of ATLAS. The BIL chamber was constructed at IHEP in Protvino, Russia. The tubes for the BML and BOL chambers were wired at CERN \([47]\), and were assembled into full chambers at INFN in Frascati, Italy, (BML) and at NIKHEF in Amsterdam, The Netherlands, (BOL). The RPC chambers, constructed at INFN in Rome, Italy, measure the 'second coordinate' \((x, \text{along the MDT wires})\) and are used in the trigger, but the layout differs from ATLAS. The muons in ATLAS will come from the center of the detector, but here we use cosmic muons, which traverse the set-up in the opposite direction. In DATCHA no magnetic field is present and the muons follow a straight path, apart from multiple scattering. In addition a scintillating hodoscope is installed for the timing of the passing muons. A concrete + iron absorber is installed to remove the low energy muons. The DATCHA set-up was assembled at CERN and became operational in the summer of 1997.

5.2.1 Scintillating hodoscope

The 3 \(\times 1\) m\(^2\) scintillating hodoscope serves to measure the time of passage of the muon. It is installed directly below the BIL chamber and consists of two layers of fourteen 0.2 \(\times 1\) m\(^2\) plastic scintillation counters. The short sides of the counters are in the \(x\)-direction and the two layers are

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1. DATCHA is the acronym for Demonstration of ATlas CHamber Alignment.
shifted by about 7 cm in x. This leads to a measurement of the x-coordinate with a precision of a few cm. The counters are read out with photo-multipliers on opposite sides (in z) for the two layers, which allows to correct for the signal propagation time in the scintillators. Passage of a muon causes a hit in one counter in each of the layers. The mean time of arrival of the signal of the counters of the two layers is taken as the time of arrival of the muon. The resolution of this mean time is 1.0 ns. Systematic variations are smaller than 0.5 ns over the entire surface. The time difference between the two layers allows to calculate the z-position with a precision of 15 cm, assuming an inverse signal propagation speed of 6.3 ns / m.

5.2.2 Resistive plate chambers

Several Resistive Plate Chambers (RPCs) are installed for second coordinate (x) measurement and for triggering. A single RPC is 2 x 2 m$^2$ and has strips with a pitch of 31 mm giving a coordinate measurement precision of 31 mm / $\sqrt{12} = 9$ mm. They are continually flushed with an Argon/Freon/Isobutane 56/40/4 gas mixture and are operated at a high-voltage of 7000 V. Three RPCs are combined to cover a region of 2 x 6 m$^2$ and are installed directly above the BOL chamber (RPC 1 in figure 5-2). One 2 x 2 m$^2$ RPC is installed directly below the BIL chamber (RPC 2 in figure 5-2). These RPCs measure only the x-coordinate. Four additional RPCs are combined below the absorber (RPC 3 in figure 5-2) to measure both the x- and z-coordinates over a surface of 2 x 4 m$^2$. The location along the wire at which the muon traverses the MDT chambers is derived from the available RPC x-measurements and has a precision of about 4 cm, which includes RPC misalignments and multiple scattering in the absorber.

5.2.3 Muon trigger

The cosmic muon trigger is derived from a coincidence between hits in the two hodoscope layers, the RPC above the BOL chamber and the RPC underneath the 0.8 m concrete + 1.6 m steel absorber. The energy cut on the muons due to the absorber is about 3 GeV and the trigger rate is around 5 Hz. A typical run collects 300,000 events in about 17 hours. In the offline analysis we select the high quality trigger events by requiring:

- Unambiguous hits in the scintillating hodoscope (90%), and
- One continuous sequence of strips hit in at least 2 of the 3 RPC chambers (96%), and
- Match between the x-coordinate of the RPCs and the x-coordinate of the hodoscope (88%),

which reduces the sample to 76% of the recorded events.

5.2.4 Muon drift tube chambers

Three MDT chambers (BIL, BML and BOL) are installed to make up one large muon barrel tower. The geometry corresponds to the third barrel tower in ATLAS$^1$, which has an average angle of 117° with the colliding beams. Table 5-1 gives an overview of the nominal geometrical parameters of the chambers used in DATCHA. The third wire locator$^2$ of the BML and BOL chambers is located halfway along the wires, above the middle cross-plate.
Table 5-1 Nominal geometrical parameters of the MDT chambers installed in DATCHA.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>BIL</th>
<th>BML</th>
<th>BOL</th>
</tr>
</thead>
<tbody>
<tr>
<td># tubes per layer</td>
<td>32</td>
<td>48</td>
<td>72</td>
</tr>
<tr>
<td># layers per multilayer</td>
<td>3</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Total # of tubes</td>
<td>192</td>
<td>288</td>
<td>576</td>
</tr>
<tr>
<td>Z-pitch (mm)</td>
<td>30.100</td>
<td>30.050</td>
<td>30.075</td>
</tr>
<tr>
<td>Y-pitch (mm)</td>
<td>26.067</td>
<td>26.024</td>
<td>26.046</td>
</tr>
<tr>
<td>Layer stacking (Z-pitches)</td>
<td>0, -\frac{1}{2}, 0; 0, -\frac{1}{2}, 0</td>
<td>0, +\frac{1}{2}, 0; 0, +\frac{1}{2}, 0</td>
<td>-1\frac{1}{2}, -1, -\frac{1}{2}, 0; 0, +\frac{1}{2}, +1, +1\frac{1}{2}</td>
</tr>
<tr>
<td>\Delta Y multilayers (mm)</td>
<td>232.134</td>
<td>332.048</td>
<td>458.138</td>
</tr>
<tr>
<td>Tube length (mm)</td>
<td>2500</td>
<td>4000</td>
<td>5700</td>
</tr>
<tr>
<td>Wire tension (g)</td>
<td>250</td>
<td>300</td>
<td>250</td>
</tr>
<tr>
<td># wire locators</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Z center of chamber (mm)</td>
<td>-2471.286</td>
<td>-3841.714</td>
<td>-5545.000</td>
</tr>
<tr>
<td>Y center of chamber (mm)</td>
<td>+4883.000</td>
<td>+7543.000</td>
<td>+10951.000</td>
</tr>
</tbody>
</table>

The operation point of the MDT chambers in DATCHA is listed in table 5-2. This is different from the one foreseen for ATLAS (listed in table 2-2). The operating point is much less critical than in ATLAS, because in DATCHA the occupancy is much lower, which relaxes the requirements on the maximum drift time. For the chosen operation point it is 1200 ns. The H$_2$O admixture helps to reduce almost all discharge effects which were present in about 10% of the tubes in the BML and BOL chambers.

Fast shaping FBPANIC-04 pre-amplifiers, which are a modified version of the pre-amplifiers originally developed for the muon drift chambers of the LEP L3 experiment, are mounted on hedgehog boards at the read-out ends of the MDTs. Discriminators and multiplexers are combined on a ‘BIMUX’ board, which fits onto the pre-amplifier board. A thick copper-clad ground plate is mounted in between the two boards to minimise electromagnetic interference. One pre-amplifier/BIMUX board unit has 32 channels and reads out a maximum of four layers of eight tubes. The BIMUX board has two types of output: four timing outputs and a tube address output. The eight tubes in one layer are OR-red into one timing output and are led into one channel of a CAMAC LeCroy 2277 Time to Digital Converter (TDC). Each TDC channel does the time stamping for a maximum of eight leading and eight trailing edges in 1 ns bins. The tube address output gives a maximum of four tube addresses per layer of eight tubes, i.e. per TDC output. The tube addresses are led via two levels of multiplexing (the DETCOM and FBROC from the L3 experiment) into a

Table 5-2 MDT operation point in DATCHA.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas mixture</td>
<td>Ar / CO$_2$ / H$_2$O 80 / 20 / 0.25</td>
</tr>
<tr>
<td>Pressure</td>
<td>2 bar absolute</td>
</tr>
<tr>
<td>High voltage</td>
<td>3150 V</td>
</tr>
<tr>
<td>Gas renewal rate</td>
<td>1 volume / day</td>
</tr>
<tr>
<td>Gas gain</td>
<td>$4 \times 10^4$</td>
</tr>
<tr>
<td>Discriminator threshold</td>
<td>15$^\text{th}$ electron (60 mV)</td>
</tr>
</tbody>
</table>

1. The DATCHA MDT geometry deviates from the final ATLAS geometry (listed in table 2-1) because the latter has been further optimised after construction of DATCHA.
2. This third wire-locator is absent in the final design for all MDT chambers.

5.2 Description of the DATCHA set-up
CAMAC CAEN input-output register. The CAMAC modules are read out into a VME crate, controlled by a FIC 8234 processor running the OS-9 operating system. The data is stored on a disk of a SUN workstation and is copied to tape via CERN’s central data recording (CDR) facility.

The MDT high-voltage is distributed on the far end of each tube via 4 x 8 channel hedgehog boards. The high-voltage is generated locally on the chamber by one Cockroft-Walton supply per multilayer.

5.2.5 Detector control

The MDT chambers are controlled via a Controller Area Network (CAN) fieldbus that is commanded from a SUN workstation. CAN nodes connected to the high-voltage generators allow to ramp up and down the high-voltage, to monitor the currents and voltages and to set limits outside which the high-voltage is automatically switched off. The Detector Control Card (DCC) controls the MDT front-end electronics. It has a GPCAN (general purpose CAN) card with a 87c592 microcontroller with CAN interface. It is used to set the discriminator thresholds, to mask channels on the BIMUXs, and to send calibration pulses to the pre-amplifiers. The CAN bus is also used to read out temperature sensors on the chambers.

Figure 5-3 shows the average temperature of the BOL chamber as a function of time for a period of several days. The day-night temperature variations are clearly visible. Figure 5-4 shows the temperature profile along a tube of the BOL chamber. The temperature at the read-out (RO) side is significantly higher than on the high-voltage (HV) side due to the large amount of heat that is produced in the front-end electronics. This effect will be much smaller in ATLAS, because the power consumption of the electronics will be an order of magnitude smaller. The small temperature rise at the HV side is due to heat production in the Rasnik cameras.

![Figure 5-3 Temperature of the BOL chamber as a function of time for a period of 3 days. The time axis starts at midnight.](image)

![Figure 5-4 Temperature profile along a tube of the BOL chamber.](image)
5.2.6 Alignment systems

The DATCHA set-up is equipped with 16 Rasnik alignment monitors (see figure 5-5) which can be divided into two different types of systems. Each chamber has four Rasnik monitors to measure chamber deformations, the ‘in-plane’ system. The three chambers are interconnected at the four corners by Rasnik monitors, the ‘projective’ system.

The in-plane system is described in paragraph 2.5.1. For DATCHA the in-plane system is more relevant than for ATLAS, because in DATCHA the BML and BOL chambers have an additional wire locator halfway along the wire. As a consequence, the position of the middle cross-plate directly influences the positions of the wires and therefore needs to be known to the order of 10 μm, to be compared to the order 0.1 mm requirement for ATLAS coming from the requirement on the concentricity of the wire and tube.

The projective system is described in paragraph 2.5.2. In DATCHA four Rasniks are used for a single tower, whereas in ATLAS typically two physical towers will be combined into one logical projective alignment tower. In DATCHA the projective alignment components are mounted on the cross-plates, whereas in ATLAS they will be mounted on the tubes on the outside of the chambers. The BIL chamber holds the four projective Rasnik masks, the BML chamber the lenses and the BOL chamber the sensors. All MDT chambers in DATCHA have cut-outs around the optical paths of the projective systems.

All 16 Rasniks are read-out via a multiplexer controlled by a PC and the images are analysed online on the same PC\(^1\), where one image takes typically 5.5 seconds to analyse. During a data-taking run, all systems are read-out typically once every 10 minutes.

5.3 Simulation of the DATCHA set-up

A simulation program for the DATCHA set-up has been developed by P. Hendriks [48] inside the Arve framework [49], the official Atlas Reconstruction and Visualisation Environment at the time of the DATCHA data taking. It produces Monte Carlo events suitable for verifying muon reconstruction, MDT calibration algorithms and geometry reconstruction using muon tracks. The latter

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1. A 200 MHz Pentium-II running Microsoft Windows NT.
is covered in paragraph 5.6, and the first two topics are covered in detail in reference [50] but are not treated in this thesis.

The program implements the complete DATCHA set-up, i.e. detectors and inactive material, in its nominal geometry:

- **Detectors:**
  - MDT chambers including their inactive material (tube walls, gas and long-beams);
  - RPCs;
  - Scintillating hodoscope.

- **Inactive material:**
  - A crude approximation of the RPC support structure;
  - The iron + concrete absorber that serves as a momentum cut (3 GeV/c) in the trigger.

A cosmic ray generator is used as a particle source. It simulates cosmic ray muons in a $\mu$:$\mu$ ratio of 5:4. Their origin is uniformly distributed in a plane above the detector, while their angular distribution is proportional to $\cos^2(\theta)$, where $\theta$ is the angle between the vertical (y-) axis and the muon. The $\mu$ momentum distribution is taken proportional to $1/p^2$.

The particle interactions are mostly handled by Gismo [51], the domain inside Arve that defines the particle properties and material definitions. It provides the core simulation functionality and implements multiple scattering and continuous energy loss in materials. In addition to Gismo, the generation of $\delta$-rays is implemented such that the effect is in principle identical to the one provided by GEANT [52].

Responses of all detectors to the cosmic muons and $\delta$-electrons are generated and digitised. The RPCs produce a list of strip numbers that were hit. The hodoscope gives the numbers of the scintillator counters that were hit.

Simulation of the response of the MDT chambers is implemented in more detail. At the passage of a (charged) particle through an MDT, the distance of the track to the MDT wire is calculated. This distance is smeared with a Gaussian where the width is a simplified function of the distance: starting from 150 $\mu$m at the wire, it decreases linearly to 80 $\mu$m at 5 mm from the wire, after which it remains constant. The next step in the digitisation process consists of converting the drift distance $r$ to a time $t$ by inverting an $r$-$t$ relation. This drift time is then corrected for the time needed by the signal to propagate along the MDT wire, and for the time of flight of the original particle from the MDT wire to the hodoscope. In both cases the velocity is assumed to be the speed of light. In addition to the (smeared) drift distances, the truth information (type of particle, distance to wire + left/ right, propagation time along the wire, time of flight) is stored in the data file. The multiplexed read-out of the MDT chambers is simulated in the analysis framework, which is described in the next paragraph.
5.4 Analysis framework

The framework for the data analysis, ‘mutdat’, was taken from the software used by ATLAS to analyze the CERN muon test-beam data, and was developed by N. Hessey [53]. It is adapted and extended to match the DATCHA set-up. It is written in the C programming language and uses the KUIP package [54] as a command line interpreter. It reads the data from file, decodes it, and makes it accessible to the user, who has to implement the actual analysis routines. A graphical display was added in the FORTRAN programming language using the HIGZ and HPILOT graphics packages [55].

For DATCHA a separate track reconstruction package was developed in FORTRAN, which implements the track reconstruction described in paragraph 4.3. It is interfaced to the framework and included as a separate library. The MDT calibration algorithms, described in paragraph 4.2, are also interfaced to and included in the framework. Simulation of the DATCHA multiplexed MDT read-out is written in C++ and is applied automatically when simulated events are read in.

The primary task of the framework is the data pre-processing. RPC strip numbers and hodoscope counter numbers are converted into three dimensional hit positions. MDT hits arrive in two separate data streams (tube addresses and time stamps) that need to be matched to assign the correct drift times to the correct tubes.

5.5 Global track reconstruction and sagitta calculation

The MDTs are calibrated and the local tracks are reconstructed as explained in chapter 4, with the following differences:

- The time-of-flight of the muon is calculated w.r.t. the impact point of the muon in the hodoscope (see paragraph 5.2.1);
- The \( x \)-coordinate (along the tubes) is measured by the RPC chambers (see paragraph 5.2.2).

Details on the MDT calibration and the track reconstruction performance of the DATCHA set-up can be found in reference [50]. After reconstructing the track segments in the BIL, BML and BOL chambers, they are combined into a ‘global’ track. Because the tracks are straight, the \( \text{angle differences} \) of the track segments are used to decide which segments belong to the same global track. The relative position of the track segments is not used since the global chamber positions are poorly known. For each track segment in the BIL chamber, a segment is searched for in the BML chamber with an angle that is equal to the BIL segment within the cut (50 mrad). When found, and the angle of a BOL segment equals the angles of both other segments within this cut, a global track is formed if the r.m.s. of the segment angles is within a second cut (10 mrad). Figure 5-6 shows a fully reconstructed DATCHA event. As an example, figure 5-7 shows the distribution of the angle difference between the track segments in the BIL and BOL chambers. The r.m.s. is much larger than the precision of the reconstructed segment angles (0.26 mrad and 0.18 mrad for BIL and BOL respectively), and is consistent with the multiple scattering in the BML chamber. Figure 5-8 shows the distribution of the r.m.s. of the three segment angles of a global track.
Figure 5-6 Event display of the 'mutdat' analysis program showing a fully reconstructed DATCHA event. Besides the global layout of the BIL, BML and BOL chambers, and the globally reconstructed track of the muon traversing them, zooms are shown of the local track segments per chamber, including the list of hits per multilayer. A second level zoom per multilayer is shown around the track segments to better visualise the hits-on-track. The sequential tube number within a layer is added in the tubes for debugging purposes.

1. With the muon energy cut in DATCHA of about 3 GeV, and 6 x 1.2 mm = 7.2 mm of aluminium traversed on average in the BML chamber, i.e. about 0.1 x X₀, the r.m.s. multiple scattering angle is about \( \sqrt{0.1 \cdot 13.6 / 3000} = 1.5 \) mrad.
Figure 5-7 Distribution of the angle difference of the track segments in the BIL and BOL chambers.

Figure 5-8 Distribution of the r.m.s. of the angles of the three track segments of a global track.

Once a global track is formed, the ‘sagitta’ is calculated. The sagitta (‘arrow’) of a curved track (the ‘bow’) is the amount of deviation from a straight line (the ‘string’) and is, in the presence of a magnetic field, a measure of the particle momentum. In the absence of a magnetic field, as in DATCHA, the tracks are straight on average, and the average sagitta is a measure for the misalignment of the chambers.

Expressing the misalignment in terms of the sagitta has the advantage of being directly related to the momentum measurement. As illustrated in figure 5-9, in DATCHA we define the sagitta as the distance from the support point of the track segment in the BML chamber to the line connecting the support points of the track segments in the BIL and BOL chambers. This line can be considered a track as described in paragraph 4.3.2 where the two support points can be considered two hits with zero measured radius and error \( \sigma_d \) (equation 4-22). It is assumed that the angle differences between the global track and the track segments are small. For this ‘IO’ (Inner-Outer) track, the support point \((z_{IO}^{10}, y_{IO}^{10})\), the angle \(\theta_{IO}^{10}\), the positional error \(\sigma_d^{10}\) and the angular error \(\sigma_{\theta}^{10}\) are given by:

Figure 5-9 Illustration of the muon track sagitta in DATCHA.
where the upper indices I, M and O refer to the track segments in the BIL, BML and BOL chamber respectively. The variable $L$ is the distance between the support points of the track segments in the BIL and BOL chambers. The precision of the sagitta measurement is the quadratic sum of the positional error of the BML track segment and the spatial error (perpendicular to the IO track) of the point on the IO track that is closest to the support point on the BML segment. The latter contribution depends on where along the IO track the point of closest approach (PCA) is located, because the error contribution due to $\sigma_\theta$ is proportional to the distance from the PCA to $(z_0^I, y_0^I)$. Realising that the sagitta is equal to the distance from the BML ‘hit’ to the IO track, we use equations 4-13 to calculate the sagitta $s$ and its error:

$$s = \left( y_0^M - y_0^O \right) \cos(\theta^I) - \left( z_0^I - z_0^O \right) \sin(\theta^I)$$

$$\sigma_s^2 = \left( \sigma_d^M \right)^2 + \left( \sigma_d^I \right)^2 + \left( \sigma_\theta \right)^2 \left[ \left( z_0^I - z_0^O \right) \cos(\theta^I) + \left( y_0^I - y_0^O \right) \sin(\theta^I) \right]^2$$

To get more insight in the error term between $\{}$, we introduce two new parameters $\lambda^M$ and $\lambda^I$, defined as the relative $y$-position in between the BIL and BOL track segments of $y_0^M$ and $y_0^I$ respectively, such that $\lambda = 0$ corresponds to the BIL segment, and $\lambda = 1$ corresponds to the BOL:

$$\lambda^M = \frac{y_0^M - y_0^I}{y_0^M - y_0^O} \quad \text{and} \quad \lambda^I = \frac{y_0^O - y_0^I}{y_0^O - y_0^O} = \frac{1}{L^2} = \frac{1 + \left( \sigma_d \right)^2}{\left( \sigma_d \right)^2}$$

Using these definitions the error on the sagitta reads (after some math):

$$\sigma_s^2 = (\sigma_d^M)^2 + (\sigma_d^I)^2 + (\sigma_\theta)^2 \left\{ \lambda^I - \lambda^M \right\} \frac{s z_0^O - z_0^I}{L_y^O - y_0^O}$$

$$= (\sigma_d^M)^2 + \left( \frac{\sigma_d^I}{\sigma_d^M} \right)^2 \left( \frac{\sigma_d^O}{\sigma_d^M} \right)^2 + \left( \frac{\sigma_\theta^I}{\sigma_d^M} \right)^2 \left( \frac{\sigma_\theta^O}{\sigma_d^M} \right)^2 \left\{ \lambda^I - \lambda^M \right\} \frac{s z_0^O - z_0^I}{L_y^O - y_0^O}$$
The y-coordinates of the track segment support points are close to the respective chamber y-positions, so $\lambda^M$ is determined mainly by the layout of the detector. The value of $\lambda^{I0}$ depends only on the ratio of BIL and BOL track segment positional errors. It is reasonable to assume that these errors are inversely proportional to the square root of the number of layers in a chamber, so $\lambda^{I0}$ depends mainly on the ratio of the number of layers in the BIL and BOL chambers. The last term within the { of equation 5-5 is usually small and can be neglected. Table 5-3 shows the values of $\lambda^M$ and $\lambda^{I0}$ for ATLAS and DATCHA for above mentioned assumptions. We see that the contribution of the last error term in equation 5-5 is small compared to the other two terms. We conclude that the error on the sagitta has only a minor dependence on the positioning of the chambers in space. It can be calculated to a few percent accuracy with:

$$\sigma_s = \sqrt{\left(\frac{\sigma^M}{\sigma_d}\right)^2 + \left(\frac{\sigma^{I0}}{\sigma_d}\right)^2 + \left(\frac{\lambda^{I0}}{\sigma_d}\right)^2}.$$  \hspace{1cm} 5-6$$

Note also that the contribution from the middle chamber is about two times more important than the contributions from the inner and outer chambers, so the measurement accuracy is enhanced most efficiently by improving the middle chambers, for example by adding layers of tubes.

Figure 5-10 shows the distribution of the sagitta and its error for one DATCHA run. The r.m.s. of the sagitta is much larger than its error due to the large multiple scattering. The average value of the sagitta is non-zero because of the misalignment of the chambers.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
          & DATCHA  & ATLAS  \\
          & B-IMO-L & B-IMO-L & B-IMO-S \\
\hline
$\lambda^M$ & 0.44    & 0.48    & 0.59    \\
$\lambda^{I0}$ & 0.57    & 0.43    & 0.43    \\
$(\lambda^M - \lambda^{I0})^2$ & 0.018   & 0.003   & 0.026   \\
\hline
\end{tabular}
\caption{Values of $\lambda^M$ and $\lambda^{I0}$ based on the geometry of DATCHA and ATLAS (muon layout P-03, February 2002).}
\end{table}

\begin{figure}[h]
\centering
\begin{subfigure}{0.49\textwidth}
\centering
\includegraphics[width=\textwidth]{sagitta_distribution_a.png}
\caption{(a) Nent = 49198, Mean = 1.6, RMS = 1.5}
\end{subfigure}
\begin{subfigure}{0.49\textwidth}
\centering
\includegraphics[width=\textwidth]{sagitta_error_distribution_b.png}
\caption{(b) Nent = 49198, Mean = 34, RMS = 3.1}
\end{subfigure}
\caption{Distribution of the sagitta of one run in DATCHA (a) and its error (b).}
\end{figure}

5.5 Global track reconstruction and sagitta calculation
5.6 Geometry reconstruction using straight muon tracks

5.6.1 Geometrical model

The positions of the MDT wires are the only geometrically relevant objects as far as the track reconstruction is concerned. The wire positions are determined by the wire locators and the gravitational wire sag in between those locators. The wire locators are positioned by the cross-plates, and the gravitational sag is calculated using the wire tension. The gravitational wire sag is already taken into account in the muon track reconstruction. Hence the basic geometrical units to be positioned in space are the cross-plates.

Figure 5-5 in paragraph 5.2.6 shows the cross-plates as well as the coordinate system, where it should be stated that the middle cross-plates of the three chambers are located at \( x = 0 \). Out of the six degrees of freedom per cross-plate (three dimensional position and three rotation angles) only three influence the wire positions to first order: the \( y \) and \( z \) positions and the rotations \( \alpha \) around the \( x \)-axis. The BIL middle cross-plate is irrelevant, since BIL does not have a central wire locator, so eight cross-plates are relevant geometrical objects.

To get an absolute reference in space the angle \( \alpha \) of one cross-plate has to be fixed (we take the BML middle one) and the positions \( y \) and \( z \) of two cross-plates have to be fixed (we take the two BIL outer ones). The number of parameters left to be determined is thus \( 3 \times 8 - 1 \times 2 = 19 \), which we choose to be the deviations \( \Delta y \), \( \Delta z \) and \( \Delta \alpha \) from the ideal cross-plates geometry. In the notation we use the upper index ‘+’ for the cross-plate at \( x < 0 \) (read-out), ‘0’ for the one at \( x = 0 \) (middle), and ‘+’ for the one at \( x > 0 \) (high-voltage). The lower index ‘i’ stands for BIL, ‘m’ for BML and ‘o’ for BOL. The 19 parameters are:

- BIL: \( \Delta \alpha_i^+ \), \( \Delta \alpha_i^- \) (2 parameters),
- BML: \( \Delta \alpha_m^+ \), \( \Delta \alpha_m^- \), \( \Delta y_m^0 \), \( \Delta z_m^0 \), \( \Delta \alpha_m^+ \), \( \Delta z_m^0 \), \( \Delta y_m^0 \) (8 parameters),
- BOL: \( \Delta \alpha_o^+ \), \( \Delta z_o^+ \), \( \Delta y_o^0 \), \( \Delta \alpha_o^+ \), \( \Delta z_o^0 \), \( \Delta y_o^+ \) (9 parameters).

5.6.2 Determination of the geometrical parameters

The parameters are determined by minimising the \( \chi^2 \) in an iterative procedure as outlined in appendix C. The \( \chi^2 \) is made up of the angular and positional information contained in the track segments in each of the three chambers. The total \( \chi^2 \) is the sum of the \( \chi^2 \)'s of the track segments in all three possible pairs of chambers:

\[
\chi^2 = \chi^2_{i,m} + \chi^2_{o,m} + \chi^2_{i,o} .
\]

This \( \chi^2 \) consists of 8 terms, depending on the chambers concerned and where the track traverses the chambers. \( \chi^2_{i,m} \) and \( \chi^2_{i,o} \) each have 2 terms, and \( \chi^2_{m,o} \) has 4 terms.

The local (i.e. at a certain \( x \)) displacement and rotation is obtained by linear interpolation in \( x \) between the two cross-plates concerned. We define \( \hat{x} \) as being the fractional position in \( x \) of the track in a chamber:
\[ \hat{x} = \frac{2x}{x^+ - x^-} \text{ for } x^- \leq x \leq x^+ \text{ so that } -1 \leq \hat{x} \leq 1. \]

The interpolated parameters are different for the three chambers:

\[ \Delta \alpha_i(\hat{x}) = \frac{1 - \hat{x}_i}{2} \cdot \Delta \alpha_i^- + \frac{1 + \hat{x}_i}{2} \cdot \Delta \alpha_i^+, \quad 5-7 \]

\[ \Delta \alpha_m(\hat{x}) = \begin{cases} \hat{x}_m \cdot \Delta \alpha_m^- & \text{for } \hat{x}_m \geq 0, \\ -\hat{x}_m \cdot \Delta \alpha_m^+ & \text{for } \hat{x}_m < 0 \end{cases}, \quad 5-8 \]

\[ \Delta \alpha_o(\hat{x}) = \begin{cases} (1 - \hat{x}_o) \cdot \Delta \alpha_o^- + \hat{x}_o \cdot \Delta \alpha_o^+ & \text{for } \hat{x}_o \geq 0, \\ (1 + \hat{x}_o) \cdot \Delta \alpha_o^- - \hat{x}_o \cdot \Delta \alpha_o^+ & \text{for } \hat{x}_o < 0 \end{cases}. \quad 5-9 \]

The interpolations for y and z are the same and are given by simply replacing the \( \alpha \)'s above by y and z respectively. Although these equations are exact for a wire, they are only exact for a track segment if the rotation angle is the same all over the track segment. This is true if there is no torque in the chamber and no rotation of the complete chamber around the y and z axes and no chamber sag in y and z. Any of these cases will make the rotation angle vary along the track segment, since the track segment extends in x. The exact change in the track segment angle can be obtained by re-fitting the segment (as described in paragraph 4.3.2) using displaced wire positions. This, however, requires the availability of the hits and is rather cpu intensive. We use three simplifications:

1. We correct the measured track angles by taking two 'super points' on the track, each halfway (in y) either multilayer. These points we translate and rotate using the exact interpolation formulas. From the two displaced points we calculate the new track segment position and angle. These new track segments are treated as 'measurement points'.

2. In calculating the second derivatives of the \( \chi^2 \) to the parameters, we use the above simple interpolation formulas, i.e. we ignore the extension of the track in x.

3. In each iteration the calculation of the corrections to the angles is separated from the calculation of the corrections to the positions.

With these simplifications, the angle differences of the (new) measurements should be zero on average, meaning that the 'model' predicts zero differences. The \( \chi^2 \) of the angles then becomes:

\[ \chi^2_{\alpha} = \sum_{k=1}^{N} \left\{ \left( \frac{\theta_{\text{new},i}^k - \theta_{\text{new},m}^k}{\sigma_{\alpha,i,m}^k} \right)^2 + \left( \frac{\theta_{\text{new},i}^k - \theta_{\text{new},o}^k}{\sigma_{\alpha,i,o}^k} \right)^2 + \left( \frac{\theta_{\text{new},o}^k - \theta_{\text{new},m}^k}{\sigma_{\alpha,o,m}^k} \right)^2 \right\}, \quad 5-10 \]
where the summation is over \( N \) global tracks denoted by upper index \( k \). The \( \theta_{\text{new}} \)s are the angles (as defined in paragraph 4.3.2) of the new track segments, and the \( \sigma_{\alpha} \)s are the errors on the measured angles, which are dominated by the multiple scattering.

In the positions \( \chi^2 \) we use the distances between the track segments in the chambers. Apart from multiple scattering, the angles of the track segments are the same for one track. In the calculation of the distance between two track segments we therefore use the average of all three (corrected) angles \( \bar{\theta}_{\text{new}}^k \) and the distance is given (here for the BIL and BML segments) by:

\[
d_{i,m}^k = \left| (y_{0,i} - y_{0,m}) \cos(\bar{\theta}_{\text{new}}^k) - (z_{0,i} - z_{0,m}) \sin(\bar{\theta}_{\text{new}}^k) \right|,
\]

where the \( y_0 \)s and \( z_0 \)s are the coordinates of the support points (as defined in paragraph 4.3.2) of the track segments. As with the angles, these distances should be zero on average, and the \('model'\) predicts zero. The \( \chi^2 \) of the positions then becomes:

\[
\chi^2_{\text{pos}} = \sum_{k=1}^{N} \left[ \left( \frac{d_{i,m}^k}{\sigma_{\text{pos},i,m}^k} \right)^2 + \left( \frac{d_{i,o}^k}{\sigma_{\text{pos},i,o}^k} \right)^2 + \left( \frac{d_{o,m}^k}{\sigma_{\text{pos},o,m}^k} \right)^2 \right].
\]

Here, the errors \( \sigma \) are given (for the BIL-BML segments) by:

\[
(\sigma_{\text{pos},i,m}^k)^2 = \left( (z_{0,i} - z_{0,m})^2 + (y_{0,i} - y_{0,m})^2 \right) (\sigma_{\alpha,i,m}^k)^2 + (\sigma_{d,i}^k)^2 + (\sigma_{d,m}^k)^2,
\]

where \( \sigma_d \) is the positional error of the track segment (equation 4-22). The total error is dominated by the \( \sigma_{\alpha} \) term.

The algorithm is implemented in the \('geofit'\) program using the C++ programming language. Details on the algorithm can be found in reference [56].

### 5.6.3 Monte Carlo results

The algorithm as described in the previous paragraph is tested on Monte Carlo (MC) events generated by the simulation described in paragraph 5.3. First the MDTs are calibrated with the \('mutdat'\) program (see paragraph 5.4) using the MC events and the ideal geometry. A distorted geometry is then simulated by moving and rotating the cross-plates (with a fixed amount), and displacing all hits accordingly, before doing the track reconstruction. The thus reconstructed 50,000 global tracks are then used by the \('geofit'\) program to reconstruct the (distorted) geometry. The geometrical parameters reconstructed by the program are then compared to the parameters used to distort the geometry.

Table 5-4 lists the input distortion parameters, the reconstructed parameters, their difference and the pull. In this table, the \( \Delta z \) and \( \Delta y \) parameters have been converted into parameters in the sagitta direction (i.e., perpendicular to the tracks; subscript \( s \)), which is the relevant direction, and the direction parallel to the tracks (subscript \( p \)), which is the non relevant direction. This is done because
the $\Delta \gamma$ and $\Delta z$ parameters (of the same cross-plate) are 99% correlated due to the fact that the tracks have an average angle of about $-117$ degrees and a small spread (3 degrees). The correlation between the converted parameters is less than 25%. The cross-plate angles are reconstructed within $2 - 20 \mu$rad. The cross-plate positions are reconstructed with high accuracy in the (relevant) sagitta direction (within $8 - 65 \mu$m) and with low accuracy in the (non relevant) parallel direction (within $20 - 675 \mu$m). The average of the pull is $-0.5$, and its r.m.s. is 1.5, which is reasonable.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Input value</th>
<th>Reconstructed value</th>
<th>Difference</th>
<th>Pull</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \alpha_i^-$ (mrad)</td>
<td>1.000</td>
<td>$1.003\pm0.011$</td>
<td>0.003</td>
<td>0.30</td>
</tr>
<tr>
<td>$\Delta \alpha_i^+$ (mrad)</td>
<td>-1.000</td>
<td>$-1.008\pm0.011$</td>
<td>-0.008</td>
<td>-0.72</td>
</tr>
<tr>
<td>$\Delta \alpha_m^-$ (mrad)</td>
<td>-2.000</td>
<td>$-1.999\pm0.019$</td>
<td>0.001</td>
<td>0.04</td>
</tr>
<tr>
<td>$\Delta \alpha_m^+$ (mrad)</td>
<td>2.000</td>
<td>$2.017\pm0.019$</td>
<td>0.017</td>
<td>0.90</td>
</tr>
<tr>
<td>$\Delta \alpha_o^-$ (mrad)</td>
<td>3.000</td>
<td>$2.983\pm0.016$</td>
<td>-0.017</td>
<td>-1.10</td>
</tr>
<tr>
<td>$\Delta \alpha_o^0$ (mrad)</td>
<td>1.000</td>
<td>$1.009\pm0.008$</td>
<td>0.009</td>
<td>1.09</td>
</tr>
<tr>
<td>$\Delta \alpha_o^+$ (mrad)</td>
<td>-1.000</td>
<td>$-0.998\pm0.016$</td>
<td>0.002</td>
<td>0.12</td>
</tr>
<tr>
<td>$\Delta s_m$ (mm)</td>
<td>-2.236</td>
<td>$-2.224\pm0.019$</td>
<td>0.012</td>
<td>0.64</td>
</tr>
<tr>
<td>$\Delta s_m^0$ (mm)</td>
<td>-0.891</td>
<td>$-0.899\pm0.010$</td>
<td>-0.008</td>
<td>-0.84</td>
</tr>
<tr>
<td>$\Delta s_m^+$ (mm)</td>
<td>-2.220</td>
<td>$-2.238\pm0.019$</td>
<td>-0.019</td>
<td>-1.01</td>
</tr>
<tr>
<td>$\Delta s_o^-$ (mm)</td>
<td>1.345</td>
<td>$1.407\pm0.022$</td>
<td>0.062</td>
<td>2.81</td>
</tr>
<tr>
<td>$\Delta s_o^0$ (mm)</td>
<td>1.766</td>
<td>$1.719\pm0.014$</td>
<td>-0.046</td>
<td>-3.31</td>
</tr>
<tr>
<td>$\Delta s_o^+$ (mm)</td>
<td>1.328</td>
<td>$1.263\pm0.022$</td>
<td>-0.065</td>
<td>-2.96</td>
</tr>
<tr>
<td>$\Delta p_m^-$ (mm)</td>
<td>-0.016</td>
<td>$0.003\pm0.264$</td>
<td>0.020</td>
<td>0.08</td>
</tr>
<tr>
<td>$\Delta p_m^0$ (mm)</td>
<td>-0.454</td>
<td>$-0.679\pm0.141$</td>
<td>-0.225</td>
<td>-1.60</td>
</tr>
<tr>
<td>$\Delta p_m^+$ (mm)</td>
<td>-2.253</td>
<td>$-2.288\pm0.267$</td>
<td>-0.036</td>
<td>-0.13</td>
</tr>
<tr>
<td>$\Delta p_o^-$ (mm)</td>
<td>-0.437</td>
<td>$-0.708\pm0.315$</td>
<td>-0.270</td>
<td>-0.86</td>
</tr>
<tr>
<td>$\Delta p_o^0$ (mm)</td>
<td>3.144</td>
<td>$2.856\pm0.198$</td>
<td>-0.288</td>
<td>-1.45</td>
</tr>
<tr>
<td>$\Delta p_o^+$ (mm)</td>
<td>1.799</td>
<td>$1.125\pm0.318$</td>
<td>-0.674</td>
<td>-2.12</td>
</tr>
</tbody>
</table>

### 5.6.4 Real data results

Nine data runs were taken in December 1997, where the BML chamber is moved in $z$ or $y$, or rotated around the $y$-axis in between runs. The 300,000 events of each data run were collected in about 17 hours of running. After processing with 'mutdat', about 60,000 reconstructed global tracks are available for geometry reconstruction with 'geofit'. The errors on the reconstructed cross-plate angles are $10 - 20 \mu$rad. The errors on the reconstructed cross-plate positions are $10 - 30 \mu$m in the
sagitta distribution of all individual runs before geometry corrections.

Figure 5-11 Sagitta distribution of all individual runs before geometry corrections.

![Sagitta distribution of all individual runs before geometry corrections.](image)

combined sagitta distribution of all runs after geometry corrections.

Figure 5-12 Combined sagitta distribution of all runs after geometry corrections.

![Combined sagitta distribution of all runs after geometry corrections.](image)

The most relevant check on the consistency of the geometry fit is provided by the sagittas. The average value should be zero within the error when the corrected geometry is used as input for the track reconstruction. The cross-plate parameters found in the geometry fit are used as input parameters for the track reconstruction program, which corrects the position of each hit before the track fit is performed. Figure 5-11 shows the sagitta distributions of all runs, without the corrections from the geometry fit. For clarity, each distribution is shown separately in the same figure, instead of added. The shifts of the distributions due to the BML displacements are clearly visible, despite the overlap due to the large multiple scattering. Figure 5-12 shows the sagitta distribution of all runs added after the corrections from the geometry fit are applied to each run. The average of this distribution is zero within the error. Figure 5-13 shows the distributions of the angle differences between the pairs of track segments after corrections for the geometry. All runs are added in the same histogram. They are all consistent with zero.

Both the Monte Carlo results and the real data results provide a consistency check for the geometry reconstruction. Both indicate that the geometry is correctly reconstructed.
Figure 5-13 Angle differences between the track segment pairs after corrections from the geometry fit. Nine runs are added.
5.7 Alignment system sagitta compared to muon track sagitta

In DATCHA, the track sagitta is the most relevant parameter related to the muon momentum measurement in ATLAS. The relevant test on the precision of the alignment system is therefore expressed in track sagittas. For several controlled displacements of the BML chamber, a data run is taken and the average sagitta as predicted from the alignment systems is compared to the average sagitta of the tracks. The latter sagitta is calculated as explained in paragraph 5.5. The alignment sagitta is determined by calculating the projections into the sagitta direction of the displacements as measured by the Rasnik systems.

For this test, global tracks are selected with an r.m.s. of the track segment angles below 2.5 mrad. This cut is based on the distribution of the r.m.s. (figure 5-8). The average muon track sagitta and its error are taken from the mean of a Gaussian fit to the central $\pm \sigma$ of the track sagitta distribution. With about 60,000 entries (total) and a width of about 1.0 mm, the statistical error on the average track sagitta is about $5 \mu m$.

The contributions of the in-plane systems to the average sagittas are the same for all runs with a run-to-run r.m.s. of $1 \, \mu m$, and are therefore ignored in the current analysis. The contributions from the projective systems are calculated per data run in two steps. First, the Rasnik readings are converted into the displacements of the Rasnik lenses (i.e. the four corners of the BML chamber) in terms of the Rasnik coordinate system $(x_{Ras}, y_{Ras}, z_{Ras})$. In this step only the Rasnik measurements and the Rasnik geometry are used. In the second step the lens displacements are converted into the global coordinate system via the rotation matrix $R_{mask}$ of the Rasnik masks (since the masks define the Rasnik coordinate system), and then into the sagitta direction via the averages of the cosines and sines of the muon track angles $\theta$. The second step only depends on the track angles and the Rasnik geometry, and is summarised in three linear transformation coefficients $(c_{xs}, c_{ys}, c_{zs})$. The sagitta of one Rasnik system is then:

$$s_{Ras} = \begin{pmatrix} c_{xs} & c_{ys} & c_{zs} \end{pmatrix} \begin{pmatrix} x_{Ras}^{\text{lens}} \\ y_{Ras}^{\text{lens}} \\ z_{Ras}^{\text{lens}} \end{pmatrix} = \left( \begin{array}{c} c_{xs} \\ c_{ys} \\ c_{zs} \end{array} \right) \begin{pmatrix} A_{Ras} \\ A_{Ras} + 1 \\ A_{Ras} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

with

$$\left( \begin{array}{c} c_{xs} \\ c_{ys} \\ c_{zs} \end{array} \right) = \begin{pmatrix} 0 & -\cos(\theta) \\ \sin(\theta) \end{pmatrix} R_{mask}$$

where $x_{Ras}$, $y_{Ras}$ and $A_{Ras}$ are the measured Rasnik $x$, $y$, and magnification respectively. The distance from the lens to the sensor $b_0$ and the distance from the lens to the mask $v_0$ are both for nominal positions of the chambers. Table 5-5 lists the values of the linear transformation coefficients $c_{xs}$, $c_{ys}$ and $c_{zs}$ for the four projective systems. The systems are indicated by the side of the chamber (High-Voltage or Read-Out) and by an index number ($1 = \text{closest to } z = 0$). The Rasnik $x$-coordinate points almost in the sagitta direction and is by far the most relevant.

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The Rasnik y-coordinate points almost parallel to the MDT wires, and still has an 11% contribution in the sagitta direction. However, the y contributions on the HV and RO cross-plates cancel each other, in case of a full chamber displacement along the wires, because all Rasnik y-axes point inward (toward the middle cross-plate). The contribution of the displacement of the lens along the Rasnik optical axis ($c_z$) is non-zero because the optical axis is not parallel to the average track angle.

The Rasnik sagitta that is used in the comparison is the average of the $s_{Ras}$ of the four projective systems. The r.m.s. of the Rasnik sagitta is 20 - 30 μm within one run. This is an estimate for the statistical uncertainty of a single set of Rasnik measurements, and is within the target precision of 30 μm. However, most of the r.m.s. on the sagitta comes from the r.m.s. of the magnification. If it is excluded from the sagitta calculation (i.e. $c_z = 0$), the r.m.s. of the Rasnik sagitta goes down by an order of magnitude to 1.5 - 2.5 μm!

Within one run, the r.m.s. of the Rasnik magnification is $2 \times 10^{-4}$ for the projective systems at the HV side and around $8 \times 10^{-4}$ for the RO side. This difference is due to the extra air turbulence that is generated by the heat of the read-out electronics. This effect is expected to be much lower in ATLAS, since the power consumption (and therefore the heat production) of the final read-out electronics will be an order of magnitude smaller. Even without the extra heat production, the r.m.s. of the Rasnik magnification is still an order of magnitude larger than the precision obtained for a short range (225 mm) system (see figure 2-16 in paragraph 2.4.4). This is most probably related to the air turbulence of the long range (7 m) projective systems. An r.m.s. of $2 \times 10^{-4}$ on the magnification corresponds to an r.m.s. of about 0.8 mm on the displacement of the lens along the optical axis. The set-up is certainly much more stable than that, so it is better not to use the Rasnik magnification to measure the lens z-displacement. The magnification, however, is still very useful to provide the scaling factor for the Rasnik x and y displacements (see equation 5-14).

Figure 5-14 shows the average sagittas measured by Rasnik as a function of the average sagittas measured by the muon tracks for the all BML displacements. The Rasnik sagittas have been calculated without the magnification ($c_z = 0$). With 40 - 75 Rasnik measurement sets taken per run, the statistical error on the Rasnik sagitta is 0.2 - 0.3 μm. The residuals are shown relative to a straight line fit, which accounts for the uncertainty of the Rasnik magnifications (the sensor pixel size was not precisely known), and for the uncertainty in the rotation of the projective masks around the optical axis. The slope therefore does not need to be exactly 1. The errors on the residuals are the quadratic sums of the errors on the track sagittas and Rasnik sagittas. There is an arbitrary offset between the Rasnik sagittas and the track sagittas because the Rasniks have not been calibrated. The r.m.s. of the residuals is 11 μm, which is comfortably within the target precision of 30 μm.

The DATCHA experiment has demonstrated that the alignment systems can track changes in the MDT chamber geometry within the required precision. However, since the alignment systems were not calibrated, is not yet demonstrated that they can provide the absolute correction to the track sagitta, which is what is needed in ATLAS.
Figure 5-14 Average sagitta predicted from the Rasnik projective systems (a) and the straight line fit residuals (b) as a function of the average sagitta of the muon tracks.