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On the Origin of Samuelson's Multiplier-Accelerator Model

Arnold Heertje and Peter Heemeijer

It is not always easy to pinpoint the intellectual provenance of a scientific theory or model. Many ideas of colleagues and predecessors influence a particular scholar who is on the brink of developing a new approach. Such influences are not only unavoidable but also necessary for the inventor to collect the knowledge and inspiration allowing him to formulate ideas of his own; ascribing a theory to a single person would in most cases be a simplification. As compared to the exact sciences, in economics this process is even more complicated, as observations are often ill structured and subjective, especially when the field of research does not lend itself to a mathematically inspired analysis. We may quote Paul Samuelson (1959, 183) in this respect: "Scientific theories are like children in that they have a life of their own. But, unlike children, they may have more than one father."

In this article we intend to establish who exactly were the "fathers" of an important and well-established model written by the same Paul Samuelson. We will investigate its intellectual genesis and see to what extent the historical records conform to common belief. As we will show, there is little factual support for Samuelson's suggestion ascribing the model mainly to Alvin Hansen, his mentor in the days of the creation of the model. Instead, the evidence indicates that Roy Harrod played

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the major role in the development of research leading to the multiplier-accelerator model. In the end, though, we will not propose to affix Harrod's name to the model, concluding that there appears to be no reason to change the custom of associating only Samuelson's name with it.

1. Samuelson's Model

When it comes to scientific research in the area of business-cycle movements, the thirties were a very productive period. At the beginning of the decade, the Great Depression shook the very foundations of the modern world, causing great damage to virtually all advanced countries and laying a responsibility on the international economics community to explain these frightening recurring downward movements in the level of production and to possibly find a cure for them. Several renowned economists went to work and constructed theories of every shape and form, ranging from the monetary overinvestment theory by Friedrich von Hayek and the theory of mass psychology by A. C. Pigou to the theory of innovative entrepreneurship by Joseph Schumpeter.

This development culminated in the publication of two books by the League of Nations that presented a theoretical survey of all advancements up to then in the explanation of the driving forces behind the business cycle. The first, written by Gottfried Haberler (1937), had a thorough and verbal character. The second, which complemented Haberler's work with the necessary statistical testing, was provided by the Dutch pioneer of econometrics, Jan Tinbergen (1939).

Although great progress had been made up to that point, none of the existing business-cycle theories was yet able to explain, in a brief and intelligible way, the movement from boom to depression and back in a fully endogenous manner.¹ They all used external shocks or at least stimulating and depressing movements in variables that were not explained by the respective models themselves. This theoretically handicapped situation ended when early in 1939 a new model sequence was developed by Paul Samuelson (1939a) that was able to produce cyclical movements using only an exogenous flow of constant governmental

1. There was at the time some mathematical literature on endogenous theories by Tinbergen, Ragnar Frisch, and others, but owing to its complicated and highly abstract nature, most economists were not acquainted with its content. One article by E. A. Radice (1939) is worth pointing out here, since it seems to have a close, although not exact, bearing on the model we are discussing.

expenditure. Still in his early twenties and studying as a research fellow at Harvard University, Samuelson revolutionized research into business cycles and raised our understanding of changes in the scale of economic activity to a new level.

And along with it rose the star of the student Paul Samuelson himself. Mathematically able as he was, writing the short 1939 article containing the relatively simple model (which will be described below) was a task of little effort. "I wrote it all up in a day and it brought me exaggerated fame," he said in a recent letter to the authors. This fame, exaggerated as it may have been by the excitement generated by any scientific breakthrough, firmly attached Samuelson's name to the multiplier-accelerator model, disregarding the possibility that others had made any significant contributions.

The main reason for the eventual success of the model was its great accessibility. Being a straight "dynamization" of the Keynesian model in its most elementary form, the variables and equations were easily understood by contemporary economists and so were the implications following from them. Samuelson's model is written as follows:

$$Y_t = C_t + I_t + G_t. \quad (1)$$

$$C_t = \alpha Y_{t-1}. \quad (2)$$

$$I_t = \beta(C_t - C_{t-1}). \quad (3)$$

$$G_t = G_0. \quad (4)$$

These equations contain two additions to the Keynesian ones. First, the time indices indicate that the model has been dynamized. Second, the level of investment is no longer exogenous but is induced by the marginal rate of consumption. This interaction is referred to as the "accelerator" or "relation."² The whole system of equations was given the name "multiplier-accelerator model," for obvious reasons. The system forms a differential model of the second order with constant coefficients and produces cyclical movements in national income for some values of α and β . The behavior of the model has been described to a great extent (see, for example, Hicks 1950), and, being primarily interested in its historical origin, we will not pursue the analytical details here further.

2. The accelerator was originally developed by Albert Aftalion (1913, 272–73), but in somewhat vague terms. Roy Harrod (1936, 58) redefined it as the "relation" in *The Trade Cycle*.

2. Harrod's Contributions

And it seems that this historical origin is a book written a few years earlier by Sir Roy Forbes Harrod, an economist working on the other side of the ocean, whose field of research covered business-cycle theory. Simply called *The Trade Cycle*, his book proved to be an interesting contribution to the theory when published in 1936.

The most important part of this contribution consisted of the idea of combining the effects of the multiplier and the relation, as Harrod called the accelerator. Both concepts occupy a prominent place in his analysis. Already in the preface, he appoints them as two of his three main inspirations for the theory he wishes to develop (the third being recent additions to the theory of imperfect competition). Explicitly and on several occasions he points out the vital importance that the connection between the multiplier and the relation assumes in his business-cycle theory. This is most evidently formulated in his important chapter 2: "It is the contention of this essay that by a study of the interconnections between the Multiplier and Relation the secret of the trade cycle may be revealed" (Harrod 1936, 70). And even more strongly: "The essence of the theory enunciated is that the cycle results from the joint operation of the Relation and the Multiplier" (102).

Since it is the interaction between the accelerator and the multiplier that is being described by Samuelson's model sequence, it is important to examine whether Harrod's "joint operation" is based on just good scientific intuition or on the knowledge of an explicit mechanism of interaction. The following quotation may provide an answer to this question. Harrod is discussing a decrease in net investment:

This, in accordance with the Multiplier, entails a further slowing down in the rate of increase of consumption. This, in accordance with the Relation, entails an absolute fall in net investment. This, in accordance with the Multiplier, entails an absolute fall in income and consumption. This, in accordance with the Relation, entails that net investment is rapidly reduced to a very low level, if not to zero. (98)

Again, we can be positive about Harrod's analysis. The above quotation suggests that, as far as a verbal analysis can go, the writer did have a definite understanding of the interaction of the two variables.

What then stops us now from concluding outright that it was Harrod who laid the verbal basis for Samuelson's model? It is not in his understanding of the interaction between the multiplier and accelerator that we can find shortcomings, but in the fact that Harrod himself did not see the explanatory possibilities of this interaction. The problem will be evident to any reader of his work: the movement of production between the turning points is made perfectly clear, but when it comes to explaining the turning points themselves, the analysis seems to drown in a massive swamp of exogenous variables. Three "static determinants," four "dynamic determinants": Harrod makes an impressive and, in its own right, quite successful attempt to put them all together in a correct and intelligible way, but in doing so he himself takes away the central importance of the multiplier and accelerator and he himself prevents his theory from being an endogenous and thus for the time a revolutionary one.

3. Hansen's Contributions

As has been said above, at the time of the publication of his original article in 1939, Samuelson was only in his early twenties. He was therefore supervised by the more senior Alvin Hansen, an established Keynesian economist. When we take a closer look at the article, it becomes clear that Hansen's contribution to the ideas presented in it must have been great. Samuelson refers to the model as a whole as the "Hansen analysis" and to the vital second and third equation of it as the "Hansen assumptions." Moreover, when he discusses the importance of adding the accelerator to the model, Samuelson states that "Professor Hansen has developed a new model sequence which ingeniously combines the *multiplier* analysis with that of the acceleration principle or *relation*" (75).

Thus it seems that Samuelson's mentor provided important ideas underlying the analysis in the original article. To determine exactly, though, how great Hansen's contributions to the multiplier-accelerator model have been, particularly compared to those of Harrod, it is necessary to investigate his own work on the subject.

Searching in that manner for thoughts regarding the combination of multiplier and accelerator, Hansen's voluminous pre-1939 oeuvre contains one relevant entry. It is his 1938 book *Full Recovery or Stagnation?* the last one published before Samuelson's article. Dealing directly with business-cycle theory, mainly applied to the situation in the United States, the work did indeed seem to provide the perfect opportunity for

Hansen to publish novel ideas of his connected to the combination of multiplier and accelerator. An investigation of its contents with respect to such ideas allows us to restrict our attention to two chapters: the second one, which happens to contain a review of Roy Harrod's *Trade Cycle*,³ and the eighteenth one, containing some thoughts about the possible implications of combining the two concepts.

Let us start with the eighteenth chapter. Contrary to what might be expected, given the sketch of Hansen's role thus far, the writer does not dedicate a large part of it to the possibility of combining the multiplier and accelerator. Instead, his comments on this subject do not fill even two pages⁴ and appear in the general analysis as only casual thoughts. Hansen brings in the multiplier and accelerator to describe the process of expansion of production, which can indeed be seen as similar to Samuelson's model. But he does not continue to describe the decline of production, nor does he indicate how the turning points must be explained. Instead, his conclusion—"All that can be said is that the expansionist process, once started, will continue up to a certain point and thereafter decline" (295)—actually partially contradicts Samuelson's model, since this statement is only true of specific values of the coefficients α and β .

Let us continue with the review in the second chapter. Any doubts about Hansen's contributions so far may prove unfounded if in his review he were to identify and criticize the main defects in Harrod's analysis, which were treated in the last section.

A close inspection, though, reveals nothing of the kind. Hansen does have a very skeptical, even cynical, approach to Harrod's work. He describes reading it as "sometimes forbidding, if indeed not at times terrifying" (Hansen 1938, 35) and appears to disagree with almost every aspect of the analysis. He criticizes Harrod for ambiguities in the use of the exogenous "determinants" (38–39), and he points out that Harrod's statement about the recession always leading to a complete collapse of productivity, or the "bottom," as Harrod calls it, is false (52–58). These comments are fully justified. But on the other hand he blames Harrod for paying too much attention to the relation (50) and even for endeavoring to create an endogenous business-cycle model (37). And above all, he nowhere in the review writes anything to accentuate the fundamental

3. In fact, a reproduction of a review first published in the *Quarterly Journal of Economics* (Hansen 1937).

4. Pages 295 and 296.

importance of the combination of the multiplier and relation. Instead he appears even to trivialize this importance (39, 40).

What then stops us now from concluding outright that Hansen cannot have been responsible for supplying Samuelson with the fundamental ideas underlying the multiplier-accelerator model, contrary to the quotations from Samuelson's 1939 article? Hansen's book is not one in which the author shows himself to be in possession of the necessary knowledge; rather, the book establishes only the fact that as a mentor Hansen could have somehow informally inspired Samuelson to perform his 1939 analysis. This may sound rather speculative, but in the following sections it will be shown to be quite relevant.

4. Samuelson's Interpretation of Events

Now that some light has been shed on the contributions by the two most likely intellectual predecessors of the multiplier-accelerator model, we can proceed by describing and evaluating Samuelson's interpretation of the events leading to his original discovery. The success of the 1939 article firmly attached Samuelson's name to the model to the exclusion of Harrod, Hansen, and others who might have lent their ideas to the analysis. The author himself has made it quite clear on several occasions, though, that this common belief does not do justice to the man whom he sees as primarily responsible for the creation of the model: his mentor Alvin Hansen.

As has been said above, already in the original article Samuelson pointed toward Hansen as the one who first came up with a rough version of the model sequence. Some twenty years later he published a short article (Samuelson 1959) in which he once more stressed "the all-important role in the formulation of the model that was played back in 1938 by Alvin Hansen" (183). Clarifying this statement, he tells us that it was Hansen who "recognized how 'undynamic' was the bulk of the *General Theory*, particularly from the standpoint of economists who had long been working with the acceleration principle" and that it was he who "worked out a determinate numerical example of developing national income" (183). Finally, in recent correspondence with the authors, Samuelson maintains very definitely his original intention of reattaching Hansen's name to the model. In particular, he insists that Hansen demonstrated to his students the model sequence for specific values of the coefficients: "To prove his point, he generated a numerical example." When

asked about the time and place of this example, he added: "Perhaps it was in his workshop that he produced this sequence."

Samuelson's opinion about Hansen's contributions thus is very straightforward. His opinion about Harrod's contributions, though, is less so. In the original article there is no mention of Harrod whatsoever. This in itself may indicate that Samuelson was unfamiliar at the time with Harrod's work, although that would be improbable given the fact that Harrod's contributions were discussed in his second 1939 article, which elaborated on the combination of multiplier and accelerator. In it, Harrod is, along with Hansen, acknowledged as among the founders of the idea of combining the multiplier and accelerator. Also, Samuelson criticizes Harrod for the unnecessary complication caused by the static and dynamic determinants, much as we did in the second section of this paper. Without naming Harrod as his direct theoretical predecessor, his overall valuation of his work in this article is, however, favorable: "On the whole, Harrod's intuition surpasses his reasoned conclusions—of what investigator worth his salt is this not true?"

In his 1959 article, Samuelson seems to have distributed the credits in another way. Hansen is described as having played an "all-important role," as cited before, while Harrod is only remembered indirectly for his effort "to bring the accelerator into the modern income analysis" (183). For this reason, according to Samuelson, "[Hansen] reviewed favorably Harrod's 1936 *Trade Cycle*." (It has been shown in the third section, however, that Hansen [1938] actually went to some length to ensure the reader of his distaste for Harrod's analysis.) In his recent correspondence with the authors, Samuelson upholds this opinion: he values Harrod's contributions to the research on the accelerator but does not speak of his ideas on the combination with the multiplier. Also, regarding Hansen's opinion of Harrod's research, he maintains that "[Hansen] reviewed it . . . , and with guarded admiration."

5. Contradictions

When comparing the last section with the identification of the theoretical contributions of Harrod and Hansen, as contained respectively in sections 2 and 3, it appears that Samuelson's opinion about the origin of the multiplier-accelerator model is not wholly in agreement with the written facts. More strongly even, the idea that Hansen was the greatest contributor and Harrod was only of secondary importance—an idea

that Samuelson has upheld through the years—seems to contradict the conclusion we have reached, assigning to Harrod a primary role of importance and to Hansen a secondary one at best.

Fortunately, there is a way out of this puzzling contradiction. Hansen may not have written anything important about the combination of multiplier and accelerator before the publication of Samuelson's original article, but he did do so shortly afterward. In a book from 1941 he devoted a whole chapter⁵ to the multiplier, the accelerator, and the combination of the two. In it, he discusses these subjects as elements of the model sequence designed by Samuelson only two years before. Hansen explains the working of the model and gives several specific examples, both verbally and numerically. What is important here is not to show that Hansen did after all provide Samuelson with a specific form of the multiplier-accelerator model. This remains implausible, because all determinate numerical examples in Hansen's chapter are identical to the examples from Samuelson's 1939 article.⁶ Rather, the fact that Hansen's 1941 book must have been written shortly after the publication of Samuelson's article suggests that he was indeed closely involved with his pupil's research, which supports the hypothesis that Hansen may informally have been of great help to Samuelson.

This line of reasoning may seem slightly far-fetched, but becomes credible when reviewing Samuelson's recollection of the events leading to his 1939 article. Suppose that Hansen did indeed come up with a "determinate numerical example" and that he did so "in his workshop." It is then quite possible that this example was actually the result of an interaction between Hansen and several of his students, among whom was Samuelson. Since this hypothesis does not require Hansen to have had any knowledge of the model before he entered the supposed workshop, but does explain Samuelson's gratitude toward his teacher who was the catalyst for ideas he very probably came up with himself, it takes away the most important contradiction we have been concerned with. Hansen's exact influence in this workshop, though, is impossible to determine, given the informal nature of the meeting, so we must still disagree with Samuelson when it comes to appointing Hansen as the most important intellectual father of the multiplier-accelerator model. The fact

5. Chapter 12.

6. In his 1951 book, Hansen refers to the chapter as "based on the brilliant article by Paul Samuelson" (175 n. 1), indicating clearly the modesty of his own contributions.

that Hansen did not change or expand Samuelson's version of the model is further evidence in this respect.

Having found a likely explanation of Samuelson's opinion of Hansen's role, it is not so difficult to explain his opinion of Harrod's. If Samuelson was personally inspired by Hansen to build his revolutionary model, which brought him worldwide fame, then it is not strange that he would wish for the credit to go largely to Hansen, even if the real magnitude of his help had been small. In contrast, Harrod must have been a more distant figure, working across the ocean at a time when communications were less developed than they are now. It is therefore understandable that Samuelson never fully accentuated the extent to which Harrod paved the way toward the discovery of the multiplier-accelerator model. Based on our analysis, such an accentuation is justified, though, forcing us to insist that Harrod's theoretical contributions were greater than Samuelson's opinion may have suggested.

6. Conclusion

To clarify the historical and intellectual origins of the multiplier-accelerator model, we have stated the contributions to the ideas underlying it of the two economists most directly tied to the subject.

As we have shown, Harrod's theoretical contributions to the model were quite significant. His 1936 book *The Trade Cycle* is the first known attempt at a business-cycle theory to be clearly and explicitly based on the combination of multiplier and accelerator. He may therefore be seen as an important contributor to Samuelson's analysis, which followed three years later, although he cannot claim full credit for the multiplier-accelerator model, since his theories were too clouded by what are essentially redundant exogenous variables.

Hansen's theoretical contributions to the model were less significant. His 1938 book *Full Recovery or Stagnation?* the one work relevant to Samuelson's model, contains only some casual thoughts about combining the multiplier and accelerator, thoughts that are quite superficial and sometimes even in disagreement with the conclusions derived from Samuelson's model sequence. Furthermore, Hansen's review of *The Trade Cycle*, contained in the same work, effectively disproves that he understood the fundamental importance such a combination might have had in business-cycle theory.

Second, we have evaluated Samuelson's opinion about the intellectual origin of the multiplier-accelerator model and, based on that opinion, his proposed change (i.e., attaching Hansen's name to the model) in what is now commonly believed about the origin.

Hansen's role seems to have been overestimated by Samuelson. No written record published before Samuelson's original article shows that Hansen was able to provide his students with ideas leading directly to the relevant combination of multiplier and accelerator. This observation apparently contradicts Samuelson's own opinion. The contradiction was resolved, however, by supposing that Hansen supervised a workshop with a number of young researchers (among whom was Samuelson) that resulted in the discovery of a numerical example of the multiplier-accelerator model, but that the real influence exerted by Hansen himself in the workshop must have been minimal. This hypothesis seems to be in accordance with Hansen's 1941 and 1951 works. Since it is impossible, though, to determine the real influence Hansen had on the results of this informal meeting (assuming it did take place), it cannot serve as a basis for changing the idea that there is insufficient evidence supporting the acknowledgment of Hansen as one of the founders of the multiplier-accelerator model.

Harrod's role seems to have been underestimated by Samuelson. In his 1959 article and in his recent correspondence with the authors, he credits Harrod only for his work on the accelerator, while it has been shown that for his 1936 contributions, Harrod should have been recognized as an important secondary founder of the multiplier-accelerator model.

In what way now should the credit for the discovery of the multiplier-accelerator model finally be distributed? Theoretically speaking, it was Harrod who provided the most important contribution to the model. His interesting 1936 work *The Trade Cycle* contained valuable ideas regarding the combination of multiplier and accelerator. The analysis needed too much exogeneity, however, and thus we cannot attach Harrod's name to the model. Informally speaking, it appears that Hansen provided the most important contribution to the model. It was one of his workshops, or so Samuelson has written us, in which a specific form of the multiplier-accelerator model was developed. No written record of his hand bears clear witness, though, of this discovery or of the knowledge that may have led to it, forcing us to conclude that Hansen's name should not be attached to the multiplier-accelerator model.

So it seems that all that can be said about the origin of the multiplier-accelerator model is that the profession after all is probably right in attributing it to Samuelson alone. This may indeed be justified, but the analysis that has led us to this conclusion is far from trivial. We can only but agree with G. L. S. Shackle (1967, 270) that it was Samuelson who “brought rigour and system into the field of Multiplier-Accelerator interaction.”

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