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Implementation of Quantum Search Algorithm using Classical Fourier Optics

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We report on an experiment on Grover's quantum search algorithm showing that *classical waves* can search a N-item database as efficiently as quantum mechanics can. The transverse beam profile of a short laser pulse is processed iteratively as the pulse bounces back and forth between two mirrors. We directly observe the sought item being found in $\sim\sqrt{N}$ iterations, in the form of a growing intensity peak on this profile. Although the lack of quantum entanglement limits the *size* of our database, our results show that entanglement is neither necessary for the algorithm itself, nor for its efficiency.

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Quantum computers [1,2] hold the promise of performing tasks [3,4] that are either impossible or much less efficient without the use of quantum mechanics. One such task is quantum searching, introduced by Grover [4,5]. Consider using a phone book with N entries to find the name of a person whose phone number you have. Classically, this would require $\sim N$ consultations of the phone book. Grover's algorithm finds the desired entry with only $\sim\sqrt{N}$ consultations, using quantum mechanics. Here we show experimentally that *classical waves* can find a "needle in a haystack" as efficiently as quantum mechanics can. Although some previous experiments [6–10] have demonstrated various aspects of quantum searching, all of them have been limited to four entries [6–8] or a single query [8–10]. Our experiment closely follows Grover's algorithm, implementing for the first time an iterative search on a 32-item database, using classical waves. It provides a striking demonstration that the algorithm itself requires only wave properties [11] but no entanglement [12].

In Grover's (first) algorithm [4] each database item is associated with a quantum state. Initially the system is prepared in a superposition of all N quantum states. The algorithm then amplifies the probability amplitude of the state being sought, in an iterative way. The item has been found once the probability amplitude of this "target state" is near unity. Ideally this requires $(\pi/4)\sqrt{N}$ iterations of the following two steps. In the first step a so-called "oracle" marks the item by inverting the phase of the associated quantum state [5]. In the second step the amplitudes of all states are inverted about the average amplitude (IAA operation), converting phase information into amplitude information.

The above protocol maps onto our classical-wave experiment as follows (see Fig. 1). A complex electric field amplitude $E(x)$, viz. a transverse laser beam profile plays the role of the quantum probability amplitudes. The continuous coordinate x labels the items of the database, corresponding to all possible quantum states. By spatial filtering we initialize the beam profile $|E(x)|^2$ as a smooth, near-Gaussian, distribution with a 1.33 mm diameter (FWHM; full width at half maximum). A single, ~300 ps laser pulse (wavelength 532 nm) enters a standing-wave cavity of 2.02 m optical path length through input mirror M_1 (transmission 2%). The pulse travels back and forth between the cavity mirrors in 13.5 ns, each roundtrip representing one iteration of the search algorithm. Inside the cavity an "oracle plate" [5] marks the item by imprinting a phase profile on the beam, $E(x) \rightarrow E(x)\exp[i\Phi_o(x)]$, where $\Phi_o(x) = \phi$ in a narrow area around the "item position" x_o and $\Phi_o(x) = 0$ elsewhere. Next, the IAA operation is performed by the sequence $F\Phi_f FF\Phi_f F$, where F denotes a Fourier transform and Φ_f denotes a phase plate like the oracle, but now imprinting a phase profile $\Phi_f(x')$ in the Fourier plane. The Fourier transforms replace the Walsh-Hadamard transforms [13] in the original proposal [4] and are experimentally performed by

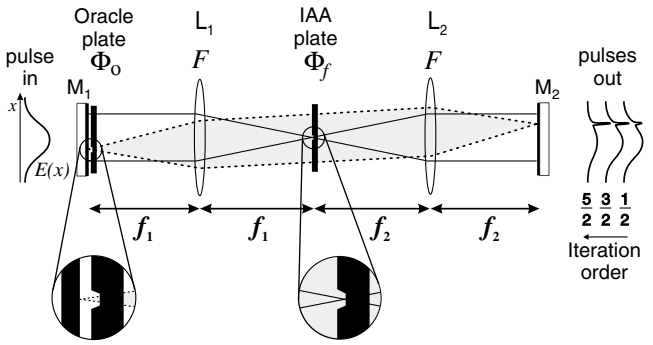

FIG. 1. Cavity implementing Grover's algorithm using optical interference. We launch a short laser pulse with a Gaussian transverse beam profile $E(x)$, x representing the data register, into the cavity formed by mirrors $M_{1,2}$. A line shaped depression in the oracle plate marks the item by imprinting a phase profile $\Phi_o(x)$. The sequence $F\Phi_f FF\Phi_f F$ performs the inversion about average (IAA) as required by Grover's algorithm. Here F denotes a Fourier transform, performed by the lenses $L_{1,2}$ (focal lengths $f_1 = 400$ mm, $f_2 = 600$ mm). The IAA plate imprints a phase profile $\Phi_f(x')$ in the Fourier plane of the oracle. The enlargements show cuts of the phase plates perpendicular to the lines. As the pulse bounces back and forth, the transverse beam profile is processed iteratively and light is concentrated into the shaded mode. A high intensity peak, growing on the beam profile in the output plane, indicates the sought item.

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spherical, achromatic doublet lenses [14]. Since F^2 is a spatial inversion and $\Phi_f(x') = \Phi_f(-x')$, the IAA operation reduces to $F^{-1}\Phi_f^2 F$. Thus the amplitude amplifying Grover iterator is $\Phi_o^2 F^{-1}\Phi_f^2 F$. Note that $F\Phi_f F$ can be recognized as phase contrast imaging.

We observe the progress of the search algorithm iteration by iteration, using the 2% transmission of mirror M_2 after each cavity roundtrip. This light is imaged onto a 55 μm wide movable slit and the transmitted light is collected on a photodiode. The photodiode signal is amplified and recorded by a digitizing oscilloscope. The light pulses are short compared to the roundtrip time, so that a train of output pulses is obtained, one pulse per iteration. In Fig. 2 we show two typical time traces. The trace in Fig. 2A has been recorded in an "empty cavity," leaving the oracle and IAA plates inside the cavity, but moving the phase-shifting lines on the plates out of the beam. We observe an exponentially decaying peak amplitude, with a roundtrip loss of about 0.25, due to reflections. Next, we move the IAA phase line into the beam focus, put the oracle line in an arbitrary position in the beam, and place the detection slit in the image of the oracle line. We then observe a peak amplitude that *grows* during the first few iterations, even though the total optical energy decreases. This is shown in Fig. 2B and is a direct observation of amplitude amplification.

We have measured the entire beam profile by recording traces as in Fig. 2B for many different detection slit positions. We combined the peak values at the same time from different traces into a transverse beam profile. A sequence of such profiles for consecutive roundtrips shows how the algorithm proceeds. In Figs. 3A–3C we show three such sequences for increasing widths of the oracle line. Consecutive profiles within a sequence have been multiplied by a factor 0.75^{-1}, in order to compensate for optical losses.

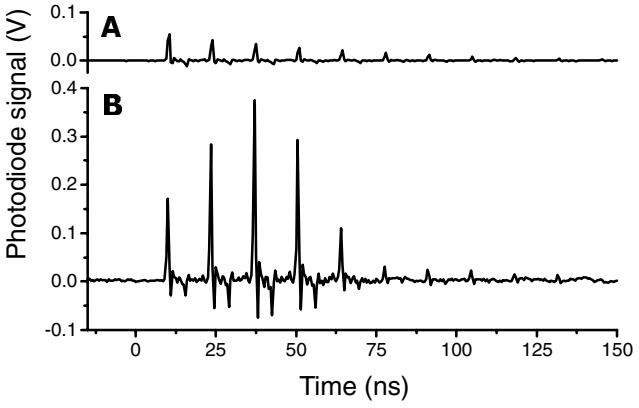

FIG. 2. Amplitude amplification as observed in trains of light pulses. Trace *A* shows the pulse train coupled out of a bare cavity, i.e., with the phase lines shifted out of the beam. The peak amplitudes decay exponentially with 25% roundtrip loss. For trace *B* the phase lines were moved into the beam and the pulses were recorded behind a narrow slit placed in the image of the oracle phase line. The energy in the pulses increases, even though the total energy decays.

We clearly observe the solution growing as a high intensity peak in the transverse beam profile. The position of this peak is the position x_o of the sought item, i.e., the phase line in the oracle, imaged by the intracavity telescope. In the quantum case it would of course be impossible to watch the solution grow as the algorithm proceeds, because a measurement would cause the wave function to collapse.

On the basis of Grover's algorithm we expect the peak height to reach a maximum after $(\pi/4)\sqrt{N/m}$ roundtrips, where m is the number of marked items [15,16], and to oscillate through a sequence of maxima and minima with a period of $(\pi/2)\sqrt{N/m}$. In an ideal, loss-free system, these cycles of finding and "unfinding" would continue indefinitely. This period assumes that the phase shifts ϕ have their ideal values. Since we use the plates in double pass inside the cavity, this ideal value is $\pm\pi/2$, whereas our measured value is $\phi = -1.1 \pm 0.2$ rad. This increases the optimum number of iterations to $[\pi/(4\sin 1.1)]\sqrt{N/m}$. Although ϕ may deviate from $\pi/2$, a "phase matching" condition [17,18] requires that the two phase shifts of the oracle and IAA plates must be approximately equal.

The ratio N/m can be interpreted as the size of the database for a single item search. Alternatively, the same N/m also describes a search for m adjacent items in a larger database of size N. The maximum database size is determined by optical diffraction, which limits the effective number of positions x that can be resolved. For our

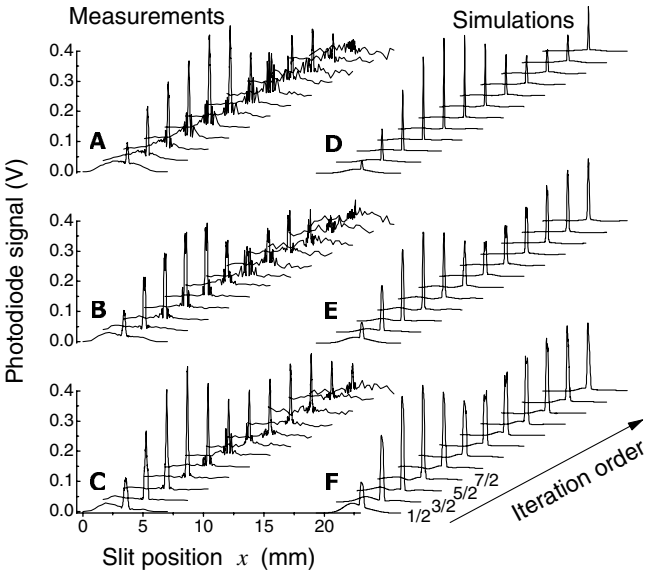

FIG. 3. Iterative progress of the search algorithm as shown by measured and simulated beam profiles. Oracle lines of widths (*A*) 42 μm, (*B*) 84 μm, and (*C*) 126 μm were used, corresponding to databases of 31.7, 15.8, and 10.6 items, respectively. Light coupled out of the cavity was recorded after each roundtrip through a scanning slit. The peak growing in the first iterations reveals the position of the sought item. The traces on the right (D–F) have been simulated using realistic experimental parameters, corresponding to those on the left.

cavity with a numerical aperture NA = 0.03, the limit on the resolution is given by Rayleigh's criterion as $0.61\lambda/\mathrm{NA} \approx 10\ \mu$m. For our 1.33 mm input beam, the maximum database size is then ~133.

We can estimate N/m as the ratio of the input beam diameter to the oracle linewidth. The phase shifting lines have been produced as the shadows of thin metal wires (50, 100, and 200 μm diameter) while evaporating a thin layer of SiO onto a BK7 substrate. A phase-contrast image revealed line cross sections that are well approximated by trapezoids, with flat inner regions of 42, 84, and 126 μm, for the oracle plate and 136 μm for the IAA plate. The deviations are probably due to details of the evaporation procedure. Using the 1.33 mm diameter (FWHM) of the input beam, we get expected ratios $N/m = 31.7$, 15.8, and 10.6. We can compare this to the N/m values as obtained from the position of the first maximum in the search, bearing in mind that the first image, having made 1/2 roundtrip, should be counted as 1/2 iteration. For the data shown in Figs. 3A–3C we estimate the maximum peak at 5, 3.5, and 3 iterations, leading to $N/m = 32$, 15.8, and 11.6, respectively, in good agreement with the expected numbers. The results thus confirm the $\sqrt{N/m}$ scaling behavior as expected from Grover's algorithm.

The prime significance of the N/m values is in the scaling of the searching period as $\sqrt{N/m}$. The absolute values of the expected N/m depend on our chosen definition for the input beam diameter (FWHM) and thus may seem somewhat arbitrary. Therefore we also compare our results to a simulation, shown in Figs. 3D–3F. We simulate the phase plates by multiplying the beam profile by the experimentally determined trapezoidal phase profiles $\Phi_o(x)$ and $\Phi_f(x)$. We describe the lenses by a Fourier transform. The results of the simulation agree well with the experiment, producing the maximum peak at the same number of iterations as the experiment. An important difference between the experiment and the simulation is due to optical losses in the experiment. As mentioned earlier, the experimental data have been scaled to compensate for the losses, which amplifies the noise in the last few iterations shown. Apart from this noise, we also see the development of side peaks. These are probably due to diffraction effects accumulating as the iterations progress, e.g., due to slight misalignments of our optical cavity.

Keeping the resolution at ~10 μm and extending the experiment to 2D, $E(x, y)$, it should be feasible to perform database searches of up to 10^6 items experimentally, assuming a beam diameter of 1 cm. This is equivalent to about 20 qubits, so that we gain experimental access to problems that are as yet inaccessible for true quantum computers. These include quantum counting [16,19], estimation of the mean and median of a population [15], and the synthesis of arbitrary superposition states [20]. Theoretical studies have investigated fault tolerance [21,22] and noise [23] in Grover's algorithm, predicting damping of the cycles of finding and "unfinding," as we also see in the experiment. The problem of "phase matching" [17,18] can also be directly translated into optics as differential phase shifts provided by the oracle and IAA plate. These issues are as yet impossible to investigate experimentally with present-day quantum computers. Our classical-wave experiment can bridge this gap. Note that it is complementary to a theoretical proposal by Farhi and Gutmann [24] to search a digital database in analog time, rather than using discrete iterations. In our case, an analog database is searched using discrete iterations.

Some classical-wave analogies of quantum information processing [25–27], as well as a hybrid quantum-classical approach [28] have been proposed previously. Some elements of Grover's algorithm have been demonstrated with classical waves [8]. The latter experiment demonstrated an oracle and IAA operation for a four-item database. Iterations were neither present nor necessary, since for $N = 4$ a single query reveals the sought item. A four-item database search has also been demonstrated using NMR techniques [6,7]. Electronic wave packets in Rydberg atoms have been used to store and retrieve numbers [9] and an equivalent experiment has been reported recently with classical light waves [10]. However, it has been pointed out that the Rydberg-atom experiment lacked the IAA operation [29], which is a crucial ingredient of the quantum search algorithms. In our present experiment, Grover's second algorithm [5] can be recognized in the first transmitted pulse, which is essentially a phase-contrast image of the oracle. Since the contrast would be relatively low, the light pulse must contain sufficiently many photons to build up good readout statistics. By contrast, using Grover's first algorithm, the item could in principle be found with near certainty by sending a single photon through the oracle $O(\sqrt{N})$ times.

It should be clear that our optical system is not a universal quantum computer. Essentially we have mapped the 2^n-dimensional Hilbert space of n qubits by the Hilbert space of a single photon in a superposition of 2^n transverse modes. It is well known [11,30] that this unary mapping comes at the cost of an exponential overhead in some physical resource. Previous classical analogies required an exponential number of components such as beam splitters [8,25–27]. The efficiency of a true quantum computer in implementing the transforms has been attributed to entanglement, i.e., to the tensor product structure of the Hilbert space. Despite the lack of entanglement in our present experiment, the Fourier transform is performed efficiently using only a single lens, independently of the size of the database. The lack of entanglement does, however, limit the *size* of the database, which scales linearly with the beam diameter D, or $\propto D^2$ for a 2D version. Thus the equivalent number of qubits scales only as $\propto \log D$. Even if we set D equal to the size of the universe, $\sim 10^{26}$ m, this would yield only 206 equivalent qubits. This limitation exists for any database containing classical information. On the other hand, since Grover's algorithm provides only a

$\sqrt{N}$ speedup, a quantum computer implementing Grover's algorithm becomes exponentially slow for an exponentially large database. Thus our experiment shows that quantum entanglement is not needed to implement the algorithm or to improve the efficiency. Its only role in this case is to allow for a larger database size.

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