Random walks in stochastic surroundings
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Citation for published version (APA):

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Summary

In this thesis, random walks in different types of stochastic surroundings are studied. Chapter 1 surveys the results of the thesis and gives some background on the topic. In Chapters 2–4, edge-reinforced random walks are considered. The process is a nearest-neighbor random walk on a locally finite graph. All edges are given strictly positive numbers as weights. In each step, the random walker traverses an edge incident to her current location with probability proportional to its weight. Each time an edge is traversed, its weight is increased by 1. Chapter 2 contains limit theorems for the normalized occupation time $\alpha_n$ spent on the edges of a finite graph and for the joint distribution of $\alpha_n$ and the normalized cycle numbers. In both cases, the limiting distributions are given explicitly. This improves a result stated by Coppersmith and Diaconis.

As a corollary it is proved in Chapter 3 that edge-reinforced random walk on a finite graph has the same distribution as a random walk in random environment where the environment is given by random weights on the edges distributed according to the limiting distribution of $\alpha_n$. Furthermore, Chapter 3 contains a characterization of edge-reinforced random walk. Let $Z$ be a nearest-neighbor random walk on a 2-edge-connected graph. Suppose $Z$ is partially exchangeable in the sense that the probability of a finite path depends only on the starting point and the number of transition counts for all non-directed edges. Furthermore, assume that the conditional probabilities to traverse edge $e$ in the next step depend only on the current location $v$, the edge $e$, the local time accumulated at the vertex $v$, and the number of times $e$ has been traversed in the past. If in addition some natural technical conditions hold, then $Z$ is either an edge-reinforced random walk or a non-reinforced random walk. In case the graph $G$ is not 2-edge-connected, $G$ decomposes into 2-edge-connected components such that any process $Z$ with the above properties is essentially an edge-reinforced or a non-reinforced random walk on any 2-edge-connected component of $G$.

In Chapter 4, directed-edge-reinforced random walk is studied. The process is defined on a directed locally finite graph. All directed edges are given weights. In each step, the process traverses a directed edge pointing from her current location to a nearest-neighbor vertex. Each time a directed edge is traversed, its weight is increased by 1. It is shown that on any graph the process has the same distribution as a random walk in random environment where the environment is given by independent Dirichlet distributed transition probabilities at the vertices. A characterization of recurrence and transience for a random walk in random environment on $\mathbb{Z} \times G$ for any finite graph $G$ with all edges directed is obtained. Whether recurrence or transience occurs depends on the sign of two Lyapunov exponents of certain i.i.d. random matrices. It is shown that a symmetry of the distribution of the environment is sufficient for recurrence. In particular, recurrence for directed-edge-reinforced random walk on $\mathbb{Z} \times G$ with all initial weights equal follows.
Chapters 5 and 6 contain work on random walks in a random scenery. The stochastic surroundings do not influence the transition probabilities, but the walker observes the scenery. A scenery is an i.i.d. coloring $\xi := (\xi(z); z \in \mathbb{Z})$ of the integers with finitely many colors. Let $S := (S_k; k \in \mathbb{N}_0)$ be a recurrent random walk on $\mathbb{Z}$, independent of $\xi$. The scenery observed along the random walk path is the process $\chi := (\xi(S_k); k \in \mathbb{N}_0)$. Suppose the random walk has i.i.d. finitely supported increments. Assume that maximal jump lengths to the left and to the right agree and the random walk can reach every integer with positive probability. Assume furthermore that there are strictly more colors than single steps for the random walk. In Chapter 6, it is shown that a finite piece of scenery of length $l$ around the origin can be reconstructed up to reflection and a small translation from the first $p(l)$ observations $\chi_0, \chi_1, \ldots, \chi_{p(l)-1}$ with high probability; here $p$ is a polynomial and the probability that the reconstruction is done correctly converges to 1 as the length $l$ tends to infinity.

In Chapter 5, the scenery reconstruction problem is studied in the case there are some errors in the observations. Let $\xi$ and $S$ satisfy the same assumptions as above. The observations with errors are defined as follows: At time $k$ the random walker observes the color $\xi(S_k)$ at her present location with probability $1 - \delta$, whereas she observes an error $Y_k$ with probability $\delta$. The occurrences of the errors are i.i.d. and $Y := (Y_k; k \in \mathbb{N}_0)$ is assumed to be stationary and ergodic. It is proved that the reconstruction is possible for all $\delta$ sufficiently small. More precisely, almost all sceneries can be reconstructed from the observations with errors up to reflection and translation for almost all random walk paths.

Chapter 7 contains work on up-right paths in a Poissonian field. In this model, the stochastic surroundings are given by a Poisson process with constant intensity 1 in the plane and the walk is determined by the stochastic surroundings. Given a fixed configuration $\omega$ of the Poisson process, an up-right path is a path from $(0,0)$ to $(1,1)$ which connects piecewise linearly points in $\omega$ in such a way that the path moves only upwards and to the right. If there are precisely $n$ points in $[0,1]^2$, then these points induce a permutation $\pi$ on $\{1,2,\ldots,n\}$ (one orders the $x$- and $y$-coordinates of the $n$ points and sets $\pi(i)$ equal to the order number of the $y$-coordinate of the point with the $i$th $x$-coordinate). Conditioned on having $n$ points in $[0,1]^2$, the induced permutation $\pi$ is chosen uniformly at random from the permutation group $S_n$.

An up-right path induces an increasing subsequence in $\pi$, i.e. a sequence $1 \leq i_1 \leq i_2 \leq \ldots \leq i_k \leq n$ with $\pi(i_1) < \pi(i_2) < \ldots < \pi(i_k)$. The length of a longest increasing subsequence of $\pi$ is denoted by $L_n(\pi)$. In the terminology of up-right paths, $L_n(\pi)$ equals the maximum number of points in an up-right path. The object of interest is the asymptotic behavior of $L_n(\pi)$ as $n$ tends to infinity. Chapter 7 contains a moderate deviation principle for the lower tail probabilities of $L_n$. This concerns the regime between the lower tail large deviation regime and the central limit regime. This result together with an article of Löwe and Merkl provides a complete picture of the moderate deviation principle for $L_n$. 